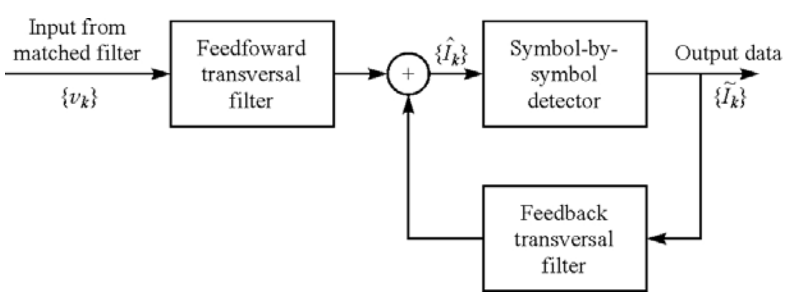
Nonlinear Equalization

Decision-Feedback Equalizer



$$\hat{I}_k = \sum_{j=-K_1}^{0} c_j v_{k-j} + \sum_{j=1}^{K_2} c_j \tilde{I}_{k-j}$$

Coefficient Optimization

- Minimize MSE

$$J(K_1, K_2) = E|I_k - \hat{I}_k|^2$$

- Feedforward filter

$$\sum_{j=-K_1}^{0} \psi_{lj} c_j = f^*_{-l} \qquad l = -K_1, \ldots, -1, 0$$

$$\psi_{lj} = \sum_{m=0}^{-l} f^*_m f_{m+l-j} + N_0 \delta_{lj} \qquad l, j = -K_1, \ldots, -1, 0$$

- Feedback filter

$$c_k = -\sum_{j=-K_1}^{0} c_j f_{k-j} \qquad k = 1, 2, \ldots, K_2$$

No ISI from previous symbols if $K_2 \geqslant L$

Minimum MSE

$$J_{\min}(K_1) = 1 - \sum_{j=-K_1}^{0} c_j f_{-j}$$

- If $K_1 \to \infty$

$$J_{\min} = \exp\left\{\frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} \ln\left[\frac{N_0}{X(e^{j\omega T}) + N_0}\right] d\omega\right\}$$

$$\gamma_\infty = \frac{1 - J_{\min}}{J_{\min}}$$

$$= -1 + \exp\left\{\frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} \ln\left[\frac{N_0 + X(e^{j\omega T})}{N_0}\right] d\omega\right\}$$

- If $X(e^{j\omega T}) = 1 \quad J_{\min} = N_0/(1 + N_0)$

$$\gamma_\infty = 1/N_0$$

Example: 2 taps

- For the channel $F(z) = f_0 + f_1 z^{-1}$

$$J_{\min} = \exp\left\{\frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} \ln\left[\frac{N_0}{1 + N_0 + 2|f_0||f_1|\cos(\omega T + \theta)}\right] d\omega\right\}$$
$$= N_0 \exp\left[-\frac{1}{2\pi}\int_{-\pi}^{\pi} \ln(1 + N_0 + 2|f_0||f_1|\cos\omega)\, d\omega\right]$$
$$= \frac{2N_0}{1 + N_0 + \sqrt{(1 + N_0)^2 - 4|f_0 f_1|^2}}$$

- Special case $|f_0| = |f_1| = \sqrt{\frac{1}{2}}$

$$J_{\min} = \frac{2N_0}{1 + N_0 + \sqrt{(1 + N_0)^2 - 1}}$$
$$\approx 2N_0, \qquad N_0 \ll 1$$

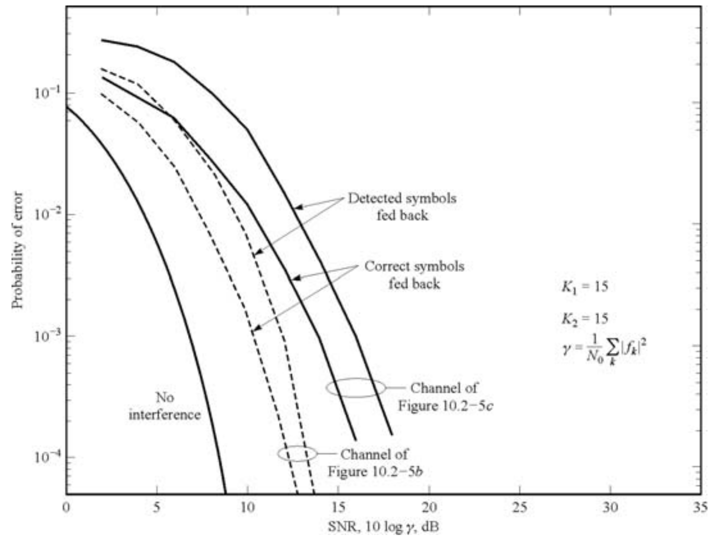
$$\gamma_\infty \approx \frac{1}{2N_0}, \qquad N_0 \ll 1$$

