Fundamental properties of polarization-mode-dispersion

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Introduction

- Because various excited modes cause different group velocities, the single-mode fiber is employing.
- But duo to imperfectly symmetry or environment perturbation, birefringence in single-mode fiber is caused. In a sense, there exists two-mode for single-mode fiber
- Polarization-mode dispersion(PMD) is appealing.

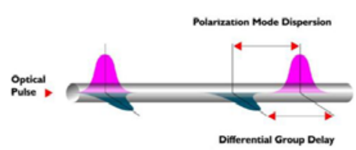
Differential group delay(DGD)

- The magnitude of PMD dependson the difference $\beta_x - \beta_y$

the differential group delay t_m is

$$t_m = \left| \frac{d(\beta_1 - \beta_2)}{d\omega} \right| \text{ secs/per unit length}$$

where β_1, β_2 are propagation constant of two orthogonally polarization mode



Plane wave and *Polarization states*

- *Plane wave*: the orientation of electric field is constant and its magnitude and sign vary in time.
- Condition on above, three states of polarization are considered.

1. Linear Linear polarization
2. Circular polarization
3. Elliptical polarization

Linear polarization

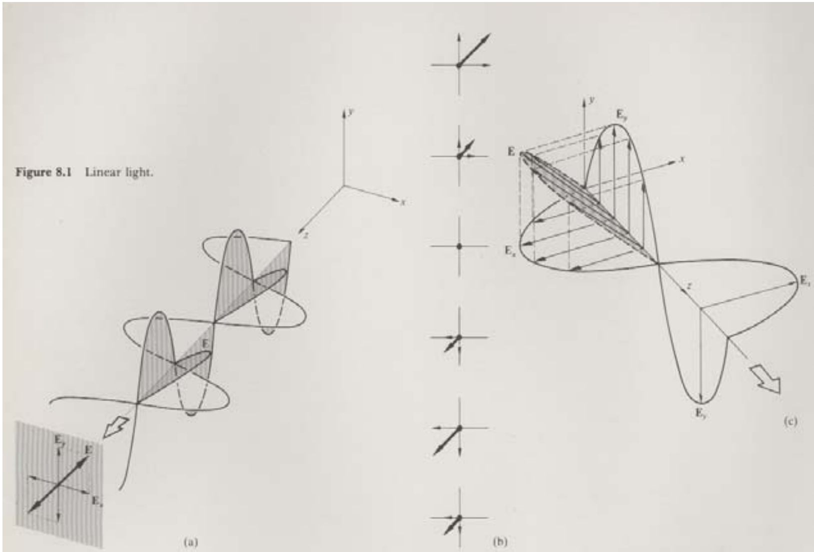
$$\overrightarrow{E_x} = \vec{i}a_1 \cos(kz - \omega t)$$

$$\overrightarrow{E_y} = \vec{j}a_2 \cos(kz - \omega t)$$

when $a_1 = a_2$, we have

$$\vec{E} = \left[\vec{i}a_1 + \vec{j}a_2\right]\cos(kz - \omega t)$$

the direction of $\vec{E}$ is always the same with $\vec{i}a_1 + \vec{j}a_2$



Circular polarization

$\vec{E} = \vec{E_x} + \vec{E_y}$

$\vec{E_x} = \vec{i} a_1 \cos(kz - \omega t)$

$\vec{E_y} = \vec{j} a_2 \cos(kz - \omega t + \varepsilon)$

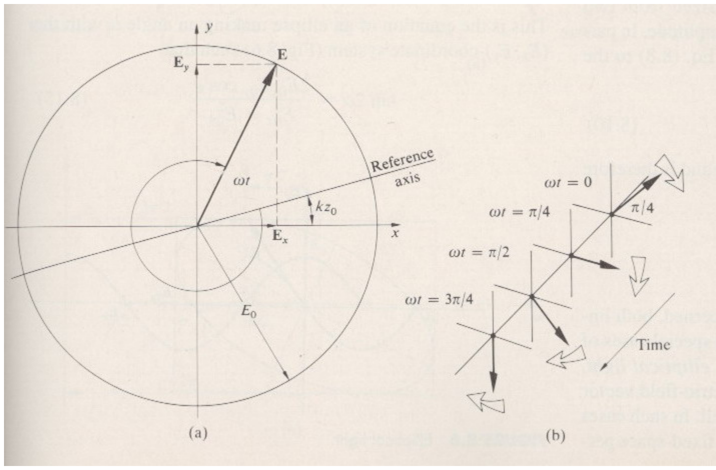
when $a_1 = a_2$ and $\varepsilon = \pm \frac{\pi}{2}$, we have

$|\vec{E}| = \sqrt{a_1^2 + a_2^2} = \text{constant}$

we find that $\vec{E}$ varies with time but amplitude is constant

NOTE: $\varepsilon = -\frac{\pi}{2}$: right circular light

$\varepsilon = \frac{\pi}{2}$: left- circular light



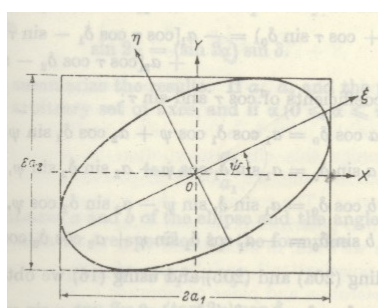
Elliptical polarization

$\vec{E} = \vec{E_x} + \vec{E_y}$

$\vec{E_x} = \vec{i} a_1 \cos(kz - \omega t + \varepsilon_1)$

$\vec{E_y} = \vec{j} a_2 \cos(kz - \omega t + \varepsilon_2)$

we have ellipticity.



