CHAPTER 10 FEEDBACK

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10.1 The General Feedback Structure

Feedback amplifier
- Signal-flow diagram of a feedback amplifier

- Open-loop gain: $A$
- Feedback factor: $\beta$
- Loop gain: $A\beta$
- Amount of feedback: $1 + A\beta$
- Gain of the feedback amplifier (closed-loop gain): $A_f \equiv \frac{x_o}{x_s} = \frac{A}{1 + A\beta}$

- Negative feedback:
  - The feedback signal $x_f$ is subtracted from the source signal $x_s$
  - Negative feedback reduces the signal that appears at the input of the basic amplifier
  - The gain of the feedback amplifier $A_f$ is smaller than open-loop gain $A$ by a factor of $(1 + A\beta)$

- The loop gain $A\beta$ is typically large ($A\beta \gg 1$):  
  - The gain of the feedback amplifier (closed-loop gain) $A_f \approx 1 / \beta$
  - Closed-loop gain is almost entirely determined by the feedback network \(\rightarrow\) better accuracy of $A_f$
  - $x_f = x_s(A\beta)/(1 + A\beta) \approx x_s \rightarrow$ error signal $x_i = x_s - x_f$
Example

The feedback amplifier is based on an opamp with infinite input resistance and zero output resistance
(a) Find an expression for the feedback factor
(b) Find the condition under which $A_f$ is almost entirely determined by the feedback network
(c) If the open-loop gain $A = 10000 \, \text{V/V}$, find $R_2/R_1$ to obtain a closed-loop gain $A_f$ of $10 \, \text{V/V}$
(d) What is the amount of feedback in decibel?
(e) If $V_s = 1 \, \text{V}$, find $V_o$, $V_f$, and $V_i$
(f) If $A$ decreases by 20%, what is the corresponding decrease in $A_f$?

\[ \beta = \frac{R_1}{R_1 + R_2} \]
\[ A_f = \frac{A}{1 + AR_1/(R_1 + R_2)} \]
\[ A\beta \gg 1 \]
\[ R_1 / R_2 = 9 \]
\[ 1 + A\beta = 1001 (60 \, \text{dB}) \]
\[ V_o = 100V; V_f = 0.999V; V_i = 0.001V \]
\[ A_f = 9.9975 \]
10.2 Some Properties of Negative Feedback

Gain desensitivity

- The negative feedback reduces the change in the closed-loop gain due to open-loop gain variation

\[ \frac{dA_f}{A_f} = \frac{1}{A} \frac{dA}{(1 + A\beta)^2} \]

- Desensitivity factor: \(1 + A\beta\)

Bandwidth extension

- High-frequency response of a single-pole amplifier:

\[ A(s) = \frac{A_M}{1 + s/\omega_H} \rightarrow A_f(s) = \frac{A_M}{1 + s/(\omega_H(1 + A_M\beta))} \]

- Low-frequency response of an amplifier with a dominant low-frequency pole:

\[ A(s) = \frac{sA_M}{s + \omega_L} \rightarrow A_f(s) = \frac{sA_M}{s + \omega_L/(1 + A_M\beta)} \]

- Negative feedback:
  - Reduces the gain by a factor of \((1 + A_M\beta)\)
  - Extends the bandwidth by a factor of \((1 + A_M\beta)\)
Interference reduction

- The signal-to-noise ratio:
  - The amplifier suffers from interference introduced at the input of the amplifier
  - Signal-to-noise ratio: \( S/I = V_s/V_n \)

- Enhancement of the signal-to-noise ratio:
  - Precede the original amplifier \( A_1 \) by a clean amplifier \( A_2 \)
  - Use negative feedback to keep the overall gain constant

\[
V_0 = V_s \frac{A_1 A_2}{1 + A_1 A_2 \beta} + V_n \frac{A_1}{1 + A_1 A_2 \beta} \rightarrow S = \frac{V_s}{V_n} A_2
\]
Reduction in nonlinear distortion

