Logic Synthesis & Verification, Fall 2011

National Taiwan University

Problem Set 5

Due on 2011/12/21 before lecture

1 [Factored Form]

Prove the following theorem about the optimality of factored form.

Theorem 1. *Let $\rho(f)$ be the minimum number of literals in any factored form of the completely specified function f. If*

$$f = \sum_{i=1}^{n} \prod_{j=1}^{m} f_{ij}$$

with $f_{ij} \perp f_{kl}$ for all $i \neq k$ or $j \neq l$, then

$$\rho(f) = \sum_{i=1}^{n} \sum_{j=1}^{m} \rho(f_{ij}).$$

2 Weak Division

Given an expression F and a divisor G, suppose $F = G \cdot H + R$ by weak division. Prove that H and R are unique.

3 [Kernelling and Factoring]

Let $F = aefh + aegh + aei + befh + begh + bei + cdefh + cdegh + cdei$.

(a) Compute `KERNEL`$(0, F)$ with literals ordered alphabetically. Draw the kernelling tree (as in the slides) and list the kernels and their corresponding co-kernels.

(b) Apply `GFACTOR` on F by using the largest level-0 kernels as the divisors and using weak division. (In case that there are several choices of divisors, using one of them is sufficient.)

4 [Extraction by Rectangle Covering]

Given two algebraic expressions

$$F = ac + ad + af + be + bc + bd + bf, \text{ and}$$
$$G = acd + ae + bcd + be,$$

use rectangle covering to extract a common sub-expression with the largest value.

5 [Technology Mapping]

(a) Given the subject graph of Figure 1 and pattern graphs of Figure 2, formulate the technology mapping problem (for validity instead of optimality) in terms of a CNF formula.
(Bonus) Formulate the optimality problem as MAXSAT or ILP.
(b) Given the subject graph of Figure 1, perform dynamic programming to find optimum tree mapping using the pattern graphs of Figure 2. Show intermediate costs and the final tree covering.
(c) Modify the subject graph of Figure 1 and pattern graphs of Figure 2 by inserting an inverter pair (i.e., a buffer) to every NAND-to-NAND wire. Per-

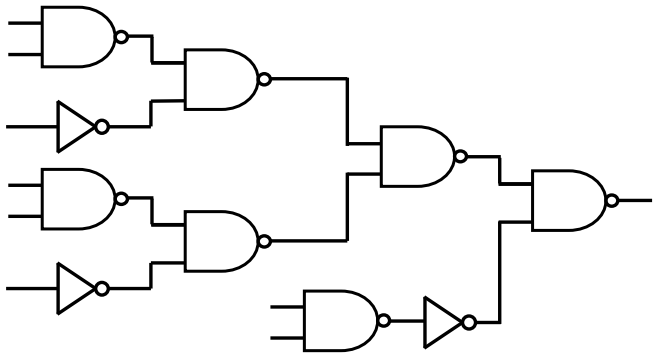

Fig. 1. Subject graph

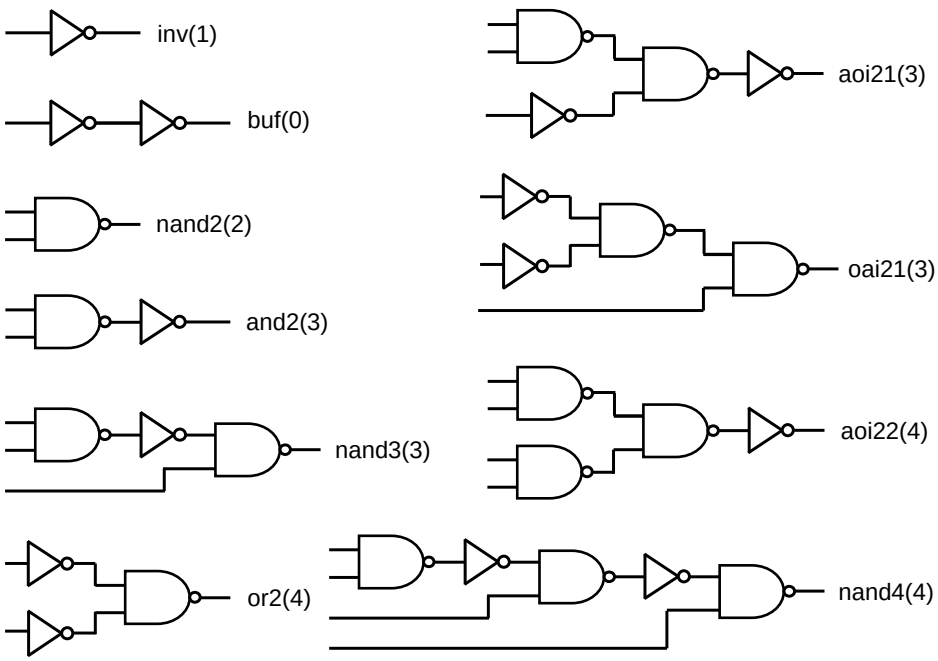

Fig. 2. Pattern graphs.