# Switching Circuits \＆ Logic Design 

Jie－Hong Roland Jiang
江介宏
Department of Electrical Engineering National Taiwan University


Fall 2014

## §5 Karnaugh Maps

K－map Walks and Gray Codes


## Outline

$\square$ Minimum forms of switching functions
-Two- and three-variable Karnaugh maps
-Four-variable Karnaugh maps
-Determination of minimum expressions using essential prime implicants
-Five-variable Karnaugh maps
-Other uses of Karnaugh maps
-Other forms of Karnaugh maps

## Limitations of Algebraic Simplification

- Two problems of algebraic simplification

1. Not systematic
2. Difficult to check if a minimum solution is achieved
$\square$ The Karnaugh map method overcomes these limitations

- Typically for Boolean functions with $\leq 5$ variables
- The Quine-McCluskey method can deal with even larger functions
- (Subject of Unit 6, skipped)


## Minimum Forms of Switching Functions

$\square$ Correspondence between Boolean expressions and logic circuits
■ SOP (POS) can be implemented with two-level AND-OR (OR-AND) gate circuits

- Reducing the number of terms and literals of an SOP expression corresponds to reducing the number of gates and gate inputs
- Combine terms by $\mathrm{XY}{ }^{\prime}+\mathrm{XY}=\mathrm{X}$
$\square$ Eliminate redundant terms by consensus theorem
Minimum SOP is not necessarily unique
- An SOP may be minimal (locally) but not minimum (globally)


## ㅁ.g.,

$F=a^{\prime} b^{\prime} c^{\prime}+a^{\prime} b^{\prime} c+a^{\prime} b c^{\prime}+a b^{\prime} c+a b c^{\prime}+a b c$
$=a^{\prime} b^{\prime}+b^{\prime} c+b c^{\prime}+a b$ (minimal but not minimum)
$=a^{\prime} b^{\prime}+b^{\prime}+a c$ (minimum)

## Two-Variable Karnaugh Maps


minterm locations

| AB | F |
| :---: | :---: |
| 00 | 1 |
| 01 | 1 |
| 10 | 0 |
| 11 | 0 |



$$
F=A^{\prime} B^{\prime}+A^{\prime} B=A^{\prime}\left(B^{\prime}+B\right)=A^{\prime}
$$

## Three-Variable Karnaugh Maps

- 3-variable K-map

minterm locations

| ABC | $F$ |
| :--- | :--- |
| 000 | 0 |
| 001 | 0 |
| 010 | 1 |
| 011 | 1 |
| 100 | 1 |
| 101 | 0 |
| 110 | 1 |
| 111 | 0 |



$$
\begin{aligned}
F & =A^{\prime} B C^{\prime}+A^{\prime} B C+A B^{\prime} C^{\prime}+A B C^{\prime} \\
& =A^{\prime} B+A C^{\prime}+B C^{\prime} \\
& =A^{\prime} B+A C^{\prime}
\end{aligned}
$$

## Three-Variable Karnaugh Maps

- 3-variable K-map (zeros omitted)

B

$B C^{\prime}$

$A C^{\prime}$


## Three-Variable Karnaugh Maps

- 3-variable K-map



## Three-Variable Karnaugh Maps

- 3-variable K-map



## Three-Variable Karnaugh Maps

- 3-variable K-map

$$
\begin{aligned}
G & =\left(m_{1}+m_{3}+m_{5}\right)^{\prime} \\
& =\left(M_{0} M_{2} M_{4} M_{6} M_{7}\right)^{\prime}
\end{aligned}
$$




## Three-Variable Karnaugh Maps

- 3-variable K-map



## Three-Variable Karnaugh Maps

- 3-variable K-map

$$
F=a^{\prime} b^{\prime}+b c^{\prime}+a c=a^{\prime} c^{\prime}+b^{\prime} c+a b
$$



## Four-Variable Karnaugh Maps

$\square 4$-variable K-map
$F=a c d+a^{\prime} b+d^{\prime}$

| $C D A B$ | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 0 | 4 | 12 | 8 |
| 01 | 1 | 5 | 13 | 9 |
| 11 | 3 | 7 | 15 | 11 |
| 10 | 2 | 6 | 14 | 10 |


| $\mathrm{cd} \mathrm{ab}$ |  | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 1 | 1 | 1 | 1 |
| 01 |  | 1 |  |  |
| 11 |  | 1 | 1 | $1)$ |
| 10 | 1 | 1 | 1 | 1 |

minterm locations

## Four-Variable Karnaugh Maps

## -4-variable K-map



## Four-Variable Karnaugh Maps

## $\square 4$-variable K-map



## Four-Variable Karnaugh Maps

Simplify incompletely specified function

- All the 1's must be covered, but X's are optional and are set to 1's only if they will simplify the expression


