Logic Synthesis & Verification, Fall 2014

National Taiwan University

Problem Set 1

Due on 2014/10/15 before lecture

1 [Boolean Algebra Definition]

(15%) Does $(\{0,1\}, \oplus, \cdot, 0, 1)$, where $\oplus$ and $\cdot$ stand for Boolean XOR and AND operations, respectively, form a Boolean algebra? Which of the five postulates of Boolean algebra are satisfied and which are not?

2 [Boolean Algebra Properties]

(20%) Prove the following equalities using ONLY the five postulates of Boolean algebra (or other properties that you have proven using the postulates). Please specify clearly which postulate is applied in each step of your derivation.

(a) $a + (b + c) = (a + b) + c$
(b) $(a + b)' = a' \cdot b'$

3 [Relation over Boolean Algebra]

(20%) Define the relation $\leq$ in a Boolean algebra with carrier $\mathbb{B}$ as follows

$$a \leq b \text{ if and only if } a \cdot b' = 0$$

for all $a, b \in \mathbb{B}$, where b' is the inverse element of b. Prove that the following properties hold for all $a, b, c \in \mathbb{B}$:

(a) $a \cdot b \leq a \leq a + c$
(b) $a \leq b$ and $a \leq c$ if and only if $a \leq b \cdot c$

4 [Boolean Functions]

(10%) How many Boolean functions of n variables are there under a Boolean algebra with $|\mathbb{B}| = m$? Please explain your answer.

5 Alternative Views on Boolean Functions

(15%) Given a three-variable Boolean function f_1 over $\mathbb{B} = \{0, 1\}$ with

$$f_1 = (1)x'y'z' + (0)x'y'z + (1)x'yz' + (0)x'yz + (1)xy'z' + (1)xy'z + (0)xyz' + (1)xyz,$$

define a two-variable Boolean function $f_2(y, z)$ taking values over $\{0, 1, x, x'\}$ as an alternative representation of f_1.

6 [Boolean Functions]

(20%) Let g and h be single-variable Boolean functions. For each of the following cases, express $f(0)$ and $f(1)$ as simplified formulas involving $g(0), g(1), h(0)$, and $h(1)$.

(a) $f(x) = g(h(x))$
(b) $f(x) = g(g'(x))$