

# Introduction to Electronic Design Automation

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# Logic Synthesis

High-level synthesis



Logic synthesis



Physical design

Part of the slides are by courtesy of Prof. Andreas Kuehlmann

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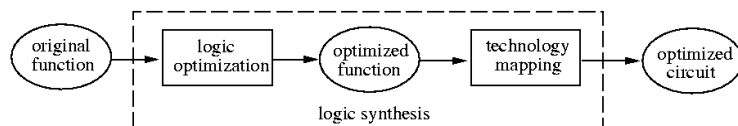
## Logic Synthesis

### □ Course contents

- Overview
- Boolean function representation
- Logic optimization
- Technology mapping

### □ Reading

- Chapter 6



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## High-Level to Logic Synthesis

### □ Hardware is normally partitioned into two parts:

- **Data path:** a network of functional units, registers, multiplexers and buses.
- **Control:** the circuit that takes care of having the data present at the right place at a specific time (i.e. FSM), or of presenting the right instructions to a programmable unit (i.e. microcode).

### □ High-level synthesis often focuses on data-path optimization

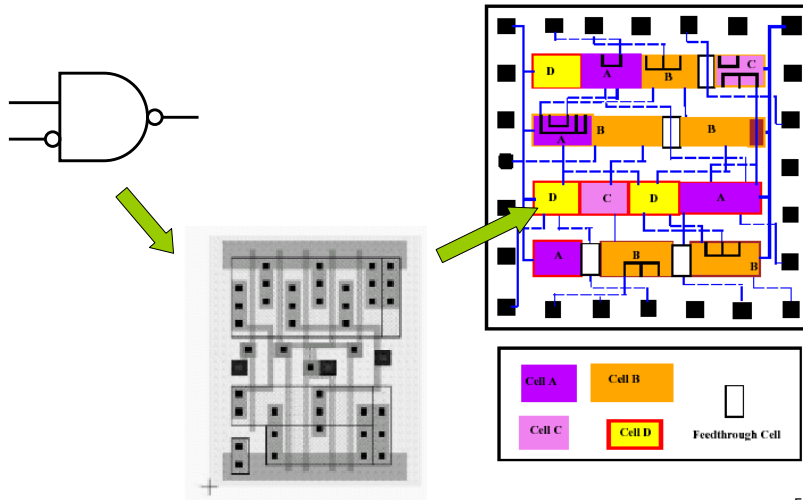
- The control part is then realized as an FSM

### □ Logic synthesis often focuses on control-logic optimization

- Logic synthesis is widely used in application-specific IC (ASIC) design, where standard cell design style is most common

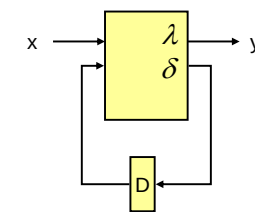
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## Standard-Cell Based Design



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## Transformation of Logic Synthesis



**Given:** Functional description of finite-state machine  $F(Q, X, Y, \delta, \lambda)$  where:

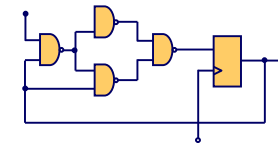
$Q$ : Set of internal states

$X$ : Input alphabet

$Y$ : Output alphabet

$\delta: X \times Q \rightarrow Q$  (next state function)

$\lambda: X \times Q \rightarrow Y$  (output function)



**Target:** Circuit  $C(G, W)$  where:

$G$ : set of circuit components  $g \in \{\text{gates, FFs, etc.}\}$

$W$ : set of wires connecting  $G$

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## Boolean Function Representation

Logic synthesis translates **Boolean functions** into **circuits**

We need representations of Boolean functions for two reasons:

- to represent and manipulate the actual circuit that we are implementing
- to facilitate *Boolean reasoning*

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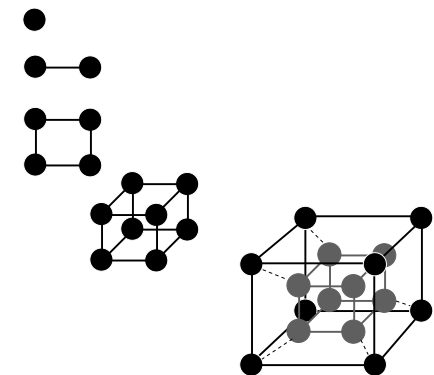
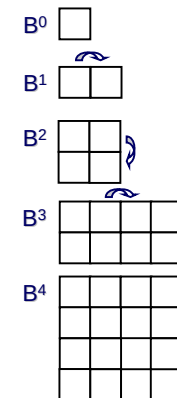
## Boolean Space

$B = \{0, 1\}$

$B^2 = \{0, 1\} \times \{0, 1\} = \{00, 01, 10, 11\}$

Karnaugh Maps:

Boolean Lattices:



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## Boolean Function

- A Boolean function  $f$  over input variables:  $x_1, x_2, \dots, x_m$ , is a mapping  $f: \mathbf{B}^m \rightarrow Y$ , where  $\mathbf{B} = \{0,1\}$  and  $Y = \{0,1,d\}$ 
  - E.g.
    - The output value of  $f(x_1, x_2, x_3)$ , say, partitions  $\mathbf{B}^m$  into three sets:
      - **on-set** ( $f=1$ )
        - E.g.  $\{010, 011, 110, 111\}$  (characteristic function  $f^1 = x_2$ )
      - **off-set** ( $f=0$ )
        - E.g.  $\{100, 101\}$  (characteristic function  $f^0 = x_1 \neg x_2$ )
      - **don't-care set** ( $f=d$ )
        - E.g.  $\{000, 001\}$  (characteristic function  $f^d = \neg x_1 \neg x_2$ )
- $f$  is an **incompletely specified function** if the don't-care set is nonempty. Otherwise,  $f$  is a **completely specified function**
  - Unless otherwise said, a Boolean function is meant to be completely specified

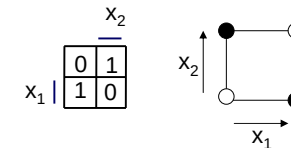
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## Boolean Function

- A Boolean function  $f: \mathbf{B}^n \rightarrow \mathbf{B}$  over variables  $x_1, \dots, x_n$  maps each Boolean valuation (truth assignment) in  $\mathbf{B}^n$  to 0 or 1

### Example

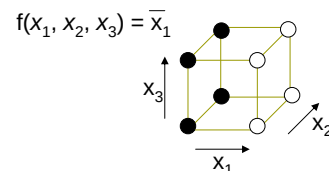
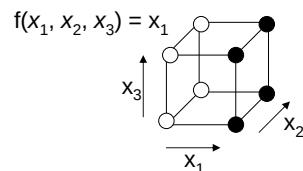
$f(x_1, x_2)$  with  $f(0,0) = 0$ ,  $f(0,1) = 1$ ,  $f(1,0) = 1$ ,  $f(1,1) = 0$



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## Boolean Function

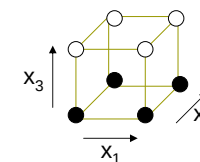
- **Onset** of  $f$ , denoted as  $f^1$ , is  $f^1 = \{v \in \mathbf{B}^n \mid f(v)=1\}$ 
  - If  $f^1 = \mathbf{B}^n$ ,  $f$  is a **tautology**
- **Offset** of  $f$ , denoted as  $f^0$ , is  $f^0 = \{v \in \mathbf{B}^n \mid f(v)=0\}$ 
  - If  $f^0 = \mathbf{B}^n$ ,  $f$  is **unsatisfiable**. Otherwise,  $f$  is **satisfiable**.
- $f^1$  and  $f^0$  are sets, not functions!
- Boolean functions  $f$  and  $g$  are **equivalent** if  $\forall v \in \mathbf{B}^n. f(v) = g(v)$  where  $v$  is a truth assignment or Boolean valuation
- A **literal** is a Boolean variable  $x$  or its negation  $x'$  (or  $x, \neg x$ ) in a Boolean formula



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## Boolean Function

- There are  $2^n$  vertices in  $\mathbf{B}^n$
- There are  $2^{2^n}$  distinct Boolean functions
  - Each subset  $f^1 \subseteq \mathbf{B}^n$  of vertices in  $\mathbf{B}^n$  forms a distinct Boolean function  $f$  with onset  $f^1$



| $x_1 x_2 x_3$ | $f$ |
|---------------|-----|
| 0 0 0         | 1   |
| 0 0 1         | 0   |
| 0 1 0         | 1   |
| 0 1 1         | 0   |
| 1 0 0         | 1   |
| 1 0 1         | 0   |
| 1 1 0         | 1   |
| 1 1 1         | 0   |

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## Boolean Operations

Given two Boolean functions:

$$f : \mathbf{B}^n \rightarrow \mathbf{B}$$

$$g : \mathbf{B}^n \rightarrow \mathbf{B}$$

□  $h = f \wedge g$  from **AND** operation is defined as

$$h^1 = f^1 \cap g^1; h^0 = \mathbf{B}^n \setminus h^1$$

□  $h = f \vee g$  from **OR** operation is defined as

$$h^1 = f^1 \cup g^1; h^0 = \mathbf{B}^n \setminus h^1$$

□  $h = \neg f$  from **COMPLEMENT** operation is defined as

$$h^1 = f^0; h^0 = f^1$$

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## Cofactor and Quantification

