CHAPTER 9 FREQUENCY RESPONSE

Chapter Outline
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Frequency response of amplifiers

- **Midband:**
  - The frequency range of interest for amplifiers
  - Large capacitors can be treated as short circuit and small capacitors can be treated as open circuit
  - Gain is constant and can be obtained by small-signal analysis

- **Low-frequency band:**
  - Gain drops at frequencies lower than $f_L$
  - Large capacitors can no longer be treated as short circuit
  - The gain roll-off is mainly due to coupling and by-pass capacitors

- **High-frequency band:**
  - Gain drops at frequencies higher than $f_H$
  - Small capacitors can no longer treated as open circuit
  - The gain roll-off is mainly due to parasitic capacitances of the MOSFETs and BJTs
9.1 Low-Frequency Response of the CS and CE Amplifiers

The CS amplifier

Small-signal analysis:

\[ V_g = V_{sig} \frac{R_i}{R_G + \frac{1}{sC_{c1}} + R_{sig}} = V_{sig} \frac{R_G}{R_G + R_{sig}} \frac{s}{s + \frac{1}{C_{c1}(R_G + R_{sig})}} \]

\[ I_d = \frac{V_g}{1 + \frac{1}{g_m sC}} = g_m V_g \frac{s}{s + \frac{g_m}{C_S}} \]

\[ V_o = I_o R_L = -I_d \frac{R_D}{R_D + \frac{1}{sC_{c2}} + R_L} = -I_d \frac{R_D R_L}{R_D + R_L s + \frac{1}{C_{c2}(R_D + R_L)}} \]

\[ \frac{V_o}{V_{sig}} = A_m \left( \frac{s}{s + \omega_{p1}} \right) \left( \frac{s}{s + \omega_{p2}} \right) \left( \frac{s}{s + \omega_{p3}} \right) \]

\[ A_m = \frac{g_m R_G (R_D || R_L)}{R_G + R_{sig}} \]

\[ \omega_{p1} = \frac{1}{C_{c1}(R_G + R_{sig})} \]

\[ \omega_{p2} = \frac{g_m}{C_S} \]

\[ \omega_{p3} = \frac{1}{C_{c2}(R_D + R_L)} \]
Determining the lower 3-dB frequency

- Coupling and by-pass capacitors result in a high-pass frequency response with three poles
- If the poles are sufficiently separated
  - Bode plot can be used to evaluate the response for simplicity
  - The lower 3-dB frequency is the highest-frequency pole
  - $\omega_{p2}$ is typically the highest-frequency pole due to small resistance of $1/g_m$
- If the poles are located closely
  - The lower 3-dB frequency has to be evaluated by the transfer function which is more complicated

Determining the pole frequency by inspection

- Reduce $V_{sig}$ to zero
- Consider each capacitor separately
  - (treat the other capacitors as short circuit)
- Find the total resistance between the terminals

Selecting values for coupling and by-pass capacitors

- These capacitors are typically required for discrete amplifier designs
- $C_S$ is first determined to satisfy needed $f_L$
- $C_{C1}$ and $C_{C2}$ are chosen such that poles are 5 to 10 times lower than $f_L$
The CE amplifier

- Small-signal analysis

- Considering the effect of each capacitor separately
  - Considering only $C_{C1}$:

\[
V_x = V_{\text{sig}} \frac{R_g \parallel r_x}{R_g \parallel r_x + R_{\text{sig}} + \frac{1}{sC_{C1}}}
\]

\[
V_o = -g_m V_x (R_C \parallel R_L)
\]

\[
\frac{V_o}{V_{\text{sig}}} = \frac{-g_m (R_B \parallel r_x)(R_C \parallel R_L)}{R_B \parallel r_x + R_{\text{sig}}} \frac{s}{s + \frac{1}{C_{C1}(R_g \parallel r_x + R_{\text{sig}})}}
\]

\[
\omega_{p1} = \frac{1}{C_{C1}(R_g \parallel r_x + R_{\text{sig}})}
\]

\[
A_M = -\frac{g_m (R_B \parallel r_x)(R_C \parallel R_L)}{R_B \parallel r_x + R_{\text{sig}}}
\]
Consider only $C_E$:

$$I_b = V_{\text{sig}} \frac{R_B}{R_B + R_{\text{sig}}} \left(\frac{1}{R_b + R_{\text{sig}} + (\beta + 1)\left(r_e + \frac{1}{sC_E}\right)}\right)$$

$$V_o = -\beta I_b (R_C \parallel R_L)$$

$$V_o = -\frac{R_B}{R_B + R_{\text{sig}}} \frac{\beta (R_C \parallel R_L)}{s} + \frac{s}{C_E \left(r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1}\right)}$$

$$\omega_{p2} = \frac{1}{C_E \left(r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1}\right)}$$