Note: Circular and linear states are special cases of elliptical state

Jones space versus *Stokes space*

- Consider a complex vector in *Jones space*

$$\vec{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{ox} e^{i\varepsilon_1 x} \\ E_{oy} e^{i\varepsilon_2 y} \end{bmatrix}$$

Can mapping to the elliptical polarization.

(a) Linear polarization($m \in Z$)

$$\frac{E_{oy}}{E_{ox}} = (-1)^m \frac{a_2}{a_1}$$

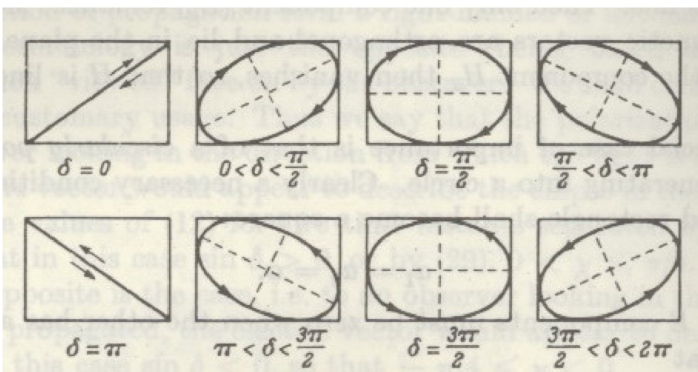
(b) Right-handed circularly polarization($a_1 = a_2$, $\delta = \pi/2$)

$$\frac{E_{oy}}{E_{ox}} = \frac{a_2}{a_1} e^{-i\pi/2} = -i \frac{a_2}{a_1}$$

(c) Left-handed circularly polarization($a_1 = a_2$, $\delta = -\pi/2$)

$$\frac{E_{oy}}{E_{ox}} = \frac{a_2}{a_1} e^{i\pi/2} = i \frac{a_2}{a_1}$$

$$\delta \equiv \varepsilon_2 - \varepsilon_1$$



Jones space versus *Stokes space*

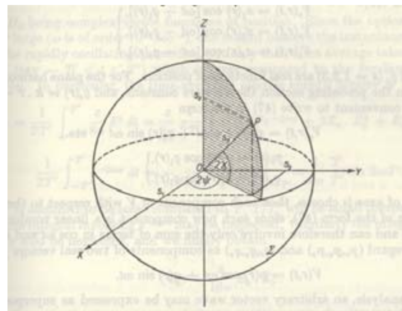
We can map these to a new parameter called Stokes parameter, that is

$$s_0 = a_1^2 + a_2^2 = |E_x|^2 + |E_y|^2$$

$$s_1 = a_1^2 - a_2^2 = |E_x|^2 - |E_y|^2$$

$$s_3 = 2a_1a_2\cos\delta = 2\,\mathrm{Re}\left(E_xE_y^*\right)$$

$$s_4 = 2a_1a_2\sin\delta = 2\,\mathrm{Im}\left(E_xE_y^*\right)$$



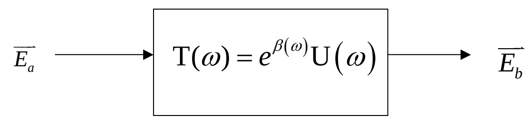
What is it meaning for?

We usually plot vector that has three dimension unit vector . $\vec{s} = (s_1, s_2, s_3)$

Existence of *principal states* of polarization

- The orientation of eigenmodes is random and hard to be identified when several fiber pieces are concatenated to form a long fiber.
- The new model of polarization dispersion for long fiber is manifested and the states of polarization is called *principal states* of polarization, but restricted with
  (a) No polarization-dependent loss
  (b) The bandwidth of source is narrow

The system model



Note: $U^H U = I$ (No polarization-dependent loss)

We want to find that there are two orthogonal states of polarization having the property that the output state of polarization is invariant with first order of frequency conditional on zero dispersion.