Example: Exponential Decay

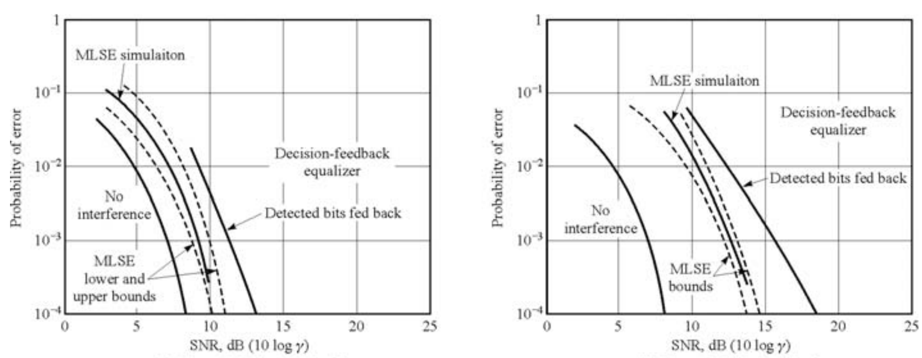
$$f_k = \sqrt{1 - a^2}\, a^k, \qquad k = 0, 1, \ldots$$

$$\gamma_\infty = -1 + \exp\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi} \ln\left[\frac{1 + a^2 + (1 - a^2)/N_0 - 2a\cos\omega}{1 + a^2 - 2a\cos\omega}\right] d\omega\right\}$$
$$= -1 + \frac{1}{2N_0}\left\{1 - a^2 + N_0(1 + a^2) + \sqrt{[1 - a^2 + N_0(1 + a^2)]^2 - 4a^2 N_0^2}\right\}$$
$$\approx \frac{(1 - a^2)[1 + N_0(1 + a^2)/(1 - a^2)] - N_0}{N_0}$$
$$\approx \frac{1 - a^2}{N_0}, \qquad N_0 \ll 1$$

Error probability

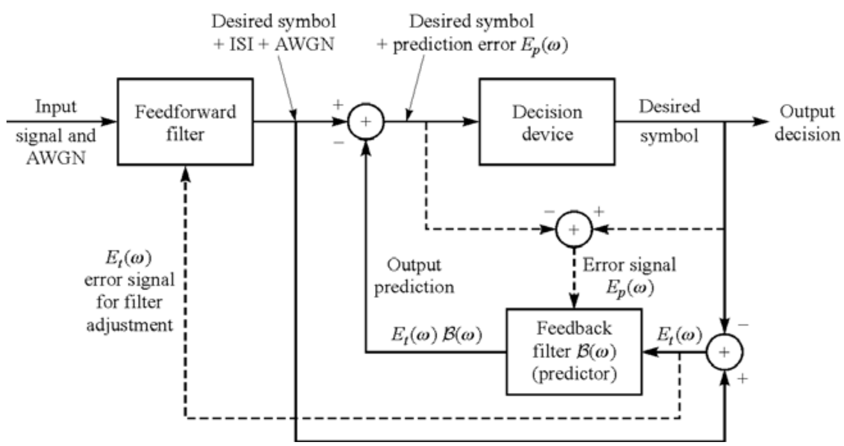


Error Probability (cont.)



(a) Channel of Figure 10.2−5b (b) Channel of Figure 10.2−5c

Predictive DFE



Design of Predictive DFE

- Noise spectrum at the output of feedforward filter

$$\frac{N_0 X(e^{j\omega T})}{|N_0 + X(e^{j\omega T})|^2}, \qquad |\omega| \leqslant \frac{\pi}{T}$$

- ISI spectrum

$$\left|1 - \frac{X(e^{j\omega T})}{N_0 + X(e^{j\omega T})}\right|^2 = \frac{N_0^2}{|N_0 + X(e^{j\omega T})|^2}, \qquad |\omega| \leqslant \frac{\pi}{T}$$

- Noise + ISI

$$|E_t(\omega)|^2 = \frac{N_0}{N_0 + X(e^{j\omega T})}, \qquad |\omega| \leqslant \frac{\pi}{T}$$

Design of Predictive DFE (cont.)

- With predictor

$$\mathcal{B}(\omega) = \sum_{n=1}^{\infty} b_n e^{-j\omega nT}$$

- Output error is

$$E_p(\omega) = E_t(\omega) - E_t(\omega)\mathcal{B}(\omega) = E_t(\omega)[1 - \mathcal{B}(\omega)]$$

with MSE of

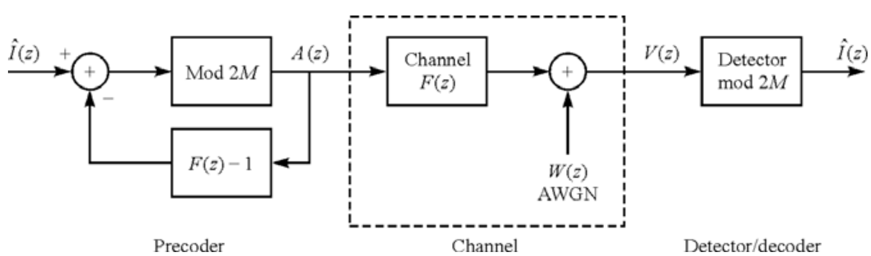
$$J = \frac{1}{2\pi}\int_{-\pi/T}^{\pi/T} |1 - \mathcal{B}(\omega)|^2 |E_t(\omega)|^2 d\omega$$

