

Logic Synthesis & Verification, Fall 2013

National Taiwan University

Problem Set 1

Due on 2013/10/9 before lecture

1 [Boolean Algebra Definition]

Consider the five-tuple $(\{1, b, a, 0\}, f_{\vee}, f_{\wedge}, 0, 1)$ with binary operators f_{\vee} and f_{\wedge} defined below. Does the five-tuple form a Boolean algebra? Which postulates of Boolean algebra are satisfied and which are not?

f_{\vee}	1	b	a	0
1	1	1	1	1
b	1	b	1	b
a	1	1	a	a
0	1	b	a	0

f_{\wedge}	1	b	a	0
1	1	b	a	0
b	b	b	0	0
a	a	0	a	0
0	0	0	0	0

2 [Boolean Algebra Properties]

Prove the following equalities using ONLY the five postulates of Boolean algebra (or other properties that you have proven using the postulates). Please specify clearly which postulate is applied in each step of your derivation.

- (a) $(a')' = a$
- (b) $a \cdot b + (a' \cdot c + b \cdot c) = a \cdot b + a' \cdot c$

3 [Minterm Canonical Form]

Prove the theorem of minterm canonical form using Boole's expansion theorem.

4 [Boolean Functions]

Let $f(x, y)$ be a Boolean function for $\mathbb{B} = \{0, 1, a, a'\}$ with the following partial function table.

2 **Problem Set 1**

x	0	0	0	1	1	1	1	a	a	a	a'	a'	a'	a'
y	0	1	a	a'	0	1	a	a'	0	1	a	a'	0	1
f	1	1	1	1	1	1	1	1	1	1	1	1	1	1

- (a) How many Boolean functions are consistent with the above function table? Please explain.
- (b) Please complete the above function table and list all possibilities if more than one.

5 Boolean Algebra Application

Let Boolean function $f(x, y, z) = xy + x'y' + y'z'$ for $\mathbb{B} = \{0, 1\}$.

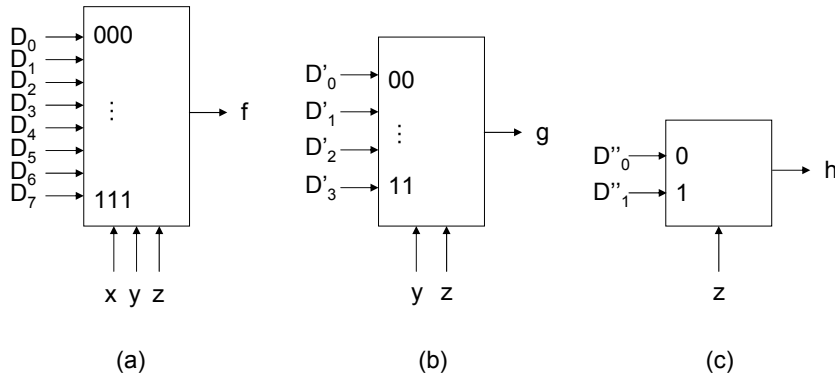


Fig. 1. Mux implementation of Boolean functions

- (a) Consider the multiplexer implementation of f in Figure 1 (a). What are the values of D_i ?
- (b) Consider implementing $f(x, y, z)$ by another Boolean function $g(y, z)$ using the multiplexer of Figure 1 (b).
 - What is the new Boolean algebra? Please define the five-tuple $(\mathbb{B}, +, \cdot, \underline{0}, \underline{1})$.
 - What are the possible values of variables y and z ? Why the multiplexer assumes y and z have only values $\{0, 1\}$?
 - Please explain in what sense $f(x, y, z)$ and $g(y, z)$ can be equivalent. What should the values D'_i be?
- (c) Consider implementing $f(x, y, z)$ by yet another Boolean function $h(z)$ using the multiplexer of Figure 1 (c). What is the new Boolean algebra? Please define the the five-tuple $(\mathbb{B}, +, \cdot, \underline{0}, \underline{1})$. What are the values D''_i ?