1.

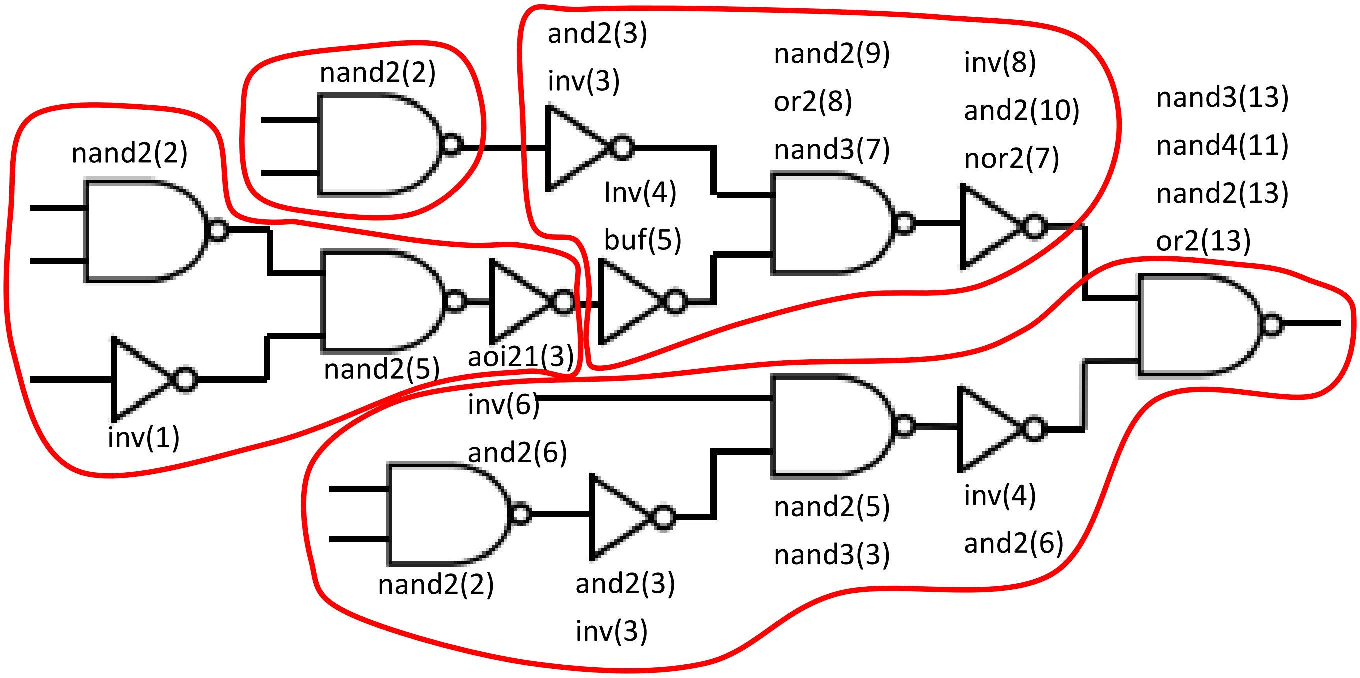
(a)

i. Every node in the subject graph must be covered by some pattern graph(s)

ii. Every input of a match must be the output of some pattern graph(s)

