

Logic Synthesis and Verification

Jie-Hong Roland Jiang
江介宏

Department of Electrical Engineering
National Taiwan University



Fall 2012

1

Sequential Synthesis

part of the following slides are by
courtesy of Andreas Kuehlmann

2

Motivation

- Pure combinational optimization can be **suboptimal** since relations across register boundaries are disregarded

3

Overview of Circuit Optimization



4

Sequential Optimization Techniques

- Clock skew scheduling
 - balance path delays by adjusting the relative clocking schedule of individual registers
- Retiming
 - balance path delays by moving registers within circuit topology
 - can be interleaved with combinational optimization techniques
- Architectural restructuring
 - add sequential redundancy
 - fixed: does not change input/output behavior
 - flexible: change input output behavior
- System-level optimization

5

Integration in Design Flow

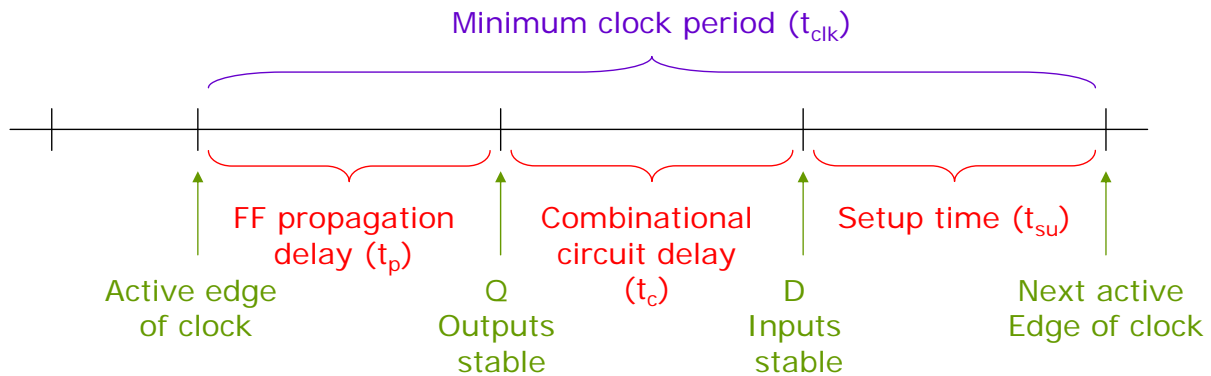
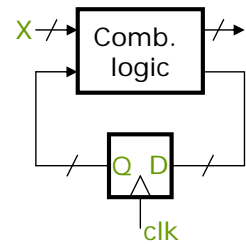
- Optimization space
 - significantly more optimization freedom at a higher level for improving performance, power, area, etc.
- Distance from physical implementation
 - difficult to accurately model impacts on final implementation
 - difficult to mathematically characterize optimization space
- Verification challenge
 - departure from combinational comparison model would impede formal equivalence checking
 - different simulation behaviors cause acceptance problems
- **Necessity of tight tool integration!**

6

Sequential Timing Constraints

Minimum clock period

- $t_{clk}(\min) = \max\{t_p, t_x\} + t_c + t_{su}$, where t_x is the time after the active clock edge at which the X inputs are stable

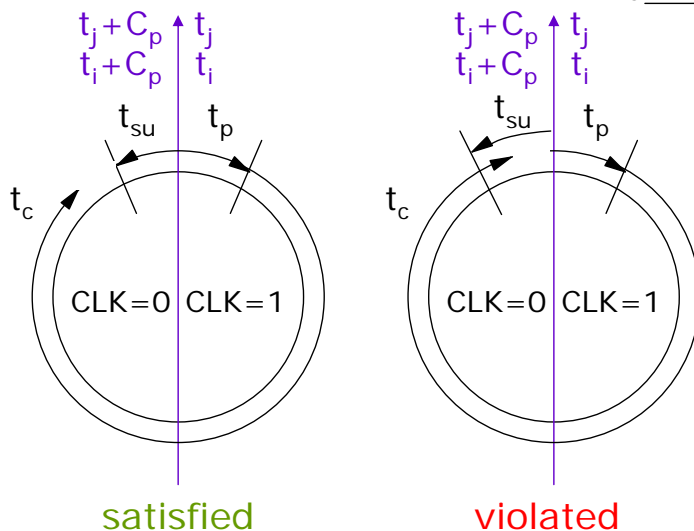
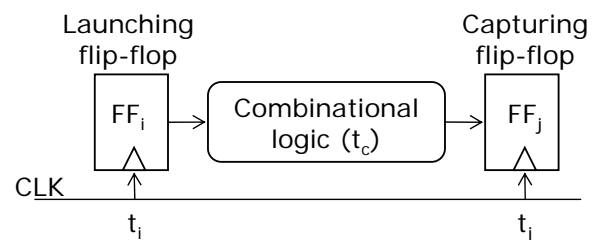


