

Logic Synthesis and Verification

Jie-Hong Roland Jiang
江介宏

Department of Electrical Engineering
National Taiwan University



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Two-Level Logic Minimization (1/2)

Reading:
Logic Synthesis in a Nutshell
Section 3 (§3.1-§3.2)

most of the following slides are by
courtesy of Andreas Kuehlmann

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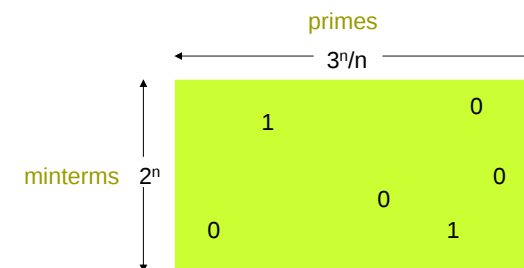
Quine-McCluskey Procedure

- Given G and D (covers for $\mathfrak{F} = (f, d, r)$ and d , respectively), find a minimum cover G^* of primes where:
 $f \subseteq G^* \subseteq f + d$ (G^* is a prime cover of $\mathfrak{F}$)
- Q-M Procedure:
 1. Generate all primes of $\mathfrak{F}$, $\{P_i\}$ (i.e. primes of $(f + d) = G + D$)
 2. Generate all minterms $\{m_i\}$ of $f = G \wedge \neg D$
 3. Build Boolean matrix B where
$$B_{ij} = 1 \text{ if } m_i \in P_j \\ = 0 \text{ otherwise}$$
 4. Solve the minimum column covering problem for B (unate covering problem)

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Complexity

- $\sim 2^n$ minterms; $\sim 3^n/n$ primes



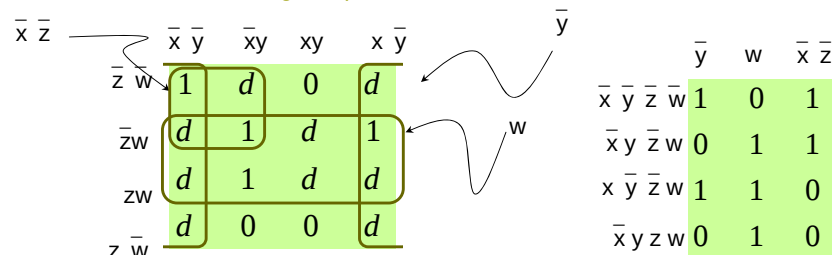
- There are $O(2^n)$ rows and $\Omega(3^n/n)$ columns. Moreover, minimum covering problem is NP-complete. (Hence the complexity can probably be double exponential in size of input, i.e. difficulty is $O(2^{3^n})$)

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Two-Level Logic Minimization

Example

Karnaugh map



$$F = \overline{x}yzw + \overline{x}yz\overline{w} + x\overline{y}zw + x\overline{y}z\overline{w} \quad (\text{cover of } \mathfrak{I})$$

$$D = \overline{y}z + xyw + \overline{x}yzw + x\overline{y}w + \overline{x}yz\overline{w} \quad (\text{cover of } d)$$

Primes: $\overline{y} + w + \overline{x}\overline{z}$

Covering Table

Solution: $\{1, 2\} \Rightarrow \overline{y} + w$ is a minimum prime cover (also $w + \overline{x}\overline{z}$)

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Covering Table

	$\overline{y}$	w	$\overline{x}\overline{z}$	Primes of f+d
$\overline{x}\overline{y}\overline{z}\overline{w}$	1	0	1	
$\overline{x}y\overline{z}w$	0	1	1	
$x\overline{y}\overline{z}w$	1	1	0	
$\overline{x}yzw$	0	1	0	Row singleton (essential minterm)
	Essential prime			

- Definition. An **essential prime** is a prime that covers an onset minterm of f not covered by any other primes.

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Covering Table Row Equality

Row equality:

- In practice, many rows in a covering table are identical. That is, there exist minterms that are contained in the same set of primes.

Example

m_1 0101101
 m_2 0101101

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Covering Table Row and Column Dominance

Row dominance:

- A row i_1 whose set of primes is contained in the set of primes of row i_2 is said to **dominate** i_2 .

Example

i_1 011010
 i_2 011110

- i_1 dominates i_2
- Can remove row i_2 because have to choose a prime to cover i_1 , and any such prime also covers i_2 . So i_2 is automatically covered.

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Covering Table Row and Column Dominance

Column dominance:

- A column j_1 whose rows are a superset of another column j_2 is said to **dominate** j_2 .

Example

j_1	j_2
1	0
0	0
1	1
0	0
1	1

- j_1 dominates j_2
- We can remove column j_2 since j_1 covers all those rows and more. We would never choose j_2 in a minimum cover since it can always be replaced by j_1 .

