CHAPTER 4 BIPOLAR JUNCTION TRANSISTORS (BJTs)

Chapter Outline
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4.5 Small-Signal Operation and Models
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4.1 Device Structure and Physical Operation

Physical structure of bipolar junction transistor (BJT)

- Both electrons and holes participate in the conduction process for bipolar devices
- BJT consists of two \textit{pn} junctions constructed in a special way and connected in series, back to back
- The transistor is a three-terminal device with \textit{emitter}, \textit{base} and \textit{collector} terminals
- From the physical structure, BJTs can be divided into two groups: \textit{npn} and \textit{pnp} transistors

Modes of operation

- The two junctions of BJT can be either forward or reverse-biased
- The BJT can operate in different modes depending on the junction bias
- The BJT operates in active mode for amplifier circuits
- Switching applications utilize both the cutoff and saturation modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>EBJ</th>
<th>CBJ</th>
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<tbody>
<tr>
<td>Cutoff</td>
<td>Reverse</td>
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<td>Active</td>
<td>Forward</td>
<td>Reverse</td>
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<tr>
<td>Saturation</td>
<td>Forward</td>
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</table>
Operation of the *npn* transistor in the active mode

- Electrons in emitter regions are injected into base due to the forward bias at EBJ
- Most of the injected electrons reach the edge of CBJ before being recombined if the base is narrow
- Electrons at the edge of CBJ will be swept into collector due to the reverse bias at CBJ
- **Emitter injection efficiency** ($\gamma$) = $i_{En}/(i_{En} + i_{Ep})$
- **Base transport factor** ($\alpha_T$) = $i_{Cn}/i_{En}$
- **Common-base current gain** ($\alpha$) = $i_{Cn}/i_{E} = \gamma \alpha_T < 1$
- Terminal currents of BJT in active mode:
  - $i_E$(emitter current) = $i_{En}$(electron injection from E to B) + $i_{Ep}$(hole injection from B to E)
  - $i_C$(collector current) = $i_{Cn}$(electron drift) + $i_{CBO}$(CBJ reverse saturation current with emitter open)
  - $i_B$(base current) = $i_{B1}$(hole injection from B to E) + $i_{B2}$(recombination in base region)
Terminal currents:

- Collector current: \( i_C \approx i_{C_n} = A_E q D_{mb} n_B(x) / dx = A_E q D_{mb} n_B(0)/W = \frac{A_E q D_{mb} n_B^2}{N_B W} e^{\frac{q V}{kT}} = I_s e^{\frac{q V}{kT}} \)

- Base current:
  - Hole injection into emitter due to forward bias: \( i_{B1} = -A_E q D_{pe} p_E(x) / dx = \frac{A_E q D_{pe} n_i^2}{N_E L_{pe}} e^{\frac{q V}{kT}} \)
  - Electron-hole recombination in base: \( i_{B2} = Q_n / \tau_n = \left( A_E q \times \frac{1}{2} n_B(0) W \right) / \tau_n = \frac{A_E q W n_i^2}{2 N_B \tau_n} e^{\frac{q V}{kT}} \)
  - Total base current: \( i_B = i_{B1} + i_{B2} = I_s \left( \frac{D_{pe} N_B W}{D_{mb} N_E L_{pe}} + \frac{1}{2} \frac{W^2}{D_{mb} \tau_n} \right) e^{\frac{q V}{kT}} \approx \frac{i_C}{\beta} \)

- Emitter current: \( i_E = i_C + i_B = \frac{\beta + 1}{\beta} i_C = \frac{i_C}{\alpha} \frac{I_s}{\alpha} e^{\frac{q V}{kT}} \)

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Large-signal model and current gain for BJT in active region

- **Common-emitter** current gain: \( \beta \equiv \frac{i_C}{i_B} = (1 - \alpha) \)

- **Common-base** current gain: \( \alpha = \frac{\beta}{\beta + 1} \)

**The structure of actual transistors**

- In modern process technologies, the BJT utilizes a vertical structure.
- Typically, \( \alpha \) is smaller and close to unity while \( \beta \) is large.