7

Sequential Timing Constraints

Setup-time constraint

- $C_p \geq t_p + t_c^{\max} + t_{su}$



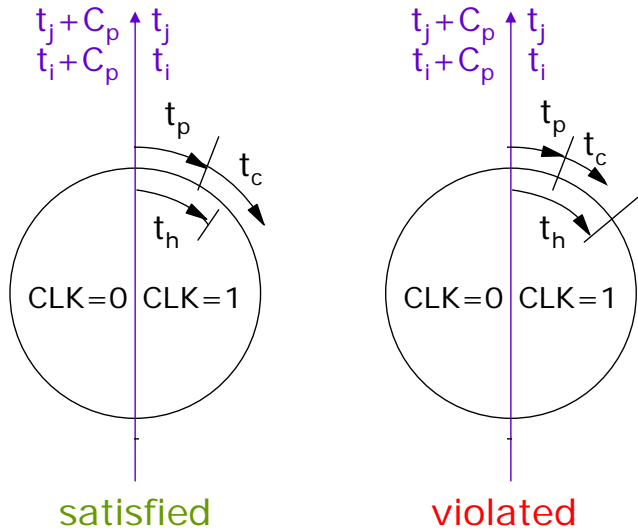
C_p : clock period

8

Sequential Timing Constraints

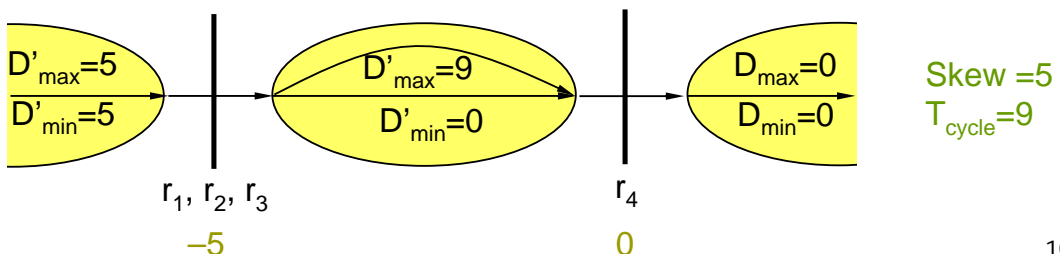
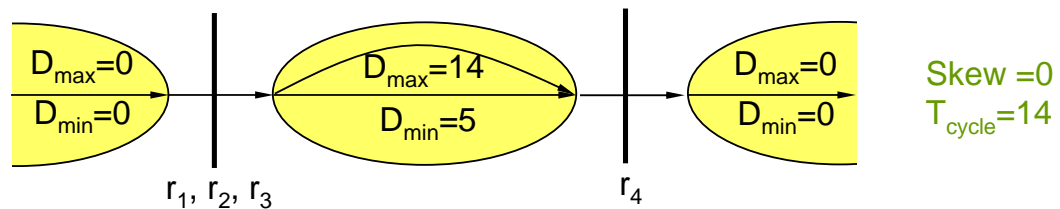
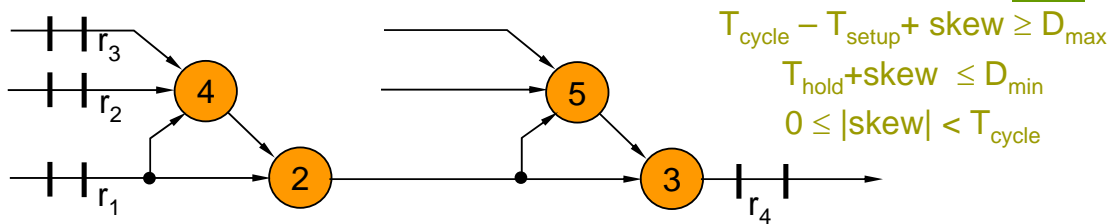
Hold-time constraint

$$t_p + t_c^{\min} \geq t_h$$



9

Clock Skew Scheduling



10

Clock Skew Scheduling

- By controlling clock delays on registers, clock frequency may be increased
 - Do not change transition and output functions (not the case in retiming)
 - Good for functional verification
 - May require sophisticated timing verification

- Clock skew: clock signal arrives at different registers at different times
 - Positive skew: the sending register gets the clock earlier than the receiving register
 - Negative skew: the receiving register gets the clock earlier than the sending register