Consider only $C_{C2}$:

$$V_\pi = V_{\text{sig}} \frac{R_B \parallel r_\pi}{R_B \parallel r_\pi + R_{\text{sig}}}$$

$$V_o = -g_m V_\pi \frac{R_C}{R_C + \frac{1}{sC_{C2}} + R_L}$$

$$V_o = -\frac{g_m (R_B \parallel r_\pi)(R_C \parallel R_L)}{R_B \parallel r_\pi + R_{\text{sig}}} \frac{s}{s + \frac{1}{C_{C2}(R_C + R_L)}}$$

$$\omega_{p3} = \frac{1}{C_{C2}(R_C + R_L)}$$
Determining the lower 3-dB frequency

- Coupling and by-pass capacitors result in a high-pass frequency response with three poles
  \[ \frac{V_o}{V_{sig}} = -A_m \left( \frac{s}{s + \omega_{p1}} \right) \left( \frac{s}{s + \omega_{p2}} \right) \left( \frac{s}{s + \omega_{p2}} \right) \]

- The lower 3-dB frequency is simply the highest-frequency pole if the poles are sufficiently separated
- The highest-frequency pole is typically \( \omega_{p2} \) due to the small resistance of \( R_E \)
- An approximation of the lower 3-dB frequency is given by
  \[ f_L \approx \frac{1}{2\pi} \left( \frac{1}{C_{C1}R_{C1}} + \frac{1}{C_{E}R_{E}} + \frac{1}{C_{C2}R_{C2}} \right) \]

Selecting values for the coupling and by-pass capacitors

- These capacitors are typically required for discrete amplifier designs
- \( C_E \) is first determined to satisfy needed \( f_L \)
- \( C_{C1} \) and \( C_{C2} \) are chosen such that poles are 5 to 10 times lower than \( f_L \)
9.2 Internal Capacitive Effects and the High-Frequency Model

The MOSFET device

There are basically two types of internal capacitance in the MOSFET

- Gate capacitance effect: the gate electrode forms a parallel-plate capacitor with gate oxide in the middle
- Junction capacitance effect: the source/body and drain/body are pn-junctions at reverse bias

The gate capacitive effect

- MOSFET in triode region:
  \[ C_{gs} = C_{gd} = \frac{1}{2} W L C_{ox} + C_{ov} \]
- MOSFET in saturation region:
  \[ C_{gs} = \frac{2}{3} W L C_{ox} + C_{ov} \quad C_{gd} = C_{ov} \]
- MOSFET in cutoff region:
  \[ C_{gs} = C_{gd} = C_{ov} \quad C_{gb} = W L C_{ox} \]
- Overlap capacitance:
  \[ C_{ov} = W L_{ov} C_{ax} \]

The junction capacitance

- Junction capacitance includes components from the bottom side and from the side walls
- The simplified expression are given by

\[
C_{sb} = \frac{C_{sb0}}{\sqrt{1 + V_{SB} / V_0}} \quad C_{dh} = \frac{C_{dh0}}{\sqrt{1 + V_{DH} / V_0}}
\]
The high-frequency MOSFET model

\[ g_m = \mu_n C_{ox} \frac{W}{L} |V_{ov}| = \sqrt{2\mu_n C_{ox} \frac{W}{L}} I_D \]
\[ g_{mb} = \chi g_m \]
\[ r_o = |V_A| / I_D \]
\[ C_{gs} = \frac{2}{3} W L C_{ox} + W L_{on} C_{ox} \]
\[ C_{gd} = W L_{ov} C_{ox} \]
\[ C_{sb} = \frac{C_{sb0}}{\sqrt{1 + |V_{SB}| / V_o}} \]
\[ C_{db} = \frac{C_{db0}}{\sqrt{1 + |V_{DB}| / V_o}} \]

Simplified high-frequency MOSFET model

- Source and body terminals are shorted
- \( C_{gs} \) plays an important role in the amplifier frequency response
- \( C_{db} \) is neglected to simplify the analysis
The unity-gain frequency \((f_T)\)

- The frequency at which the current gain becomes unity
- Is typically used as an indicator to evaluate the high-frequency capability
- Smaller parasitic capacitances \(C_{gs}\) and \(C_{gd}\) are desirable for higher unity-gain frequency

\[
I_o = g_m V_{gs} - sC_{gd} V_{gs} \approx g_m V_{gs}
\]

\[
V_{gs} = \frac{I_i}{s(C_{gs} + C_{gd})}
\]

\[
\frac{I_o}{I_i} = \frac{g_m}{s(C_{gs} + C_{gd})}
\]

\[
f_T = \frac{1}{2\pi} \frac{g_m}{C_{gs} + C_{gd}}
\]

