Logic Synthesis & Verification, Fall 2012

National Taiwan University

Problem Set 1

Due on 2012/10/9

Please drop your solution in the instructor's mailbox in EE2 Building by 18:00.

1 [Boolean Algebra Definition]

Does $(\{0,1\}, \oplus, \cdot, 0, 1)$, where $\oplus$ and $\cdot$ stand for Boolean XOR and AND operations, respectively, form a Boolean algebra? Which postulates of Boolean algebra are satisfied and which are not?

2 [Boolean Algebra Properties]

Prove the following equalities using only the postulates of Boolean algebra.

(a) $(a + b)' = (a' \cdot b')$
(b) $a + a' \cdot b = a + b$

(Please specify clearly which postulate is applied in each step of your derivation.)

3 [Relation over Boolean Algebra]

Define the relation $\leq$ on a Boolean algebra with carrier $\mathbf{B}$ as follows

$$a \leq b \text{ if and only if } a \cdot b' = 0$$

for every $a, b \in \mathbf{B}$, where b' is the inverse element of b.

(a) Show that this relation induces a partial order on $\mathbf{B}$, that is,
   (i) reflexive: $a \leq a$
   (ii) antisymmetric: if $a \leq b$ and $b \leq a$, then $a = b$
   (iii) transitive: if $a \leq b$ and $b \leq c$, then $a \leq c$
(b) What does "$\leq$" correspond to in the subset algebra (the algebra of classes)?
(c) Let $(\mathbf{B}, +, \cdot, 0, 1)$ be a Boolean algebra, and let a and b be two distinct elements of $\mathbf{B}$ with $a \leq b$. Show that the system $([a, b], +, \cdot, a, b)$ is also a Boolean algebra, where interval $[a, b] = \{x \mid x \in \mathbf{B} \text{ and } a \leq x \leq b\}$.

4 [Boolean Functions]

Let g and h be single-variable Boolean functions. For each of the following cases, express $f(0)$ and $f(1)$ as simplified formulas involving $g(0), g(1), h(0)$, and $h(1)$.

(a) $f(x) = g(h(x))$
(b) $f(x) = g(g'(x))$

5 Boolean Operations

Given an arbitrary Boolean function $f(x_1, \ldots, x_i, \ldots, x_n)$ in switching algebra, what is the smallest (in terms of onset sizes) Boolean function $g(x_1, \ldots, x_i, \ldots, x_n)$ such that $g(x_1, \ldots, 0, \ldots, x_n) = g(x_1, \ldots, 1, \ldots, x_n)$ and $(f \wedge \neg g) = 0$? What is the Boolean difference of g over variable x_i? Express g using f, Boolean connectives $(\wedge, \vee, \neg)$, and/or quantifiers $(\exists, \forall)$.