11

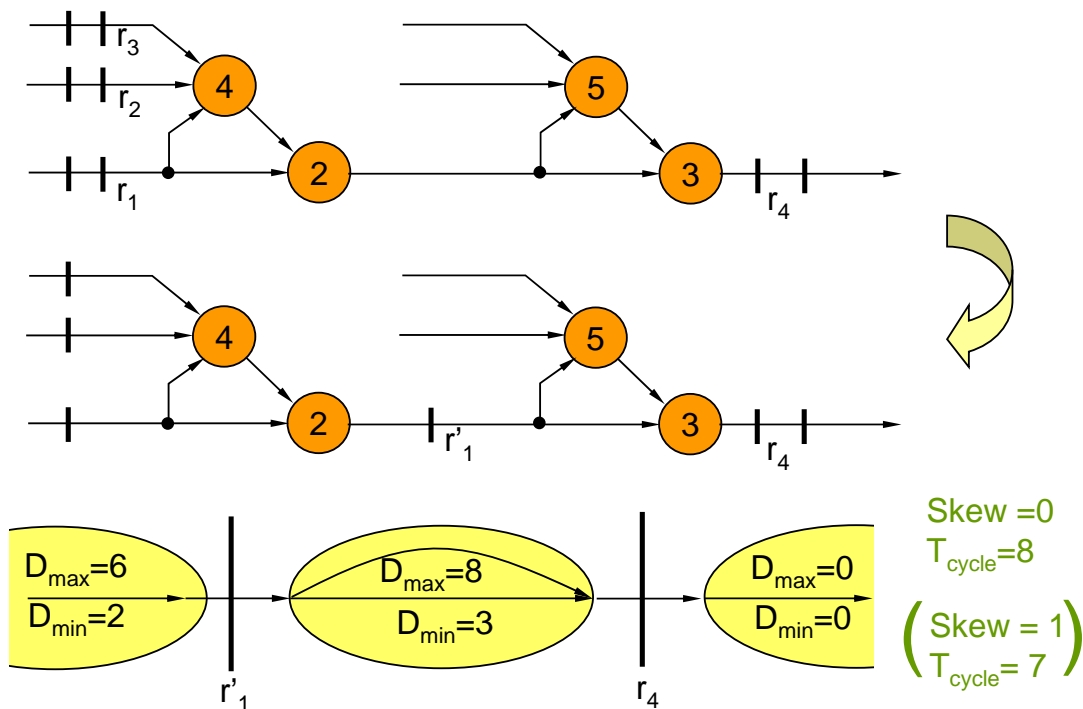
Clock Skew Scheduling

- Pros
 - Inexpensive “post synthesis” technique to further reduce clock period
 - Combinational design model is preserved

- Cons
 - Setup **and** hold time constraints must be obeyed
 - including hold time constraints from scan chain
 - Interleaving with combinational optimizations impossible
 - Replication of clocking tree required

12

Retiming

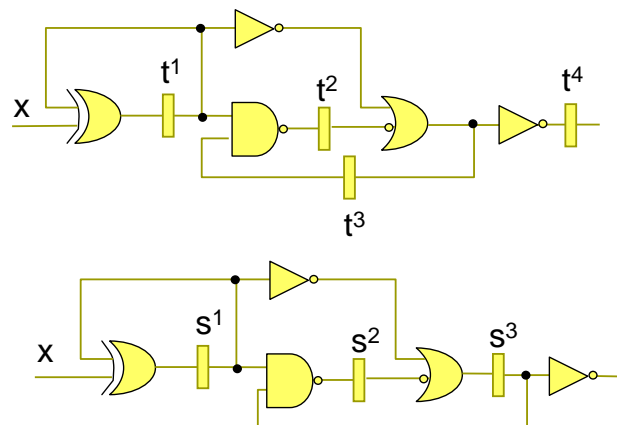


13

Retiming

□ Optimize sequential circuits by repositioning registers

- Move registers so that clock cycle decreases or register count decreases
- Input-output behavior is preserved; however, transition and output functions are changed due to the register movement