- Optimal Predictor of

$$\mathcal{B}(\omega) = 1 - \frac{G(\omega)}{g_0}$$

$$G(\omega)G^*(-\omega) = \frac{1}{|E_t(\omega)|^2}$$

Tomlinson-Harashima Precoding



- Precoder output

$$a_k = I_k - \sum_{j=1}^{L} f_j a_{k-j} + 2Mb_k$$

$$A(z) = I(z) - [F(z) - 1]A(z) + 2MB(z)$$

f_0 = 0 for simplicity

$$A(z) = \frac{I(z) + 2MB(z)}{F(z)}$$

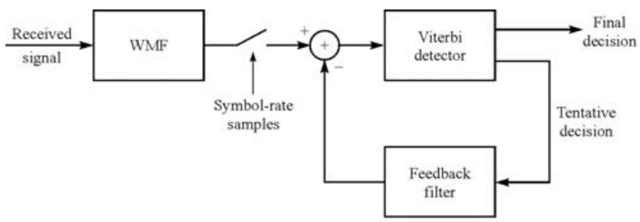
Tomlinson-Harashima Precoding (cont.)

- The output is

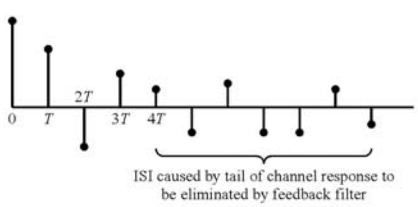
$$V(z) = A(z)F(z) + W(z)$$
$$= [I(z) + 2MB(z)] + W(z)$$

No ISI at output

Reduced Complexity ML Detector



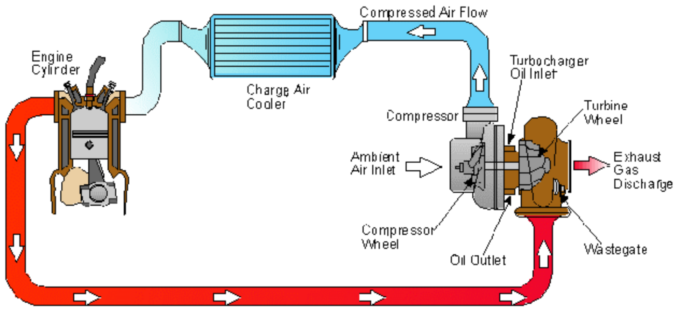
(a) Block diagram of symbol detector



(b) Channel response

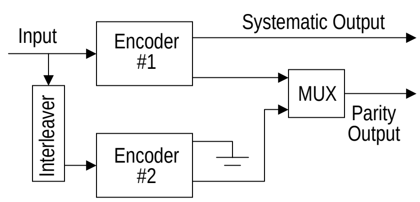
Turbo Principle

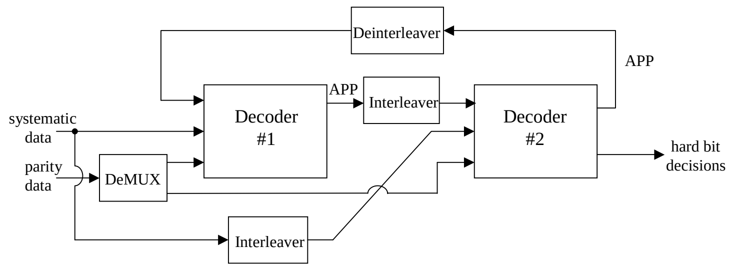
Turbo codes get their name because the decoder uses feedback, like a turbo engine.



Source: M. Valenti

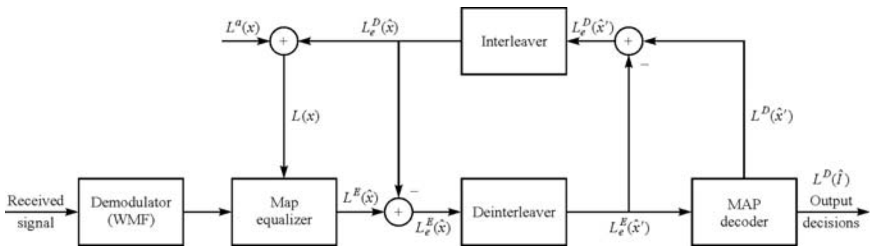
Turbo codes





Source: M. Valenti

Turbo Equalization



$$L(x) = \ln\left[\frac{\Pr(u_k = 1 | \text{channel output})}{\Pr(u_k = -1 | \text{channel output})}\right]$$

Performance of Turbo Equalization