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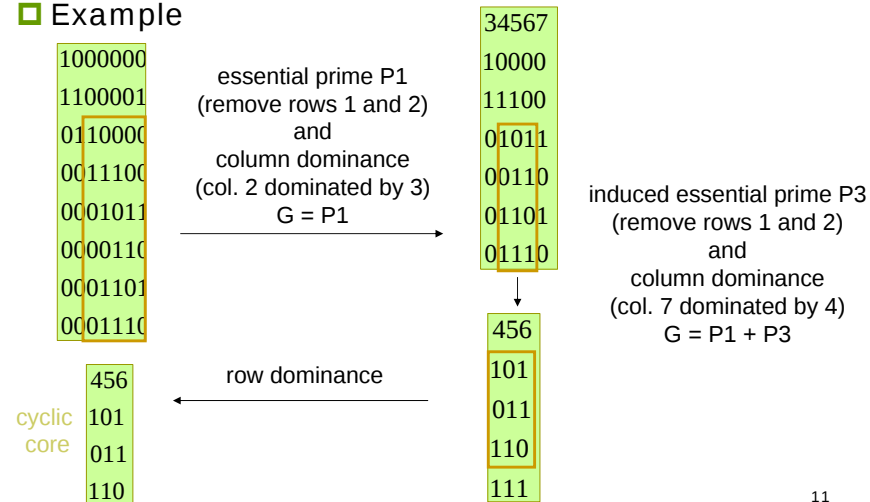
Covering Table Table Reduction

- Remove all rows covered by essential primes (columns in row singletons). Put these primes in the cover G .
 - Group identical rows together and remove dominated rows.
 - Remove dominated columns. For equal columns, keep one prime to represent them.
 - Newly formed row singletons define **induced essential primes**.
 - Go to 1 if covering table decreased.
- The resulting reduced covering table is called the **cyclic core**. This has to be solved (**unate covering problem**). A minimum solution is added to G . The resulting G is a minimum cover.

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Covering Table Table Reduction

Example



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Solving Cyclic Core

- Best known method (for unate covering) is **branch and bound** with some clever bounding heuristics
- Independent Set Heuristic:**
 - Find a maximum set I of "independent" rows. Two rows B_{i1}, B_{i2} are independent if **not** $\exists j$ such that $B_{i1j} = B_{i2j} = 1$. (They have **no column in common.**)
 - Example**
A covering matrix B rearranged with independent sets first

B=	<table><tr><td>1 11</td><td></td></tr><tr><td>1111</td><td>1 1</td></tr></table>	1 11		1111	1 1	<table><tr><td>0</td></tr></table>	0	Independent set $\mathcal{I}$ of rows
	1 11							
1111	1 1							
0								
<table><tr><td>A</td></tr></table>	A	<table><tr><td>C</td></tr></table>	C					
A								
C								

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Solving Cyclic Core

Lemma:

$$|\text{Solution of Covering}| \geq |\mathcal{J}|$$

m_1 must be covered by one of the three columns

m_1	1 1 1		
m_2		1 1 1 1	
m_3			1 1
	A		C

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Solving Cyclic Core

Heuristic algorithm:

- Let $\mathcal{J} = \{l_1, l_2, \dots, l_k\}$ be the independent set of rows

- choose $j \in l_i$ such that column j covers the most rows of A . Put P_j in G
- eliminate all rows covered by column j
- $\mathcal{J} \leftarrow \mathcal{J} \cup \{l_i\}$
- go to 1 if $|\mathcal{J}| > 0$
- If B is empty, then done (in this case achieve minimum solution because of the lower bound of previous lemma attained - **IMPORTANT**)
- If B is not empty, choose an independent set of B and go to 1

1 1 1	1 1 1 1	1 1	0
A			C

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Prime Generation for Single-Output Function

Tabular method

(based on consensus operation, or ∇):

- Start with minterm canonical form of F
- Group pairs of adjacent minterms into cubes
- Repeat merging cubes until no more merging possible; mark ($\checkmark$) + remove all covered cubes.
- Result: set of primes of f .

Example

$$F = x' y' + w x y + x' y z' + w y' z$$

$$F = x' y' + w x y + x' y z' + w y' z$$

$w' x' y' z'$ $\checkmark$	$w' x' y'$ $\checkmark$	$x' y'$
$w' x' y' z$ $\checkmark$	$w' x' y' z'$ $\checkmark$	$x' z'$
$w' x' y' z'$ $\checkmark$	$x' y' z'$ $\checkmark$	
$w' x' y' z$ $\checkmark$	$x' y' z$ $\checkmark$	
$w' x' y' z'$ $\checkmark$	$w x' y'$ $\checkmark$	
$w' x' y' z$ $\checkmark$	$w x' z'$ $\checkmark$	
$w x' y' z$ $\checkmark$	$w y' z$ $\checkmark$	
$w x' y' z'$ $\checkmark$	$w y' z'$ $\checkmark$	
$w x y z'$ $\checkmark$	$w x y$ $\checkmark$	
$w x y' z$ $\checkmark$	$w x z$ $\checkmark$	
$w x y z$ $\checkmark$		