14

Retiming

□ Pros

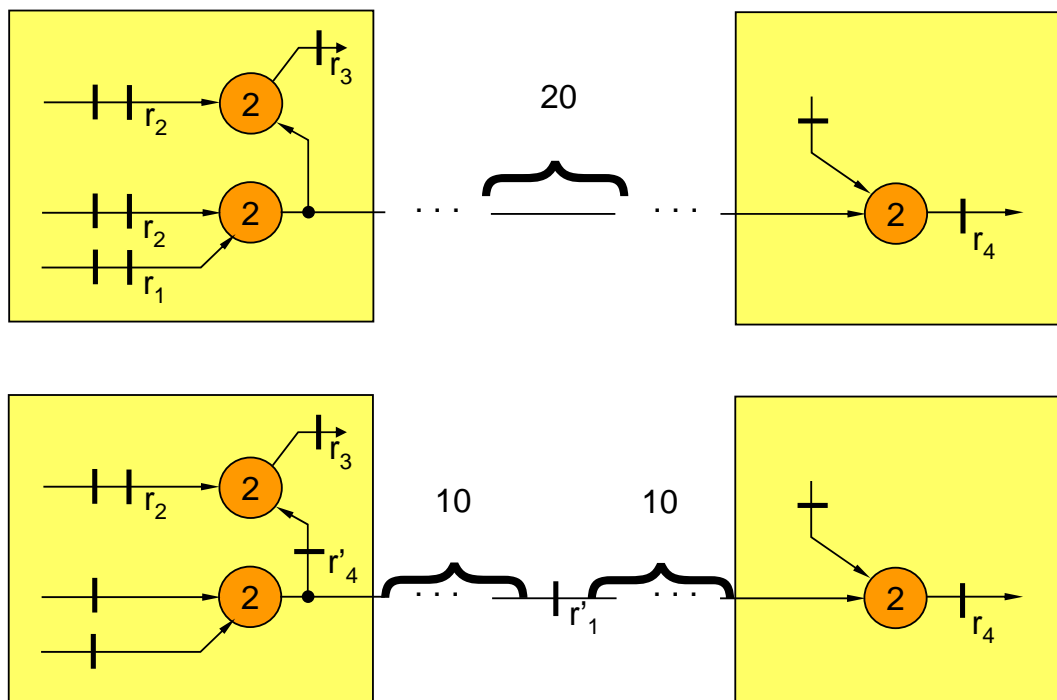
- Only setup time constraint (0 clock skew)
- Simple integration with other logical (e.g. combinational) or physical optimizations
 - E.g., iterative retiming and resynthesis
- Easy combination with clock skew scheduling to obtain global optimum

□ Cons

- Change combinational model of design
 - Severe impact on verification methodology
- Inaccurate delay model
- Computation of equivalent reset state required

15

Architectural Retiming



16

Architectural Retiming

□ Pros

- Smooth extension of regular retiming
- Potential to alleviate global performance bottlenecks by adding sequential redundancy and pipelining

□ Cons

- Significant change of design structure
 - substantial impact on verification methodology
- Flexible architectural restructuring changes I/O behavior
 - existing RTL specification methods not always applicable

17

Verification Issues

□ Timing verification unchanged

□ Functional verification affected

- Except for clock skew scheduling, sequential optimization **does** change register (transition) functions
- Traditional combinational equivalence checking not applicable
- Simulation runs not recognizable by designers - acceptance problems
- Solution:
 - preserve retime function (mapping function) from synthesis for:
 - reducing sequential EC problem back to combinational case
 - *no false positives possible!*
 - modifying simulation model to reproduce original simulation output

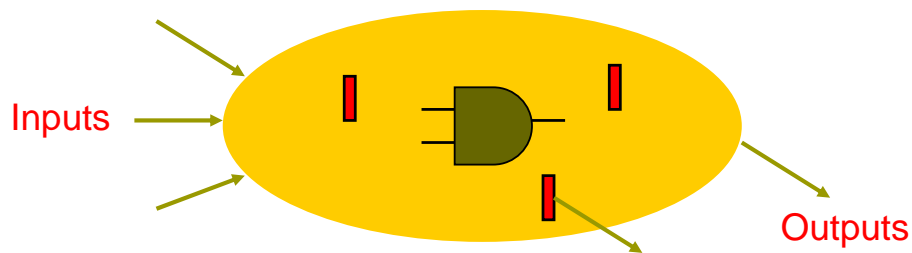
18

Retiming Circuits

□ Objectives:

- Reduce clock cycle time
- Reduce register count (area)
- Reduce power, etc.

□ Input: A netlist of gates and registers



19

Retiming Circuits

□ Circuit represented as retiming graph $G(V, E)$

[Leiserson and Saxe 1983, 1991]

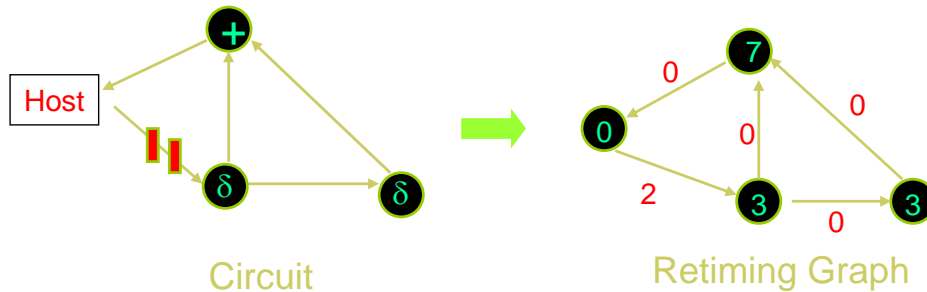
- V : vertex set representing logic gates
- E : edge set representing connections
- $d(v)$ = delay of gate/vertex v , ($d(v) \geq 0$)
- $w(e)$ = number of registers on edge e , ($w(e) \geq 0$)