- The unity-gain frequency can also be expressed as

\[
f_T = \frac{1}{2\pi} \sqrt{\frac{2\mu_n C_{ox} (W / L) I_D}{C_{gs} + C_{gd}}} \approx \frac{3}{2\pi L} \sqrt{\frac{\mu_n I_D}{2C_{ox}WL}}
\]

\[
f_T = \frac{1}{2\pi} \frac{\mu_n C_{ox} (W / L) V_{OV}}{C_{gs} + C_{gd}} \approx \frac{3\mu_n V_{OV}}{4\pi L^2}
\]

- The unity-gain frequency is strongly influenced by the channel length
- Higher unity-gain frequency can be achieved for a given MOSFET by increasing the bias current or the overdrive voltage

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The BJT Device

- High-frequency hybrid-\(\pi\) model:
  - The base-charging or diffusion capacitance \(C_{de}\):
    \[
    C_{de} = \tau_p g_m = \tau_F \frac{I_c}{V_T}
    \]
  - The base-emitter junction capacitance \(C_{je}\):
    \[
    C_{je} \approx 2C_{j0}
    \]
  - The collector-base junction capacitance \(C_{\mu}\):
    \[
    C_{\mu} = \frac{C_{\mu0} m}{(1 + \frac{V_{CB}}{V_{0c}})^m}
    \]

- The cutoff (unity-gain) frequency:
  \[
  I_c = (g_m - sC_{\mu})V_{\pi}
  \]
  \[
  V_{\pi} = I_b (r_{\pi} \| C_{\pi} \| C_{\mu}) = \frac{I_b}{1/r_{\pi} + s(C_{\pi} + C_{\mu})}
  \]
  \[
  h_{fe} \equiv \frac{I_c}{I_b} = \frac{g_m - sC_{\mu}}{1/r_{\pi} + s(C_{\pi} + C_{\mu})}
  \]
  \[
  h_{fe} \approx \frac{g_m r_{\pi}}{1 + s(C_{\pi} + C_{\mu})r_{\pi}} = \frac{\beta_0}{1 + s(C_{\pi} + C_{\mu})r_{\pi}}
  \]
  \[
  f_T = \frac{1}{2\pi C_{s} + C_{\mu}} = \frac{1}{2\pi (C_{s} + C_{\mu})V_T}
  \]

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9.3 High-Frequency Response of the CS and CE Amplifiers

The common-source amplifier

- Midband gain:
  \[ A_M = -\frac{R_G}{R_G + R_{sig}} (g_m R'_L) \]

- Frequency response:
  \[
  \frac{V_{sig} - V_{gs}}{R_{sig}} = \frac{V_{gs}}{R_G} + sC_{gs} V_{gs} + sC_{gd} (V_{gs} - V_o) \\
  sC_{gd} (V_{gs} - V_o) = g_m V_{gs} + \frac{V_o}{R'_L}
  \]

  \[
  \begin{align*}
  \frac{V_o}{V_{sig}} &= -
  
  1 + s \left( \frac{R_{sig} R_G}{R_{sig} + R_G} \right) \left[ \left( \frac{R'_L}{R_{sig} + R_G} + 1 + g_m R'_L \right) C_{gd} + C_{gs} \right] + s^2 \left( \frac{R'_L R_{sig} R_G}{R_{sig} + R_G} C_{gd} C_{gs} \right)
  \end{align*}
  \]

- The common-source amplifier has one zero and two poles at higher frequencies
- The amplifier gain falls off at frequencies beyond midband
- The amplifier bandwidth is defined by the 3-dB frequency which is typically evaluated by the dominant pole (the lowest-frequency pole) in the transfer function
Simplified analysis technique

- Assuming the gain is nearly constant ($\approx -g_m R'_L$)
- Find the equivalent capacitance of $C_{gd}$ at the input (with identical $I_{gd}$)
  \[
  I_{gd} = s C_{gd} (V_{gs} - V_o) = s C_{gd} [V_{gs} - (-g_m R'_L) V_{gs}]
  \]
  \[
  = s C_{gd} (1 + g_m R') V_{gs}
  \]
  \[
  I_{gd} = s C_{eq} V_{gs}
  \]
  \[
  C_{eq} = (1 + g_m R'_L) C_{gd}
  \]

**Miller effect**

- Neglect the small current $I_{gd}$ at the output

\[
V_o = \frac{-A_M}{1 + s \{R_{sig}' [(1 + g_m R'_L) C_{gd} + C_{gs}]\}} = -\frac{A_M}{1 + s \frac{1}{\omega_H}}
\]

\[
A_M = -\frac{R_G}{R_G + R_{sig}} (g_m R'_L)
\]

\[
\omega_H = \left( R_{sig}' [(1 + g_m R'_L) C_{gd} + C_{gs}] \right)^{-1}
\]

- The dominant pole is normally determined by $C_{eq}$
- The frequency response of the common-source amplifier is approximated by a STC
The common-emitter amplifier