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4-5
Operation of the $npn$ transistor in the saturation mode

- Saturation mode: both EBJ and CBJ are forward biased
- Carrier injection from both emitter and collector into base
- Base minority carrier concentration change accordingly → leading to reduced slope as $v_{BC}$ increases
- Collector current drops from the value in active mode for negative $v_{CB}$
- For a given $v_{BE}$, $i_C$ drops sharply to zero at $v_{CB}$ around $-0.5$ V and $v_{CE}$ around 0.2 V
- BJT in saturation: $V_{CE_{sat}} = 0.2$ V
- Current gain reduces (from $\beta$ to $\beta_{forced}$): $\beta_{forced} = \left| \frac{i_C}{i_B} \right|_{saturation} \leq \beta$

\[ n_0 \exp\left(\frac{v_{BE}}{V_T}\right) \quad n_0 \exp\left(\frac{v_{BC}}{V_T}\right) \]

$v_{BC}$ increases

$\alpha I_E$

$i_E = I_E$

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Ebers-Moll model

- In EM model, the EBJ and CBJ are represented by two back to back diodes $i_{DE}$ and $i_{DC}$
- Current transported from one junction to the other is presented by $\alpha_F$ (forward) and $\alpha_R$ (reverse)
- EM model can be used to describe the BJT in any of its possible modes of operation
- EM model is used for more detailed dc analysis
- The diode currents: $i_{DE} = I_{SE}(e^{\gamma_{SE}/V_T} - 1)$  $i_{DC} = I_{SC}(e^{\gamma_{SC}/V_T} - 1)$  $\alpha_F I_{SE} = \alpha_R I_{SC} = I_S$
- The terminal currents: $i_E = i_{DE} - \alpha_R i_{DC}$  $i_C = -i_{DC} + \alpha_F i_{DE}$  $i_B = i_E - i_C$

Application of the EM model

- The forward active mode:
  $i_C \approx I_S e^{\gamma_{SE}/V_T} + I_S \left( \frac{1}{\alpha_R} - 1 \right)$
  $i_B \approx \frac{I_S}{\beta_F} e^{\gamma_{SE}/V_T} - I_S \left( \frac{1}{\beta_F} + \frac{1}{\beta_R} \right)$
  $i_E \approx \frac{I_S}{\alpha_F} e^{\gamma_{SE}/V_T} + I_S \left( 1 - \frac{1}{\alpha_F} \right)$
- The saturation mode:
  $i_C = I_S e^{\gamma_{SE}/V_T} - I_{SC} e^{\gamma_{SC}/V_T}$
  $i_E = I_S e^{\gamma_{SE}/V_T} - I_S e^{\gamma_{SC}/V_T}$
  $i_B = I_S (1 - \alpha_F) e^{\gamma_{SE}/V_T} + I_S (1 - \alpha_R) e^{\gamma_{SC}/V_T}$
The cutoff mode

- **$I_{CBO}$ (CBJ reverse current with emitter open-circuited)**
  - $I_{CBO} = (1 - \alpha_R \alpha_F)I_{SC}$
  - Both EBJ and CBJ are reverse-biased
  - In real case, reverse current depends on $v_{CB}$

- **$I_{CEO}$ (CBJ reverse current with base open-circuited)**
  - $I_{CEO} = I_{CBO}/(1 - \alpha_F)$
  - $\alpha_F$ is always smaller than unity such that $I_{CEO} > I_{CBO}$
  - CBJ current flows from (C to B) so CBJ is reverse-biased
  - EBJ current flows from (E to B) so EBJ is slightly forward-biased

\[i_C = I_{CBO} = (1 - \alpha_R \alpha_F)I_{SC}\]

\[i_C = I_{SC} + \alpha_F i_{DE} = I_{SC}(1 - \alpha_R \alpha_F)/\alpha_F \rightarrow I_{CEO} = I_{CBO}/(1 - \alpha_F)\]
The *pnp* transistor

- Transistor structure:
  - emitter and collector are *p*-type
  - base is *n*-type
- Operation of *pnp* is similar to that of *nnp*

**Operation of pnp in the active mode**

- Collector current: \( i_c = I_s e^{v_{EB}/V_T} \)
- Base current: \( i_B = i_C / \beta \)
- Emitter current: \( i_E = i_C + i_B \)

**Large-signal model and current gain for BJT in active region**

**Exercise 4.1 (Textbook)**
**Exercise 4.3 (Textbook)**
4.2 Current-Voltage Characteristics

Circuit symbols, voltage polarities and current flow
- Terminal currents are defined in the direction as current flow in active mode
- Negative values of current or voltage mean in opposite polarity (direction)

Summary of the BJT current-voltage relationships in the active mode
- The terminal currents for a BJT in active mode solely depend on the junction voltage of EBJ
- The ratios of the terminal currents for a BJT in active mode are constant
- The current directions for npn and pnp transistors are opposite