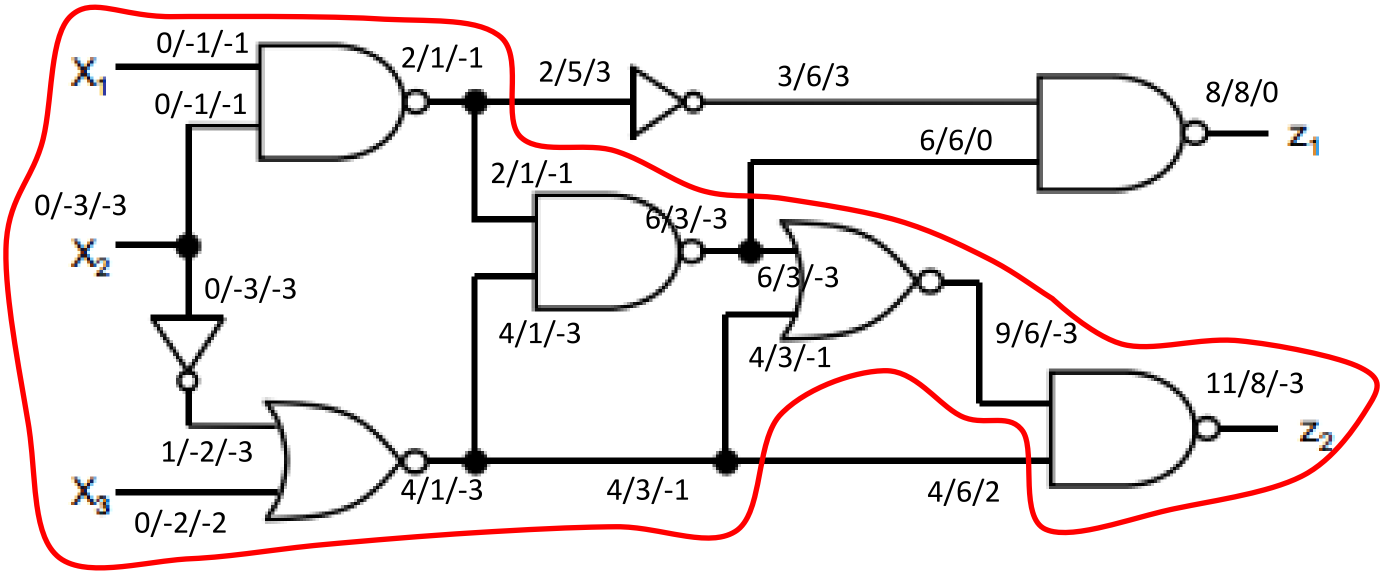
iii. Primary outputs must be produced by some pattern graph(s)

(b)

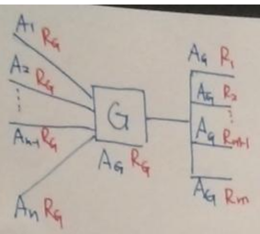


2.

(a)



(b)



$A_G = \underset{n}{\mathrm{Max}}\, A_n + D(G)$

$R_G = \underset{m}{\mathrm{Min}}\, R_m - D(G)$

Slack of inputs $= \{R_G - A_1, R_G - A_2, \ldots, R_G - A_n\}$

Slack of outputs $= \{R_1 - A_G, R_2 - A_G, \ldots, R_m - A_G\}$

If one output has a slack of C, i.e., $R_i - A_G = C \quad (1 \le i \le m)$

$\Rightarrow R_G = \underset{m}{\mathrm{Min}}\, R_m - D(G) \le R_i - D(G) \Rightarrow R_G - A_G \le C - D(G)$

Say that $\underset{n}{\mathrm{Max}}\, A_n = A_j \quad (1 \le j \le n) \Rightarrow R_G - (A_j + D(G)) \le C - D(G)$

$\Rightarrow R_G - A_j \le C$ #

If one output has a slack C there exists one input with slack $\le C$

$\Rightarrow$ If a primary output has a slack C, then there exists at least one primary input with slack $\le C$

If one input has a slack of C, i.e., $R_G - A_j = C \quad (1 \le j \le n)$

$\Rightarrow A_G = \underset{n}{\mathrm{Max}}\, A_n + D(G) \ge A_j + D(G) \Rightarrow R_G - A_G \le C - D(G)$

Say that $\underset{m}{\mathrm{Max}}\, R_m = R_i \quad (1 \le i \le n) \Rightarrow (R_i - D(G)) - A_G \le C - D(G)$

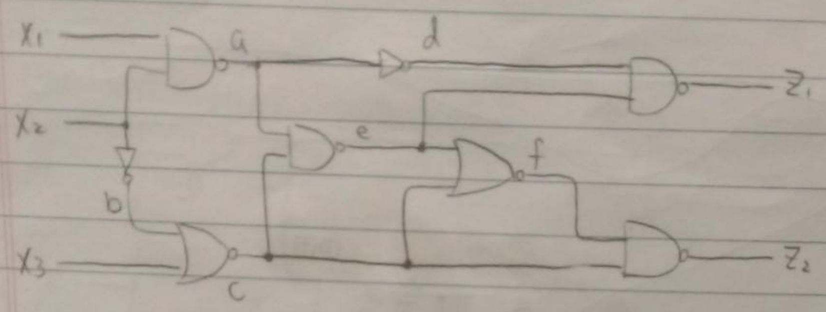
$\Rightarrow R_i - A_G \le C$ #

If one input has a slack C there exists one output with slack $\le C$

$\Rightarrow$ If a primary input has a slack C, then there exists at least one primary output with slack $\le C$

(c)

(c)