- Simplified analysis for frequency response
  - Miller effect: replacing $C_\mu$ with $C_{eq}$
  - The response is approximated by a STC

\[
\frac{V_o}{V_{sig}} = \frac{A_M}{1 + \frac{\omega}{\omega_H}}
\]

\[
A_M = -\frac{R_B}{R_B + R_{sig}} \frac{r_x}{r_x + R_B \| R_{sig}} \mu g_m R'_L
\]

\[
\omega_H = \left\{ \left[ C_x + (1 + g_m R'_L) C_\mu \right] \left[ r_x + R_B \| R_{sig} \right] \right\}^{-1}
\]

\[
R'_L = r_x \| (R_B \| R_{sig})
\]
9.4 Useful Tools for the Analysis of the High-Frequency Response of Amplifiers

Determining the upper 3-dB frequency

- General transfer function of the amplifier

\[ A(s) = A_M F_H(s) = A_M \frac{(1 + s / \omega_{Z1})(1 + s / \omega_{Z2}) \cdots (1 + s / \omega_{Zn})}{(1 + s / \omega_{P1})(1 + s / \omega_{P2}) \cdots (1 + s / \omega_{Pn})} \]

- The upper 3-dB frequency

\[ A(s) = A_M F_H(s) = A_M \frac{(1 + s / \omega_{Z1})(1 + s / \omega_{Z2})}{(1 + s / \omega_{P1})(1 + s / \omega_{P2})} \]

\[ |F_H(j\omega_H)|^2 = \frac{1}{2} \frac{1 + \omega_H^2 \left( \frac{1}{\omega_{Z1}^2} + \frac{1}{\omega_{Z2}^2} \right) + \omega_H^4 \left( \frac{1}{\omega_{Z1}^2} - \frac{1}{\omega_{Z2}^2} \right)}{1 + \omega_H^2 \left( \frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2} \right) + \omega_H^4 \left( \frac{1}{\omega_{P1}^2} - \frac{1}{\omega_{P2}^2} \right)} \approx \frac{1}{2} \frac{1 + \frac{1}{\omega_{Z1}^2} + \frac{1}{\omega_{Z2}^2}}{1 + \frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2}} \]

\[ \omega_H \approx \frac{1}{\sqrt{\left( \frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2} \right) - 2 \left( \frac{1}{\omega_{Z1}^2} + \frac{1}{\omega_{Z2}^2} \right)}} \]

- Dominant-pole response

  - One of the poles is of much lower frequency than any of the other poles and zeros
  - Dominant-pole: the lowest-frequency pole is at least 4× away from the nearest pole or zero

\[ \omega_H \approx \frac{1}{\sqrt{\left( \frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2} \right) - 2 \left( \frac{1}{\omega_{Z1}^2} + \frac{1}{\omega_{Z2}^2} \right)}} \approx \omega_{P1} \]
Open-circuit time constant method to evaluate amplifier bandwidth

- General transfer function of the amplifier
  \[ F_H(s) = \frac{(1+s/\omega_{z1})(1+s/\omega_{z2}) \cdots (1+s/\omega_{zm})}{(1+s/\omega_{p1})(1+s/\omega_{p2}) \cdots (1+s/\omega_{pn})} = \frac{1+a_1s+a_2s^2+\cdots+a_ms^n}{1+b_1s+b_2s^2+\cdots+b ns^n} \]
  \[ b_i = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots + \frac{1}{\omega_{pn}} \]

- Open-circuit time constant (exact solution):
  \[ b_i = \sum_{i=1}^{n} C_i R_i \]

- Dominant pole approximation:
  \[ b_i = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots + \frac{1}{\omega_{pn}} \approx \frac{1}{\omega_{p1}} \]
  \[ \omega_H \approx \omega_{p1} \approx \frac{1}{b_1} = \frac{1}{\sum_i C_i R_i} \]

Miller’s Theorem

- A technique to replace the bridging capacitance
- The equivalent input and output impedances are:
  \[ V_2 = KV_1 \]
  \[ Z_1 = \frac{Z}{1-K} \]
  \[ Z_2 = \frac{Z}{1-1/K} \]

\[ Z_1 = Z/(1-K), \quad Z_2 = Z/\left(1 - \frac{1}{K}\right) \]

\[ V_1 \quad \downarrow \quad Z \quad \uparrow \quad V_2 = KV_1 \]

\[ Z_1 \quad \downarrow \quad V_1 \quad \uparrow \quad Z_2 \quad V_2 = KV_1 \]
**Time-constant method**

