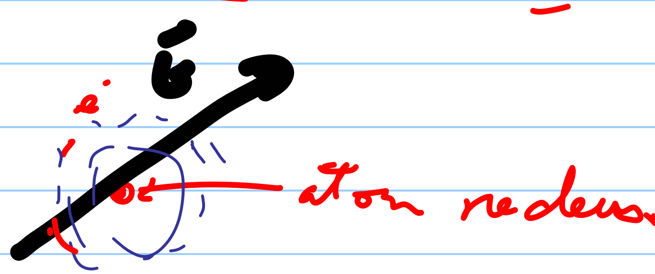
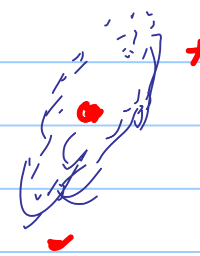
Note Title 3/17/2005

$$\tilde{E} = E_x \hat{x} + E_y \hat{y} + E_z \hat{z}$$





$$\tilde{P} = \varepsilon_0 \varepsilon_r \tilde{E}$$

$$P_x = \varepsilon_0 \varepsilon_r E_x + \chi^{(2)}_{xxx} E_x^2 + \chi^{(2)}_{xyy} E_y^2$$

$$+ \chi^{(2)}_{xxy} E_x E_y + \ldots$$

3 3 3 2

$$P_4 = k\,2\,E_1 E_2 E_3\, e^{j\theta_-}$$

$$\theta_- = (\beta_1 + \beta_2 - \beta_3 - \beta_4)\,z - (\omega_1 + \omega_2 - \omega_3 - \omega_4)\,t$$

$\rightarrow \Delta\beta$

$$\omega_1 + \omega_2 - \omega_3 - \omega_4 = 0$$

$$\omega_4 = \omega_1 + \omega_2 - \omega_3$$

$$\frac{dE_4}{dz} = 2k\,\bar{E}_1 \bar{E}_2 \bar{E}_3\, e^{j\theta_-}$$

← Attenuation coefficient

$$E_1(z) = \bar{E}_1(0)\, e^{-\alpha z/2}$$

$$E_2(z) = \bar{E}_2(0)\, e^{-\alpha z/2}$$

What αz for E_3 & E_4

$$E_4 e^{-\alpha z/2} \propto \int_0^L e^{-\alpha z + j\Delta\beta z}\, dz$$

$$= \frac{1 - e^{-(\alpha - j\Delta\beta)L}}{\alpha - j\Delta\beta}$$

Fiber dispersion:

$$E \propto e^{-j(\beta z - \omega t)}$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_g} = \frac{\omega n}{c_0}$$

n ← refractive index, $n(\omega)$

Taylor Series

$$\beta = \beta_0 + \frac{d\beta}{d\omega}(\omega - \omega_0) + \frac{1}{2}\frac{d^2\beta}{d\omega^2}(\omega-\omega_0)^2 + \frac{1}{6}\frac{d^3\beta}{d\omega^2}(\omega-\omega_0)^3 + \cdots$$

$$\frac{1}{v_g} = \frac{d\beta}{d\omega}$$

↑ Group velocity

$$D = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right) \qquad ps/nm/km$$

↑ Dispersion coefficient.

$$\Delta\beta = \beta_2 + \beta_2 - \beta_1 - \beta_4$$

$$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$$

$$\omega_2 \quad \omega_2 \quad \omega_1 \quad \omega_{221} = 2\omega_2 - \omega_1$$

$$\beta(\omega) = -\frac{\lambda^2}{4\pi c} D (\omega - \omega_2)^2$$

$$\uparrow \; \omega_0$$

$$\Delta\beta = 0 + 0 + \frac{\lambda^2}{4\pi c} D (\omega_1 - \omega_2) + \frac{\lambda^2}{4\pi c} D (2\omega_2 - \omega_1 - \omega_2)^2$$

$$= \frac{\lambda^2}{2\pi c} D (\omega_1 - \omega_2)^2$$