Given a Boolean function:

$$f : \mathbf{B}^n \rightarrow \mathbf{B}, \text{ with the input variable } (x_1, x_2, \dots, x_i, \dots, x_n)$$

□ **Positive cofactor on variable  $x_i$**

$$h = f_{x_i} \text{ is defined as } h = f(x_1, x_2, \dots, 1, \dots, x_n)$$

□ **Negative cofactor on variable  $x_i$**

$$h = f_{\neg x_i} \text{ is defined as } h = f(x_1, x_2, \dots, 0, \dots, x_n)$$

□ **Existential quantification over variable  $x_i$**

$$h = \exists x_i. f \text{ is defined as } h = f(x_1, x_2, \dots, 0, \dots, x_n) \vee f(x_1, x_2, \dots, 1, \dots, x_n)$$

□ **Universal quantification over variable  $x_i$**

$$h = \forall x_i. f \text{ is defined as } h = f(x_1, x_2, \dots, 0, \dots, x_n) \wedge f(x_1, x_2, \dots, 1, \dots, x_n)$$

□ **Boolean difference over variable  $x_i$**

$$h = \partial f / \partial x_i \text{ is defined as } h = f(x_1, x_2, \dots, 0, \dots, x_n) \oplus f(x_1, x_2, \dots, 1, \dots, x_n)$$

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## Boolean Function Representation

□ Some common representations:

■ Truth table

■ Boolean formula

□ SOP (sum-of-products, or called disjunctive normal form, DNF)

□ POS (product-of-sums, or called conjunctive normal form, CNF)

■ BDD (binary decision diagram)

■ Boolean network (consists of nodes and wires)

□ Generic Boolean network

■ Network of nodes with generic functional representations or even subcircuits

□ Specialized Boolean network

■ Network of nodes with SOPs (PLAs)

■ And-Inv Graph (AIG)

□ Why different representations?

■ Different representations have their own strengths and weaknesses (no single data structure is best for all applications)

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## Boolean Function Representation Truth Table

□ Truth table (function table for multi-valued functions):

The **truth table** of a function  $f : \mathbf{B}^n \rightarrow \mathbf{B}$  is a tabulation of its value at each of the  $2^n$  vertices of  $\mathbf{B}^n$ .

In other words the truth table lists all **minterms**

$$\text{Example: } f = a'b'c'd + a'b'cd + a'bc'd + ab'c'd + ab'cd + abc'd + abcd' + abcd$$

The truth table representation is

- impractical for large  $n$

- canonical

If two functions are equal, then their **canonical** representations are isomorphic.

|   | abcd | f |    | abcd | f |
|---|------|---|----|------|---|
| 0 | 0000 | 0 | 8  | 1000 | 0 |
| 1 | 0001 | 1 | 9  | 1001 | 1 |
| 2 | 0010 | 0 | 10 | 1010 | 0 |
| 3 | 0011 | 1 | 11 | 1011 | 1 |
| 4 | 0100 | 0 | 12 | 1100 | 0 |
| 5 | 0101 | 1 | 13 | 1101 | 1 |
| 6 | 0110 | 0 | 14 | 1110 | 1 |
| 7 | 0111 | 0 | 15 | 1111 | 1 |

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## Boolean Function Representation Boolean Formula

- A **Boolean formula** is defined inductively as an expression with the following formation rules (syntax):

|             |                     |                 |
|-------------|---------------------|-----------------|
| formula ::= | '(' formula ')'     |                 |
|             | Boolean constant    | (true or false) |
|             | <Boolean variable>  |                 |
|             | formula "+" formula | (OR operator)   |
|             | formula "." formula | (AND operator)  |
|             | ¬ formula           | (complement)    |

### Example

$$f = (x_1 \cdot x_2) + (x_3) + \neg(\neg(x_4 \cdot (\neg x_1)))$$

typically "." is omitted and '(', ')' are omitted when the operator priority is clear, e.g.,  $f = x_1 x_2 + x_3 + x_4 \neg x_1$

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## Boolean Function Representation Boolean Formula in SOP

- Any function can be represented as a **sum-of-products (SOP)**, also called **sum-of-cubes** (a **cube** is a product term), or **disjunctive normal form (DNF)**

### Example

$$\varphi = ab + a'c + bc$$

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## Boolean Function Representation Boolean Formula in POS

- Any function can be represented as a **product-of-sums (POS)**, also called **conjunctive normal form (CNF)**
  - Dual of the SOP representation

### Example

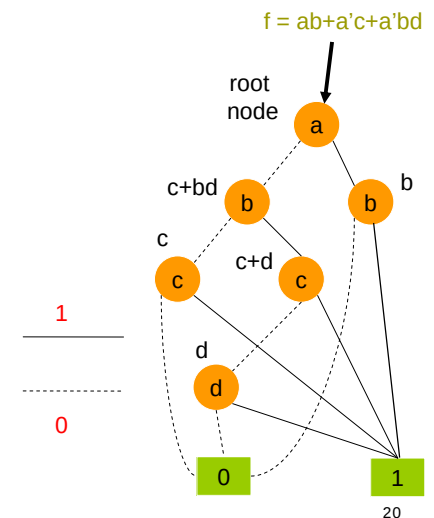
$$\varphi = (a+b'+c) (a'+b+c) (a+b'+c') (a+b+c)$$

- Exercise: Any Boolean function in POS can be converted to SOP using De Morgan's law and the distributive law, and vice versa

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## Boolean Function Representation Binary Decision Diagram

- BDD – a graph representation of Boolean functions
  - A **leaf node** represents constant 0 or 1
  - A **non-leaf node** represents a decision node (multiplexer) controlled by some variable
  - Can make a BDD representation **canonical** by imposing the **variable ordering** and **reduction** criteria (ROBDD)



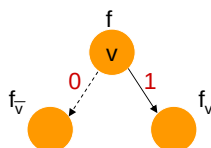
## Boolean Function Representation Binary Decision Diagram

- Any Boolean function  $f$  can be written in term of **Shannon expansion**

$$f = v f_v + \neg v f_{\neg v}$$

- Positive cofactor:  $f_{x_i} = f(x_1, \dots, x_i=1, \dots, x_n)$
- Negative cofactor:  $f_{\neg x_i} = f(x_1, \dots, x_i=0, \dots, x_n)$

- BDD is a compressed Shannon cofactor tree:
  - The two children of a node with function  $f$  controlled by variable  $v$  represent two sub-functions  $f_v$  and  $f_{\neg v}$



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## Boolean Function Representation Binary Decision Diagram

- Reduced** and **ordered** BDD (**ROBDD**) is a **canonical** Boolean function representation

- Ordered**:

- cofactor variables are in the **same order along all paths**

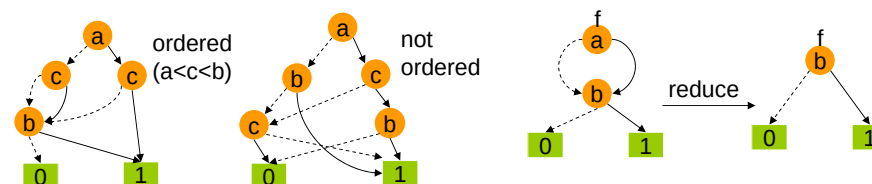
$$x_{i_1} < x_{i_2} < x_{i_3} < \dots < x_{i_n}$$

- Reduced**:

- any node with two identical children is removed

- two nodes with isomorphic BDD's are merged

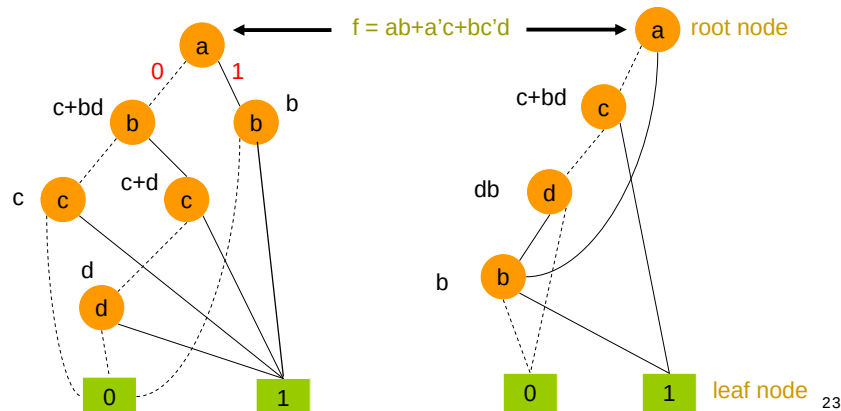
These two rules make any node in an ROBDD represent a distinct logic function



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## Boolean Function Representation Binary Decision Diagram

- For a Boolean function,
  - ROBDD is unique with respect to a given variable ordering
  - Different orderings may result in different ROBDD structures



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## Boolean Function Representation Boolean Network

- A **Boolean network** is a directed graph  $C(G, N)$  where  $G$  are the gates and  $N \subseteq (G \times G)$  are the directed edges (nets) connecting the gates.

Some of the vertices are designated:

**Inputs:**  $I \subseteq G$

**Outputs:**  $O \subseteq G$

$$I \cap O = \emptyset$$

Each gate  $g$  is assigned a Boolean function  $f_g$  which computes the output of the gate in terms of its inputs.