- A technique used to determine the coefficients of the transfer function from the circuit

\[ F_H(s) = \frac{\left(1 + \frac{s}{\omega_{z1}}\right)\left(1 + \frac{s}{\omega_{z2}}\right)\cdots\left(1 + \frac{s}{\omega_{zn}}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)\cdots\left(1 + \frac{s}{\omega_{pn}}\right)} = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_n s^n}{1 + b_1 s + b_2 s^2 + \cdots + b_n s^n} \]

- Determining \( b_1 \):
  \[ b_1 = C_1 R^0_{11} + C_2 R^0_{22} + C_3 R^0_{33} \]

  - Set all independent sources zero
  - \( R^0_{ii} \): the equivalent resistance in parallel with \( C_i \) by treating the other capacitors as open circuit

- Determining \( b_2 \):
  \[ b_2 = C_1 R^0_{11} C_2 R^1_{22} + C_1 R^0_{11} C_3 R^1_{33} + C_2 R^0_{22} C_3 R^2_{33} \]

  - Set all independent sources zero
  - \( R^1_{ii} \): the equivalent resistance in parallel with \( C_i \) by treating \( C_j \) short and the other capacitors open
9.5 A Closer Look at the High-Frequency Response

Exact analysis by transfer function

- Transfer function of the amplifier:
  - Step 1: define nodal voltages
  - Step 2: find branch currents
  - Step 3: KCL equations
    \[
    \frac{V'_{\text{sig}} - V_{gs}}{R'_{\text{sig}}} = sC_{gs}V_{gs} + sC_{gd}(V_{gs} - V_o)
    \]
    \[
    sC_{gd}(V_{gs} - V_o) = g_mV_{gs} + V_o / R' + sC_L V_o
    \]
  - Step 4: transfer function by solving the linear equations
    \[
    \frac{V_o}{V'_{\text{sig}}} = - \frac{g_mR'_L[1 - s(C_{gd} / g_m)]}{1 + s[(C_{gs} + C_{gd}(1 + g_mR'_L)]R'_{\text{sig}} + (C_L + C_{gd})R'_L] + s^2[(C_L + C_{gd})C_{gs} + C_LC_{gd}]}R'_{\text{sig}}R'_L
    \]
    \[
    \frac{V_o}{V'_{\text{sig}}} = \frac{\left(\frac{g_mR'_L}{R_G + R'_{\text{sig}}}ight)}{1 + s[(C_{gs} + C_{gd}(1 + g_mR'_L)]R'_{\text{sig}} + (C_L + C_{gd})R'_L] + s^2[(C_L + C_{gd})C_{gs} + C_LC_{gd}]}R'_{\text{sig}}R'_L
    \]

- Poles and zero:
  \[
  D(s) = \left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right) = 1 + s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + s^2\left(\frac{1}{\omega_{p1}\omega_{p2}}\right) \approx 1 + s\left(\frac{1}{\omega_{p1}}\right) + s^2\left(\frac{1}{\omega_{p1}\omega_{p2}}\right)
  \]
  \[
  \omega_{p1} = \frac{1}{[C_{gs} + C_{gd}(1 + g_mR'_L)]R'_{\text{sig}} + (C_L + C_{gd})R'_L}
  \]
  \[
  \omega_{p2} = \frac{[C_{gs} + C_{gd}(1 + g_mR'_L)]R'_{\text{sig}} + (C_L + C_{gd})R'_L}{[(C_L + C_{gd})C_{gs} + C_LC_{gd}]}R'_LR'_{\text{sig}}
  \]
  \[
  \omega_{z1} = g_m / C_{gd}
  \]

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Analysis by using open-circuit time constants

Open-circuit time constants:

\[
R_{gs} = R'_{gs}
\]

\[
R_{gd} = R'_{gd}(1 + g_m R'_L) + R'_L
\]

\[
R_{Cl} = R'_L
\]

\[
\tau_H = C_{gs} R_{gs} + C_{gd} R_{gd} + C_L R_{Cl} = C_{gs} R'_{gs} + C_{gd} [R'_{gs}(1 + g_m R'_L) + R'_L] + C_L R'_L
\]

\[
\omega_H = \frac{1}{\tau_H} = \frac{1}{C_{gs} R_{gs} + C_{gd} R_{gd} + C_L R_{Cl}} = \frac{1}{C_{gs} R'_{gs} + C_{gd} [R'_{gs}(1 + g_m R'_L) + R'_L] + C_L R'_L}
\]
Analysis by using Miller’s Theorem

- Bridging capacitance $C_{gd}$ is replaced by $C_1$ and $C_2$

$$K \equiv \frac{V_a}{V_{gs}} \approx -g_m R_L'$$

$$C_1 = C_{gd} (1 - K) = C_{gd} (1 + g_m R_L')$$

$$C_2 = C_{gd} (1 - K^{-1}) = C_{gd} (1 + \frac{1}{g_m R_L'}) \approx C_{gd}$$

$$K \equiv \frac{V_a}{V_{gs}} \approx -g_m R_L'$$

- Transfer function and $\omega_H$:

$$\frac{V_a}{V_{sig}} = \left( -g_m R_L' \frac{R_G}{R_G + R_{sig}} \right) \frac{1}{(1 + s / \omega_{p1})(1 + s / \omega_{p2})}$$

$$\omega_H = \frac{1}{\sqrt{1/\omega_{p1}^2 + 1/\omega_{p2}^2}} \approx \omega_{p1}$$

