Introduction to Four-Wave Mixing and its Application

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Outline

- Origin of Four-Wave Mixing (FWM)
- Theory of Four-Wave Mixing
- Phase Matching
- Parametric Amplification
- FWM Applications

Origin of Four-Wave Mixing (FWM)

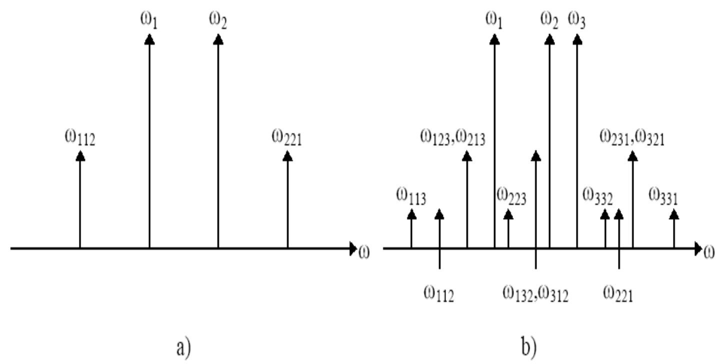
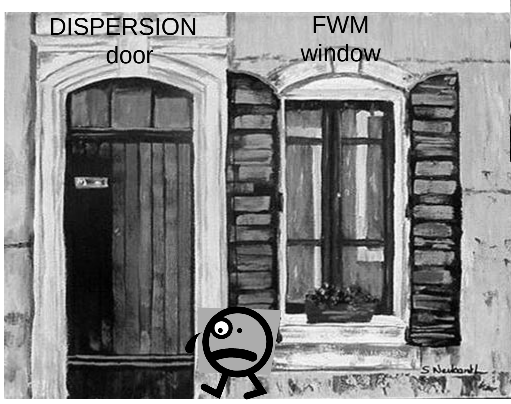


Figure 1. Additional frequencies generated through FWM in the partially degenerate (a) and non-degenerate case (b).

$\omega_{ijk} = \omega_i + \omega_j - \omega_k$ Phase matching condition



GOD
of
PHASE MATCHING

Theory of Four-Wave Mixing

Polarization Vector P:

$$\mathrm{P} = \varepsilon_0 \left(\chi^{(1)} \cdot \mathrm{E} + \chi^{(2)} : \mathrm{EE} + \chi^{(3)} \vdots \mathrm{EEE} + \cdots \right)$$

$\chi^{(j)}$: jth order susceptibility

$$\mathrm{P}_{\mathrm{NL}} = \varepsilon_0 \chi^{(3)} \vdots \mathrm{EEE}$$

$$\mathrm{E} = \frac{1}{2} \hat{x} \sum_{j=1}^{4} E_j \exp\left[i \left(k_j z - \omega_j t \right) \right] + c.c.$$

$$\mathrm{P}_{\mathrm{NL}} = \frac{1}{2} \hat{x} \sum_{j=1}^{4} P_j \exp\left[i \left(k_j z - \omega_j t \right) \right] + c.c.$$

SPM XPM

$$P_4 = \frac{3\varepsilon_0}{4} \chi^{(3)}_{xxxx} \Big[|E_4|^2 E_4 + 2\left(|E_1|^2 + |E_2|^2 + |E_3|^2 \right) E_4$$
$$+ 2E_1 E_2 E_3 \exp(i\theta_+) + 2E_1 E_2 E_3^* \exp(i\theta_-) + \cdots \Big]$$

FWM

$$\theta_+ = \left(k_1 + k_2 + k_3 - k_4 \right) z - \left(\omega_1 + \omega_2 + \omega_3 - \omega_4 \right) t$$
$$\theta_+ = \left(k_1 + k_2 - k_3 - k_4 \right) z - \left(\omega_1 + \omega_2 - \omega_3 - \omega_4 \right) t$$

