

# Logic Synthesis & Verification, Fall 2014

National Taiwan University

## Problem Set 1 Solution

### 1 [Boolean Algebra Definition]

|                                     |  |
|-------------------------------------|--|
| <input checked="" type="checkbox"/> | $B = \{0, 1\}$   |
| <input checked="" type="checkbox"/> | ① closed, ② commutative law  |
| <input checked="" type="checkbox"/> | ③ $\begin{array}{ c c c c } \hline 0 & 1 & \cdot & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 1 & 0 & 1 \\ \hline 1 & 0 & 0 & 1 & 0 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline \end{array}$ 由左方的表格可知①和②皆符合 |
| <input checked="" type="checkbox"/> | ④ distributive law<br>$(a+b)(a+c) = (a'b+ab')(ac+ac')$<br>$= a'bc + ab'c$ → 两式不相等<br>$a(b+c) = a'bc + a \cdot (b+c)' \Rightarrow$ ∵ 不遵守 distributive law   |
| <input checked="" type="checkbox"/> | ⑤ Identities<br>$0 \oplus a = 0 \cdot a' + 0' \cdot a = a$ 符合<br>$1 \cdot a = a$ 符合<br>$⑥ Complement$<br>$a \oplus a' = 1$ 符合<br>$a \cdot a' = 0$ 符合   |
| <input checked="" type="checkbox"/> | ∴ $\{0, 1\}$ , ④, ⑤ 不是 Boolean Algebra, 因为它满足 1, 2, 4, 5 但不满足 3  |

### 2 [Boolean Algebra Properties]

|                                     |   |  |
|-------------------------------------|---|--|
| <input checked="" type="checkbox"/> | $a + (b+c) = x, (a+b)+c = y$  | $*1 \quad x \stackrel{id.}{=} x \cdot 1 \stackrel{com.}{=} x \cdot (x+x') \stackrel{dis.}{=} x \cdot x + x \cdot x' \stackrel{id.}{=} x \cdot x = x$ |
|                                     | $ax = a[a+(b+c)] \stackrel{dis.}{=} aa+a(b+c) \stackrel{id.}{=} a+ab+ac \stackrel{id.}{=} a$  | $\stackrel{com.}{=} x \cdot x + 0 = x \cdot x$   |
|                                     | $a \oplus b = a[(a+b)+c] \stackrel{dis.}{=} a(a+b)+ac \stackrel{id.}{=} a+ac \stackrel{id.}{=} a$   | $*2 \quad x + (x \cdot y) \stackrel{id.}{=} (x \cdot 1) + (x \cdot y) \stackrel{dis.}{=} x \cdot (1+y) \stackrel{id.}{=} x \cdot 1 = x$              |
|                                     | $a' \cdot x = a'[a+(b+c)] \stackrel{dis.}{=} a'a+a'(b+c) \stackrel{com.}{=} 0 + a'(b+c) \stackrel{id.}{=} a'(b+c)$  | $*3 \quad x \cdot (x+y) \stackrel{id.}{=} (x+0) \cdot (x+y) \stackrel{dis.}{=} x + (0 \cdot y) \stackrel{id.}{=} x + 0 \stackrel{id.}{=} x$          |
|                                     | $a' \cdot y = a'[(a+b)+c] \stackrel{dis.}{=} a'(a+b)+a'c \stackrel{dis.}{=} (a'a+a'b)+a'c \stackrel{com.}{=} (0+a'b)a'c \stackrel{id.}{=} a'b+a'c \stackrel{dis.}{=} a'(b+c)$   | $*4 \quad 1+x \stackrel{id.}{=} (1+x) \cdot 1 \stackrel{com.}{=} (1+x)(x+x') \stackrel{dis.}{=} x+(1-x) \stackrel{id.}{=} x+x' \stackrel{com.}{=} 1$ |
|                                     | $\therefore ax = ay, a'x = a'y$   | $*5 \quad 0 \cdot y \stackrel{id.}{=} 0 \cdot y + 0 \stackrel{com.}{=} 0 \cdot y + y \cdot y' \stackrel{dis.}{=} y \cdot (1+y')$                     |
|                                     | $x = x \cdot 1 \stackrel{com.}{=} x \cdot (a+a') \stackrel{dis.}{=} ax + a'x = ay + a'y \stackrel{dis.}{=} y(a+a') \stackrel{com.}{=} y \cdot 1 \stackrel{id.}{=} y \stackrel{id.}{=} y \cdot y' \stackrel{com.}{=} 0$  |  |
|                                     | $\therefore a + (b+c) = (a+b)+c$ (acc.)   |  |
| <input checked="" type="checkbox"/> | $(b) (a+b) \cdot (a'b') = [(a+b) \cdot a'] \cdot b' \stackrel{dis.}{=} (aa'+a'b) \cdot b' \stackrel{com.}{=} (0+a'b) \cdot b' \stackrel{id.}{=} (a'b)b' \stackrel{com.}{=} a'b = 0$<br>from duality, $(a \cdot b) + (a'b') = 1$<br>substitute $a \rightarrow a'$ , $b \rightarrow b' \Rightarrow (a'b') + (a+b) = 1 \therefore (a'b')$ is complement of $(a+b)$<br>$\therefore (a+b)' = (a'b')$ |  |

