Logic Synthesis & Verification, Fall 2012

National Taiwan University

Problem Set 3

Due on 2012/11/07 before lecture

1 [Unate Functions]

(20%) Prove or disprove the following statements.

(a) (10%) If functions f and g are positive and negative unate, respectively, in variable x_i, then $f \vee \neg g$ is positive unate in variable x_i.
(b) (10%) Every prime cube of a unate function must be essential.

2 [Generalized Cofactor]

(20%) Prove or disprove the following equalities.

(a) (5%) $\neg f = g \cdot co(\neg f, g) + \neg g \cdot \neg co(\neg f, \neg g)$
(b) (5%) $co(co(f, g), h) = co(f, g \cdot h)$
(c) (5%) $co(f \cdot g, h) = co(f, h) \cdot co(g, h)$
(d) (5%) $co(f', g) = co(f, g)'$

3 [Operation on Cube Lists]

(15%)

(a) (5%) Given two cubes c_1 and c_2 over variables $x_1, \cdots, x_k$, how do you derive a cover for $c_1 \wedge \neg c_2$?
(b) (10%) Following the algorithm in Slide 35 (for operation on cube lists), please show detailed steps in adding the cube $(0-11--0)$ to the following orthogonal cube list.

$$\begin{pmatrix} 0 & 1 & - & - & 1 & 1 & 0 \\ - & 0 & - & 1 & 0 & - & - \end{pmatrix}$$

4 [Column Covering]

(20%)

(a) (10%) Given a Boolean matrix, devise a procedure that converts the column covering problem to a CNF formula for SAT solving. Please show your conversion with an example. (Note that the column covering needs not be minimum.)

(b) (10%) Show an algorithm that uses SAT solving to find the minimum column cover.
There will be a 10% extra bonus if you can formulate it using **partial Max-SAT**, defined as follows.

Definition 1 (partial Max-SAT). *Given a CNF formula $\Phi = C_1^h \wedge \cdots \wedge C_m^h \wedge C_1^s \wedge \cdots \wedge C_n^s$, **partial Max-SAT** is the problem of finding a truth assignment that satisfies all the **hard clauses** $C_1^h, \ldots, C_m^h$ and simultaneously satisfies the maximum number of **soft clauses** $C_1^s, \ldots, C_n^s$.*

5 [Prime Generation]

(25%) Let

$$
\begin{aligned}
F &= a'b'e + a'b'c' + a'bc + abc' + abe + ab'c, \\
D &= a'bc'e' + abce', \\
R &= a'b'ce' + a'bc'e + ab'c',
\end{aligned}
$$

be the covers of the onset, don't-care set, and offset of an incompletely specified function, respectively. Use Quine-McCluskey's method to minimize it with the following steps.

(a) (5%) Generate all prime implicants by unate recursive paradigm.
(b) (5%) Derive the Boolean matrix. (Order the rows and columns alphabetically with $a < b < c < e$ and $x' < x$. E.g., $a'b'c$ is listed before $a'bc'$.)
(c) (5%) Which prime implicants are essential? Reduce the Boolean matrix based on the prime implicants.
(d) (5%) Reduce iteratively the Boolean matrix using row equality, row dominance, column dominance, and induced n-ary essential primes until no more reduction is possible. Show intermediate steps.
(e) (5%) Solve the cyclic core, if any, using the independent set heuristic algorithm.