

Special Topics on Applied Mathematical Logic

Spring 2012

Lecture 1

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Course Info (1/4)

- Instructors
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- TA
 - TBA

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Course Info (2/4)

- Course webpage
 - <http://cc.ee.ntu.edu.tw/~jhjiang/instruction/courses/spring12-logic/logic.html>
- Contact list
 - NTU email addresses of enrolled students will be used for contact

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Course Info (3/4)

- Grading policy
 - Homework 30%
 - Around 8 homework assignments in total
 - Discussions are encouraged, but solutions should be done independently
 - Late homework (20% off per day)
 - Peer-assisted grading
 - Midterm 30%
 - In-class exam on 4/16
 - Quiz 10%
 - In replacement of final exam
 - For materials covered after midterm
 - Project 30%
 - Three possible types of projects (survey, implementation, research)
 - Suggested topics to be announced later

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Course Info (4/4)

- Prerequisite
 - Discrete math?
- Textbooks
 - A Mathematical Introduction to Logic (H. Enderton)
 - Proofs and Types (J.-Y. Girard, Y. Lafont, and P. Taylor)

Introduction

About the Course

- Cover selected topics on mathematical logic and its applications in computer science
 - This year we cover
 - Basics of mathematical logic (particularly, first-order logic)
 - Textbook: A Mathematical Introduction to Logic, H. Enderton
 - Computer-science oriented logic
 - Textbook: Proofs and Types, J.-Y. Girard, Y. Lafont, and P. Taylor
- Aim at strengthening the capability of logical reasoning
 - Logic might be a weak subject for non-phonetic (non-alphabetic) language speakers
- What do you gain from the course?
 - Logical thinking
 - Formalism and preciseness

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Importance of Mathematical Logic

- Foundations of mathematics
 - Almost all ordinary mathematics can be formalized in terms of sets
 - Algebra, analysis, arithmetic, geometry, ...
- Applications in computer science
 - Programming language, verification, computability, ...
 - Hardware/software design specification
 - Model checking
 - Type inference in programming language
 - Theorem proving for large knowledge bases
 - Satisfiability modulo theories
 - Logic synthesis

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A Brief History

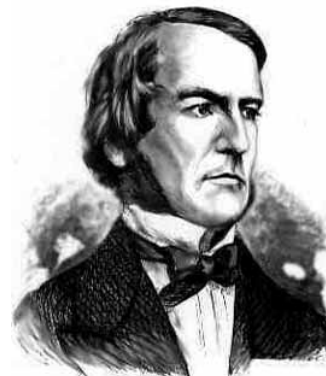
- Arithtotle (384-322 BC)
 - Theory of syllogisms
- Euclid (fl. 300 BC)
 - The fifth postulate of geometry
 - Consistency of axioms
 - Crisis of mathematical foundations



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A Brief History (cont'd)

- Leibniz (1646-1716)
 - *Calculus ratiocinator*
(inference engine / calculating machine)
- Boole (1815-1864)
 - *Law of thoughts*



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Subjects in Mathematical Logic

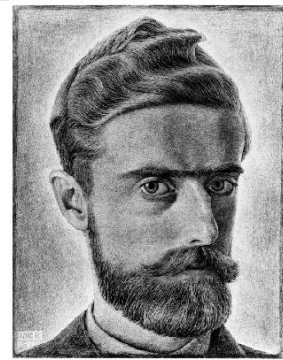
- Subjects
 - Set theory
 - Cantor's idea on sizes of infinite sets
 - Continuum Hypothesis (CH)
 - Zermelo-Fraenkel set theory
 - Axiom of Choice (AC)
 - Russell's paradox
 - » A set S containing exactly the sets that are not members of themselves ?!
 - Proof theory
 - Hilbert's program
 - Gödel's incompleteness theorem
 - Model theory
 - Löwenheim-Skolem theorem
 - Gödel's completeness theorem
 - Recursion theory
 - Turing's machine and computability