- Open-circuit time constant analysis based on Miller’s equivalent circuit:

$$R_{C_{in}} = R_{sig}'$$

$$R_{C_L} = R_L'$$

$$\tau_H = C_{in} R_{C_{in}} + C_L R_{C_L} = [C_{gs} + C_{gd} (1 + g_m R_L')] R_{sig}' + (C_L + C_{gd}) R_L'$$

$$\omega_H = \frac{1}{\tau_H} = \frac{1}{[C_{gs} + C_{gd} (1 + g_m R_L')] R_{sig}' + (C_L + C_{gd}) R_L'}$$

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CS amplifier with a small source resistance

- The case where source resistance is zero
  \[ R'_{\text{sig}} = R_{\text{sig}} \approx 0 \]
  \[ V'_{\text{sig}} = V_{\text{sig}} \]

- The transfer function of the amplifier:
  \[ \frac{V_{\text{a}}}{V_{\text{sig}}} = -(g_m R') \frac{1 - s(C_{gd} / g_m)}{1 + s[(C_L + C_{gd})R'_L]} \]
  - Midband gain:
    \[ A_M = -g_m R'_L \]
  - The transfer function has one pole and one zero
    \[ \omega_{p1} = 1/[(C_L + C_{gd})R'_L] \]
    \[ \omega_{z1} = g_m / C_{gd} \]

  \[ \Rightarrow \] Midband gain and zero are virtually unchanged
  \[ \Rightarrow \] The lowest-frequency pole no longer exists

- Gain-bandwidth product:
  - Gain rolls off beyond \( f_H \) (−20 dB/decade)
  - The gain becomes 0 dB at \( f_i \):
    \[ f_i = |A_M| f_{3dB} = \frac{g_m}{2\pi(C_L + C_{gd})} \]

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9.6 High-Frequency Response of the CG and Cascode Amplifiers

**High-frequency response of the CG amplifier**

- The frequency response by neglecting $r_o$:
  \[
  \frac{V_o}{V_{sig}} = \frac{A_M}{(1 + s / \omega_{p1})(1 + s / \omega_{p2})}
  \]

  - Midband gain:
    \[
    A_M \approx \frac{g_m R_L}{1 + g_m R_{sig}} \approx g_m R_L / (1 + g_m R_{sig})
    \]

  - Poles:
    \[
    \omega_{p1} = \frac{1}{C_{gs} [R_{sig} || (1 / g_m)]}
    \]
    \[
    \omega_{p2} = \frac{1}{(C_{gd} + C_L) R_L}
    \]

- The upper 3-dB frequency $\omega_H \approx \omega_{p2}$ - Typically higher than the CS amplifier

- The frequency response including $r_o$:
  - Open-circuit time constant method
    \[
    R_{in} = \frac{(r_o + R_L)}{(1 + g_m r_o)}
    \]
    \[
    R_o = r_o + R_{sig} + g_m r_o R_{sig}
    \]
    \[
    R_{gs} = R_{sig} || R_{in}
    \]
    \[
    R_{gd} = R_o || R_L
    \]
    \[
    \omega_H = \frac{1}{[C_{gs} R_{gs} + (C_{gd} + C_L) R_{gd}]}
    \]
High-frequency response of the cascode amplifier

- Open-circuit time constant method:
  - Capacitance $C_{gs1}$ sees a resistance $R_{sig}$
  - Capacitance $C_{gd1}$ sees a resistance $R_{gd1}$
    \[ R_{gd1} = (1 + g_{m1}R_{d1})R_{sig} + R_{d1} \]
  - Capacitance $(C_{db1} + C_{gs2})$ sees a resistance $R_{d1}$
    \[ R_{d1} = r_{o1} \parallel r_{o2} = r_{o1} \parallel [(r_{o2} + R_L)/g_{m2}r_{o2}] \]
  - Capacitance $(C_{L} + C_{gd2})$ sees a resistance $R_{L} \parallel R_{o}$
    \[ R_{o} = r_{o2} + r_{o1} + g_{m2}r_{o2}\]
  - Effective time constant
    \[ \tau_{H} = C_{gs1}R_{sig} + C_{gd1}[(1 + g_{m1}R_{d1})R_{sig} + R_{d1}] + (C_{db1} + C_{gs2})R_{d1} + (C_{L} + C_{gd2})(R_{L} \parallel R_{o}) \]