<table>
<thead>
<tr>
<th>npn transistor</th>
<th>pnp transistor</th>
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<tbody>
<tr>
<td>$i_c = I_S e^{v_{BE}/V_T}$</td>
<td>$i_c = I_S e^{v_{BE}/V_T}$</td>
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<tr>
<td>$i_B = \frac{i_c}{\beta}$</td>
<td>$i_B = \frac{i_c}{\beta}$</td>
</tr>
<tr>
<td>$i_E = \frac{i_c}{\alpha}$</td>
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</table>

$\beta = \frac{\alpha}{1-\alpha}$

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Current-voltage characteristics of BJT

The $i_C-v_{CB}$ characteristics

As CBJ reverse bias increases, the effective base width $W_{eff}$ reduces due to the increasing depletion.

For a constant junction voltage $v_{BE}$:
- The slope of $n_B(x)$ increases $\rightarrow i_C$ increases
- Charge storage $Q_n$ reduces $\rightarrow i_B$ decreases
- Current gain $\alpha$ and $\beta$ increases

Early voltage ($V_A$) is used for the linear approximation of Early Effect

Linear dependence of $i_C$ on $v_{CE}$: $i_C = I_s e^{v_{BE}/V_T} (1 + v_{CE}/V_A)$

Exhibit finite output resistance: $r_o \equiv \left[ \frac{\partial i_C}{\partial v_{CE}} \right]_{v_{BE} = \text{constant}}^{-1} \approx \frac{V_A}{I_c}$

The $i_C-v_{CE}$ characteristics

Saturation region

Active region

Early voltage ($V_A$)

Linear dependence of $i_C$ on $v_{CE}$

Exhibit finite output resistance
Common-base output characteristics

- $i_C$ versus $v_{CB}$ plot with various $i_E$ as parameter is known as common-base output characteristics.
- The slope indicates that $i_C$ depends to a small extent on $v_{CB}$ → Early effect.
- $i_C$ increases rapidly at high $v_{CB}$ → breakdown.
- BCJ is slightly forward-biased for $-0.4V < v_{CB} < 0$.
  - No significant change is observed in $i_C$.
  - The BJT still exhibits I-V characteristics as in the active mode.
- BCJ turns on strongly and the $i_C$ starts to decrease for $v_{BC} < -0.4V$.
  - I-V characteristics in the saturation mode and $v_{CESat}$ is considered a constant ($\approx 0.2$ V).
- Current gain ($\alpha$): large-signal $\alpha \equiv i_C/i_E$ and small-signal (incremental) $\alpha \equiv \Delta i_C/\Delta i_E$.
Common-emitter output characteristics (I)

- $i_C$ versus $v_{CE}$ plot with various $v_{BE}$ as parameter
- Common-emitter current gain is defined as $\beta = i_C / i_B$
- The BCJ turns on with a positive $v_{BC}$ at low $v_{CE}$
  - BJT operates in saturation mode
- The $i_C$ curve has a finite slope due to Early effect
- The characteristics lines meet at $v_{CE} = -V_A$
- $V_A$ is called the Early Voltage (~ 50 to 100 V)

Common-emitter output characteristics (II)

- Plot of $i_C$ versus $v_{CE}$ with various $i_B$ as parameter
- BJT in active region acts as a current source with high (but finite) output resistance
- The cutoff mode in common-emitter configuration is defined as $i_B = 0$
- Current gain: large-signal $b_{dc} = i_C / i_B$ and $b_{ac} = \Delta i_C / \Delta i_B$
Saturation of common-emitter configuration

- In saturation region, it behaves as a closed switch with a small resistance $R_{CEsat}$.
- The saturation $I-V$ curve can be approximated by a straight line intersecting the $v_{CE}$ axis at $V_{CEoff}$.
- The saturation voltage $V_{CEsat} \approx V_{CEoff} + I_{Sat}R_{CEsat}$.
- $V_{CEsat}$ is normally treated as a constant of 0.2 V for simplicity regardless the value of $i_C$.
- Incremental $\beta$ in saturation is lower than that in active region: $\beta_{forced} \equiv I_{Sat}/I_B < \beta$.
- Overdrive factor $\equiv \beta/\beta_{forced}$.

\[ V_{CEsat} \approx V_{CEoff} + I_{Sat}R_{CEsat} \]

\[ \beta_{forced} \equiv I_{Sat}/I_B < \beta \]

\[ \beta/\beta_{forced} \]

\[ i_C \]