20

Retiming Circuits

Example

- **Synchronous circuit** assumption: every cycle of a circuit has at least one register, i.e., no combinational loop



The host node represents the environment that interacts with the circuit via the primary inputs and outputs

Operation	delay
δ	3
+	7

21

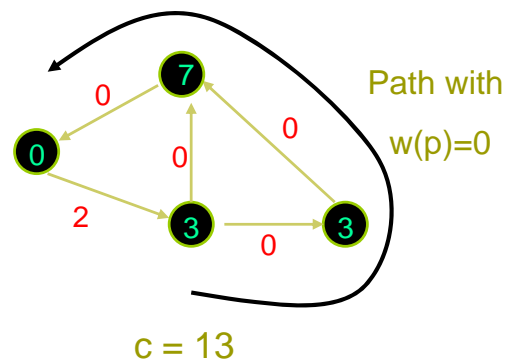
Retiming Circuits

- For a path $p : v_0 \xrightarrow{e_0} v_1 \xrightarrow{e_1} \dots v_{k-1} \xrightarrow{e_{k-1}} v_k$

- Path delay $d(p) = \sum_{i=0}^k d(v_i)$ (includes endpoints)
- Path weight $w(p) = \sum_{i=0}^{k-1} w(e_i)$

- Minimum clock cycle

$$c = \max_{p: w(p)=0} \{d(p)\}$$

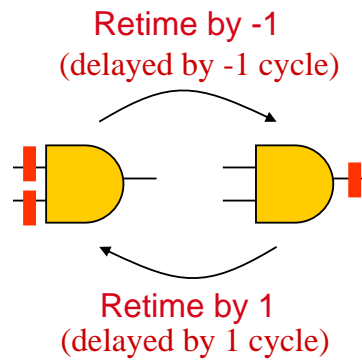


22

Retiming Circuits

Atomic operation

- Move registers across a gate in a forward or backward direction



- Does not affect gate functionality, but timing

23

Retiming Circuits

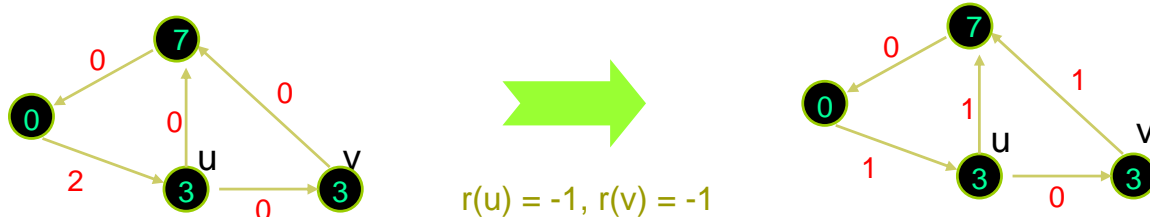
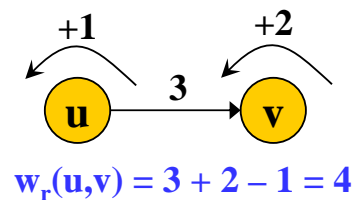
- Retiming can be formalized with a **retime function** $r: V \rightarrow \mathbb{Z}$, where \mathbb{Z} is the set of integers

- I.e., a retime function performs integer labeling on vertices

- Weight update after retiming with r

- $w_r(e) = w(e) + r(v) - r(u)$, for edge $e = (u, v)$
- $w_r(p) = w(p) + r(t) - r(s)$, for path p from s to t

- A retiming with some r is **legal** if $w_r(e) \geq 0, \forall e \in E$



24

Min-Cycle Retiming

□ Problem Statement: (minimum cycle retiming)