- The amplifier transfer characteristic is linearized through the application of negative feedback
  - \( \beta = 0.01 \)
  - \( A \) changes from 1000 to 100

\[
A_{f1} = \frac{1000}{1+1000 \times 0.01} = 90.9
\]
\[
A_{f2} = \frac{100}{1+100 \times 0.01} = 50
\]
10.3 The Four Basic Feedback Topologies

**Voltage amplifiers**

- The most suitable feedback topologies is voltage-mixing and voltage-sampling one
- Known as series-shunt feedback

![Diagram of voltage amplifiers]

- Example:

![Example diagrams of voltage amplifiers]

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Current amplifiers

- The most suitable feedback topologies is current-mixing and current-sampling one
- Known as shunt-series feedback

Example:
Transconductance amplifiers

- The most suitable feedback topologies is voltage-mixing and current-sampling one
- Known as series-series feedback

Example:
Transresistance amplifiers

- The most suitable feedback topologies is current-mixing and voltage-sampling one
- Known as shunt-shunt feedback

Example:

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10.4 The Feedback Voltage Amplifier (Series-Shunt)

Ideal case

- Input resistance of the feedback amplifier \( R_g = (1 + A\beta) R_i \)
- Output resistance of the feedback amplifier \( R_{of} = \frac{R_o}{1 + A\beta} \)
- Voltage gain of the feedback amplifier \( A_f = \frac{A}{1 + A\beta} \) V/V

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The practical case
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Analysis techniques

(a) The $A$ circuit is

where $R_{11}$ is obtained from

and $R_{22}$ is obtained from

and the gain $A$ is defined $A \equiv \frac{V_o}{V_i}$

(b) $\beta$ is obtained from

$\beta \equiv \frac{V_f}{V_o} \bigg|_{I_1 = 0}$
Example

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Example
10.5 The Feedback Transconductance Amplifier (Series-Series)

**Ideal case**

- **Input resistance of the feedback amplifier** \( R_{\text{if}} = (1 + A\beta)R_i \)

- **Output resistance of the feedback amplifier** \( R_{\text{of}} = (1 + A\beta)R_o \)

- **Transconductance gain of the feedback amplifier** \( A_f = \frac{A}{1 + A\beta} \ \Omega^{-1} \)
The practical case
Analysis techniques

(a) The $A$ circuit is

\[ V_i \quad \text{Basic amplifier} \quad I_o \]

where $R_{11}$ is obtained from

\[ R_{11} \quad \text{Feedback network} \quad 2 \]

and $R_{22}$ is obtained from

\[ R_{22} \quad \text{Feedback network} \quad 2 \]

and the gain $A$ is defined $A \equiv \frac{I_o}{V_i}$

(b) $\beta$ is obtained from

\[ I_i = 0 \quad V_f \quad \text{Feedback network} \quad I_o \]

\[ \beta \equiv \frac{V_f}{I_o} \bigg|_{I_i = 0} \]
10.6 The Feedback Transresistance Amplifier (Shunt-Shunt)

Ideal case

- Input resistance of the feedback amplifier $R_{if} = \frac{R_i}{(1 + A\beta)}$
- Output resistance of the feedback amplifier $R_{of} = \frac{R_o}{(1 + A\beta)}$
- Transresistance gain of the feedback amplifier $A_f = \frac{A}{1 + A\beta}$
10.7 The Feedback Current Amplifier (Shunt-Series)

Ideal case

- Input resistance of the feedback amplifier: \( R_f = \frac{R_i}{1 + A\beta} \)
- Output resistance of the feedback amplifier: \( R_{of} = (1 + A\beta)R_o \)
- Current gain of the feedback amplifier: \( A_f = \frac{A}{1 + A\beta} \ A/A \)
10.9 Determining the Loop Gain

An Alternative Approach for Finding Loop Gain

- Open-loop analysis with equivalent loading:
  - Remove the external source
  - Break the loop with equivalent loading
  - Provide test signal $V_t$
  - Loop gain: $A\beta = \frac{V_r}{V_i}$

- Equivalent method for determining loop gain:
  - Usually convenient to employ in simulation
  - Remove the external source
  - Break the loop
  - Provide test signal $V_t$
  - Find the open-circuit voltage transfer function $T_{oc}$
  - Find the short-circuit current transfer function $T_{sc}$
  - Loop gain: $A\beta = -\left(\frac{1}{T_{oc}} + \frac{1}{T_{sc}}\right)^{-1}$