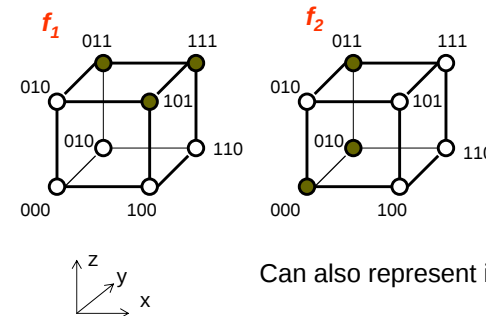
Courtesy: Maciej Ciesielski, UMASS

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Prime Generation for Multi-Output Function

- Similar to *single-output* function, except that we should include also the primes of the products of individual functions

Example



Can also represent it as:

$x y z$	$f_1 f_2$
0-0	0 1
0 1 1	1 1
1-1	1 0

$x y z$	$f_1 f_2$
0-0	0 1
0 1 -	0 1
- 1 1	1 0
1-1	1 0

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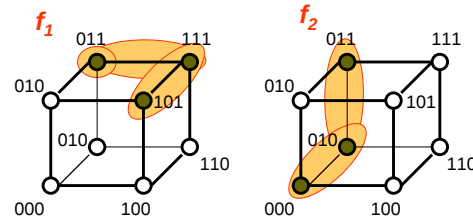
Prime Generation

Example

- Modification from single-output case: When two adjacent implicants are merged, the output parts are **intersected**

xyz	f_1	f_2
0-0	0 1	
0 1 1	1 1	
1-1	1 0	

000	01	✓	0-0	01
010	01	✓	0 1 -	01
011	11		- 1 1	10
101	10	✓	1-1	10
111	10	✓		



There are five primes for this two-output function
- What is the min cover?

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Minimize Multi-Output Cover

Example

- List multiple-output primes
- Create a covering table
- Solve

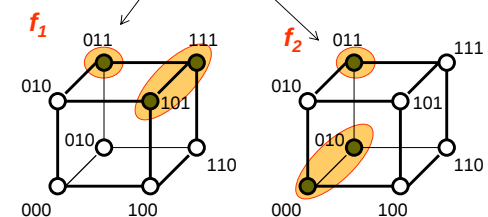
$p_1 = 0 1 1 | 11$
 $p_2 = 0-0 | 01$
 $p_3 = 0 1- | 01$
 $p_4 = -1 1 | 10$
 $p_5 = 1-1 | 10$

listed twice

	p_1	p_2	p_3	p_4	p_5
000	0 1	0 0	0 0	0 0	0 0
010	0 1	0 1	1 0	0 0	0 0
011	0 1	1 0	1 0	0 0	0 0
101	1 0	1 0	0 0	0 1	0 1
111	1 0	0 0	0 0	0 1	1 1

Min cover has 3 primes:

$F = \{p_1, p_2, p_5\}$



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Prime Generation Using Unate Recursive Paradigm

- Apply **unate recursive paradigm** with the following **merge step**

- (Assume we have just generated all primes of f_{x_i} and $f_{\neg x_i}$)

Theorem.

p is a prime of f iff p is **maximal** (in terms of containment) among the set consisting of

- $P = x_i q$, q is a prime of f_{x_i} , $q \not\subseteq f_{\neg x_i}$
- $P = x_i' r$, r is a prime of $f_{\neg x_i}$, $r \not\subseteq f_{x_i}$
- $P = q r$, q is a prime of f_{x_i} , r is a prime of $f_{\neg x_i}$

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Prime Generation Using Unate Recursive Paradigm

Example

- Assume $q = abc$ is a prime of f_{x_i} . Form $p = x_i abc$.
- Suppose $r = ab$ is a prime of $f_{\neg x_i}$. Then $x_i' ab$ is an implicant of f .

$$f = x_i abc + x_i' ab + abc + \dots$$

- Thus abc and $x_i' ab$ are implicants, so $x_i abc$ is not prime.
- Note:** abc is prime because if not, $ab \subseteq f$ (or ac , or bc) contradicting abc prime of f_{x_i} .
- Note:** $x_i' ab$ is prime, since if not then either $ab \subseteq f$, $x_i' a \subseteq f$, $x_i' b \subseteq f$. The first contradicts abc prime of f_{x_i} and the second and third contradict ab prime of $f_{\neg x_i}$.

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Summary

□ Quine-McCluskey Method:

1. Generate cover of all primes $G = p_1 + p_2 + \dots + p_{3^n/n}$
2. Make G irredundant (in optimum way)

■ Q-M is **exact**, i.e., it gives an exact minimum

□ Heuristic Methods:

1. Generate (somehow) a cover of $\mathfrak{S}$ using some of the primes $G = p_{i_1} + p_{i_2} + \dots + p_{i_k}$
2. Make G irredundant (maybe not optimally)
3. Keep best result - try again (i.e. go to 1)