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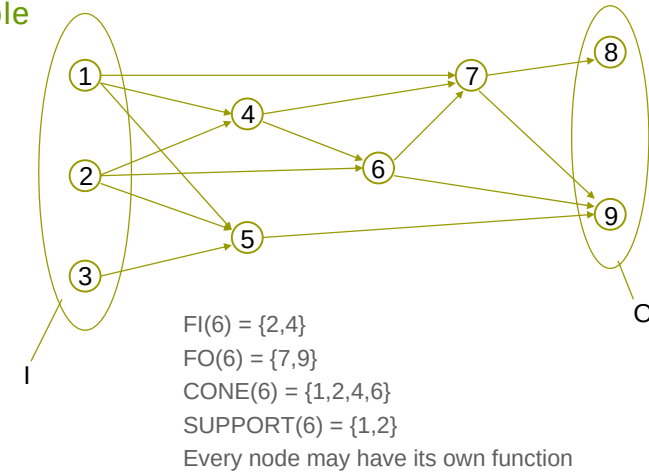
## Boolean Function Representation Boolean Network

- The **fanin**  $FI(g)$  of a gate  $g$  are the predecessor gates of  $g$ :  
 $FI(g) = \{g' \mid (g', g) \in N\}$  ( $N$ : the set of nets)
- The **fanout**  $FO(g)$  of a gate  $g$  are the successor gates of  $g$ :  
 $FO(g) = \{g' \mid (g, g') \in N\}$
- The **cone**  $CONE(g)$  of a gate  $g$  is the **transitive fanin (TFI)** of  $g$  and  $g$  itself
- The **support**  $SUPPORT(g)$  of a gate  $g$  are all inputs in its cone:  
 $SUPPORT(g) = CONE(g) \cap I$

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## Boolean Function Representation Boolean Network

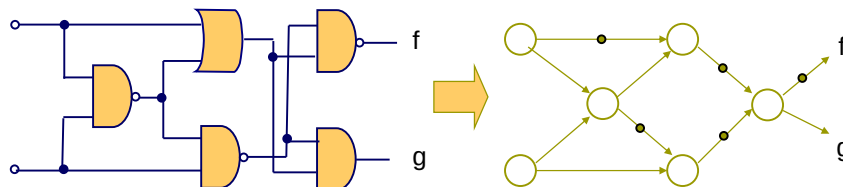
### Example



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## Boolean Function Representation And-Inverter Graph

- AND-INVERTER graphs (AIGs)
  - **vertices**: 2-input AND gates
  - **edges**: interconnects with (optional) dots representing INVs
- Hash table to identify and reuse structurally isomorphic circuits



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## Boolean Function Representation

- A **canonical form** of a Boolean function is a **unique** representation of the function
  - It can be used for verification purposes
- Example
  - Truth table is canonical
    - It grows exponentially with the number of input variables
  - ROBDD is canonical
    - It is of practical interests because it may represent many Boolean functions compactly
  - SOP, POS, Boolean networks are NOT canonical

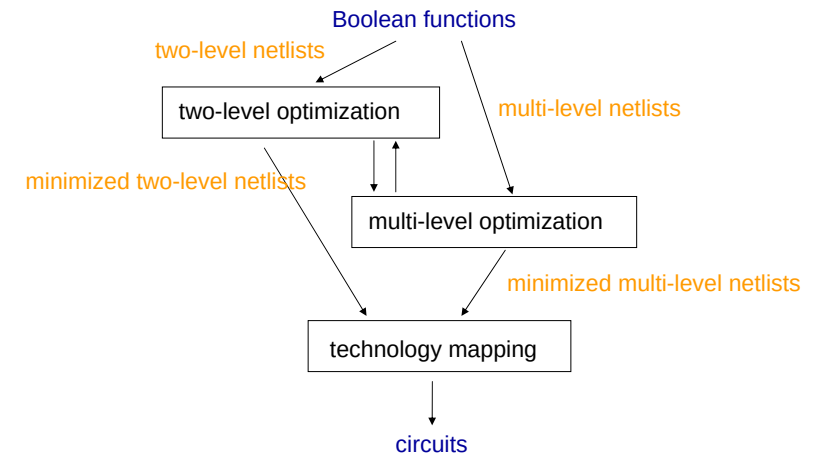
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## Boolean Function Representation

- Truth table
  - Canonical
  - Useful in representing small functions
- SOP
  - Useful in two-level logic optimization, and in representing local node functions in a Boolean network
- POS
  - Useful in SAT solving and Boolean reasoning
  - Rarely used in circuit synthesis (due to the asymmetric characteristics of NMOS and PMOS)
- ROBDD
  - Canonical
  - Useful in Boolean reasoning
- Boolean network
  - Useful in multi-level logic optimization
- AIG
  - Useful in multi-level logic optimization and Boolean reasoning

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## Logic Optimization



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## Two-Level Logic Minimization

- Any Boolean function can be realized using PLA in two levels: AND-OR (sum of products), NAND-NAND, etc.
  - Direct implementation of two-level logic using PLAs (programmable logic arrays) is not as popular as in the nMOS days
- Classic problem solved by the *Quine-McCluskey* algorithm
  - Popular cost function: #cubes and #literals in an SOP expression
    - #cubes – #rows in a PLA
    - #literals – #transistors in a PLA
  - The goal is to find a **minimal irredundant prime cover**

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## Two-Level Logic Minimization

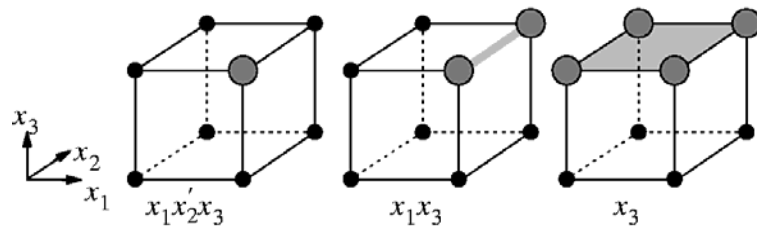
- Exact algorithm
  - Quine-McCluskey's procedure
- Heuristic algorithm
  - Espresso

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## Two-Level Logic Minimization Minterms and Cubes

- A **minterm** is a product of **every** input variable or its negation
  - A minterm corresponds to a single point in  $\mathbf{B}^n$
- A **cube** is a product of literals
  - The fewer the number of literals is in the product, the bigger the space is covered by the cube



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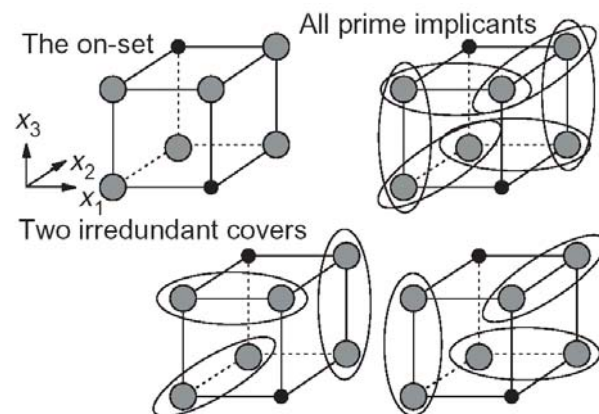
## Two-Level Logic Minimization Implicant and Cover

- An **implicant** is a cube whose points are either in the on-set or the dc-set.
- A **prime implicant** is an implicant that is not included in any other implicant.
- A set of prime implicants that together cover all points in the on-set (and some or all points of the dc-set) is called a **prime cover**.
  - A prime cover is **irredundant** when none of its prime implicants can be removed from the cover.
  - An irredundant prime cover is **minimal** when the cover has the minimal number of prime implicants. (c.f. minimum vs. minimal)

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## Two-Level Logic Minimization Cover

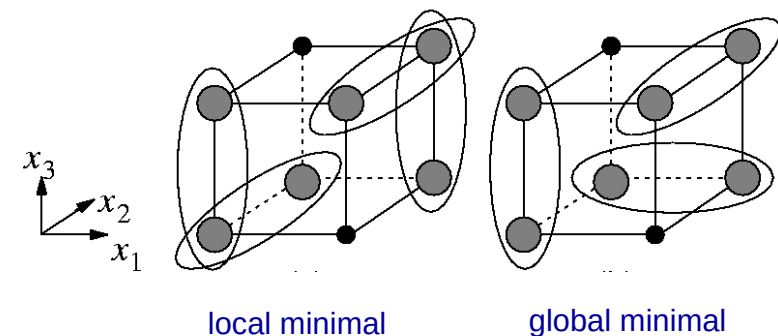
- Example
  - $f = \neg x_1 \neg x_3 + \neg x_2 x_3 + x_1 x_2$
  - $f = \neg x_1 \neg x_2 + x_2 \neg x_3 + x_1 x_3$



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## Two-Level Logic Minimization Cover

- Example



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## Two-Level Logic Minimization Quine-McCluskey Procedure

- Given  $G$  and  $D$  (covers for  $\mathfrak{Z} = (f, d, r)$  and  $d$ , respectively), find a minimum cover  $G^*$  of primes where:
  - $f \subseteq G^* \subseteq f + d$  ( $G^*$  is a prime cover of  $\mathfrak{Z}$ )
  - $f$  is the onset,  $d$  don't-care set, and  $r$  offset
- Q-M Procedure:
  1. Generate all primes of  $\mathfrak{Z}$ ,  $\{P_j\}$  (i.e. primes of  $(f + d) = G + D$ )
  2. Generate all minterms  $\{m_i\}$  of  $f = G \wedge \neg D$
  3. Build Boolean matrix  $B$  where
 
$$B_{ij} = 1 \text{ if } m_i \in P_j$$

$$= 0 \text{ otherwise}$$
  4. Solve the minimum column covering problem for  $B$  (unate covering problem)

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## Two-Level Logic Minimization Quine-McCluskey Procedure

### Generating Primes

Tabular method  
(based on *consensus* operation):

- Start with all **minterm canonical form** of  $F$
- Group *pairs* of adjacent minterms into cubes
- Repeat merging cubes until no more merging possible; mark (✓) + remove all covered cubes.
- Result: set of *primes* of  $f$ .