### 3 [Relation over Boolean Algebra]

|                                     |   |
|-------------------------------------|---|
| <input checked="" type="checkbox"/> | $a \cdot b \cdot a' = 0 \therefore a \cdot b \leq a$ , $a \cdot (a+c)' = a \cdot (a \cdot c') = 0 \therefore a \leq a+c$  |
| <input checked="" type="checkbox"/> | $a \cdot b' = 0, a \cdot c' = 0 \therefore a \cdot (b+c)' = a \cdot (bc)' = 0 \therefore a \leq b, a \leq c \Rightarrow a \leq b+c$   |
| <input checked="" type="checkbox"/> | $a \cdot (b+c)' = 0 \quad a \cdot (b'+c') = 0 \quad a \cdot b' + a \cdot c' = 0 \quad a \cdot b' = 0, a \cdot c' = 0 \therefore a \leq b \cdot c \Rightarrow a \leq b, a \leq c$<br>$\therefore a \leq b$ and $a \leq c$ iff $a \leq b \cdot c$ |

## 4 [Boolean Functions]

$$(m)^2^n$$

formula 共有  $n$  個 variable, 每個 variable 有  $x, \bar{x}$  兩種可能  $\Rightarrow$  formula 共有  $2^n$  種可能

每種 formula 可能 map 到  $m$  種 output  $\Rightarrow$  function 共  $(m)^{2^n}$  種可能

## 5 [Alternative Views on Boolean Functions]

### 5. [Alternative Views on Boolean Functions]

$$f_2(y, z) = y'z' \cdot f_2(0, 0) + y'z \cdot f_2(0, 1) + yz' \cdot f_2(1, 0) + yz \cdot f_2(1, 1)$$

Compare the corresponding minterm with  $f_1$

$$\begin{aligned} f_1 &= ((1)X'Y'Z' + (1)XY'Z') + ((0)X'Y'Z + (1)XY'Z) + ((1)X'YZ' + (0)XYZ) + ((0)X'YZ + (1)XYZ) \\ &= (1)Y'Z' + (X)Y'Z + (X')YZ' + (X)YZ \end{aligned}$$

| $y$ | $z$ | $f_2(y, z)$ |
|-----|-----|-------------|
| 0   | 0   | 1           |
| 0   | 1   | X           |
| 1   | 0   | X'          |
| 1   | 1   | X           |

The remaining entries can be determined by  
minterm canonical form

$$f_2(y, z) = (1)Y'Z' + (X)Y'Z + (X')YZ' + (X)YZ$$

## 6 [Boolean Functions]

$$(a) f(x) = g(h(x)) = h(x)g(1) + h(x)'g(0)$$

$$\Rightarrow f(0) = h(0)g(1) + h(0)'g(0)$$

$$f(1) = h(1)g(1) + h(1)'g(0)$$

$$(b) f(x) = g(g'(x)) = g'(x)g(1) + (g'(x))'g(0) = g'(x)g(1) + g(x)g(0)$$

$$\Rightarrow f(0) = g'(0)g(1) + g(0)g(0) = g'(0)g(1) + g(0) = g(1) + g(0)$$

$$f(1) = g'(1)g(1) + g(1)g(0) = g(1)g(0)$$