Saturation $\rightarrow$ Active

Incremental $\beta$ is high

Incremental $\beta$ is low

$\beta I_B$

Slope $= \frac{1}{R_{CEsat}}$

$X$

$V_{CEsat}$ $\rightarrow$

$\beta I_B$

$I_{Sat}$

\[ I_B = I_B \]

\[ 0 \rightarrow 0.1 \rightarrow 0.2 \rightarrow 0.3 \rightarrow 0.4 \rightarrow 0.5 \rightarrow 0.6 \rightarrow 0.7 \rightarrow 0.8 \rightarrow v_{CE} (V) \]
Transistor breakdown

- Transistor breakdown mechanism:
  - **Avalanche breakdown**: avalanche multiplication mechanism takes place at CBJ or EBJ
  - **Base punch-through effect**: the base width reduces to zero at high CBJ reverse bias

- In CB configuration, $BV_{CBO}$ is defined at $i_E = 0$
- The breakdown voltage is smaller than $BV_{CBO}$ for $i_E > 0$
- In CE configuration, $BV_{CEO}$ is defined at $i_B = 0$
- The breakdown voltage is smaller than $BV_{CEO}$ for $i_B > 0$
- Typically, $BV_{CEO}$ is about half of $BV_{CBO}$
- Breakdown of the BCJ is not destructive as long as the power dissipation is kept within safe limits
- Breakdown of the EBJ is destructive because it will cause permanent degradation of $\beta$

**Graphs:**

- I-V characteristics for CB and CE configurations.
- Expanded scale for $v_{CB}$ and $v_{CE}$.
- Saturation and active regions.

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Exercise 4.13 (Textbook)
Exercise 4.14 (Textbook)
Example 4.3 (Textbook)
Exercise 4.21 (Textbook)
4.3 BJT Circuits at DC

**BJT operation modes**
- The BJT operation mode depends on the voltages at EBJ and BCJ
- The I-V characteristics are strongly nonlinear
- Simplified models and classifications are needed to speed up the hand-calculation analysis

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<thead>
<tr>
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<tr>
<td>Active</td>
<td>Forward</td>
<td>Reverse</td>
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<tr>
<td>Cutoff</td>
<td>Reverse</td>
<td>Reverse</td>
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<tr>
<td>Saturation</td>
<td>Forward</td>
<td>Forward</td>
</tr>
<tr>
<td>Inverse</td>
<td>Reverse</td>
<td>Forward</td>
</tr>
</tbody>
</table>

**npn transistor**
- Inverse Mode: \( v_{BE} < 0, v_{BC} > 0 \)
- Saturation Mode: \( v_{BE} > 0, v_{BC} > 0 \)

**pnp transistor**
- Inverse Mode: \( v_{EB} < 0, v_{CB} > 0 \)
- Saturation Mode: \( v_{EB} > 0, v_{CB} > 0 \)

**Simplified models and classifications for the operation of the npn BJT**
- Cut-off mode:
  - \( i_E = i_C = i_B = 0 \)
  - \( v_{BE} < 0.5 \text{ V} \) and \( v_{BC} < 0.4 \text{ V} \)
- Active mode:
  - \( v_{BE} = 0.7 \text{ V} \) and \( i_B : i_C : i_E = 1: \beta : (1+\beta) \)
  - \( v_{CE} > 0.3 \text{ V} \)
- Saturation mode:
  - \( v_{BE} = 0.7 \text{ V} \) and \( v_{CE} = 0.2 \text{ V} \)
  - \( i_C/i_B = \beta_{\text{forced}} < \beta \)
Equivalent circuit models

**nnp**

- $I_B = 0$ + $V_{BE} < 0.4\, \text{V}$ - $I_C = 0$
- $V_{BE} < 0.5 \, \text{V}$
- $I_B > 0$ + $V_{BE} < 0.4 \, \text{V}$ - $I_C = \beta I_B$
- $V_{BE} = 0.7 \, \text{V}$
- $I_B > 0$ + $V_{BE} = 0.5 \, \text{V}$ - $I_C = \beta I_B$
- $V_{BE} = 0.7 \, \text{V}$