### Example

$$F = x' y' + w x y + x' y z' + w y' z$$

$$F = x' y' + w x y + x' y z' + w y' z$$

|                 |              |         |
|-----------------|--------------|---------|
| $w' x' y' z'$ ✓ | $w' x' y'$ ✓ | $x' y'$ |
|                 | $w' x' z'$ ✓ | $x' z'$ |
|                 | $x' y' z'$ ✓ |         |
| $w' x' y' z$ ✓  | $x' y' z$ ✓  |         |
| $w' x' y z'$ ✓  | $x' y z'$ ✓  |         |
| $w x' y' z'$ ✓  | $w x' y'$ ✓  |         |
|                 | $w x' z'$ ✓  |         |
| $w x' y' z$ ✓   | $w y' z$     |         |
| $w x' y z'$ ✓   | $w y z'$     |         |
| $w x y z'$ ✓    | $w x y$      |         |
| $w x y' z$ ✓    | $w x z$      |         |
| $w x y z$ ✓     |              |         |

Courtesy: Maciej Ciesielski, UMMASS

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## Two-Level Logic Minimization Quine-McCluskey Procedure

### Example

Karnaugh map

|                                   |                   |             |       |             |           |
|-----------------------------------|-------------------|-------------|-------|-------------|-----------|
| $\bar{x} \bar{z}$                 | $\bar{x} \bar{y}$ | $\bar{x} y$ | $x y$ | $x \bar{y}$ |           |
| $\bar{z} \bar{w}$                 | 1                 | d           | 0     | d           | $\bar{y}$ |
| $\bar{z} w$                       | d                 | 1           | d     | 1           |           |
| $z \bar{w}$                       | d                 | 1           | d     | d           | $w$       |
| $z w$                             | d                 | 0           | 0     | d           |           |
| $\bar{x} \bar{y} \bar{z} \bar{w}$ | 1                 | 0           | 1     |             |           |
| $\bar{x} y \bar{z} \bar{w}$       | 0                 | 1           | 1     |             |           |
| $x \bar{y} \bar{z} \bar{w}$       | 1                 | 1           | 0     |             |           |
| $\bar{x} y \bar{z} w$             | 0                 | 1           | 0     |             |           |

$$F = \bar{x} \bar{y} \bar{z} \bar{w} + \bar{x} y \bar{z} \bar{w} + x \bar{y} \bar{z} \bar{w} + \bar{x} y z \bar{w} \quad (\text{cover of } \mathfrak{Z})$$

$$D = \bar{y} z + x y w + \bar{x} y \bar{z} w + x \bar{y} \bar{z} w + \bar{x} y z \bar{w} \quad (\text{cover of } d)$$

Primes:  $\bar{y} + w + \bar{x} \bar{z}$

Covering Table

Solution:  $\{1, 2\} \Rightarrow \bar{y} + w$  is a minimum prime cover (also  $w + \bar{x} \bar{z}$ )

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## Two-Level Logic Minimization Quine-McCluskey Procedure

### Column covering of Boolean matrix

|                                   |           |     |                   |                                   |
|-----------------------------------|-----------|-----|-------------------|-----------------------------------|
|                                   | $\bar{y}$ | $w$ | $\bar{x} \bar{z}$ | Primes of $f + d$                 |
| $\bar{x} \bar{y} \bar{z} \bar{w}$ | 1         | 0   | 1                 |                                   |
| $\bar{x} y \bar{z} \bar{w}$       | 0         | 1   | 1                 |                                   |
| $x \bar{y} \bar{z} \bar{w}$       | 1         | 1   | 0                 |                                   |
| $\bar{x} y z \bar{w}$             | 0         | 1   | 0                 | Row singleton (essential minterm) |
|                                   |           |     |                   | Essential prime                   |

- **Definition.** An **essential prime** is a prime that covers an onset minterm of  $f$  not covered by any other primes.

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## Two-Level Logic Minimization Quine-McCluskey Procedure

### □ Row equality in Boolean matrix:

- In practice, many rows in a covering table are identical. That is, there exist minterms that are contained in the same set of primes.

#### ■ Example

|       |         |
|-------|---------|
| $m_1$ | 0101101 |
| $m_2$ | 0101101 |

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## Two-Level Logic Minimization Quine-McCluskey Procedure

### □ Row dominance in Boolean matrix:

- A row  $i_1$  whose set of primes is contained in the set of primes of row  $i_2$  is said to **dominate**  $i_2$ .

#### ■ Example

|       |        |
|-------|--------|
| $i_1$ | 011010 |
| $i_2$ | 011110 |

- $i_1$  dominates  $i_2$
- Can remove row  $i_2$  because have to choose a prime to cover  $i_1$ , and any such prime also covers  $i_2$ . So  $i_2$  is automatically covered.

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## Two-Level Logic Minimization Quine-McCluskey Procedure

### □ Column dominance in Boolean matrix:

- A column  $j_1$  whose rows are a superset of another column  $j_2$  is said to **dominate**  $j_2$ .

#### ■ Example

| $j_1$ | $j_2$ |
|-------|-------|
| 1     | 0     |
| 0     | 0     |
| 1     | 1     |
| 0     | 0     |
| 1     | 1     |

- $j_1$  dominates  $j_2$
- We can remove column  $j_2$  since  $j_1$  covers all those rows and more. We would never choose  $j_2$  in a minimum cover since it can always be replaced by  $j_1$ .

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## Two-Level Logic Minimization Quine-McCluskey Procedure

### Reducing Boolean matrix

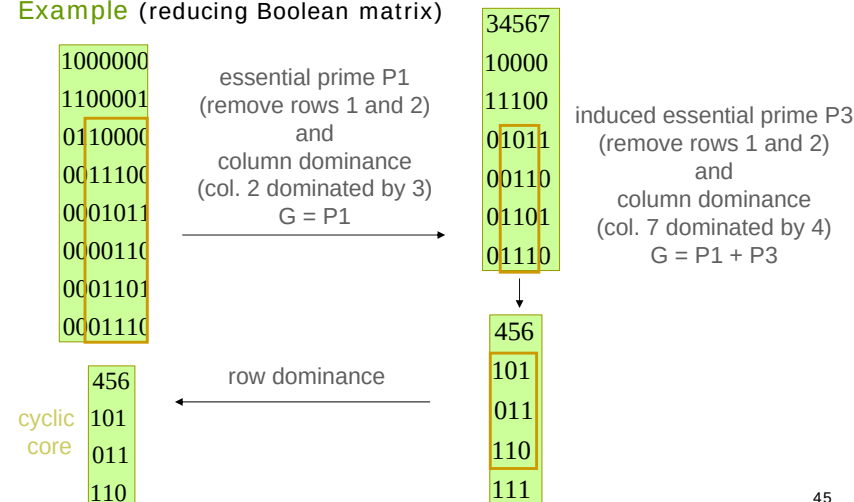
1. Remove all rows covered by essential primes (columns in row singletons). Put these primes in the cover G.
2. Group identical rows together and remove dominated rows.
3. Remove dominated columns. For equal columns, keep one prime to represent them.
4. Newly formed row singletons define **induced essential primes**.
5. Go to 1 if covering table decreased.

- The resulting reduced covering table is called the **cyclic core**. This has to be solved (**unate covering problem**). A minimum solution is added to G. The resulting G is a minimum cover.

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## Two-Level Logic Minimization Quine-McCluskey Procedure

Example (reducing Boolean matrix)



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## Two-Level Logic Minimization Quine-McCluskey Procedure

Solving cyclic core

- Best known method (for unate covering) is **branch and bound** with some clever bounding heuristics
- Independent Set Heuristic:**
  - Find a maximum set  $I$  of "independent" rows. Two rows  $B_{i1}, B_{i2}$  are independent if **not**  $\exists j$  such that  $B_{i1j} = B_{i2j} = 1$ . (They have no column in common.)

Example

A covering matrix  $B$  rearranged with independent sets first

|      |      |     |   |
|------|------|-----|---|
| 1 11 | 1111 | 1 1 | 0 |
| A    | C    |     |   |

Independent set  $\mathcal{I}$  of rows

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## Two-Level Logic Minimization Quine-McCluskey Procedure

Solving cyclic core

Heuristic algorithm:

- Let  $\mathcal{I} = \{I_1, I_2, \dots, I_k\}$  be the independent set of rows
- choose  $j \in I_i$  such that column  $j$  covers the most rows of  $A$ . Put  $P_j$  in  $G$
- eliminate all rows covered by column  $j$
- $\mathcal{I} \leftarrow \mathcal{I} \setminus \{I_i\}$
- go to 1 if  $|\mathcal{I}| > 0$
- If  $B$  is empty, then done (in this case achieve minimum solution)
- If  $B$  is not empty, choose an independent set of  $B$  and go to 1

|      |      |     |   |
|------|------|-----|---|
| 1 11 | 1111 | 1 1 | 0 |
| A    | C    |     |   |

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## Two-Level Logic Minimization Quine-McCluskey Procedure

Summary

- Calculate all prime implicants (of the union of the onset and don't care set)
- Find the minimal cover of all minterms in the onset by prime implicants
  - Construct the covering matrix
  - Simplify the covering matrix by detecting **essential** columns, **row and column dominance**
  - What is left is the **cyclic core** of the covering matrix.
    - The covering problem can then be solved by a branch-and-bound algorithm.