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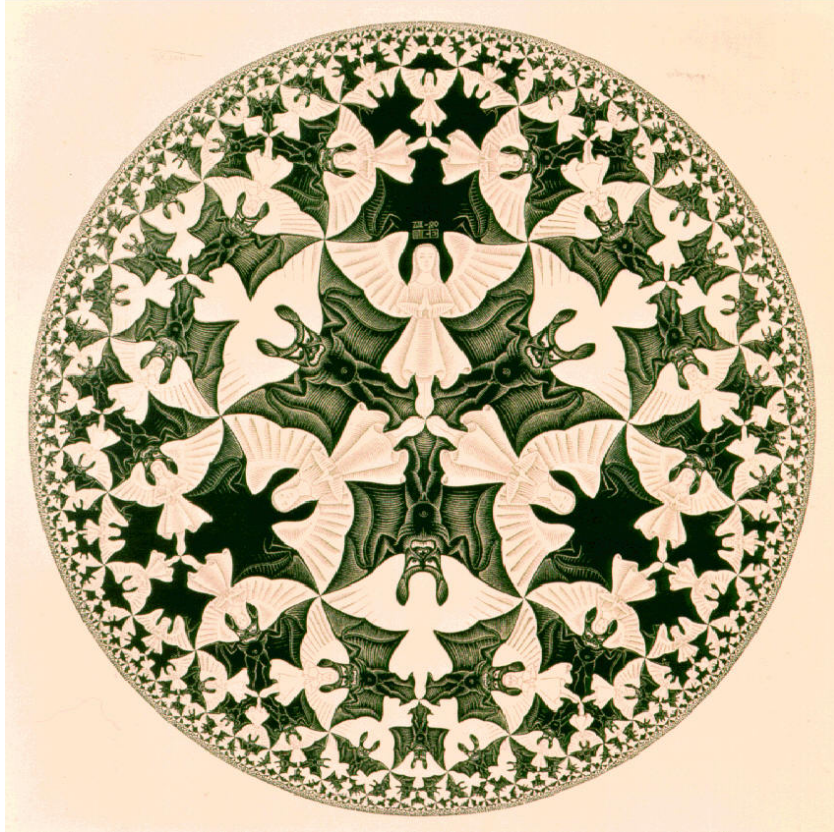
Fun Visualization

- M.C. Escher (1898-1972)
 - Dutch artist
 - Best known for his mathematically inspired art work



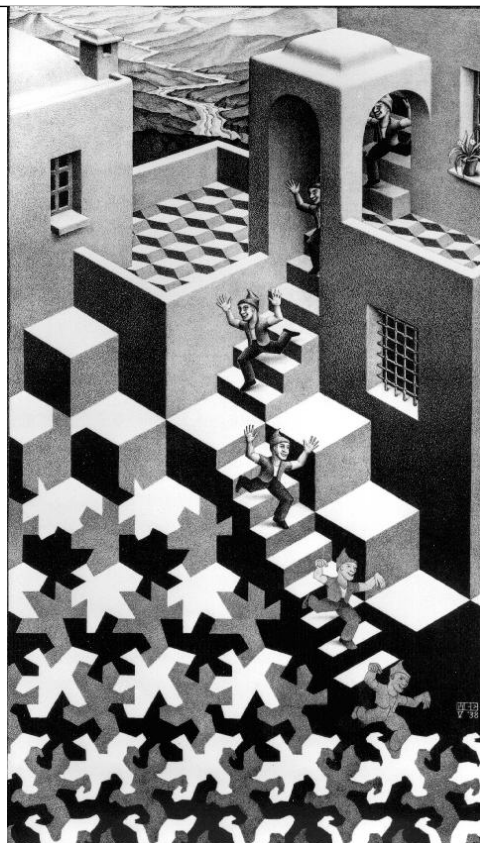
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Good vs. Evil



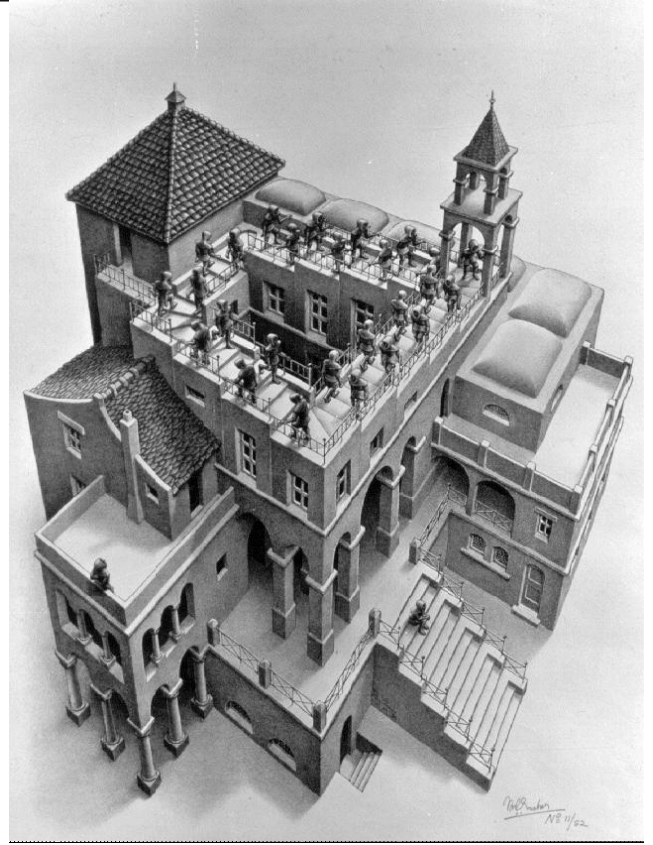
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Abstract vs. Concrete