**pnp**

- $I_B = 0$ - $V_{EB} < 0.4 \, \text{V}$ + $I_C = 0$
- $V_{EB} < 0.5 \, \text{V}$
- $I_B > 0$ - $V_{EB} < 0.4 \, \text{V}$ + $I_C = \beta I_B$
- $V_{EB} = 0.7 \, \text{V}$
- $I_B > 0$ - $V_{EB} = 0.5 \, \text{V}$ + $I_C = \beta I_B$
- $V_{EB} = 0.7 \, \text{V}$
DC analysis of BJT circuits

- Step 1: assume the operation mode
- Step 2: use the conditions or model for circuit analysis
- Step 3: verify the solution
- Step 4: repeat the above steps with another assumption if necessary

Example 4.4

\[ \beta = 100 \]

Example 4.5
Example 4.9 (Textbook)

\[ I_e = \frac{5 - (V_B + 0.7)}{1} \]

1. \( V_E = V_B + 0.7 \)
2. \( I_B = \frac{V_B}{10} \)
3. \( V_C = V_E + 0.2 \)
4. \( V_C = V_B + 0.5 \)
5. \( I_C = \frac{V_B + 0.5 - (-5)}{10} \)

Example 4.11 (Textbook)
Exercise 4.22 (Textbook)
Exercise 4.23 (Textbook)
Exercise 4.24 (Textbook)
Exercise 4.25 (Textbook)
Exercise 4.28 (Textbook)
Example 4.12 (Textbook)
4.4 Applying the BJT in Amplifier Design

BJT voltage amplifier

- A BJT circuit with a collector resistor $R_C$ can be used as a simple voltage amplifier.
- Base terminal is used as the amplifier input and the collector is considered as the amplifier output.
- The voltage transfer characteristic (VTC) is obtained by solving the circuit from low to high $v_{BE}$.

- **Cutoff mode:**
  - $0 \text{ V} \leq v_{BE} < 0.5 \text{ V}$ and $i_C = 0$
  - $v_O = v_{CE} = V_{CC}$

- **Active mode:**
  - $v_{BE} > 0.5 \text{ V}$ and $i_C = I_S \exp(v_{BE}/V_T)$
  - $v_O = V_{CC} - i_C R_C = V_{CC} - R_C I_S \exp(v_{BE}/V_T)$

- **Saturation:**
  - $v_{BE}$ further increases
  - $v_{CE} = v_{CESat} = 0.2 \text{ V}$
  - $v_O = 0.2 \text{ V}$
**Biasing the circuit to obtain linear amplification**

- The slope in the VTC indicates voltage gain.
- BJT in active mode can be used as voltage amplification.
- Point Q is known as bias point or dc operating point.
  
  \[ I_C = I_S \exp \left( \frac{V_{BE}}{V_T} \right) \]

- The signal to be amplified is superimposed on \( V_{BE} \)
  
  \[ v_{BE}(t) = V_{BE} + v_{be}(t) \]

- The time-varying part in \( v_{CE}(t) \) is the amplified signal.
- The circuit can be used as a linear amplifier if:
  - A proper bias point is chosen for gain
  - The input signal is small in amplitude

**The small-signal voltage gain**

- The amplifier gain is the slope at Q:
  
  \[ A_v \equiv \frac{dV_{CE}}{dV_{BE}} \bigg|_{v_{BE}} = -\frac{I_C}{V_T} R_C \]

  \[ 
  V_{CE} \\
  \hline 
  I_C \\
  R_C \\
  v_{BE} \\
  v_{CE} \\
  \hline 
  V_{CC} \\
  + \\
  - \\
  + \\
  - \\
  \]

  - Voltage gain depends on \( I_C \) and \( R_C \)
  - Maximum voltage gain of the amplifier

\[ A_v = -\frac{I_C}{V_T} R_C = \frac{V_{CC} - V_{CE}}{V_T} \leq \frac{V_{CC}}{V_T} = |A_{v_{max}}| \]
Determining the VTC by graphical analysis

- Provides more insight into the circuit operation
- **Load line**: the straight line represents in effect the load
  \[ i_C = \frac{(V_{CC} - V_{CE})}{R_C} \]
- The operating point is the intersection point

**Locating the bias point Q**

- The bias point (intersection) is determined by properly choosing the load line
- The output voltage is bounded by \( V_{CC} \) (upper bound) and \( V_{CE_{sat}} \) (lower bound)
- The load line determines the voltage gain
- The bias point determines the headroom or maximum upper/lower voltage swing of the amplifier
4.5 Small-Signal Operation and Models