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## Two-Level Logic Minimization Exact vs. Heuristic Algorithms

### ■ Quine-McCluskey Method:

1. Generate cover of all primes  $G = p_1 + p_2 + \dots + p_{3^n/n}$
2. Make G irredundant (in optimum way)
  - Q-M is **exact**, i.e., it gives an exact minimum

### ■ Heuristic Methods:

1. Generate (somehow) a cover of  $\mathfrak{Z}$  using some of the primes  $G = p_{i_1} + p_{i_2} + \dots + p_{i_k}$
2. Make G irredundant (maybe not optimally)
3. Keep best result - try again (i.e. go to 1)

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## Two-Level Logic Minimization ESPRESSO

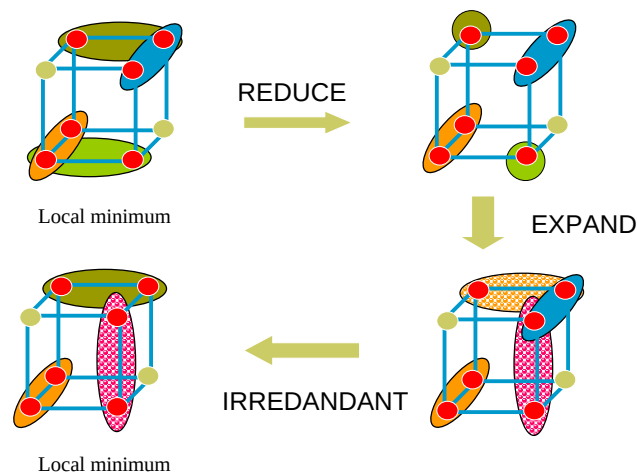
### ■ Heuristic two-level logic minimization

```

ESPRESSO( $\mathfrak{Z}$ )
{
  (F,D,R) ← DECODE( $\mathfrak{Z}$ )           //LASTGASP
  F ← EXPAND(F,R)                 G ← REDUCE_GASP(F,D)
  F ← IRREDUNDANT(F,D)            G ← EXPAND(G,R)
  E ← ESSENTIAL_PRIMES(F,D)       F ← IRREDUNDANT(F+G,D)
  F ← F-E; D ← D+E               //LASTGASP
  do{                             }while fewer terms in F
  {
    F ← REDUCE(F,D)
    F ← EXPAND(F,R)
    F ← IRREDUNDANT(F,D)
  }while fewer terms in F
  return (F,error)
}
    
```

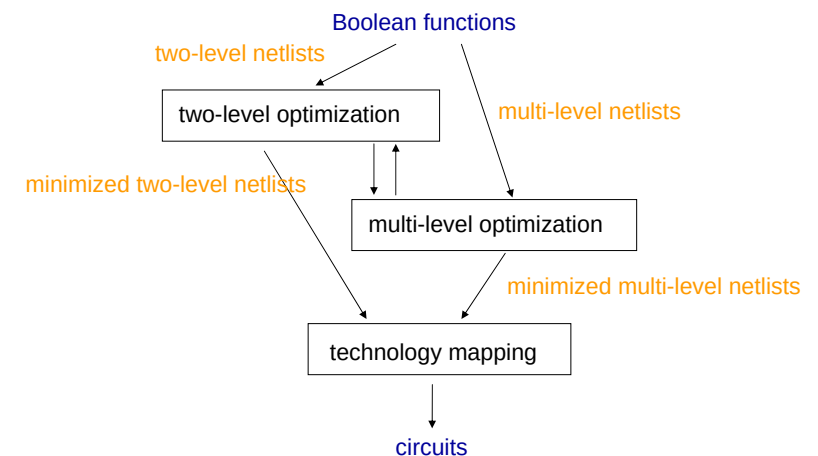
50

## Two-Level Logic Minimization ESPRESSO



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## Logic Minimization



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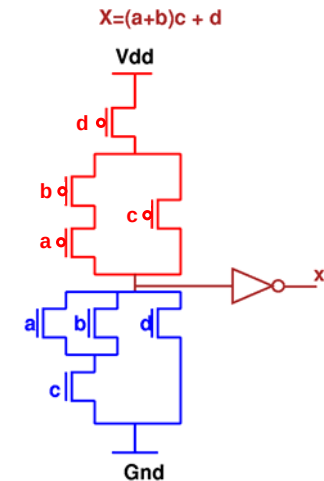
## Factor Form

- Factor forms – beyond SOP
  - Example:  
 $(ad+bc)(c+d'(e+ac'))+(d+e)fg$
- Advantages
  - good representation reflecting logic complexity (SOP may not be representative)
    - E.g.,  $f=ad+ae+bd+be+cd+ce$  has complement in simpler SOP  $f'=a'b'c'+d'e'$ ; effectively has simple factor form  $f=(a+b+c)(d+e)$
  - in many design styles (e.g. complex gate CMOS design) the implementation of a function corresponds directly to its factored form
  - good estimator of logic implementation complexity
  - doesn't blow up easily
- Disadvantages
  - not as many algorithms available for manipulation

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## Factor From

- Factored forms are useful in **estimating** area and delay in multi-level logic
  - Note: literal count  $\approx$  transistor count  $\approx$  area
    - however, area also depends on wiring, gate size, etc.
    - therefore very crude measure



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## Factor From

- There are functions whose sizes are **exponential** in the SOP representation, but **polynomial** in the factored form

### Example

Achilles' heel function

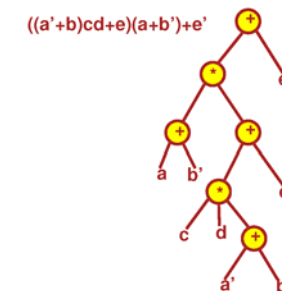
$$\prod_{i=1}^{i=n/2} (x_{2i-1} + x_{2i})$$

There are  $n$  literals in the factored form and  $(n/2) \times 2^{n/2}$  literals in the SOP form.

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## Factor Form

- Factored forms can be graphically represented as labeled **trees**, called **factoring trees**, in which each internal node including the root is labeled with either  $+$  or  $\times$ , and each leaf has a label of either a variable or its complement
- Example: factoring tree of  $((a'+b)cd+e)(a+b')+e'$



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## Multi-Level Logic Minimization

### Basic techniques in Boolean network manipulation:

- structural manipulation (change network topology)
- node simplification (change node functions)
  - node minimization using don't cares

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## Multi-Level Logic Minimization Structural Manipulation

**Restructuring Problem:** Given initial network, find **best** network.

Example:

$$f_1 = abcd + abce + ab'cd' + ab'c'd' + a'c + cdf + abc'd'e' + ab'c'df'$$

$$f_2 = bdg + b'dfg + b'd'g + bd'eg$$

minimizing,

$$f_1 = bcd + bce + b'd' + a'c + cdf + abc'd'e' + ab'c'df'$$

$$f_2 = bdg + dfg + b'd'g + d'eg$$

factoring,

$$f_1 = c(b(d+e) + b'(d'+f) + a') + ac'(bd'e' + b'df')$$

$$f_2 = g(d(b+f) + d'(b'+e))$$

decompose,

$$f_1 = c(b(d+e) + b'(d'+f) + a') + ac'x'$$

$$f_2 = gx$$

$$x = d(b+f) + d'(b'+e)$$

Two problems:

- find good **common** subfunctions
- effect the **division**

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## Multi-Level Logic Minimization Structural Manipulation

### Basic operations:

1. Decomposition (for a single function)

$$f = abc + abd + a'c'd' + b'c'd'$$

↓

$$f = xy + x'y' \quad x = ab \quad y = c+d$$

2. Extraction (for multiple functions)

$$f = (az + bz')cd + e \quad g = (az + bz')e' \quad h = cde$$

↓

$$f = xy + e \quad g = xe' \quad h = ye \quad x = az + bz' \quad y = cd$$

3. Factoring (series-parallel decomposition)

$$f = ac + ad + bc + bd + e$$

↓

$$f = (a+b)(c+d) + e$$

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## Multi-Level Logic Minimization Structural Manipulation

### Basic operations (cont'd):

4. Substitution

$$f = a + bc \quad g = a + b$$

↓

$$f = g(a+c) \quad g = a+b$$

5. Collapsing (also called elimination)

$$f = ga + g'b \quad g = c+d$$

↓

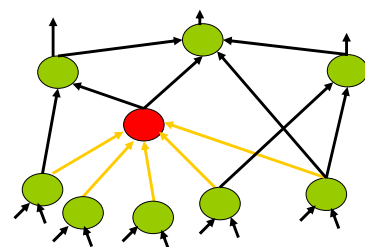
$$f = ac + ad + bc'd' \quad g = c+d$$

**Note:** "division" plays a key role in all these operations

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## Multi-Level Logic Minimization Node Simplification