- The upper 3-dB frequency:
  \[ f_{H} = \frac{1}{2\pi \tau_{H}} = \frac{1}{2\pi} \left\{ R_{sig}[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})] + R_{d1}(C_{gd1} + C_{db1} + C_{gs2}) + (R_{L} \parallel R_{o})(C_{L} + C_{gd2}) \right\}^{-1} \]
  - In the case of a large $R_{sig}$:
    - The first term dominates if the Miller multiplier is large (typically with large $R_{d1}$ and $R_L$)
    - A small $R_L$ (to the order of $r_o$) is needed for extended bandwidth
    - The midband gain drops as the value of $R_L$ decreases
    - A trade-off exists between gain and bandwidth

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In the case of a small $R_{sig}$:

- The first term becomes negligible and the third term dominates
- A large $R_L$ (to the order of $A_o r_o$) can be used to boost the amplifier gain

In the case of zero $R_{sig}$:

$$f_{H} \approx \frac{1}{2\pi \left( R_{L} \parallel R_{o} \right) \left( C_{L} + C_{gd/2} \right)}$$

$$f_{i} \approx \frac{1}{2\pi C_{L} + C_{gd/2}}$$

Summary of CS and cascode amplifiers

<table>
<thead>
<tr>
<th>Circuit</th>
<th>Common Source</th>
<th>Cascode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_i \rightarrow f_o \rightarrow R_L \parallel C_L \rightarrow V_o$</td>
<td>$g_m R'_L$</td>
<td>$-A_0 g_m R'_L$</td>
</tr>
<tr>
<td>DC Gain</td>
<td>$-g_m R'_L$</td>
<td>$-A_0 g_m R'_L$</td>
</tr>
<tr>
<td>$f_{MB}$</td>
<td>$\frac{1}{2\pi \left( C_L + C_{gd/2} \right) R'_L}$</td>
<td>$\frac{1}{2\pi \left( C_L + C_{gd} \right) A_0 R'_L}$</td>
</tr>
<tr>
<td>$f_i$</td>
<td>$\frac{g_m}{2\pi \left( C_L + C_{gd} \right)}$</td>
<td>$\frac{g_m}{2\pi \left( C_L + C_{gd} \right)}$</td>
</tr>
</tbody>
</table>
High-frequency response of the bipolar cascode amplifier

- The similar analysis technique can be applied for bipolar cascode amplifier.

\[
R'_{\text{sig}} = r_{\pi1} \parallel R_{\text{sig}} \\
R_{\pi1} = R'_{\text{sig}} \\
R_{\mu1} = R'_{\text{sig}}(1 + g_{m1}R_{c1}) + R_{c1} \\
R_{c1} = r_{o1} \left[ r_{e2} \left( \frac{r_{o2} + R_L}{r_{o2} + r_{o1} + R_L/(\beta_2 + 1)} \right) \right] \\
\tau_H = C_{\pi1}R_{\pi1} + C_{\mu1}R_{\mu1} + (C_{c1} + C_{\pi2})R_{c1} \\
+ (C_L + C_{c2} + C_{\mu2})(R_L \parallel R_o) \\
R_o = \beta_2 r_{o2} \\
f_H = \frac{1}{2\pi\tau_H} \\
A_M = -\frac{r_\pi}{r_\pi + r_x + R_{\text{sig}}(\beta r_o \parallel R_L)}
\]
9.7 High-Frequency Response of the Source and Emitter Followers

The source follower:

- Low-frequency (midband) gain and output resistance:
  \[ A_M = \frac{R_L \parallel r_o}{R_L \parallel r_o + 1/g_m} = \frac{R'_L}{R'_L + 1/g_m} \approx 1 \]
  \[ R_o = r_o \parallel (1/g_m) \]

- High-frequency characteristics:
  - High-frequency zero:
    \( \Rightarrow \) Output becomes 0 at \( s = s_Z = -g_m/C_{gs} \)
    \( \Rightarrow \) High-frequency zero: \( \omega_Z = g_m/C_{gs} \)
    \( \Rightarrow f_Z \approx f_T \) (transistor’s unity-gain frequency)
  - The 3-dB frequency \( f_H \):
    \[ R_{gd} = R_{sig} \]
    \[ R_{gs} = \frac{R_{sig} + R'_L}{1 + g_m R'_L} \]
    \[ R_{C_L} = R_L \parallel r_o \parallel (1/g_m) \]
    \[ f_H = \frac{1}{2\pi} \left( C_{gd} R_{gd} + C_{gs} R_{gs} + C_L R_{C_L} \right) \]

\[ R'_L = R_L \parallel r_o \]

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The emitter follower:

- Low-frequency (midband) gain and output resistance:
  \[ A_M \approx \frac{R_L}{R_L + r_e} \approx 1 \]
  \[ R_o = R_L \parallel r_o \parallel [r_e + R_{sig} / (1 + \beta)] \]