The collector current and the transconductance

- The total quantities (ac + dc) of the collector current:
  \[ v_{BE} = V_{BE} + v_{bc} \]
  \[ i_c = I_s e^{v_{be}/V_T} = (I_s e^{v_{be}/V_T}) e^{v_{bc}/V_T} = I_c e^{v_{bc}/V_T} \]

- Small-signal approximation: \( v_{be} \ll V_T \)
  \[ i_c = I_c + i_c \approx I_c \left( 1 + \frac{v_{be}}{V_T} \right) = I_c + \frac{I_c}{V_T} v_{be} \]
  \[ g_m = \left. \frac{\partial i_c}{\partial v_{BE}} \right|_{i_c=I_c} = \frac{I_c}{V_T} \]

- The transconductance indicates the incremental change of \( i_c \) versus change of \( v_{BE} \)
- The transconductance \( g_m \) is determined by its dc collector current \( I_c \)
- General, BJTs have relatively high transconductance compared with FETs at the same current level

The base current and the input resistance at the base

- The total quantities (ac + dc) of the base current:
  \[ i_B = \frac{I_c}{\beta} = \frac{I_s}{\beta} e^{v_{be}/V_T} = \frac{I_s}{\beta} e^{v_{be}/V_T} e^{v_{bc}/V_T} = I_b e^{v_{bc}/V_T} \]

- Small-signal approximation:
  \[ i_B = I_B + i_b \approx I_B \left( 1 + \frac{v_{be}}{V_T} \right) = I_B + \frac{I_B}{V_T} v_{be} \]
  \[ r_\pi = \frac{v_{be}}{i_b} = \frac{\beta}{g_m} = \frac{V_T}{I_B} \]

- Resistance \( r_\pi \) is the small-signal input resistance between base and emitter (looking into the base)
The emitter current and the input resistance at the emitter

- The total quantities (ac + dc) of the emitter current:
  \[ i_E = I_E + i_e = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{i}{\alpha} \]

- Small-signal approximation:
  \[ i_e = \frac{i_e}{\alpha} = \frac{g_m}{\alpha} v_{be} = \frac{I_C}{\alpha V_T} v_{be} = \frac{I_E}{V_T} v_{be} \]
  \[ r_e \approx \frac{v_{be}}{i_e} = \frac{V_T}{I_E} \approx \frac{\alpha}{g_m} \approx \frac{1}{g_m} \]

- Relation between \( r_\pi \) and \( r_e \):
  \[ r_e = \frac{\alpha}{g_m} \]
  \[ r_\pi = \frac{\beta}{g_m} \]
  \[ \rightarrow r_\pi = (1 + \beta) r_e \]

Output resistance accounting for Early effect

- Use the collector current equation with linear \( v_{CE} \) dependence:
  \[ i_C = I_S e^{v_{BE}/V_T} \left( 1 + \frac{v_{CE}}{V_A} \right) \]
  \[ r_o \equiv \left[ \frac{\partial i_C}{\partial v_{CE}} \right]_{v_{BE}=\text{constant}}^{-1} \approx \frac{V_A}{I_C} \]

- The output resistance \( r_o \) is included to represent Early Effect of the BJT
- The resulting \( r_o \) is typically a large resistance and can be neglected to simplify the analysis
**BJT small-signal models**

- Two models are exchangeable and does not affect the analysis result
- The hybrid-π model
  - Typically used as the emitter is grounded
    - Neglect $r_o$

- The T model
  - Typically used as the emitter is not grounded
    - Neglect $r_o$

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<table>
<thead>
<tr>
<th>Model Parameters in Terms of DC Bias Currents</th>
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<tbody>
<tr>
<td>$g_m = \frac{I_C}{V_T}$</td>
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<tr>
<td>$r_x = \frac{V_I}{I_B} = \beta \frac{V_I}{I_C}$</td>
</tr>
</tbody>
</table>

- In Terms of $g_m$
  - $r_e = \frac{\alpha}{g_m}$
  - $r_x = \frac{\beta}{g_m}$

- In Terms of $r_e$
  - $g_m = \frac{\alpha}{r_e}$
  - $r_x = (\beta + 1)r_e$

- Relationships between $\alpha$ and $\beta$
  - $\beta = \frac{\alpha}{1 - \alpha}$
  - $\alpha = \frac{\beta}{\beta + 1}$
4.6 Basic BJT Amplifier Configuration

Three basic configurations

- Common-Emitter (CE)
- Common-Base (CB)
- Common-Collector (CC)

