Coherent Optical Communications

Midterm Exam, due date 1pm, Monday, May 24 at Rm. 553.

Detail derivations are required for all questions. You may print your Matlab or Maple code to substitute as derivation. Mathematica worksheet can be printed out as answer too.

1. For a differential quadrature phase-shift keying (DQPSK) signal with phase error of θ_e.

   (a) Find the error probability as a function of signal-to-noise ratio (SNR) and phase error θ_e.

   (b) If $\theta_e = 12.5^\circ$, please calculate the required SNR for an error probability of 10^{-9}. Expressed in term of dB with an accuracy of ± 0.25 dB.

2. An optical pulse has an electric field of

$$e(t) = \sqrt{P_0} \exp\left(-\frac{t^2}{2T_0^2}\right).$$

   If $E(f)$ is the spectrum of the optical pulse, the root-mean-squared (RMS) bandwidth of the optical pulse of w_{rms} is defined as

$$w_{\text{rms}}^2 = \frac{\int_{-\infty}^{+\infty} f^2 |E(f)|^2 \text{df}}{\int_{-\infty}^{+\infty} |E(f)|^2 \text{df}}.$$

   Due to self-phase modulation (SPM), at the output of the optical fiber, the optical pulse becomes

$$e_o(t) = e(t) \exp\left(-j\gamma L_{\text{eff}} |e(t)|^2\right)$$

   Please find the ratio of the RMS bandwidth at the output to that in the input. Expressed your results in term of $\phi_{\max} = \gamma L_{\text{eff}} P_0$.

3. The normalized nonlinear phase noise is equal to

$$\Phi = \int_0^1 |A + b(t)|^2 \text{dt},$$

   where A is a real number and $b(t)$ is a zero-mean complex Wiener process with an auto-correlation function of

$$R_b(t, s) = E\{b(s) \cdot b^*(t)\} = \min(t, s).$$

   (a) Please find a correction factor of $\alpha(s)$ as a function of position s such that the variance of $\sigma_s^2 = \Phi - \alpha(s)|A + b(s)|^2$ is minimum.

   (b) Please find the optimal location of s such that σ_s^2 is minimum. A good approximation is sufficient for this part.

4. Please briefly comment on whether the compensation of nonlinear phase noise can improved the channel capacity. Assume that compensation is preceding or with the receiver.

5. A small dc optical signal is used to measure cross-phase modulation (XPM). Due to XPM, the electric field becomes $e(t) = E_0 \exp[-j\phi(t)]$ where E_0 is a small constant and $\phi(t)$ is phase-modulation due to XPM. If $\phi(t)$ is a stationary Gaussian random process with an autocorrelation function of $R_\phi(\tau)$, please find the autocorrelation function of the electric field of $e(t)$ in terms of $R_\phi(\tau)$.

6. The noise of a single-branch heterodyne receiver is dominated by both relative intensity noise (RIN), thermal noise, and shot noise.

   (a) Please find an expression of the signal-to-noise ratio (SNR) at the receiver.

   (b) Please find the optimum value of P_{LO} to maximize the SNR.

7. A phase noise having a probability density of

$$p_\Theta(\theta) = \frac{a}{2} e^{-a|\theta|}$$

   is added into a differential phase-shift keying (DPSK) signal with a signal-to-noise ratio (SNR) of ρ_s. The phase noise is independent of the additive Gaussian noise.

   (a) Please find the error probability of the DPSK signals with the additive phase noise.

   (b) If the phase noise is assumed to be Gaussian distributed, please find whether the Gaussian approximation underestimates or overestimates the error probability. Please focus for the case that the error probability is around 10^{-9}.