CHAPTER 11 FILTERS AND TUNED AMPLIFIERS

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11.1 FILTER TRANSMISSION, TYPES AND SPECIFICATIONS

Filter Transfer Function

- A filter is a **linear** two-port network represented by the ratio of the output to input voltage.
- Transfer function \( T(s) = \frac{V_o(s)}{V_i(s)} \).
- Transmission: evaluating \( T(s) \) for physical frequency \( s = j\omega \rightarrow T(j\omega) = |T(j\omega)|e^{j\phi(\omega)} \).
  - Gain: \( 20 \log|T(j\omega)| \) (dB)
  - Attenuation: \( -20 \log|T(j\omega)| \) (dB)
- Output frequency spectrum: \( |V_o(s)| = |T(s)| |V_i(s)| \).

Types of Filters

- **Low-pass (LP)**
- **High-pass (HP)**
- **Bandpass (BP)**
- **Bandstop (BS)**
Filter Specification

- Passband edge: $\omega_p$
- Maximum allowed variation in passband transmission: $A_{\text{max}}$
- Stopband edge: $\omega_s$
- Minimum required stopband attenuation: $A_{\text{min}}$

The first step of filter design is to determine the filter specifications. Then find a transfer function $T(s)$ whose magnitude meets the specifications. The process of obtaining a transfer function that meets given specifications is called filter approximation.
11.2 FILTER TRANSFER FUNCTION

Transfer Function

- The filter transfer function is written as the ratio of two polynomials:
  \[ T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \ldots + a_0}{s^N + b_{N-1} s^{N-1} + \ldots + b_0} \]

- The degree of the denominator \( \rightarrow \) filter order.
- To ensure the stability of the filter \( \rightarrow N \geq M \).
- The coefficients \( a_i \) and \( b_j \) are real numbers.
- The transfer function can be factored and expressed as:
  \[ T(s) = \frac{a_M (s - z_1)(s - z_2)\ldots(s - z_M)}{(s - p_1)(s - p_2)\ldots(s - p_N)} \]

- Zeros: \( z_1, z_2, \ldots, z_M \) and \( (N-M) \) zeros at infinity.
- Poles: \( p_1, p_2, \ldots, p_N \).
- Zeros and poles can be either a real or a complex number.
- Complex zeros and poles must occur in conjugate pairs.
Transfer Function Examples

- Low-Pass Filter (with Stopband Ripple)

\[ T(s) = \frac{a_4(s^2 + \omega_1^2)(s^2 + \omega_2^2)}{s^5 + b_4s^4 + b_3s^3 + b_2s^2 + b_1s + b_0} \]

- Low-Pass Filter (without Stopband Ripple)

\[ T(s) = \frac{a_4}{s^5 + b_4s^4 + b_3s^3 + b_2s^2 + b_1s + b_0} \]

- Band-Pass Filter

\[ T(s) = \frac{a_3s(s^2 + \omega_1^2)(s^2 + \omega_2^2)}{s^6 + b_5s^5 + b_4s^4 + b_3s^3 + b_2s^2 + b_1s + b_0} \]
11.3 BUTTERWORTH AND CHEBYSHEV FILTERS

The Butterworth Filter

- Butterworth filters exhibit monotonically decreasing transmission with all zeros at \( \omega = \infty \).
- Maximally flat response \( \rightarrow \) degree of passband flatness increases as the order \( N \) is increased.
- Higher order filter has a sharp cutoff in the transition band.
- The magnitude function of the Butterworth filter is:

\[
|T(j\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 (\omega/\omega_p)^{2N}}}
\]

- Required transfer functions can be defined based on filter specifications \((A_{\text{max}}\ ,\ A_{\text{min}}\ ,\ \omega_p\ ,\ \omega_s)\)

\[\text{Diagram showing Butterworth filter response.}\]
Natural Modes of the Butterworth Filter

- The natural modes lines on a circle.
- The radius of the circle is $\omega_0 = \omega_p (1/\epsilon)^{1/N}$
- Equal angle space $\pi/N$

$$\omega_p \left(\frac{1}{\epsilon}\right)^{1/N}$$
Design Procedure of the Butterworth Filters

**Design Specifications**

\[ A_{\text{max}} , A_{\text{min}} , \omega_p , \omega_s \]

**Design Procedure**

1. Determine \( \epsilon \) (from \( A_{\text{max}} \))
   \[ |T(j\omega_p)| = \frac{1}{\sqrt{1 + \epsilon^2}} \rightarrow A_{\text{max}} [\text{dB}] = 20 \log \sqrt{1 + \epsilon^2} \rightarrow \epsilon = \sqrt{10^{\frac{A_{\text{max}}}{10}} - 1} \]