- Goal: For any node of a given Boolean network, find a **least-cost** SOP expression among the set of permissible functions for the node
  - Don't care computation + two-level logic minimization

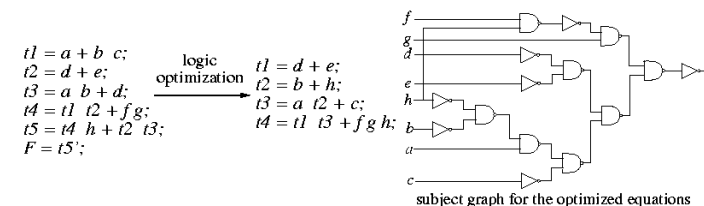


combinational Boolean network

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## Combinational Logic Minimization

- Two-level:** minimize #product terms and #literals
  - E.g.,  $F = x_1'x_2'x_3' + x_1'x_2'x_3 + x_1x_2'x_3' + x_1x_2'x_3 + x_1x_2x_3' \Rightarrow F = x_2' + x_1x_3'$
- Multi-level:** minimize the # literals (**area** minimization)
  - E.g., equations are optimized using a smaller number of literals

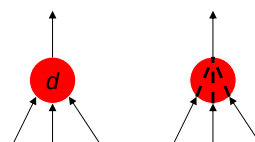


subject graph for the optimized equations

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## Timing Analysis and Optimization

- Delay model at logic level
  - Gate delay model (our focus)
    - Constant gate delay, or pin-to-pin gate delay
    - Not accurate
  - Fanout delay model
    - Gate delay considering fanout load (#fanouts)
    - Slightly more accurate
  - Library delay model
    - Tabular delay data given in the cell library
      - Determine delay from **input slew** and **output load**
      - Table look-up + interpolation/extrapolation
    - Accurate

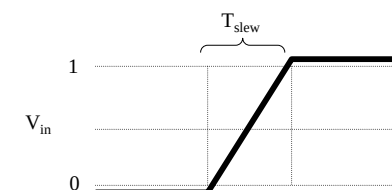
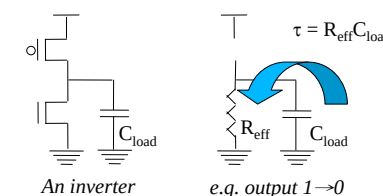


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## Timing Analysis and Optimization Gate Delay

The delay of a gate depends on:

- Output Load**
  - Capacitive loading  $\propto$  charge needed to swing the output voltage
  - Due to interconnect and logic fanout
- Input Slew**
  - Slew = transition time
  - Slower transistor switching  $\Rightarrow$  longer delay and longer output slew



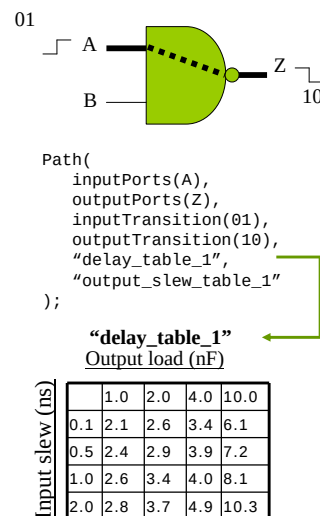
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# Timing Analysis and Optimization

## Timing Library

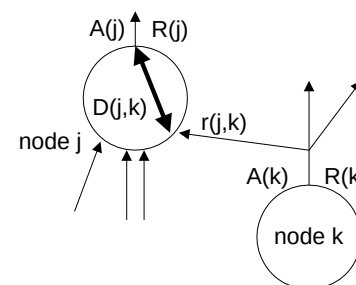
- Timing library contains all relevant information about each standard cell
  - E.g., pin direction, clock, pin capacitance, etc.
- Delay (fastest, slowest, and often typical) and output slew are encoded for each input-to-output path and each pair of transition directions
- Values typically represented as 2 dimensional look-up tables (of output load and input slew)
  - Interpolation is used



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## Static Timing Analysis

- Arrival time:** the time signal arrives
  - Calculated from input to output in the topological order
- Required time:** the time signal must ready (e.g., due to the clock cycle constraint)
  - Calculated from output to input in the reverse topological order
- Slack** = required time – arrival time
  - Timing flexibility margin (positive: good; negative: bad)

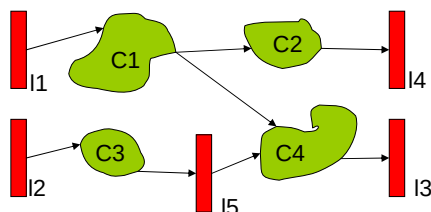


$$\begin{aligned}
 A(j): & \text{arrival time of signal } j \\
 R(k): & \text{required time for signal } k \\
 S(k): & \text{slack of signal } k \\
 D(j,k): & \text{delay of node } j \text{ from input } k \\
 A(j) = & \max_{k \in \text{FI}(j)} [A(k) + D(j,k)] \\
 r(j,k) = & R(j) - D(j,k) \\
 R(k) = & \min_{j \in \text{FO}(k)} [r(j,k)] \\
 S(k) = & R(k) - A(k)
 \end{aligned}$$

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## Static Timing Analysis

- Arrival times known at register outputs  $I_1$ ,  $I_2$ , and  $I_5$
- Required times known at register inputs  $I_3$ ,  $I_4$ , and  $I_5$
- Delay analysis gives arrival and required times (hence slacks) for combinational blocks  $C_1$ ,  $C_2$ ,  $C_3$ ,  $C_4$



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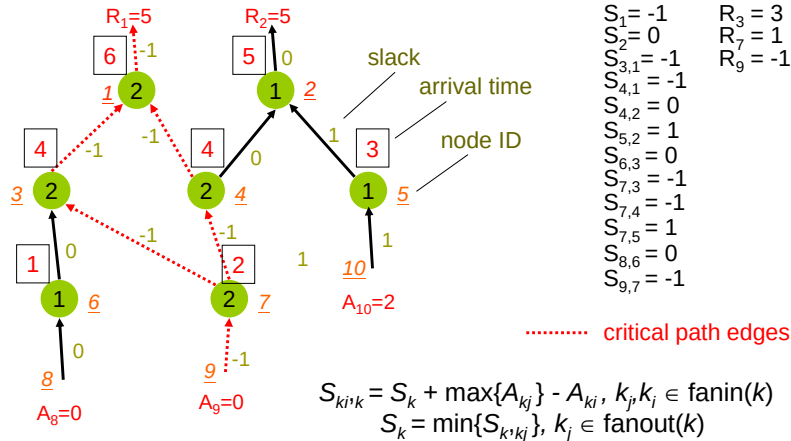
## Static Timing Analysis

- Arrival time** can be computed in the topological order from inputs to outputs
  - When a node is visited, its output arrival time is: the max of its fanin arrival times + its own gate delay
- Required time** can be computed in the reverse topological order from outputs to inputs
  - When a node is visited, its input required time is: the min of its fanout required times – its own gate delay

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## Static Timing Analysis

### Example



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## Timing Optimization

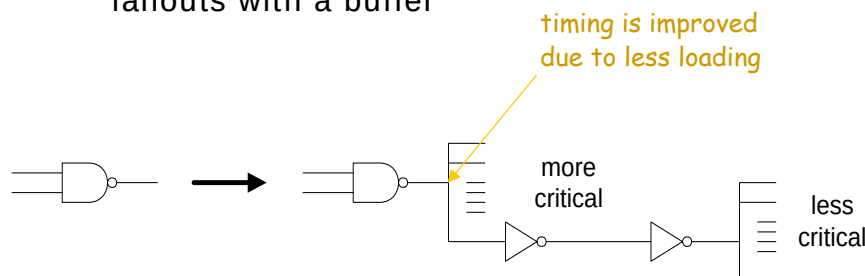
- Identify timing critical regions
- Perform timing optimization on the selected regions
  - E.g., gate sizing, buffer insertion, fanout optimization, tree height reduction, etc.

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## Timing Optimization

### Buffer insertion

- Divide the fanouts of a gate into critical and non-critical parts, and drive the non-critical fanouts with a buffer

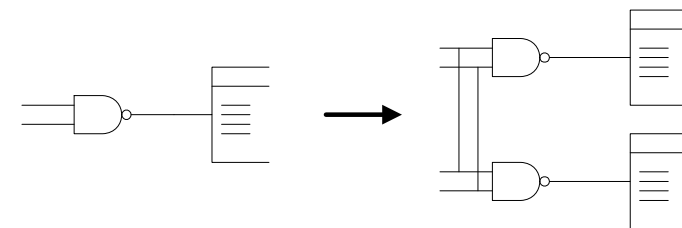


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## Timing Optimization

### Fanout optimization

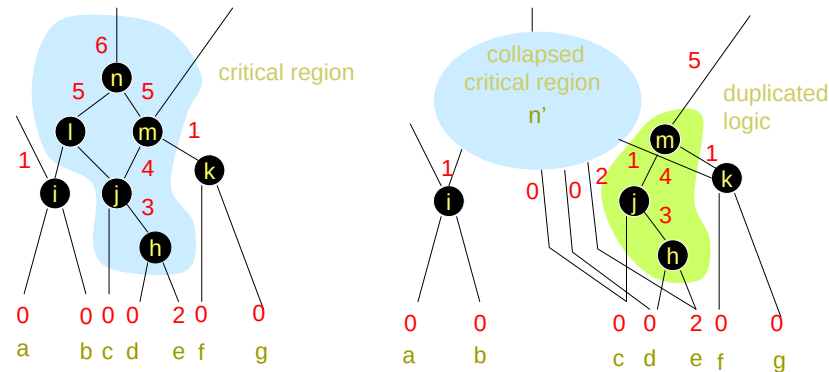
- Split the fanouts of a gate into several parts. Each part is driven by a copy of the original gate.