Poole show the *Existence* of *principal states* of polarization

Also, in the same papar, he find that

(1) the principal states $\hat{\varepsilon}_{a\pm}$ of polarization and corresponding output unit states $\hat{\varepsilon}_{b\pm}$ of polarization are have relation

$\left\langle \hat{\varepsilon}_{a+}, \hat{\varepsilon}_{a-} \right\rangle = 0,\ \left\langle \hat{\varepsilon}_{b+}, \hat{\varepsilon}_{b-} \right\rangle = 0$ $\left(\langle . \rangle \text{ represents inner product}\right)$

(2) The any input state can divide two part $\hat{\varepsilon}_{a+}$ and $\hat{\varepsilon}_{a-}$ duo to orthogonality, and corresponding propagation delay difference is $\Delta\tau = 2\sqrt{\left(\left|u_1'\right|^2 + \left|u_2'\right|^2\right)}$

where u_1, u_2 are diagnonal elements of $U(\omega)$

Connecting Stokes vector

- The rotation of the stokes vector with change frequency is directly related to difference in propagation delay time for two principle states.

$$\frac{d\hat{s}}{d\omega} = \vec{\Omega} \times \hat{s}$$

$\hat{s}$: corresponding Stokes vector related to $\vec{E}$

$\vec{\Omega}$: direction of rotation

$|\vec{\Omega}|$: rate of rotation

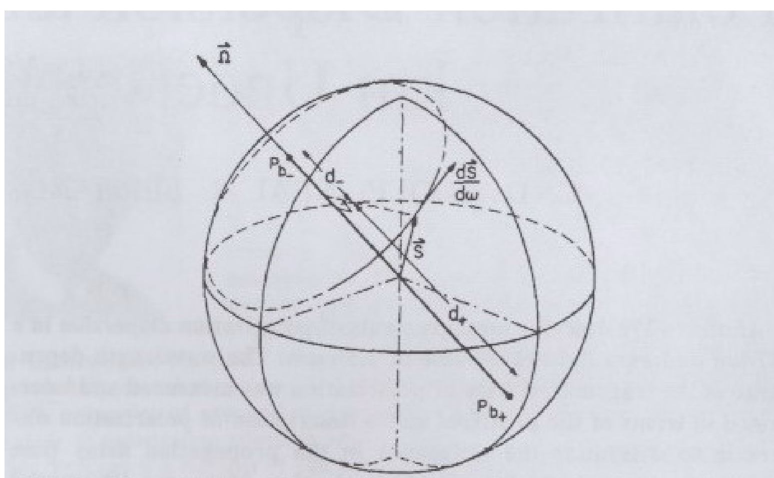


Fig. 1. Theoretical wavelength dependence of transmitted state of polarization as represented on Poincare sphere.

Statistics of polarization-mode dispersion dispersion with random mode coupling

- The objective is to find the differential delay time (δ_τ) between the principal states of polarization in single mode fiber with random mode coupling and have some assumption below

(1) assume that we have a collection of statistically equivallent fibers

(2) δ_τ is defined differential time delay between one of two principal states of polarization chosen randomly, so $\mathrm{E}\left[\delta_\tau\right]=0$

(3) κ(z) is coupling constant proportional to perturbing birefringency and assume to be W.S.S., where z is position of single mode fiber

Variance of differential delay time

$$E\left[\delta_\tau\left(z\right)^2\right]=\frac{\Delta\beta'^2}{2h^2}\left[e^{-2hz}-1+2hz\right]$$

$\Delta\beta$: difference of propagation constant

h : interprete as coupling length

$$\lim_{hz\to 0} E\left[\delta_\tau\left(z\right)^2\right]=\Delta\beta'^2 z^2$$

$$\lim_{hz\to\infty} E\left[\delta_\tau\left(z\right)^2\right]=\frac{\Delta\beta'^2 z}{h}$$

Dynamical equation for polarization dispersion

$$\frac{\partial \hat{s}}{\partial z} = W(\omega, z) \times \hat{s}$$

$$\frac{\partial \hat{s}}{\partial \omega} = \Omega(\omega, z) \times \hat{s}$$

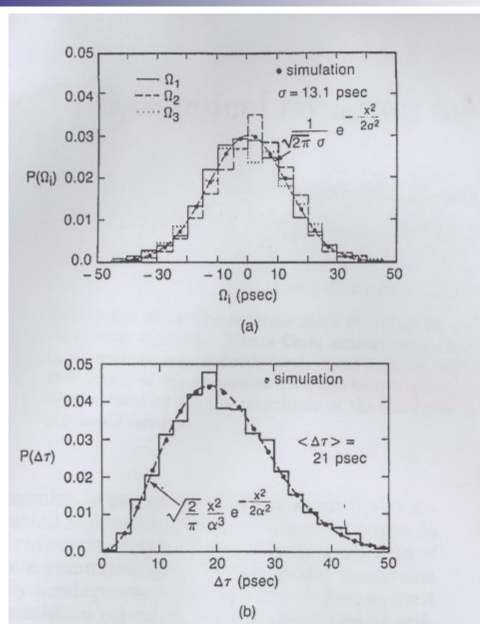
from above, we can get

$$\frac{\partial \Omega(\omega, z)}{\partial z} = \frac{\partial W(\omega, z)}{\partial \omega} + W(\omega, z) \times \Omega(\omega, z)$$

we can find that $\Omega_{1,2,3}$ are *i.i.d*. *Gaussian* r.v.

then $\delta_\tau = \sqrt{\Omega_1^{\,2} + \Omega_1^{\,2} + \Omega_1^{\,2}}$ is χ distribution

or called Maxwellian distribution



Reference