$z_1(1,t=\infty) = \bar{x}_1 + \bar{x}_2$

$z_1(1,t=5) = d(0,t=3) \cup e(0,t=3) = a(1,t=2) \cup [a(1,t=1) \cap c(1,t=1)]$

$= (x_1(0,t=0) \vee x_2(0,t=0)) \cup \phi = \bar{x}_1 + \bar{x}_2$

$z_1(1,t=4) = d(0,t=2) \cup e(0,t=2) = a(1,t=1) \cup [a(1,t=0) \cap c(1,t=0)]$

$= \phi$

$z_1(1,t=\infty) \setminus z_1(1,t=4) = \bar{x}_1 + \bar{x}_2$ satisfiable $\therefore z_1$ is not stable until $t=5$ under $(x_1,x_2,x_3) = (0,x,x)$ or $(x,0,x)$

$z_1(0,t=\infty) = x_1 \cdot x_2$

$z_1(0,t=6) = d(1,t=4) \cap e(1,t=4) = a(0,t=3) \cap [a(0,t=2) \cup c(0,t=2)]$

$= [x_1(1,t=1) \cap x_2(1,t=1)] \cap [x_1(1,t=0) \cap x_2(1,t=0) \cup \phi]$

$= x_1 x_2 \cap x_1 \cdot x_2 = x_1 \cdot x_2$

$z_1(0,t=5) = d(1,t=3) \cap e(1,t=3) = a(0,t=2) \cap [a(0,t=1) \cup c(0,t=1)]$

$= a(0,t=2) \cap \phi = \phi$

$z_1(0,t=\infty) \setminus z_1(1,t=5) = x_1 x_2$ satisfiable $\therefore z_1$ is not stable until $t=6$ under $(x_1,x_2,x_3) = (1,1,x)$

$\therefore z_1$ has longest delay 6

$z_2(1,t=\infty) = 1$

$z_2(1,t=8 = f(0,t=6) \cup c(0,t=6) = [e(1,t=3) \cup c(1,t=3)] \cup [b(1,t=3) \cup x_3(1,t=3)]$

$= \{[a(0,t=1) \cup c(0,t=1)] \cup [b(0,t=0) \cap x_3(0,t=0)]\} \cup [\bar{x}_2 + x_3]$

$= \{\phi \cup \phi \cap \bar{x}_3\} \cup [\bar{x}_2 + x_3] = \bar{x}_2 + x_3$

$z_2(1,t=\infty) \setminus z_2(1,t=8) = x_2 \bar{x}_3$ satisfiable

$\therefore z_2$ is not stable until $t=9$ under $(x_1,x_2,x_3) = (x,1,0)$

$z_2(0,t=\infty) = \phi \Rightarrow z_2$ never become false

$\therefore z_2$ has longest delay 9

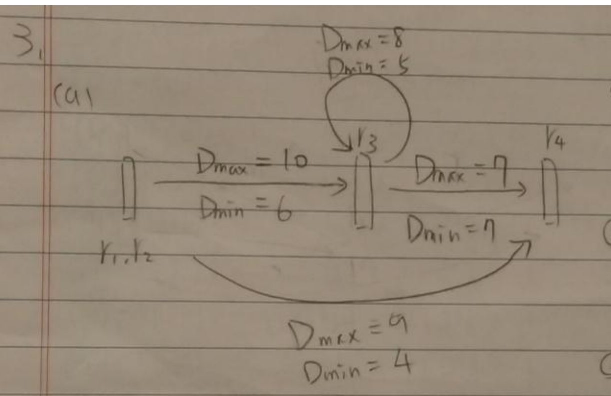
$\Rightarrow$ The longest time delay of entire circuit is 9 #

3.

(a)

3.

(a)



① $r_1, r_2 \to r_3$: $T_{cycle} + skew_{1,3} \geq 10$

$skew_{1,3} \leq 6$

$0 \leq |skew_{1,3}| \leq T_{cycle}$

② $r_3 \to r_4$: $T_{cycle} + skew_{3,4} \geq 7$

$skew_{3,4} \leq 7$

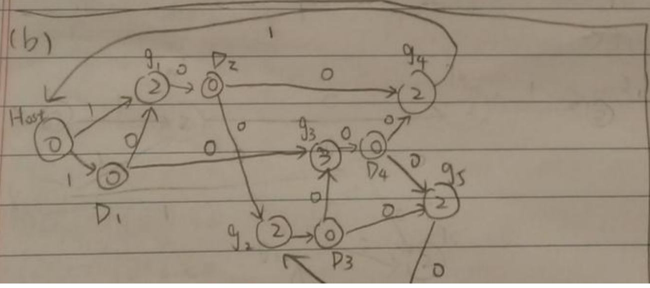
$0 \leq |skew_{3,4}| \leq T_{cycle}$

③ $r_{1,2} \to r_4$: $T_{cycle} + skew_{1,4} \geq 9$

$skew_{1,4} \leq 4$

$0 \leq |skew_{1,4}| \leq T_{cycle}$

(b)