- The value of loop gain determined using the method discussed here may differ somewhat from the value determined by the approach studied in the previous session, but the difference is usually limited to a few percent.
Example

(1) Breaking point at differential node

(2) Breaking point at single-ended node

\[ A\beta = \mu \frac{\{R_L \parallel [R_2 + R_1 \parallel (R_{id} + R)]\}}{\{R_L \parallel [R_2 + R_1 \parallel (R_{id} + R)]\} + r_o \left[ R_1 \parallel (R_{id} + R) \right] + R_{id} \frac{R_{id}}{R_2 + R_{id} + R} + R} \rightarrow \mu \frac{R_1}{R_1 + R_2} \text{(ideal op amp)} \]
**Characteristic Equation**

- The gain of a feedback amplifier can be expressed as a transfer function (function of s) by taking the frequency-dependent properties into consideration.
- The denominator determines the poles of the system and the numerator defines the zeros.
- From the study of circuit theory, the poles of a circuit are independent of the external excitation, and the poles or the natural modes can be determined by setting the external excitation to zero.
- The characteristic equation and the poles are completely determined by the loop gain.
- A given feedback loop may be used to generate a number of circuits having the same poles but different transmission zeros.
- The closed-loop gain and the transmission zeros depend on how and where the input signal is injected into the loop.

![Feedback Loop Diagram](image)

**Transfer function:**

\[
A_f(s) = \frac{x_o}{x_s} = \frac{A(s)}{1 + A(s)\beta(s)} = \frac{1 + a_1s + \ldots + a_ms^m}{1 + b_1s + \ldots + b_ns^n}
\]

**Characteristics equation:**

\[1 + b_1s + \ldots + b_ns^n = 0\]
10.10 The Stability Problem

Transfer function of the feedback amplifier

- Transfer functions:
  - Open-loop transfer function: \( A(s) \)
  - Feedback transfer function: \( \beta(s) \)
  - Closed-loop transfer function: \( A_f(s) \)
  
  \[ A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)} \]

  - For physical frequencies \( s = j\omega \)
  
  \[ A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)} \]

- Loop gain: \( L(j\omega) = A(j\omega)\beta(j\omega) = |A(\omega)\beta(\omega)| e^{j\phi(\omega)} \)

- Evaluating the close-loop stability by the frequency response of the loop gain \( L(j\omega) \):
  
  - For loop gain smaller than unity at \( \omega_{180^\circ} \):
    - Becomes positive feedback
    - Closed-loop gain becomes larger than open-loop gain
    - The feedback amplifier is still stable
  
  - For loop gain equal to unity at \( \omega_{180^\circ} \):
    - The amplifier will have an output for zero input (oscillation)
  
  - For loop gain larger than unity at \( \omega_{180^\circ} \):
    - Oscillation with a growing amplitude at the output
The Nyquist plot

- A plot used to evaluate the stability of a feedback amplifier
- Plot the loop gain $L(j\omega)$ versus frequency on the complex plane
- Magnitude decreases as frequency increases
- Phase decreases as frequency increases due to the poles (final phase depends on number of poles)
- Stability:
  - The plot does not encircle the point (-1, 0)
  - The magnitude of loop gain has to be less than unity when phase reaches $-180^\circ$
  - The system is more likely to become unstable as $\beta$ increases

Magnitude=$A_0\beta$ (increases with $\beta$)
10.11 Effect of Feedback on the Amplifier Poles

Stability and pole location

- The stability can be evaluated by the poles of the closed-loop transfer function.
- The poles have to be in the left half of the s-plane to ensure stability.
- Consider an amplifier with a pole pair at $s = \sigma_0 \pm j \omega_n$.
- The transient response contains the terms of the form $v(t) = e^{\sigma_0 t} [e^{j\omega_n t} + e^{-j\omega_n t}] = 2e^{\sigma_0 t} \cos(\omega_n t)$.
Poles of the feedback amplifier