After further simplification and approximation

$$A_1(z) = \sqrt{P_1}\exp[i\gamma(P_1 + 2P_2)z]$$
$$A_2(z) = \sqrt{P_2}\exp[i\gamma(P_2 + 2P_1)z]$$
$$B_3(z) = \left(a_3e^{gz} + b_3e^{-gz}\right)\exp(-i\kappa z/2)$$
$$B_4^*(z) = \left(a_4e^{gz} + b_4e^{-gz}\right)\exp(i\kappa z/2)$$

$$\kappa = \Delta k + \gamma(P_1 + P_2)$$
$$g = \sqrt{(\gamma P_0 r)^2 - (\kappa/2)^2}$$
$$r = 2\sqrt{(P_1P_2)}/P_0$$
$$P_0 = P_1 + P_2$$

g: parametric gain

- Not only phase matching but also matching of the group velocity (relate to dispersion) participate the FWM process.
- Complicated wave equation

Phase Matching

Energy and momentum conservation

FWM peak at

$$\kappa = \Delta k_M + \Delta k_W + \Delta k_{\mathrm{NL}} = 0 \quad \Rightarrow \quad \cos(0) = 1$$

Material dispersion | Waveguide dispersion | Nonlinear effect

Parametric Amplification

$$P_3(L) = P_3(0)\left[1 + \left(1 + \kappa^2/4g^2\right)\sinh^2(gL)\right] \quad \Rightarrow \quad \text{Amplified}$$

$$P_4(L) = P_3(0)\left(1 + \kappa^2/4g^2\right)\sinh^2(gL) \quad \Rightarrow \quad \text{New wave}$$

Gain

$$\mathrm{G_p} = P_3(L)/P_3(0) = 1 + (\gamma P_0 r/g)^2 \sinh^2(gL)$$

Bandwidth

$$\Delta\Omega_{\mathrm{A}} = \frac{1}{|\beta_2|\Omega_{\mathrm{s}}}\left[\left(\frac{\pi}{L}\right)^2 + (\gamma P_0 r)^2\right]^{1/2}$$

FWM Applications

- High-Speed Optical Multiplexing/Demultiplexing
- Parametric Amplifier
- Wavelength Conversion
- Dispersion Compensation
- FWM-Based Measurements of Fiber Parameters
- Others

High-Speed Optical Multiplexing/Demultiplexing

- Short, intense, co-propagating pulse
- Ether the data or the pump pulses must be at the zero dispersion wavelength

Partially degenerate FWM

Identical data stream at a complimentary freq.

Parametric Amplifier

Disadvantage:

1. Gain Bandwidth (~10-100GHz)
   < Raman amplified (~5THz)
2. Low efficiency, because of limited maximum pump power.

Advantage:

Spectral separation between pump and signal wavelength (~100THz)
> Raman gain (~13THz)

Wavelength Conversion

Advantage:

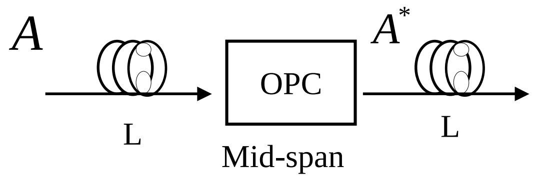
FWM conversion is coherent and maintain the phase of the wave (suitable for PSK system)

Disadvantage:

Conversion efficiency is limited by the limited maximum incident power (~15dBm)

Dispersion Compensation

- Optical phase conjugator (idler field is complex conjugate of the signal field)



Too many disadvantages:

limited small signal-pump separation
around zero dispersion wavelength
fail for third-order chromatic dispersion
polarization sensitivity

FWM-Based Measurements of Fiber Parameters

- Zero Dispersion wavelength
- Dispersion
- Second-order Refractive index
- Nonlinear Coefficient n_2 / A_{eff}

Other Applications

- Squeezing:
  noise fluctuations in some freq. range are reduced below the quantum-noise level
- Optical XOR gate
- Signal Reshaping (reducing intensity noise)

Optical XOR gate

$$E_{132} = (A_1 \bullet A_3) r(\omega_1 - \omega_3) A_2$$
$$\times \exp[j(\omega_1 + \omega_2 - \omega_3)t + (\phi_1 + \phi_2 - \phi_3)]$$