Characterizing amplifiers

- The BJT circuits can be characterized by a voltage amplifier model (unilateral model)
- The electrical properties of the amplifier is represented by $R_{in}$, $R_o$ and $A_{vo}$
- The analysis is based on the small-signal or linear equivalent circuit (dc components not included)

- Voltage gain: $A_v \equiv \frac{v_o}{v_i} = \frac{R_L}{R_L + R_o} A_{vo}$
- Overall voltage gain: $G_v \equiv \frac{v_o}{v_{\text{sig}}} = \frac{R_{in}}{R_{in} + R_{\text{sig}}} A_v = \frac{R_{in}}{R_{in} + R_{\text{sig}}} \frac{R_L}{R_L + R_{so}} A_{vo}$
The common-emitter (CE) amplifier

- Characteristic parameters of the CE amplifier
  - Input resistance: \( R_{in} = r_{\pi} \)
  - Output resistance: \( R_o = R_C \parallel r_o \approx R_C \)
  - Open-circuit voltage gain: \( A_{vo} = -g_m (R_C \parallel r_o) \approx -g_m R_C \)
  - Voltage gain: \( A_v = -g_m (R_C \parallel R_L \parallel r_o) \approx -g_m (R_C \parallel R_L) \)
  - Overall voltage gain: \( G_v = -\frac{r_\pi}{r_\pi + R_{sig}} g_m (R_C \parallel R_L \parallel r_o) \approx -g_m \frac{r_\pi}{r_\pi + R_{sig}} (R_C \parallel R_L) \)

- CE amplifier can provide high voltage gain
- Input and output are out of phase due to negative gain
- Lower \( I_C \) increases \( R_{in} \) at the cost of voltage gain
- Output resistance is moderate to high
- Small \( R_C \) reduces \( R_o \) at the cost of voltage gain
The common-emitter (CE) with an emitter resistance

- Characteristic parameters (by neglecting $r_o$)
  - Input resistance:
    $$R_{in} = (1 + \beta)(r_e + R_e) = r_\pi + (1 + \beta)R_e$$
  - Output resistance:
    $$R_o = R_C$$
  - Open-circuit voltage gain:
    $$A_{vo} = -\frac{g_mR_C}{1 + R_e/r_e} \approx -\frac{g_mR_C}{1 + g_mR_e}$$
  - Voltage gain:
    $$A_v \approx -\frac{g_mR_o R_L}{1 + g_mR_e R_L + R_C} = -\frac{g_m(R_C \parallel R_L)}{1 + g_mR_e}$$
  - Overall voltage gain:
    $$G_v \approx -\frac{r_\pi}{r_\pi + R_{sig}} \frac{g_mR_C}{1 + g_mR_e R_L + R_C} = -\frac{r_\pi}{r_\pi + R_{sig}} \frac{g_m(R_C \parallel R_L)}{1 + g_mR_e}$$

- Emitter degeneration resistance $R_e$ is adopted
- Input resistance is increased by adding $(1+\beta)R_e$
- Gain is reduced by the factor $(1+g_mR_e)$
- The overall gain is less dependent on $\beta$
- It is considered a negative feedback of the amplifier
The common-base (CB) amplifier

- Characteristic parameters of the CE amplifier (by neglecting $r_o$)
  - Input resistance: $R_{in} = r_e$
  - Output resistance: $R_o = R_C$
  - Open-circuit voltage gain: $A_{oc} = g_m R_C$
  - Voltage gain: $A_v = g_m (R_C \parallel R_L)$
  - Overall voltage gain: $G_v = \frac{r_e}{r_e + R_{sig}} g_m (R_C \parallel R_L)$

- CE amplifier can provide high voltage gain
- Input and output are in-phase due to positive gain
- Input resistance is very low
- A single CB stage is not suitable for voltage amplification
- Output resistance is moderate to high
- Small $R_C$ reduces $R_o$ at the cost of voltage gain
- The amplifier is no longer unilateral if $r_o$ is included

$\begin{align*}
\text{Input resistance: } R_{in} &= r_e \\
\text{Output resistance: } R_o &= R_C \\
\text{Open-circuit voltage gain: } A_{oc} &= g_m R_C \\
\text{Voltage gain: } A_v &= g_m (R_C \parallel R_L) \\
\text{Overall voltage gain: } G_v &= \frac{r_e}{r_e + R_{sig}} g_m (R_C \parallel R_L)
\end{align*}$
The common-collector (CC) amplifier