- Characteristic equation: \(1 + A(s) \beta(s) = 0\)
- The feedback amplifier poles are obtained by solving the characteristic equation

Amplifier with single-pole response

\[
A(s) = \frac{A_0}{1 + s / \omega_p} \rightarrow A_f(s) = \frac{A_0 / (1 + A_0 \beta)}{1 + s / \omega_p (1 + A_0 \beta)}
\]

\[
\omega_{pf} = \omega_p (1 + A_0 \beta)
\]

- The feedback amplifier is still a single-pole system
- The pole moves away from origin in the s-plane as feedback \((\beta)\) increases
- The bandwidth is extended by feedback at the cost of a reduction in gain
- **Unconditionally stable** system (the pole never enters the right-half plane)
Amplifier with two-pole response

- Feedback amplifier
  
  \[ A(s) = \frac{A_0}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})} \rightarrow A_f(s) = \frac{A(s)}{1 + A(s)\beta} = \frac{A_0}{(1 + s/\omega_{p1})(1 + s/\omega_{p2}) + A_0\beta} \]

- Still a two-pole system

- Characteristic equation
  
  \[ s^2 + s(\omega_{p1} + \omega_{p2}) + (1 + A_0\beta)\omega_{p1}\omega_{p2} = 0 \]

- The closed-loop poles are given by
  
  \[ s = -\frac{1}{2}(\omega_{p1} + \omega_{p2}) \pm \frac{1}{2}\sqrt{(\omega_{p1} + \omega_{p2})^2 - 4(1 + A_0\beta)\omega_{p1}\omega_{p2}} \]

- The plot of poles versus \( \beta \) is called a root-locus diagram

- Unconditionally stable system (the pole never enters the right-half plane)
Amplifier with three or more poles

Root-locus diagram:

- As $\beta$ increases, the two poles become coincident and then become complex and conjugate
- A value of $\beta$ exists at which this pair of complex-conjugate poles enters the right half of the $s$ plane
- The feedback amplifier is stable only if $\beta$ does not exceed a maximum value
- Frequency compensation is adopted to ensure the stability
10.12 Stability Study Using Bode Plots

Gain and phase margin

- The stability of a feedback amplifier is determined by examining its loop gain as a function of frequency \( L(j\omega) = L(j\omega)\beta(j\omega) \)
- One of the simplest means is through the use of Bode plot for \( A\beta \)
- Stability is ensured if the magnitude of the loop gain is less than unity at a frequency shift of \( 180^\circ \)
- Gain margin:
  - The difference between the value \( |A\beta| \) of at \( \omega_{180^\circ} \) and unity
  - Gain margin represents the amount by which the loop gain can be increased while maintaining stability
- Phase margin:
  - A feedback amplifier is stable if the phase is less than \( 180^\circ \) at a frequency for which \( |A\beta| = 1 \)
  - A feedback amplifier is unstable if the phase is in excess of \( 180^\circ \) at a frequency for which \( |A\beta| = 1 \)
  - The difference between the phase at a frequency \( (\omega_1) \) for which \( |A\beta| = 1 \) and \(-180^\circ \)
Effect of phase margin on closed-loop response

- Consider a feedback amplifier with a large low-frequency loop gain \( A_0 \beta >> 1 \)
- The closed-loop gain at low frequencies is approximately \( 1/\beta \)
- Denoting the frequency at which \( |A(\omega)| = 1 \) by \( \omega_1 \); \( A(j\omega_1)\beta = 1 \times e^{-j\theta} \) and \(-\theta = 180^\circ – \) phase margin

\[
A_f(j0) = \frac{A_0}{1 + A_0\beta} \approx \frac{1}{\beta}
\]
\[
A_f(j\omega_1) = \frac{A(j\omega_1)}{1 + A(j\omega_1)\beta} = \frac{1}{\beta} \frac{e^{-j\theta}}{1 + e^{-j\theta}}
\]
\[
|A_f(j\omega_1)| = \frac{1/\beta}{|1 + e^{-j\theta}|}
\]