$r(\omega_1 - \omega_3)$: conversion efficiency

Because of phase signal "0, π"

$$\phi_{132} = \phi_1 \oplus \phi_2 \oplus \phi_3$$

Conclusion

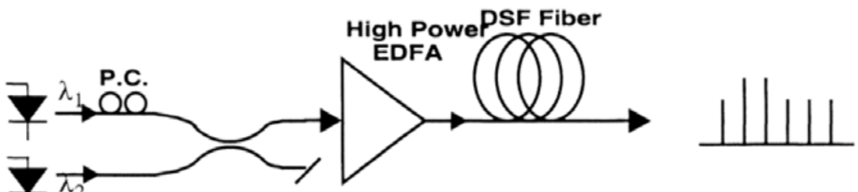
- FWM is a nonlinear process that generates a new optical freq. from many co-propagating waves.
- Efficient FWM requires high power and phase matching.
- Phase matching is most easily achieved between waves with similar freqs. and low dispersion.
- SPM and other nonlinear effects may relax the phase matching condition.

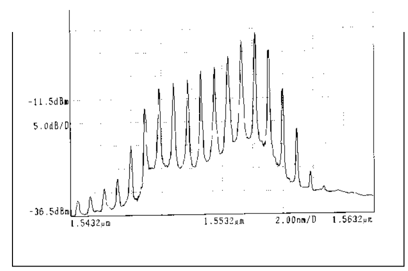
Reference

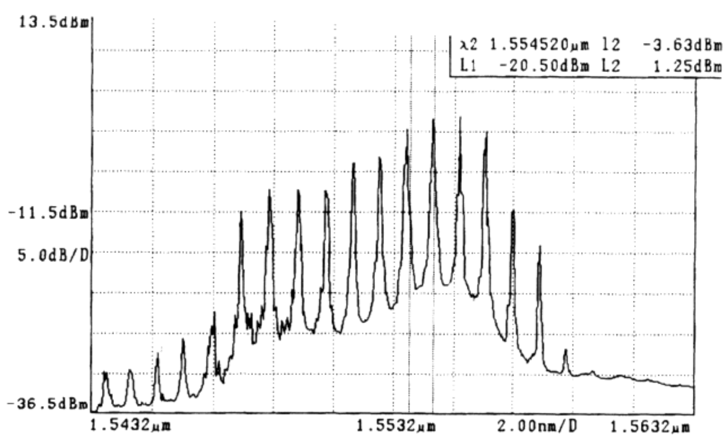
Main

- Nonlinear Fiber Optics
  -- by G.P. Agrawal
- A report of FWM in optical fiber and its metrological applications
  -- by R. Billington
- All-optical signal reshaping via FWM in optical fiber
  -- by E. Ciaramella
- Demostration of 20Gps all-optical XOR gate by FWM inSOA with RZ-DPSK mod. input
  -- by Kit Chan

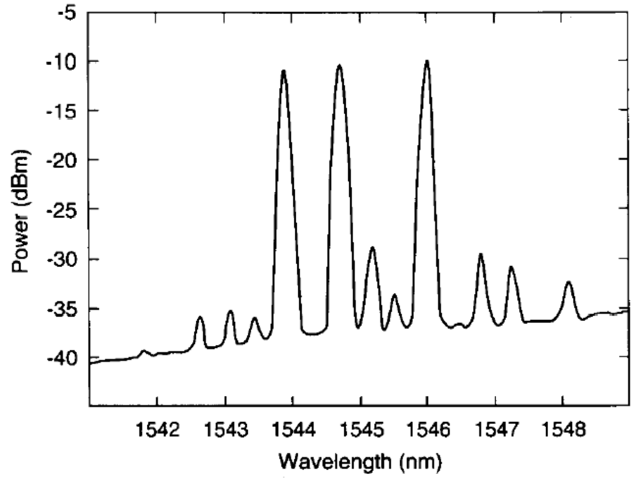
Some measurement for Four-Wave-Mixing







Typical Three Channels



FWM Efficiency