- High-frequency characteristics:
  ■ High-frequency zero:
    \[ \text{Output becomes 0 at } s = s_Z = -1/C \pi r_e \]
  ■ High-frequency zero: \( \omega_Z = 1/C \pi r_e \)
  ■ \( f_Z \approx f_T \) (transistor’s unity-gain frequency)
  ■ The 3-dB frequency \( f_{3dB} \):
    \[ R_\mu = R_{sig} \parallel [r_\pi + (\beta + 1)R'_L] \]
    \[ R_\pi = \frac{R_{sig} + R'_L}{1 + \frac{R_{sig} + R'_L}{r_\pi + r_e}} \]
    \[ f_{3dB} = \frac{1}{2\pi} \left( C_\mu R_\mu + C_\pi R_\pi \right) \]
9.8 High-Frequency Response of Differential Amplifiers

Resistively Loaded MOS Differential Amplifier:

- **Differential-mode operation:**
  - Use differential half-circuit for analysis
  - Identical to the case for common-source amplifier
  - Can be approximated by a dominant-pole system

- **Common-mode operation:**
  - Use common-mode half-circuit for analysis
  - The capacitance $C_{SS}$ is significant
  - $C_{SS}$ results in a zero at lower frequency
  - Other capacitance $\rightarrow$ high-frequency poles and zeros

\[
A_{em} = -\frac{R_D}{2R_{SS}} \frac{\Delta R_D}{R_D}
\]

\[
A_{em}(s) = -\frac{R_D}{2Z_{ss}} \frac{\Delta R_D}{R_D} = -\frac{R_D}{2} \frac{\Delta R_D}{R_D} \left( \frac{1}{R_{ss}} + sC_{ss} \right) = -\frac{R_D}{2R_{ss}} \frac{\Delta R_D}{R_D} \left( 1 + sR_{ss}C_{ss} \right)
\]

\[
\omega_Z = \frac{1}{R_{ss}C_{ss}}
\]
Common-mode rejection ratio (CMRR):
- The frequency dependence of CMRR can be evaluated
- CMRR decreases at higher frequencies due to the pole of $A_d$ and the zero of $A_{cm}$
Active-Loaded MOS Differential Amplifier:

- Differential-mode operation:
  - Equivalent capacitances:
    \[ C_m = C_{gd1} + C_{db1} + C_{gd3} + C_{gs4} \]
    \[ C_L = C_{gd2} + C_{db2} + C_{gd4} + C_{db4} + C_x \]
  - Transconductance (\( G_m \)):
    \[ V_{g3} = - \frac{g_{m1}V_{id} / 2}{g_{m3} + sC_m} \]
    \[ I_{d4} = -g_{m4}V_{g3} = \frac{g_{m4}g_{m1}V_{id} / 2}{g_{m3} + sC_m} = \frac{g_{m1}V_{id} / 2}{1 + sC_m / g_{m3}} \]
    \[ I_o = I_{d4} + I_{d2} = \frac{g_{m1}V_{id} / 2}{1 + sC_m / g_{m3}} + g_{m1}V_{id} / 2 \]
    \[ G_m = \frac{I_o}{V_{id}} = g_{m1} \frac{1 + sC_m / 2g_{m3}}{1 + sC_m / g_{m3}} \]

- The pole and zero of \( G_m(s) \) are at very high frequencies
- Minor pole and zero near unity-gain frequency

\[ f_{p2} = \frac{g_{m3}}{2\pi C_m} \approx f_T / 2 \]
\[ f_z = \frac{2g_{m3}}{2\pi C_m} \approx f_T \]

---

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The transfer function of the amplifier:

\[ V_o = G_m V_{id} \frac{R_o}{1 + sC_L R_o} = G_m V_{id} \frac{r_{o2} \parallel r_{o4}}{1 + sC_L (r_{o2} \parallel r_{o4})} \]

\[ \frac{V_o}{V_{id}} = g_m R_o \frac{1 + s \frac{C_m}{2g_{m3}}}{1 + s \frac{C_m}{g_{m3}}} \frac{1}{1 + sC_L R_o} \]

The dominant pole is typically

\[ f_{p1} = \frac{1}{2\pi C_L R_o} \]

Common-mode operation:

- By taking \( C_{SS} \) into account for the mid-band common-mode gain:

\[ A_{cm} \approx \frac{1}{2g_{m2}} \frac{R_{SS}}{1 + sR_{SS}C_{SS}} = -\frac{1 + sR_{SS}C_{SS}}{2g_{m2}R_{SS}} \]

Contributes to a zero at lower frequency

Common-mode rejection ratio (CMRR):

- CMRR decreases due to the dominant pole of \( A_d(s) \) and the zero of \( A_{cm}(s) \)