- Closed-loop gain at \( \omega_1 \) peaks by a factor of 1.3 above the low-frequency gain for phase margin of 45°
- This peaking increases as the phase margin is reduced, eventually reaching infinite when the phase margin is zero (sustained oscillations)

(1) PM = 90° \(-\theta = -90^\circ\) \(\rightarrow |A_f(\omega_1)| = 0.707 \times (1/\beta)\)
(2) PM = 60° \(-\theta = -120^\circ\) \(\rightarrow |A_f(\omega_1)| = 1 \times (1/\beta)\)
(3) PM = 45° \(-\theta = -135^\circ\) \(\rightarrow |A_f(\omega_1)| = 1.3 \times (1/\beta)\)
An alternative approach for investigating stability

- In a Bode plot, the difference between $20 \log |A(j\omega)|$ and $20 \log (1/\beta)$ is $20 \log |A\beta|$.

Example:

\[
A = \frac{10^5}{(1+jf/10^5)(1+jf/10^6)(1+jf/10^7)}
\]

\[
\phi = -\left[\tan^{-1}(f/10^5) + \tan^{-1}(f/10^6) + \tan^{-1}(f/10^7)\right]
\]

(a)

$\beta = 0.000056$

$1/\beta = 17782$ (85 dB)

$f_{o\text{-db}} = 5.6 \times 10^5$ Hz

$\phi(f_{o\text{-db}}) = -108^\circ$

PM = 72°

$-180^\circ = -90^\circ - \tan^{-1}(f_{180^\circ}/10^6) - \tan^{-1}(f_{180^\circ}/10^7)$

$f_{180^\circ} = 3.17 \times 10^6$ Hz

$|A(f_{180^\circ})| = 60$ dB

GM = 25 dB

(b)

$\beta = 0.00316$

$1/\beta = 316^\circ$ (50 dB)

$f_{o\text{-db}} > f_{180^\circ}$ (Unstable)
10.13 Frequency Compensation

Theory

- Modify the open-loop transfer function $A(s)$ so that the closed-loop amplifier is stable for a given closed-loop gain.
- The simplest method for frequency compensation is to introduce a new pole at sufficiently low frequency $f_D$.
- The disadvantage of introducing a new pole at lower frequency is the significant bandwidth reduction.
- Alternatively, the dominant pole can be shifted to a lower frequency $f_D'$ such that the amplifier is compensated without introducing a new pole.
Increase the time-constant of the dominant pole by adding additional capacitance

- Add external capacitance $C_C$ at the node which contributes to a dominant pole
- The required value of $C_C$ is usually quite large, making it unsuitable for IC implementation

Assume $R_x = 1.6$ M and $C_x = 1$ pF: $f_{p1} = 10^5$ Hz
For $f_D' = 10^3$ Hz, the required $C_C$ is 99 pF

$$f_{p1} = \frac{1}{2\pi C_x R_x}$$
$$f_D' = \frac{1}{2\pi (C_x + C_C) R_x}$$
Miller compensation and pole splitting

- Miller effect equivalently increase the capacitance by a factor of voltage gain
- Use miller capacitance for compensation can reduce the need for large capacitance

\[
V_o = \frac{(sC_f - g_m)R_1R_2}{1 + s[C_1R_1 + C_2R_2 + C_f(g_mR_1R_2 + R_1 + R_2)] + s^2[C_1C_2 + C_f(C_1 + C_2)]R_1R_2}
\]

\[
f'_{p1} = \frac{1}{2\pi} \frac{1}{C_1R_1 + C_2R_2 + C_f(g_mR_1R_2 + R_1 + R_2)} \approx \frac{1}{2\pi} \frac{1}{C_f g_m R_1 R_2}
\]

\[
f'_{p2} = \frac{1}{2\pi} \frac{g_m C_f}{C_1C_2 + C_1C_f + C_2C_f}
\]

- Pole splitting: As \(C_f\) increases, low-frequency pole reduces and high-frequency pole increases
- It is beneficial for phase margin
- The phase margin of an open-loop op amp defines the worst-case phase margin of a closed-loop amplifier with \(\beta = 1\) (loop gain \(A\beta = A\) )