④ $r_3 \to r_3$: $T_{cycle} + 0 \geq 8$

$\therefore$ minimum clock period = 8ns

$skew_{1,2} = 0$

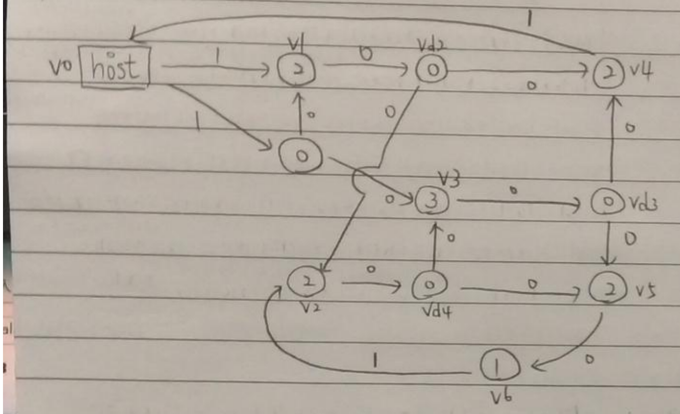
$skew_{1,3} = 2$

$skew_{1,4} = 1$

#

(b)(c)

(b)



(c)

W.

	V0	V1	V2	V3	V4	V5	V6	Vd1	Vd2	Vd3	Vd4
V0	0	1	1	1	1	1	1	1	1	1	1
V1	1	0	0	0	0	0	0	⊥	0	0	0
V2	1	⊥	0	0	0	0	0	⊥	⊥	0	0
V3	1	⊥	1	0	0	0	0	⊥	⊥	0	1
V4	1	⊥	⊥	⊥	0	⊥	⊥	⊥	⊥	⊥	⊥
V5	2	⊥	1	1	1	0	0	⊥	⊥	1	1
V6	2	⊥	1	1	1	1	0	⊥	⊥	1	1
Vd1	1	0	0	0	0	0	0	0	0	0	0
Vd2	1	⊥	0	0	0	0	0	⊥	0	0	0
Vd3	1	⊥	1	1	0	0	0	⊥	⊥	0	1
Vd4	1	⊥	1	0	0	0	0	⊥	⊥	0	0

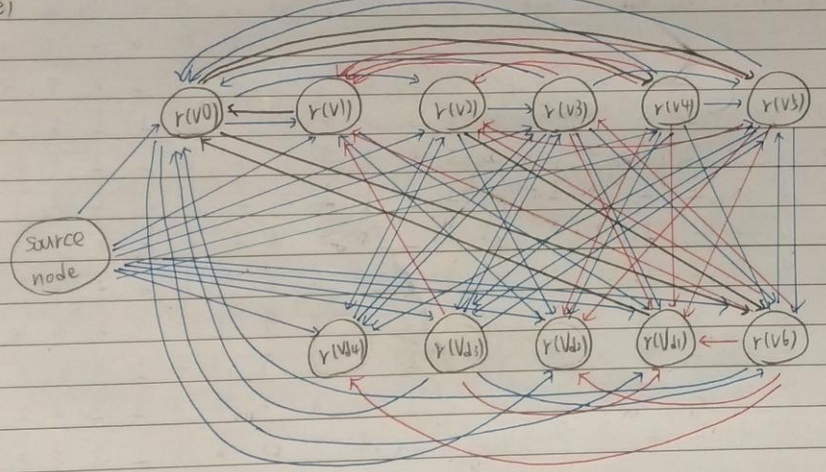
D

	V0	V1	V2	V3	V4	V5	V6	Vd1	Vd2	Vd3	Vd4
V0	0	2	4	7	9	9	10	0	2	7	4
V1	9	2	4	7	9	9	10	⊥	2	7	4
V2	7	⊥	2	5	7	7	8	⊥	⊥	5	2
V3	5	⊥	8	3	5	5	6	⊥	⊥	3	8
V4	2	⊥	⊥	⊥	2	⊥	⊥	⊥	⊥	⊥	⊥
V5	10	⊥	5	8	10	2	3	⊥	⊥	8	5
V6	8	⊥	3	6	8	8	1	⊥	⊥	6	3
Vd1	9	2	4	7	9	9	10	0	2	7	4
Vd2	7	⊥	2	5	7	7	8	⊥	0	5	2
Vd3	2	⊥	5	8	2	2	3	⊥	⊥	0	5
Vd4	5	⊥	8	3	5	5	6	⊥	⊥	3	0

