Digital Communication I

Final Exam, Jan. 7, 2005

Open books, open notes, calculator and handheld computer are allowed.

1. (40%) An equivalent discrete-time channel model consisting of two components f_0 and f_1, which are normalized to $|f_0|^2 + |f_1|^2 = 1$. The channel response is

$$F(z) = f_0 + f_1 z^{-1}.$$

   (a) If the additive white Gaussian noise has a variance of N_0, please find the SNR for the infinite-tap zero-forcing equalizer of

$$\gamma_\infty = \left[\frac{TN_0}{2\pi} \int_{-\pi/T}^{+\pi/T} \frac{d\omega}{X(e^{j\omega T})} \right]^{-1}.$$

   Hint: Express the results as function of f_0, f_1, and N_0. See Example 10.2-1.

   (b) Assumed that binary sequence of ± 1 is transmitted through the channel, if the maximum-likelihood sequence estimator (MLSE) is used as channel equalizer, please write down the formulae to calculate the branch metric in the Viterbi algorithm.

   (c) For part (b), if binary duobinary signal is transmitted through the channel, design the Viterbi algorithm for the MLSE receiver.

   (d) If Tomlinson-Harashima precoding is used for the case of part (b), please design the precoder.
   Hint: Very easy problem but only 1/3 of the grade for this part if not 100% correct.

Solutions

   (a) We need to find the inverse of

$$\frac{TN_0}{2\pi} \int_{-\pi/T}^{+\pi/T} \frac{d\omega}{X(e^{j\omega T})}$$

   i.e. $J_{\min}$ of Eq. (10-2.8) without N_0 in the denominator. Just copy Eq. (10.2-52) without N_0 in the denominator and get

$$\frac{TN_0}{2\pi} \int_{-\pi/T}^{+\pi/T} \frac{d\omega}{X(e^{j\omega T})} = \frac{N_0}{||f_0|^2 - |f_1|^2|}. \tag{1}$$

   (b) The branch metric is $(s_k \pm f_0 \pm f_1)^2$ for a total of four transitions. The answer is better to draw the trellis diagram to label the metric of each branch.

   (c) The branch metrics are $[s_k - f_0(I_k + I_{k-1}) - f_1(I_{k-1} + I_{k-2})]^2$. For an overall of 8 transitions, the branch metrics are $[s_k \pm f_0 \pm (f_0 + f_1) \pm f_1)]^2$.

   (d) Do not forget the requirement of normalization. The feedback filter in the precoder is $f_1/f_0 z^{-1}$.

2. (20%) A channel has an equivalent discrete-time channel of $F(z) = 0.8 + 0.6z^{-1}$.

   (a) Suppose we use a linear equalizer to equalize the channel. Determine the tap coefficients of c_{-1}, c_0, c_{+1} of a three tap equalizer. To simplify the computation, let the channel has zero noise.

   (b) Determine the tap weights of a decision-feedback-equalizer (DFE) with two feed-forward taps and one feed-back tap. To simplify the computation, let the channel has zero noise.

Solutions:

(a) Many solutions: Zero-forcing solution: $Q(z) = F(z)C(z) = 0.8c_{-1}z + 0.8c_0 + 0.6c_{-1} + (0.6c_0 + 0.8c_1)z^{-1} + 0.6c_1 z^{-2} = z$, we get $c_{-1} = 1.25$, $c_0 = -0.9375$, $c_1 = 0.6975$.
MSE solution #1 (not good):

$$\begin{bmatrix} 1 & 0.48 & 0 \\ 0.48 & 1 & 0.48 \\ 0 & 0.48 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0.8 \\ 0.6 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.4 \\ -0.19 \end{bmatrix}$$

MSE solution #2:

$$\begin{bmatrix} 1 & 0.48 & 0 \\ 0.48 & 1 & 0.48 \\ 0 & 0.48 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0.8 \\ 0.6 \end{bmatrix} = \begin{bmatrix} -0.46 \\ 0.95 \\ 0.144 \end{bmatrix}$$