- Characteristic parameters of the CC amplifier (by neglecting $r_o$)
  - Input resistance: $R_{in} = (1 + \beta)(r_e + R_L)$
  - Output resistance: $R_o = r_e + R_{sig} / \beta$
  - Open-circuit voltage gain: $A_{vo} = R_L / (R_L + r_e) \approx 1$
  - Overall voltage gain: $G_v = \frac{R_m}{R_m + R_{sig}} \frac{R_L}{R_L + r_e} \approx \frac{(\beta + 1)R_L}{(\beta + 1)(R_L + r_e) + R_{sig}} \approx 1$

- CC amplifier is also called emitter follower.
- Input resistance is very high
- Output resistance is very low
- The voltage gain is less than but can be close to 1
- CC amplifier can be used as voltage buffer
- It is noted that, in the analysis, the amplifier is not unilateral
4.7 Biasing in BJT Amplifier Circuits

**DC bias for BJT amplifier**
- The amplifiers are operating at a proper dc bias point
- Linear signal amplification is provided based on small-signal circuit operation
- The DC bias circuit is to ensure the BJT in **active mode** with a proper collector current $I_C$

**The classical discrete-circuit bias arrangement**
- A single power supply and resistors are needed
- Thevenin equivalent circuit:
  - $V_{BB} = V_{CC}R_2/(R_1+R_2)$
  - $R_B = R_1||R_2$
- BJT operating point: $I_C = \frac{V_{BB} - V_{BE}}{R_B / \beta + R_E(1+1/\beta)}$

- $R_C$ is chosen to ensure the BJT in active ($V_{CE} > V_{CESat}$)

**A two-power-supply version of the classical bias arrangement**
- Two power supplies are needed
- Similar dc analysis
- BJT operating point: $I_C = \frac{V_{EE} - V_{BE}}{R_B / \beta + R_E(1+1/\beta)}$
Biasing using a collector-to-base feedback resistor

- A single power supply is needed
- $R_B$ ensures the BJT in active ($V_{CE} > V_{BE} \approx 0.7 \text{V}$)
- BJT operating point: $I_C = \frac{V_{CC} - V_{BE}}{R_B / \beta + R_C (1 + 1/\beta)}$
- The value of the feedback resistor $R_B$ affects the small-signal gain

Biasing using a constant-current source

- The BJT can be biased with a constant current source $I$
- The resistor $R_C$ is chosen to operate the BJT in active mode
- The current source is typically implemented by a BJT current mirror
- Current mirror circuit:
  - Both BJT transistors $Q_1$ and $Q_2$ are in active mode
  - Assume current gain $\beta$ is very high:
    $$I = I_{REF} = \frac{V_{CC} + V_{EE} - V_{BE}}{R}$$
  - When applying to the amplifier circuit, the voltage $V$ has to be high enough to ensure $Q_2$ in active

Exercise 6.35 (Textbook)
Exercise 6.36 (Textbook)
4.8 Discrete-Circuit BJT Amplifiers

**Circuit analysis:**
- **DC analysis:**
  - Remove all ac sources (short for voltage source and open for current source)
  - All capacitors are considered open-circuit
  - DC analysis of BJT circuits for all nodal voltages and branch currents
  - Find the dc current $I_C$ and make sure the BJT is in active mode
- **AC analysis:**
  - Remove all dc sources (short for voltage source and open for current source)
  - All **large capacitors** are considered short-circuit
  - Replace the BJT with its small-signal model for ac analysis
  - The circuit parameters in the small-signal model are obtained based on the value of $I_C$
The common-emitter (CE) amplifier

The common-emitter amplifier with an emitter resistance

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The common-base (CB) amplifier

The common-collector (CC) amplifier

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The amplifier frequency response

- The gain falls off at low frequency band due to the effects of the coupling and by-pass capacitors.
- The gain falls off at high frequency band due to the internal capacitive effects in the BJTs.
- Midband:
  - All coupling and by-pass capacitors (large capacitance) are considered short-circuit.
  - All internal capacitive effects (small capacitance) are considered open-circuit.
  - Midband gain is nearly constant and is evaluated by small-signal analysis.
  - The bandwidth is defined as \( BW = f_H - f_L \).
  - A figure-of-merit for the amplifier is its gain-bandwidth product defined as \( GB = |A_M|BW \).
Exercise 6.40 (Textbook)
Exercise 6.41 (Textbook)
Exercise 6.43 (Textbook)
Exercise 6.44 (Textbook)