Logic Synthesis & Verification, Fall 2014
National Taiwan University

Problem Set 3

Due on 2014/11/14 by 17:30
(Please hand in your assignment in the instructor's mailbox in EE2.)

1   [Symmetric Functions]

(20%) Two types of variable symmetries of a Boolean function are defined as follows.

S1: $f(\ldots, x, y, \ldots) = f(\ldots, y, x, \ldots)$
S2: $f(\ldots, x, y, \ldots) = \neg f(\ldots, y, x, \ldots)$

For a Boolean function $f(x_1, \ldots, x_n)$, do S_1 and S_2 form equivalence relations over the variables $X = \{x_1, \ldots, x_n\}$? Note that an equivalence relation R must be reflexive, i.e., $(a, a) \in R$ for all $a \in X$, symmetry, i.e., $(a, b) \in R \to (b, a) \in R$, and transitive, i.e., $(a, b) \in R$ and $(b, c) \in R \to (a, c) \in R$.

2   [Unate Functions]

(10%) Show that every prime implicant of a unate function is an essential prime implicant.

3   [Generalized Cofactor]

(20%) Prove or disprove the following equalities.

(a) (5%) $\neg f = g \cdot co(\neg f, g) + \neg g \cdot \neg co(\neg f, \neg g)$
(b) (5%) $co(co(f, g), h) = co(f, g \cdot h)$
(c) (5%) $co(f \cdot g, h) = co(f, h) \cdot co(g, h)$
(d) (5%) $co(\neg f, g) = \neg co(f, g)$

4   [Unate Recursive Paradigm: Complementation]

(20%) Complement the function

$$f = a'b'c + a'cd + ab'd' + bc + bc'd + b'cd',$$

using the unate recursive paradigm.

5   [Minimum Column Covering]

(10%) Given a $m \times n$ Boolean matrix, how would you use a (CNF-based) SAT solver to solve the MINIMUM column covering problem? Specifically, how would you encode the problem into CNF formulas and apply the solver to solve them? Please have a procedure that queries the solver at most $O(\log n)$ times.

6   [Quine-McCluskey]

(20%) Given an incompletely specified function over variables a, b, c, d, e, f with onset minterms

$$\{000001, 000010, 000011, 000101, 000111, 001011, 001101,$$
$$001111, 100001, 100011, 101011, 101111, 111011, 111100, \}$$

and don't care set minterms

$$\{000000, 000110, 011111, 110011, 110110, 111010, 111111\},$$

apply the Quine-McCluskey procedure to minimize it. Identify all essential prime implicants and find all minimum sum-of-products expressions. Show intermediate results of your derivation.