Given $G(V, E)$ with delay function d and weight function w , find a legal retiming r so that

$$c = \max_{p: w_r(p)=0} \{d(p)\}$$

is minimized

□ Retiming: two important matrices

- Register weight matrix

$$W(u, v) = \min_p \{w(p) : u \xrightarrow{p} v\}$$

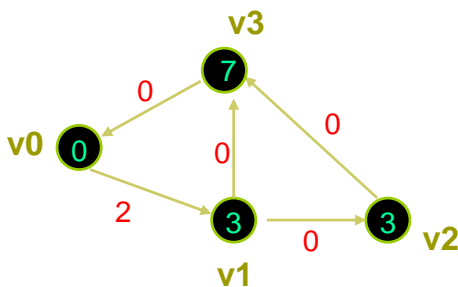
- Delay matrix

$$D(u, v) = \max_p \{d(p) : u \xrightarrow{p} v, w(p) = W(u, v)\}$$

25

Min-Cycle Retiming

□ Example



For some constant α , minimum clock cycle $c \leq \alpha \Leftrightarrow \forall p, \text{ if } d(p) > \alpha \text{ then } w(p) \geq 1$

	W				D				
	V0	V1	V2	V3	V0	V1	V2	V3	
V0	0	2	2	2	0	3	6	13	V0
V1	0	0	0	0	13	3	6	13	V1
V2	0	⊥	0	0	10	⊥	3	10	V2
V3	0	⊥	⊥	0	7	⊥	⊥	7	V3

W = register path weight matrix
(minimum # registers on all paths between u and v)
D = path delay matrix
(maximum delay on the paths between u and v with $w(p)=W(u,v)$)

Don't count paths passing through the host!

26

Min-Cycle Retiming

- Assume that we are asked to check if a retiming exists for a clock cycle α

- Legal retiming: $w_r(e) \geq 0$ for all e . Hence

$$w_r(e) = w(e) + r(v) - r(u) \geq 0, \text{ or}$$

$$r(u) - r(v) \leq w(e)$$

- For all paths $p: u \rightarrow v$ such that $d(p) \geq \alpha$, we require $w_r(p) \geq 1$.
Thus

$$1 \leq w_r(p) = \sum_{i=0}^{k-1} w_r(e_i)$$

$$= \sum_{i=0}^{k-1} [w(e_i) + r(v_{i+1}) - r(v_i)]$$

$$= w(p) + r(v_k) - r(v_0)$$

$$= w(p) + r(v) - r(u)$$

- Take the least $w(p)$ (tightest constraint) $r(u) - r(v) \leq W(u,v) - 1$

- Note:** This is independent of the path from u to v , so we just need to apply it to u, v such that $D(u,v) > \alpha$

27

Min-Cycle Retiming

Example

Assume $\alpha = 7$

Legality:

$$r(u) - r(v) \leq w(e)$$

$$r(v_0) - r(v_1) \leq 2$$

$$r(v_1) - r(v_2) \leq 0$$

$$r(v_1) - r(v_3) \leq 0$$

$$r(v_2) - r(v_3) \leq 0$$

$$r(v_3) - r(v_0) \leq 0$$

$D > 7$:

$$r(u) - r(v) \leq W(u,v) - 1$$

$$r(v_0) - r(v_3) \leq 1$$

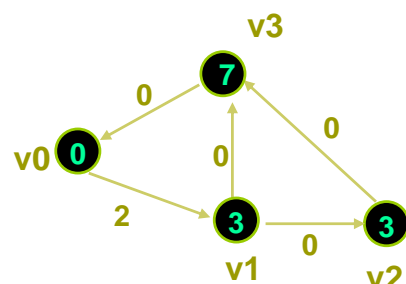
$$r(v_1) - r(v_0) \leq -1$$

$$r(v_1) - r(v_3) \leq -1$$

$$r(v_2) - r(v_0) \leq -1$$

$$r(v_2) - r(v_3) \leq -1$$

	W				D				
	V0	V1	V2	V3	V0	V1	V2	V3	
V0	0	2	2	2	0	3	6	13	V0
V1	0	0	0	0	13	3	6	13	V1
V2	0	⊥	0	0	10	⊥	3	10	V2
V3	0	⊥	⊥	0	7	⊥	⊥	7	V3



All constraints are in the difference-of-2-variable form and closely related to shortest path problem

28

Min-Cycle Retiming

Example

Legality:

$$r(u) - r(v) \leq w(e)$$

$$r(v_0) - r(v_1) \leq 2$$

$$r(v_1) - r(v_2) \leq 0$$

$$r(v_1) - r(v_3) \leq 0$$

$$r(v_2) - r(v_3) \leq 0$$

$$r(v_3) - r(v_0) \leq 0$$

$D > 7$:

$$r(u) - r(v) \leq W(u,v) - 1$$

$$r(v_0) - r(v_3) \leq 1$$

$$r(v_1) - r(v_0) \leq -1$$

$$r(v_1) - r(v_3) \leq -1$$

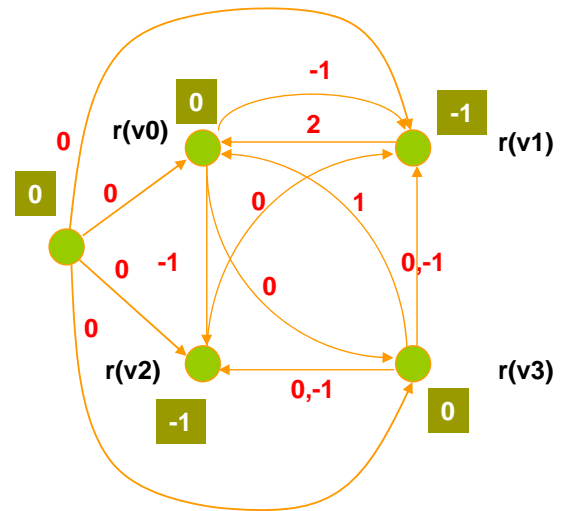
$$r(v_2) - r(v_0) \leq -1$$

$$r(v_2) - r(v_3) \leq -1$$

Search shortest path on constraint graph:
Bellman-Ford algorithm $O(|V||E|)$ or $O(|V|^3)$

A solution exists if and only if there exists **no** negative weighted cycle

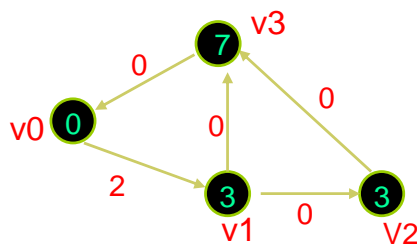
Constraint graph



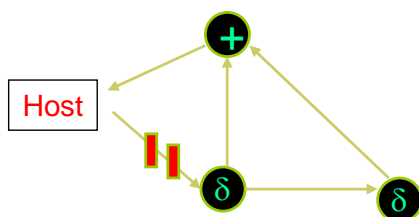
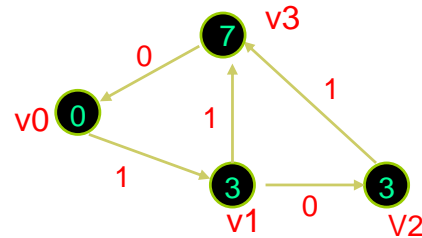
A solution is $r(v_0) = r(v_3) = 0$,
 $r(v_1) = r(v_2) = -1$

Min-Cycle Retiming

To find the minimum cycle time, do a binary search among the entries of the D matrix $O(|V||E| \log |V|)$

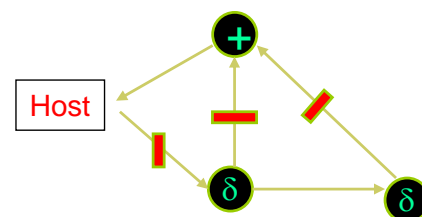


$$r(v_0) = r(v_3) = 0, \\ r(v_1) = r(v_2) = -1$$



Clock cycle
= $3+3+7=13$

Retime



Clock cycle = 7

Min-Cycle Retiming

□ Theorem: r is a legal retiming on G such that the clock cycle $c \leq \alpha$ for some constant α if and only if

1. $r(v_h) = 0$
2. $r(u) - r(v) \leq w(e)$ for every edge $e(u, v)$
3. $r(u) - r(v) \leq W(u, v) - 1$ (i.e. register count > 1) for every (u, v) with $D(u, v) > \alpha$

□ Solve the integer linear programming problem

- Bellman-Ford method $O(|V|^3)$

31

Min-Cycle Retiming

□ Algorithm of optimal retiming:

1. Compute W and D
2. Binary search the minimum achievable clock period by applying Bellman-Ford algorithm to check the satisfaction of the prior Theorem
3. Derive $r(v)$ under the minimum achievable clock period found in Step 2

□ Complexity $O(|V|^3 \lg |V|)$

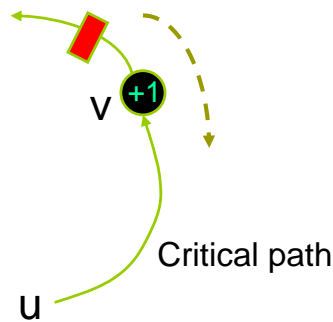
32

Min-Cycle Retiming

□ Two more algorithms:

1. Relaxation based:

- Repeatedly find critical path
- Retime vertex at end of path by +1 ($O(|V| |E| \log |V|)$)



