# Digital Modulation

**Amplitude**

**Phase**

**Frequency**

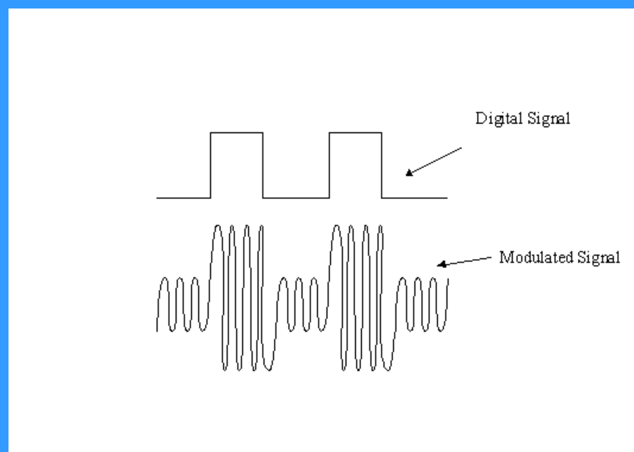
# Sinusoidal Waveform

- **The signal at frequency around $f_c$:**

  $$s(t) = A\cos(2\pi f_c t + \theta)$$

- **To encode the symbol of {0, 1},**
  - You can encode using the amplitude
  - You can encode using the phase
  - You can encode using the frequency
  - Some combination of amplitude, phase, or frequency.

# Amplitude-Shift Keying (ASK)



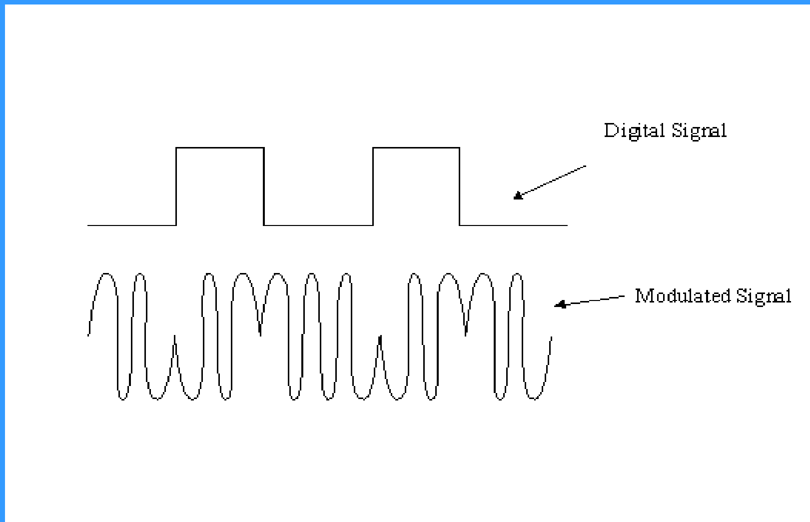

# ASK (cont.)

- Signal is

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \qquad 0 \le t \le T_b$$

$$s_2(t) = 0$$

# Phase-shift Keying (PSK)



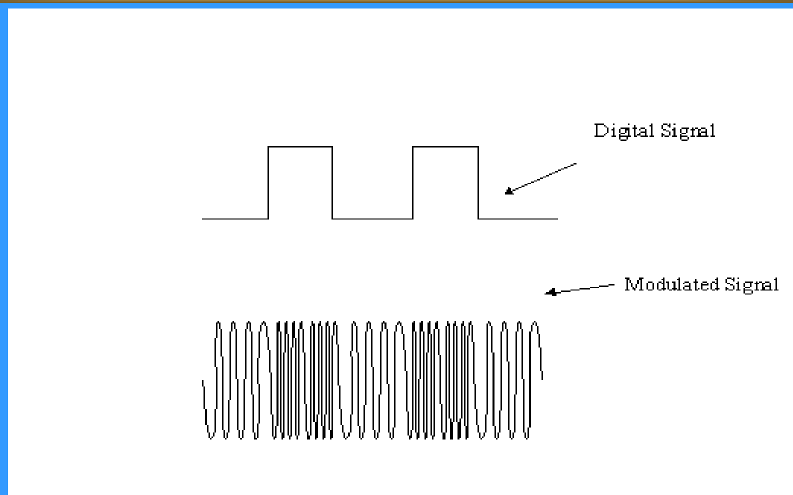

# PSK (cont.)

- Signal is

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \qquad 0 \le t \le T_b$$

$$s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \qquad 0 \le t \le T_b$$

# Frequency-shift keying (FSK)




# FSK (cont.)

- Signal is

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t) \qquad 0 \le t \le T_b$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t) \qquad 0 \le t \le T_b$$

# NRZ Baseband Systems

- NRZ Signal

- Signal is
  - $s_1(t)$ = 
  - $s_2(t)$ = 

# Gram-Schmidt orthogonalization process in linear space

If $S = \{y_1, \ldots, y_n\}$ is a linearly independent subsets of a vector space $V$, define $S' = \{x_1, \ldots, x_n\}$, where

$$\begin{aligned} x_1 &= y_1 \\ x_k &= y_k - \sum_{j=1}^{k-1} \frac{< y_k, x_j >}{||x_j||^2} x_j \qquad \text{for } 2 \leq k \leq n. \end{aligned}$$

$S'$ is an orthogonal set of non-zero vectors that can represent $S$.

**Example:**
$y_1 = (1, 1, 0)$, $y_2 = (2, 0, 1)$, $y_3 = (2, 2, 1)$.
$x_1 = (1, 1, 0)$, $x_2 = (1, -1, 1)$, $x_3 = (-1, 1, 2)/3$

Orthonormal process:
Just change $x_k \to x_k/||x_k||$ for $1 \leq k \leq n$.

# Gram-Schmidt orthogonalization process in function space

If $\{s_1(t), s_2(t), \ldots, s_N(t)\}$ are linearly independent signals, $\{\phi_1(t), \phi_2(t), \ldots, \phi_N(t)\}$ are orthonormal basic functions generated by:

$$\begin{aligned} \phi_1(t) &= \frac{s_1(t)}{\sqrt{\int_0^T s_1^2(t)dt}} \\ \phi_i(t) &= \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)dt}} \quad 2 \leq i \leq N \end{aligned}$$

where

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\phi_j(t)$$

and $s_{ij} = \int_0^T s_i(t)\phi_j(t)dt$

# Geometric Representation of Signals

The signal $\{s_i(t)\}, i = 1, 2, \ldots, M$ may be expanded by $\{\phi_j(t)\}, j = 1, 2, \ldots, N$ as

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t) \qquad 0 \le t \le T$$

where

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt \qquad \begin{array}{l} i = 1, 2, \ldots, M \\ j = 1, 2, \ldots, N \end{array}$$

It can be shown that

$$||s_i||^2 = \int_0^T s_i^2(t)dt = \sum_{j=1}^{N} s_{ij}^2$$

$$||s_i - s_k||^2 = \int_0^T [s_i(t) - s_k(t)]^2 dt = \sum_{j=1}^{N} (s_{ij} - s_{kj})^2$$