$f=\sum m(1,3,5,7,9)+\sum d(6,12,13)$

$$
f=a^{\prime} d+c^{\prime} d
$$

## Four-Variable Karnaugh Maps

Simplify product-of-sums

- Circle 0's instead of 1 's
- Apply De Morgan's law converting SOP to POS

|  | 00 | 01 | 11 | 10 | $y z$ | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00 | 1 | 1 | 0 | 1 | 00 | 1 | 1 | 0 | 1 |
| 01 | 0 | 0 | 0 | 0 | - 01 | 0 | 0 | 0 | 0 |
| 11 | 1 | 0 | 1 | 1 | 11 | 1 | 0 | 1 | 1 |
| 10 | 1 | 0 | 0 | 1 | 10 | 1 | 0 | 0 | 1 |
| $f=x^{\prime} z^{\prime}+w y z+w^{\prime} y^{\prime} z^{\prime}+x^{\prime} y$ |  |  |  |  | $f^{\prime}=y^{\prime} z+w x z^{\prime}+w^{\prime} x y$ |  |  |  |  |

# Determination of Minimum Expressions Using Essential Prime Implicants 

$\square$ Implicant

- A product term of a function
$\square$ Any single 1 or any group of 1's on a K-map combined together forms a product term


## Prime implicant

- A maximal implicant
$\square$ An implicant that cannot be combined with another term to eliminate a variable
$\square$ All of the prime implicants of a function can be obtained from a K-map by expanding the 1's as much as possible in every possible way


## Determination of Minimum Expressions Using Essential Prime Implicants

## Example



## Determination of Minimum Expressions Using Essential Prime Implicants

$\square$ Determine all prime implicants

- In finding prime implicants, don't cares are treated as 1 's. However, a prime implicant composed entirely of don't cares can never be part of the minimum solution
- Not all prime implicants are needed in forming the minimum SOP


## Example

- All prime implicants:
a'b'd, bc', ac, a'c'd, ab, b'cd (composed entirely of don't cares)
- Minimum solution:

$F=a^{\prime} b^{\prime} d+b c^{\prime}+a c$


## Determination of Minimum Expressions Using Essential Prime Implicants

## Essential prime implicant (EPI)

- A prime implicant that covers some minterm not covered by any other prime implicant
-If a single term covers some minterm and all of its adjacent 1's and X's, then the term is an EPI
- Must be present in the minimum SOP

$f=C D+B D+B^{\prime} C+A C$


$$
f=B D+B^{\prime} C+A C
$$

## Determination of Minimum Expressions Using Essential Prime Implicants

$\square$ SOP minimization

1. Select all essential prime implicants
2. Find a minimum set of prime implicants which cover the minterms not covered by the essential prime implicants
$\square$ There may be freedom left after all essential prime implicants are selected (it affects optimality especially for functions with more variables)


## Determination of Minimum Expressions Using Essential Prime Implicants

$\square$ Flowchart for determining a minimum SOP using K-map


# Determination of Minimum Expressions Using Essential Prime Implicants 

-Example

| $C D A B$ | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | $\mathrm{X}_{0}$ | $1_{4}$ |  | 18 |
| 01 |  | $1_{5}$ | $1{ }_{13}$ | 19 |
| 11 |  | $\mathrm{X}_{7}$ | $\mathrm{X}_{15}$ |  |
| 10 |  | $1{ }_{6}$ |  | $1{ }_{10}$ |

Step 1: $1_{4}$ checked
Step 2: $1_{5}$ checked
Step 3: $1_{6}$ checked
EPI $\rightarrow$ A'B selected
Step 4: $1_{8}$ checked
Step 5: $\mathbf{1}_{9}$ checked
Step 6: $1_{10}$ checked
EPI $\rightarrow$ AB'D' selected Step 7: $1_{13}$ checked
(up to this point all EPIs determined)
Step 8: AC'D selected to cover remaining 1's

## Five-Variable Karnaugh Maps

$\square 5-$ var K-map

minterm locations

## Five-Variable Karnaugh Maps

$\square 5-v a r ~ K-m a p$
Expand to eliminate $A$


## Five-Variable Karnaugh Maps



## Five-Variable Karnaugh Maps



## Other Uses of Karnaugh Maps

-Use K-map to prove the equivalence of two Boolean expressions
■ K-maps are canonical representations of Boolean functions, similar to truth tables
$\square$ Use K-map to perform Boolean operations

- AND, OR, NOT operations can be done over Kmaps (truth tables)


## Other Uses of Karnaugh Maps

-Use K-map to facilitate factoring
Identify common literals among product terms

| $C D A B$ | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 |  |  |  |  |
| 01 | 1 |  |  |  |
| 11 | 1 |  | 1 |  |
| 10 |  |  | 1 | 1 |

$$
\begin{aligned}
F & =A^{\prime} B^{\prime} C^{\prime}+A^{\prime} B^{\prime} D+A C B+A C D^{\prime} \\
& =A^{\prime} B^{\prime}\left(C^{\prime}+D\right)+A C\left(B+D^{\prime}\right)
\end{aligned}
$$

## Other Uses of Karnaugh Maps

-Use K-map to guide simplification


## Other Forms of Karnaugh Maps

$\square$ Other conventions (Veitch diagrams)


## Other Forms of Karnaugh Maps

-Other conventions ( 5 -var K-map)
$F=D^{\prime} E^{\prime}+B^{\prime} C^{\prime} D^{\prime}+B C E+A^{\prime} B^{\prime} E^{\prime}+A C D E$



## Other Forms of Karnaugh Maps

-Other conventions ( 5 -var Veitch diagram)