(d)(e)(f)

(d) Legality $= r(u) - r(v) \le w(e)$	$D > 5 = r(u) - r(v) \le W(u,v) - 1$
$r(V_0) - r(V_1) \le 1$; $r(V_0) - r(V_{d1}) \le 1$	$r(V_0) - r(V_3) \le 0$; $r(V_0) - r(V_4) \le 0$; $r(V_0) - r(V_5) \le 0$; $r(V_0) - r(V_{d3}) \le 0$; $r(V_0) - r(V_6) \le 0$
$r(V_1) - r(V_{d2}) \le 0$.	$r(V_1) - r(V_0) \le 0$; $r(V_1) - r(V_3) \le -1$, $r(V_1) - r(V_4) \le -1$; $r(V_1) - r(V_{d3}) \le -1$; $r(V_1) - r(V_5) \le -1$; $r(V_1) - r(V_6) \le -1$
$r(V_2) - r(V_{d4}) \le 0$	$r(V_2) - r(V_0) \le 0$; $r(V_2) - r(V_4) \le -1$; $r(V_2) - r(V_5) \le -1$; $r(V_2) - r(V_6) \le -1$
$r(V_3) - r(V_{d3}) \le 0$	$r(V_3) - r(V_2) \le 0$; $r(V_3) - r(V_6) \le -1$; $r(V_3) - r(V_{d4}) \le 0$
$r(V_4) - r(V_0) \le 1$	$r(V_5) - r(V_0) \le 1$; $r(V_5) - r(V_3) \le 0$; $r(V_5) - r(V_4) \le 0$; $r(V_5) - r(V_{d3}) \le 0$
$r(V_5) - r(V_6) \le 0$	$r(V_6) - r(V_0) \le 1$; $r(V_6) - r(V_3) \le 0$, $r(V_6) - r(V_4) \le 0$; $r(V_6) - r(V_5) \le 0$; $r(V_6) - r(V_{d3}) \le 0$
$r(V_6) - r(V_2) \le 1$	$r(V_{d1}) - r(V_0) \le 0$, $r(V_{d1}) - r(V_3) \le -1$; $r(V_{d1}) - r(V_4) \le -1$; $r(V_{d1}) - r(V_5) \le -1$; $r(V_{d1}) - r(V_6) \le -1$; $r(V_{d1}) - r(V_{d3}) \le -1$
$r(V_{d1}) - r(V_1) \le 0$; $r(V_{d1}) - r(V_3) \le 0$	$r(V_{d2}) - r(V_0) \le 0$; $r(V_{d2}) - r(V_4) \le -1$; $r(V_{d2}) - r(V_5) \le -1$; $r(V_{d2}) - r(V_6) \le -1$
$r(V_{d2}) - r(V_2) \le 0$; $r(V_{d2}) - r(V_4) \le 0$	$r(V_{d3}) - r(V_3) \le 0$
$r(V_{d3}) - r(V_5) \le 0$; $V(V_{d3}) - r(V_4) \le 0$	$r(V_{d4}) - r(V_2) \le 0$; $r(V_{d4}) - r(V_6) \le -1$.
$r(V_{d4}) - r(V_3) \le 0$; $r(V_{d4}) - r(V_5) \le 0$.	

(e)



—: weight = 1

—: weight = 0.

—: weight = -1

(f) No negative - weighted cycle

4.

(a)

$\delta_{1\times 2}(x, s, t_0, t_1) = (s', (t_0', t_1')) = (s, (t_0 \oplus t_1, x \oplus t_0 \oplus t_1))$

$\lambda_{1\times 2} = \lambda_1 \oplus \lambda_2 = (x \oplus s) \oplus (x \oplus (t_0 \oplus t_1)) = s \oplus t_0 \oplus t_1$

Initial state $= (s', t_0' t_1')$

(b)

$$(S' \equiv S)\big(t_0{}' \equiv (t_0 \oplus t_1)\big)$$

(c)

start from initial state 000

Iteration	1	2	3
Reached	000	000,001	000,001, 011,010
Current	000	000,001	000,001, 011,010
Next	000,001	000,001, 011,010	000,001, 011,010,

(d)

Not equivalent for both cases

Both outputs are not constant 0