MSE solution #3:

$$\begin{bmatrix} 1 & 0.48 & 0 \\ 0.48 & 1 & 0.48 \\ 0 & 0.48 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 0.34 \\ -0.71 \\ 1.14 \end{bmatrix}$$

(b) Feedforward part:

$$\begin{bmatrix} 1 & 0.48 \\ 0.48 & 0.64 \end{bmatrix}^{-1} \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 1.25 \\ 0 \end{bmatrix}$$

Feedback: $-c_0 f_1 = -0.75$.

3. (35%) Consider the adaptive finite-impulse-response (FIR) filter shown in Figure P3. The system $C(z)$ is characterized by the system function

$$C(z) = \frac{0.6}{1 - 0.8z^{-1}}.$$

The additive noise is white with variance of $\sigma_w^2 = 0.1$. Determine the optimum coefficients of the adaptive transversal (FIR) filter $B(z) = b_0 + b_1 z^{-1}$ when the adaptive algorithm is

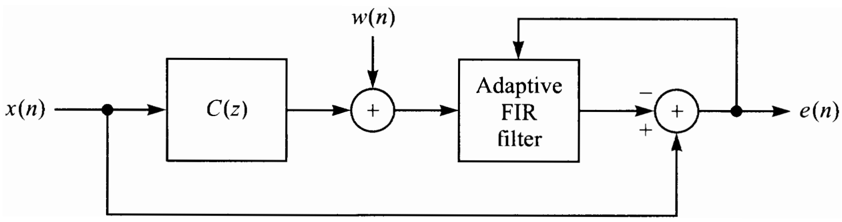

Figure P3

(a) Zero forcing algorithm to minimize the residual intersymbol interference in $e(n)$.

(b) The least-mean-square (LMS) algorithm to minimize the mean square error of $e(n)$.

(c) What is the maximum step size of Δ for the LMS algorithm such that the algorithm converges?

Solutions:

(a) $1 - 0.8z^{-1}$ or $0.61 - 0.49z^{-1}$.

(b)

$$\begin{bmatrix} 1.1 & 0.8 \\ 0.8 & 1.1 \end{bmatrix}^{-1} \begin{bmatrix} 0.6 \\ 0.48 \end{bmatrix} = \begin{bmatrix} 0.48 \\ 0.08 \end{bmatrix}$$

or the correct answer

$$\begin{bmatrix} 1.1 & 0.8 \\ 0.8 & 1.1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0.6 \end{bmatrix} = \begin{bmatrix} -0.84 \\ 1.16 \end{bmatrix}$$

(c) The eigenvalues of $\begin{bmatrix} 1.1 & 0.8 \\ 0.8 & 1.1 \end{bmatrix}$ are 1.9 and 0.3. The maximum Δ less than $2/1.9 = 1.05$.

4. (15%) In the design of finite-length linear equalizer, the mean-square-error (MSE) criterion minimizes the MSE of $J = E\{|I_k - \hat{I}_k|^2\}$. The optimal coefficients are given by

$$\mathbf{C}_{\text{opt}} = \mathbf{\Gamma}^{-1}\xi,$$

where $\mathbf{\Gamma}$ is the covariance matrix given by Eq. (10.2-44) and ξ is the correlation vector given by Eq. (10.2-45). If the linear equalizer is designed based on the tap-leakage algorithm to minimize

$$J = E\{|I_k - \hat{I}_k|^2\} + \mu \sum_{i=-K}^{K} |c_i|^2,$$

where μ is a small positive constant. Please find the optimal coefficients.

Hint: Expressed the results by $\mathbf{\Gamma}$, ξ, and μ.

Solution:

$$\mathbf{C}_{\text{opt}} = [\mathbf{\Gamma} + \mu\mathbf{I}]^{-1}\,\xi,$$