## Special Topics on Applied Mathematical Logic Spring 2012

National Taiwan University

## Problem Set 5

. Due on 2012/05/08 Drop your solution in the instructor's mail box at EE2 building by 18:00

## 1 [Models of Theories]

Exercises 1, 2, and 3 of Section 2.6.

## 2 [Natural Deduction]

- (a) Find a deduction for the statement  $((\forall x.Px) \land (\forall y.Py \Rightarrow Qy)) \Rightarrow \forall y.Qy$ .
- (b) Find a deduction for  $((A\Rightarrow B)\land (A\Rightarrow C))\Rightarrow (A\Rightarrow (B\land C))$  and its representation in  $\lambda$ -terms.
- (c) Define  $\lceil \mathsf{T} \rceil = \lambda x. \lambda y. x$  and  $\lceil \mathsf{F} \rceil = \lambda x. \lambda y. y$ . Furthermore, let  $\lceil \mathsf{if} \ t$  then u else  $v \rceil = tuv$ . Show  $\lceil \mathsf{if} \ \mathsf{T}$  then u else  $v \rceil = u$  and  $\lceil \mathsf{if} \ \mathsf{F}$  then u else  $v \rceil = v$ .