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## Timing Optimization

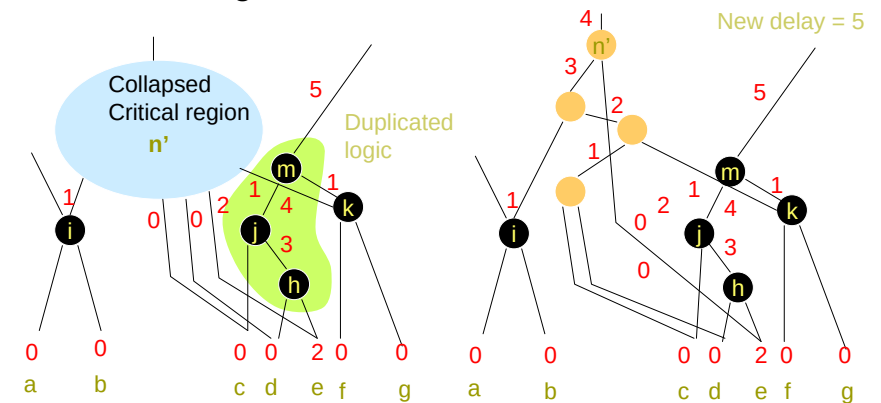
### Tree height reduction



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## Timing Optimization

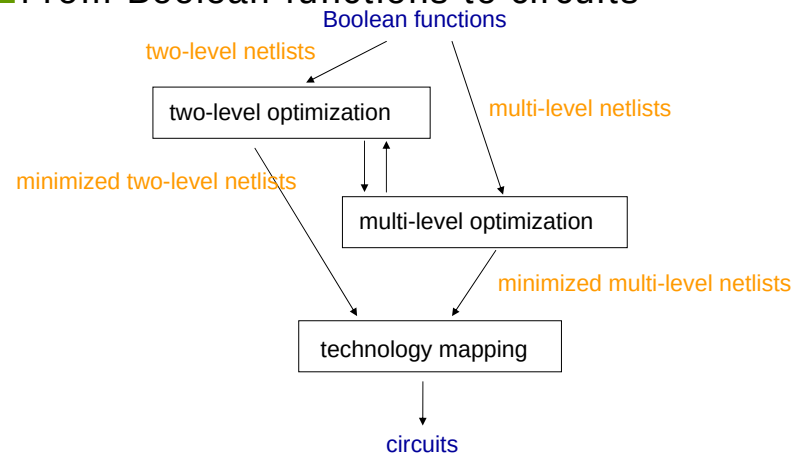
### Tree height reduction



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## Combinational Optimization

### From Boolean functions to circuits



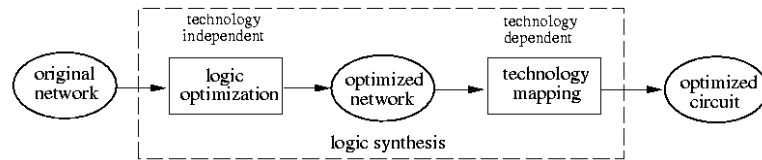
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## Technology Independent vs. Dependent Optimization

- Technology independent optimization produces a two-level or multi-level netlist where literal and/or cube counts are minimized
- Given the optimized netlist, its logic gates are to be implemented with library cells
- The process of associating logic gates with library cells is **technology mapping**
  - Translation of a technology independent representation (e.g. Boolean networks) of a circuit into a circuit for a given technology (e.g. standard cells) with optimal cost

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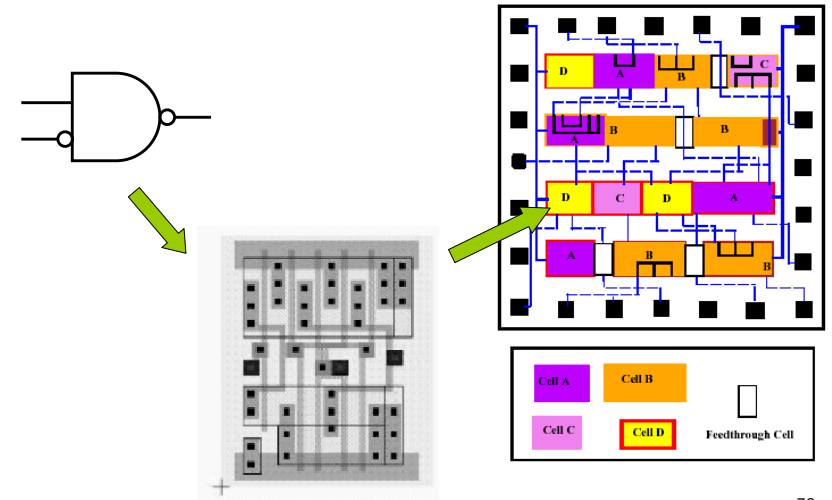
# Technology Mapping



- **Standard-cell technology mapping:** standard cell design
  - Map a function to a limited set of pre-designed library cells
- **FPGA technology mapping**
  - Lookup table (LUT) architecture:
    - E.g., Lucent, Xilinx FPGAs
    - Each lookup table (LUT) can implement all logic functions with up to k inputs ( $k = 4, 5, 6$ )
  - Multiplexer-based technology mapping:
    - E.g., Actel FPGA
    - Logic modules are constructed with multiplexers

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# Standard-Cell Based Design



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# Technology Mapping

- **Formulation:**
  - Choose base functions
    - Ex: 2-input NAND and Inverter
  - Represent the (optimized) Boolean network with base functions
    - **Subject graph**
  - Represent library cells with base functions
    - **Pattern graph**
    - Each pattern is associated with a cost depending on the optimization criteria, e.g., area, timing, power, etc.
- **Goal:**
  - Find a minimal cost covering of a subject graph using pattern graphs

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# Technology Mapping

- **Technology Mapping:** The optimization problem of finding a minimum cost covering of the subject graph by choosing from a collection of pattern graphs of gates in the library.
- A **cover** is a collection of pattern graphs such that every node of the subject graph is contained in one (or more) of the pattern graphs.
- The cover is further constrained so that each input required by a pattern graph is actually an output of some other pattern graph.

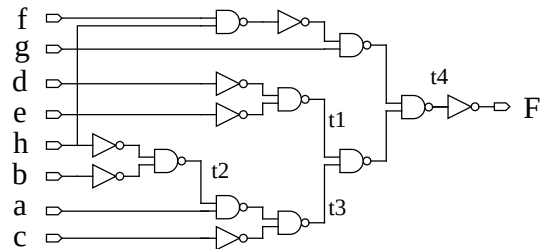
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# Technology Mapping

## Example

### Subject graph

$t1 = d + e$   
 $t2 = b + h$   
 $t3 = a t2 + c$   
 $t4 = t1 t3 + f g h$   
 $F = t4'$