2. Also, Mixed Integer Linear Program formulation

33

Min-Area Retiming

□ Goal: minimize number of registers used

$$\begin{aligned}
 \min N_r &= \sum_{e \in E} w_r(e) \\
 &= \sum_{e: u \rightarrow v} (w(e) + r(v) - r(u)) \\
 &= \sum_{e \in E} w(e) + \sum_{e: u \rightarrow v} (r(v) - r(u)) \\
 &= N + \sum_{u \rightarrow v} (r(v) - r(u)) \\
 &= N + \sum_{v \in V} [r(v)(\# \text{fanin}(v) - \# \text{fanout}(v))] \\
 &= N + \sum_{v \in V} a_v r(v)
 \end{aligned}$$

where a_v is a constant

34

Min-Area Retiming

- Minimize:

$$\sum_{v \in V} a_v r(v)$$

- Subject to:

$$w_r(e) = w(e) + r(v) - r(u) \geq 0$$

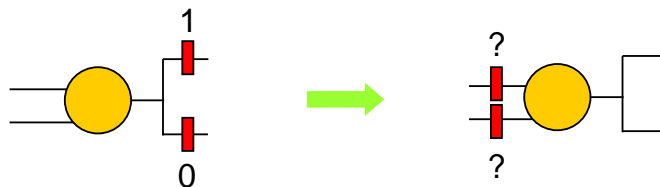
- Note: It is reducible to a flow problem

35

Retiming Issues

- Computation of equivalent initial states

- Equivalent initial states may not always exist



- General solution requires replication of logic for initialization

- Timing models

- Too far away from actual implementation

36

Retiming + Clock Scheduling

□ Mathematical formulation

- $s: E \rightarrow \mathbb{R}$, a real edge labeling
- $s(e)$ denotes the clock signal delay of all registers of e

□ In addition to the register weight matrix and delay matrix for the maximum delay, we also need the minimum paths delays

$$W(u, v) = \min_p \{w(p) : u \xrightarrow{p} v\}$$
$$D(u, v) = \max_p \{d(p) : u \xrightarrow{p} v, w(p) = w(u, v)\}$$
$$D_{\min}(u, v) = \min_p \{d(p) : u \xrightarrow{p} v, w(p) = w(u, v)\}$$

37

Retiming + Clock Scheduling

□ A valid retiming and clock skew schedule is an assignment to r and s such that:

- (1) $w_r \geq 0$
- (2) $\forall (u', u), (v, v')$:

$$w(u', u) > 0 \wedge w(v, v') > 0 \wedge W(u, v) = 0 \Rightarrow$$

$$D_{\min}(u, v) + s(u', u) - s(v, v') \geq T_{hold} \wedge$$

$$D(u, v) + s(u', u) - s(v, v') \leq T_{clock} - T_{setup}$$

□ Solution Mixed Integer Linear Program (MILP)

38

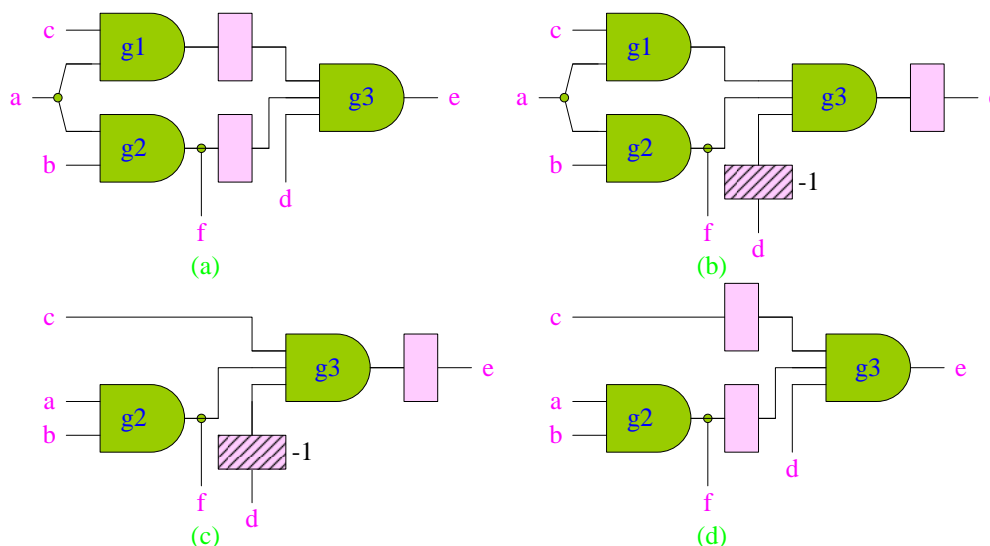
Retiming & Resynthesis

- Combine retiming and combinational optimization
 - Retime registers such that the circuit has a large combinational logic block for optimization
 - Resynthesize the combinational logic block with combinational logic minimization techniques
 - Retiming and resynthesis can be iterated
 - Can achieve any state re-encoding

39

Retiming & Resynthesis

□ Example



40