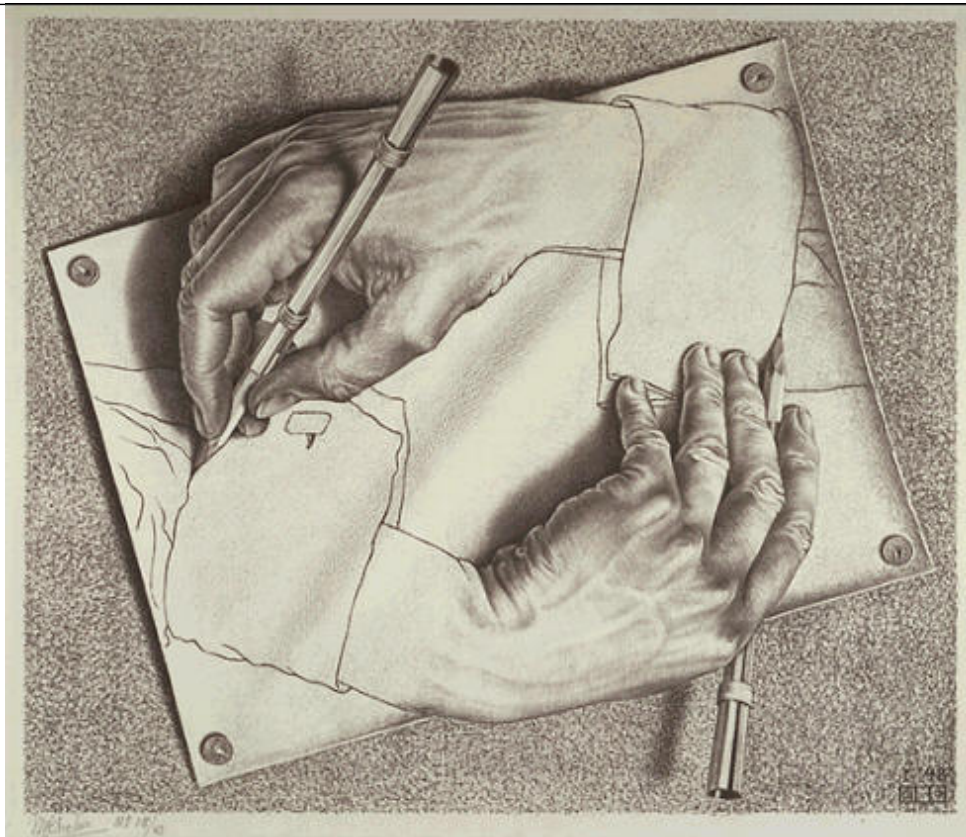


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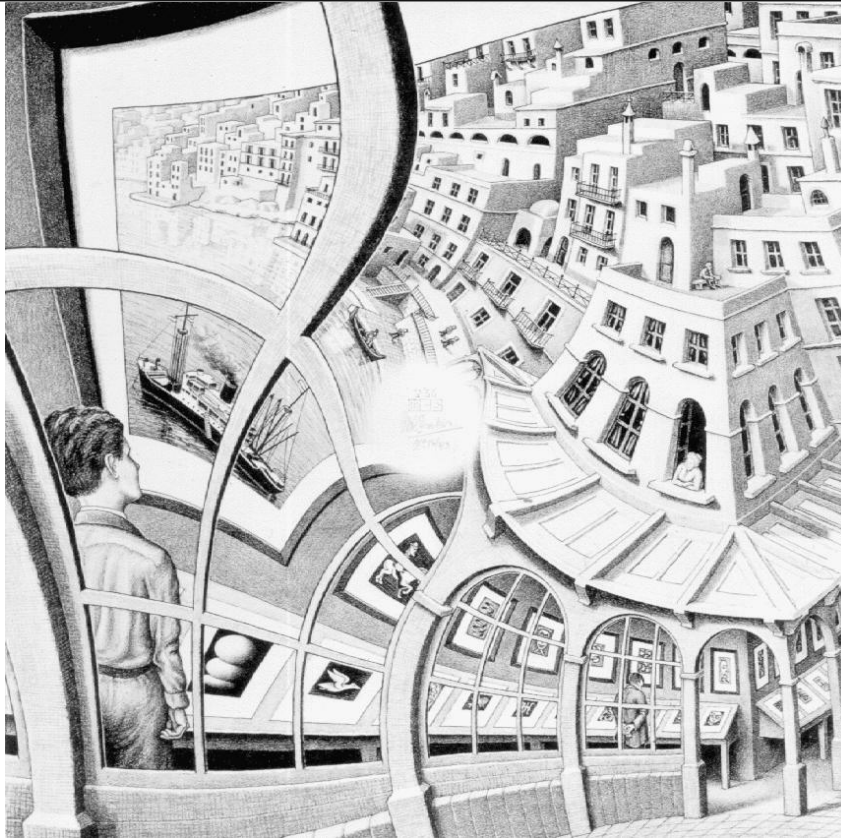
Consistency



Causality



Recursion



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Subjects to Cover

- Sentential (propositional) logic
- First-order logic
 - Models and definability
 - Soundness theorem
 - Completeness theorem
 - Compactness theorem
- Undecidability
 - First incompleteness theorem
 - Second incompleteness theorem

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Subjects to Cover

- Natural deduction
- Curry-Howard isomorphism
- Normalization theorem
- Sequent calculus
- Cut elimination
- System F
- Interactive theorem proving

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Are You Ready ?

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