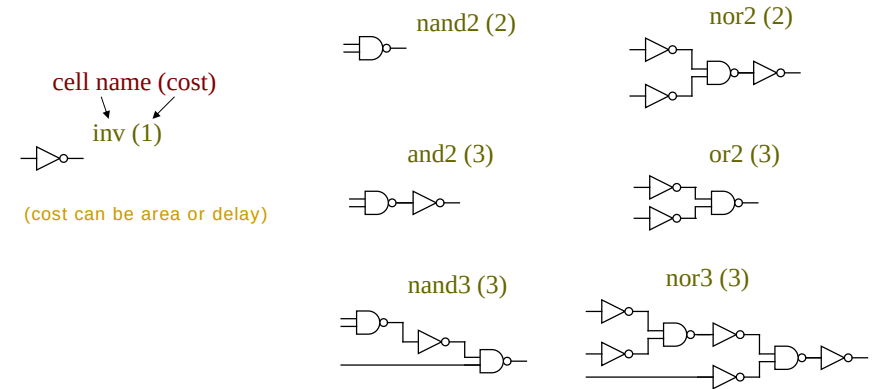


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# Technology Mapping

## Example

### Pattern graphs (1/3)

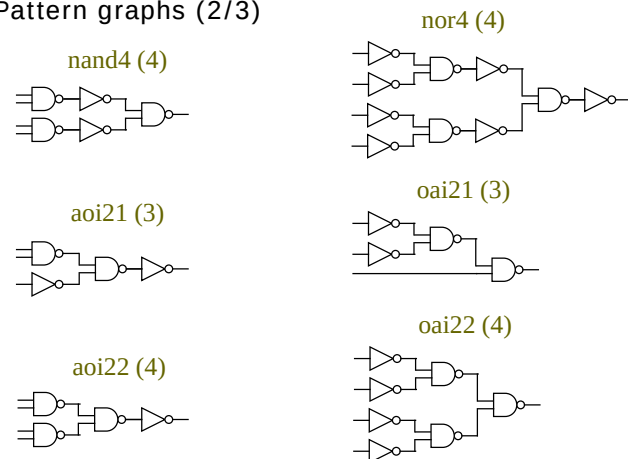


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# Technology Mapping

## Example

### Pattern graphs (2/3)

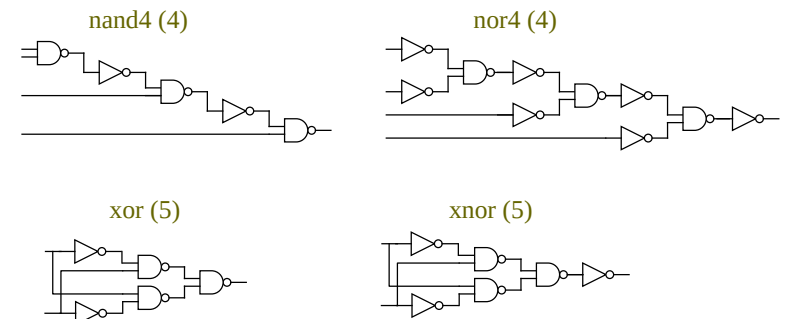


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# Technology Mapping

## Example

### Pattern graphs (3/3)



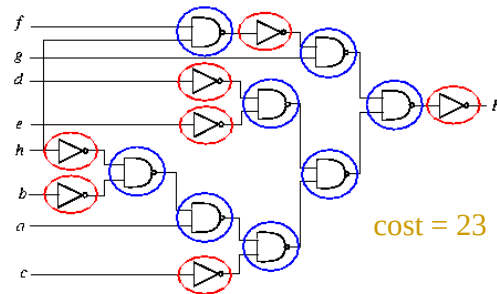
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## Technology Mapping

### Example

- A trivial covering
  - Mapped into NAND2's and INV's
    - 8 NAND2's and 7 INV's at cost of 23

$$\begin{aligned} t1 &= d + e; \\ t2 &= b + h; \\ t3 &= a \cdot t2 + c; \\ t4 &= t1 \cdot t3 + f g h; \end{aligned}$$

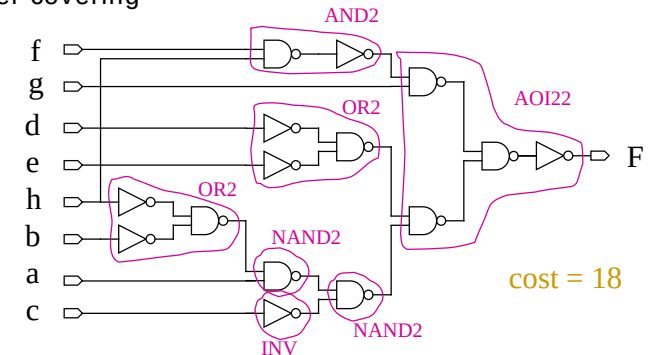


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## Technology Mapping

### Example

- A better covering



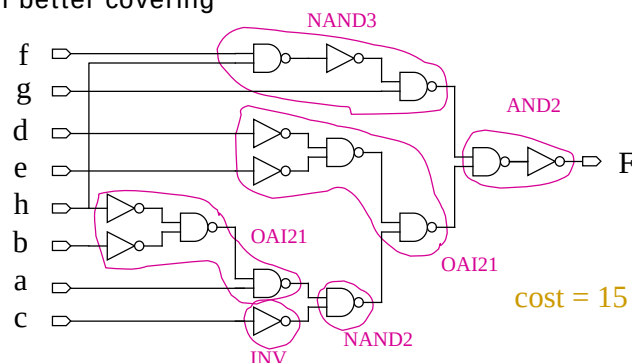
For a covering to be legal, every input of a pattern graph must be the output of another pattern graph!

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## Technology Mapping

### Example

- An even better covering



For a covering to be legal, every input of a pattern graph must be the output of another pattern graph!

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## Technology Mapping

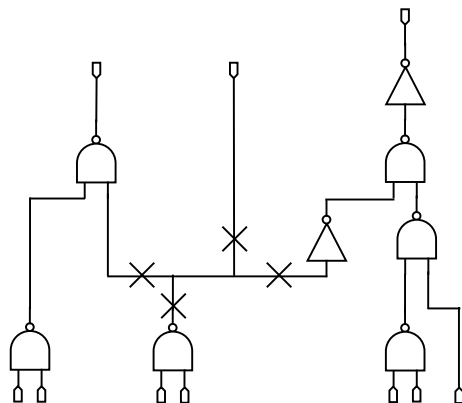
### Complexity of covering on directed acyclic graphs (DAGs)

- NP-complete
- If the subject graph and pattern graphs are trees, then an efficient algorithm exists (based on **dynamic programming**)

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## Technology Mapping DAGON Approach

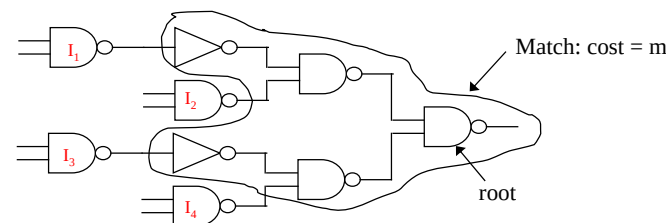
- Partition a subject graph into trees
  - Cut the graph at all multiple fanout points
- Optimally cover each tree using dynamic programming approach
- Piece the tree-covers into a cover for the subject graph



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## Technology Mapping DAGON Approach

- Principle of optimality: optimal cover for the tree consists of a match at the root plus the optimal cover for the sub-tree starting at each input of the match



$$C(\text{root}) = m + C(I_1) + C(I_2) + C(I_3) + C(I_4)$$

cost of a leaf (i.e. primary input) = 0

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## Technology Mapping DAGON Approach

- Example
  - Library

|       |   |  |            |  |
|-------|---|--|------------|--|
| INV   | 2 |  | $a'$       |  |
| NAND2 | 3 |  | $(ab)'$    |  |
| NAND3 | 4 |  | $(abc)'$   |  |
| NAND4 | 5 |  | $(abcd)'$  |  |
| AOI21 | 4 |  | $(ab+c)'$  |  |
| AOI22 | 5 |  | $(ab+cd)'$ |  |

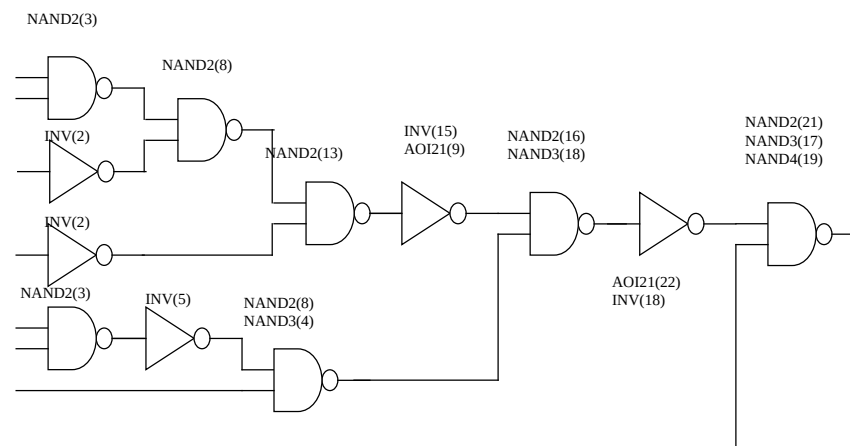
library element

base-function representation

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## Technology Mapping DAGON Approach

- Example



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## Technology Mapping DAGON Approach

- Complexity of DAGON for tree mapping is controlled by finding **all** sub-trees of the subject graph isomorphic to pattern trees
- Linear** complexity in both the size of subject tree and the size of the collection of pattern trees
  - Consider library size as constant

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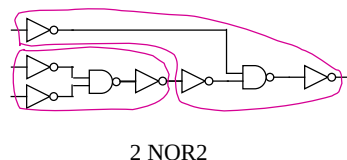
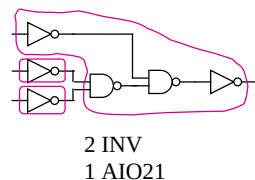
## Technology Mapping DAGON Approach

- |                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <ul style="list-style-type: none"><li>Pros:<ul style="list-style-type: none"><li>Strong algorithmic foundation</li><li>Linear time complexity<ul style="list-style-type: none"><li>Efficient approximation to graph-covering problem</li></ul></li><li>Give locally optimal matches in terms of both area and delay cost functions</li><li>Easily “portable” to new technologies</li></ul></li></ul> | <ul style="list-style-type: none"><li>Cons:<ul style="list-style-type: none"><li>With only a local (to the tree) notion of timing<ul style="list-style-type: none"><li>Taking load values into account can improve the results</li></ul></li><li>Can destroy structures of optimized networks<ul style="list-style-type: none"><li>Not desirable for well-structured circuits</li></ul></li><li>Inability to handle non-tree library elements (XOR/XNOR)</li><li>Poor inverter allocation</li></ul></li></ul> |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

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## Technology Mapping DAGON Approach

- DAGON can be improved by
  - Adding a pair of inverters for each wire in the subject graph
  - Adding a pattern of a wire that matches two inverters with zero cost



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## Available Logic Synthesis Tools

- Academic CAD tools:
  - Espresso (heuristic two-level minimization, 1980s)
  - MIS (multi-level logic minimization, 1980s)
  - SIS (sequential logic minimization, 1990s)
  - ABC (sequential synthesis and verification system, 2005-)
    - <http://www.eecs.berkeley.edu/~alanmi/abc/>

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