2. Determine the required filter order \( N \) (from \( \omega_p , \omega_s , A_{\text{min}} \))
   Attenuation \( A(\omega_s)[\text{dB}] = -20 \log \left[ 1 + \epsilon^2 (\omega_s / \omega_p)^2 \right] = 10 \log \left[ 1 + \epsilon^2 (\omega_s / \omega_p)^2 \right] \geq A_{\text{min}} \)

3. Determine the \( N \) natural modes
   \( p_1, p_2, \ldots, p_N \)

4. Determine \( T(s) \)
   \[ T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\ldots(s-p_N)} \quad \text{where} \quad \omega_0 = \omega_p (1/\epsilon)^{1/N} \]
The Chebyshev Filter

- An equiripple response in the passband.
- A monotonically decreasing transmission in the stopband.
- Odd-order filter → $|T(0)| = 1$.
- Even-order filter → maximum magnitude deviation at $\omega = 0$.

The transfer function of the Chebyshev filter is:

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 \cos^2[N \cos^{-1}(\omega/\omega_p)]}} \quad \text{for } \omega \leq \omega_p$$

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 \cosh^2[N \cosh^{-1}(\omega/\omega_p)]}} \quad \text{for } \omega \geq \omega_p$$
Design Procedure of the Chebyshev Filters

**Design Specifications**

\[ A_{\text{max}} , A_{\text{min}} , \omega_p , \omega_s \]

**Design Procedure**

1. Determine \( \varepsilon \) (from \( A_{\text{max}} \))
   \[ |T(j\omega_p)| = \frac{1}{\sqrt{1 + \varepsilon^2}} \rightarrow A_{\text{max}}(dB) = 20\log\sqrt{1 + \varepsilon^2} \rightarrow \varepsilon = \sqrt{10^{A_{\text{max}}/10} - 1} \]

2. Determine the required filter order \( N \) (from \( \omega_p , \omega_s , A_{\text{min}} \))
   Attenuation \( A(\omega_s) = 10\log[1 + \varepsilon^2 \cosh^2(N \cosh^{-1}(\omega_s / \omega_p))] \geq A_{\text{min}} \)

3. Determine the \( N \) natural modes
   \[ p_k = -\omega_p \sin\left(\frac{2k - 1}{N} \cdot \frac{\pi}{2}\right) \sinh\left(\frac{1}{N} \sinh^{-1} \frac{1}{\varepsilon}\right) + j\omega_p \cos\left(\frac{2k - 1}{N} \cdot \frac{\pi}{2}\right) \cosh\left(\frac{1}{N} \sinh^{-1} \frac{1}{\varepsilon}\right) \]

4. Determine \( T(s) \)
   \[ T(s) = \frac{K\omega_p^N}{\varepsilon 2^{N-1}(s-p_1)(s-p_2)\ldots(s-p_N)} \]
11.4 FIRST-ORDER AND SECOND-ORDER FILTER FUNCTIONS

Cascade Filter Design
- First-order and second-order filters can be cascaded to realize high-order filters.
- Cascade design is one of the most popular methods for the design of active filters.
- Cascading does not change the transfer functions of the individual blocks if the output resistance is low.

First-Order Filters
- Bilinear transfer function $T(s) = \frac{a_0 s + a_0}{s + b_0}$
First-Order Filters (Cont’d)

\[ T(s) = \frac{a_1 s + a_0}{s + \omega_0} \]

\[ T(s) = \frac{a_1 s}{s + \omega_0} \]

\[ C_1 R_1 = \frac{a_0}{a_1} \]

\[ \text{dc gain} = \frac{R_1 + R_2}{C_1 + C_2} \]

\[ \text{HF gain} = \frac{C_1}{C_1 + C_2} \]

\[ C_1 R_1 = \frac{a_1}{a_0} \]

\[ \text{dc gain} = -\frac{R_2}{R_1} \]

\[ \text{HF gain} = -\frac{C_1}{C_1 + C_2} \]

\[ CR = \frac{1}{\omega_0} \]

\[ \text{Flat gain} (\alpha_1) = 0.5 \]

\[ CR = \frac{1}{\omega_0} \]

\[ \text{Flat gain} (\alpha_1) = 1 \]
Second-Order Filters

- Biquadratic transfer function

\[ T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0/Q)s + \omega_0^2} \]

- Pole frequency: \( \omega_0 \)
- Pole quality factor: \( Q \)
- Poles:

\[ p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0 \sqrt{1 - \frac{1}{4Q^2}} \]

- Bandwidth:

\[ BW \equiv \omega_2 - \omega_1 = \frac{\omega_0}{Q} \]
Second-Order Filters (Cont’d)

(d) Notch

\[ T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \]

dc gain = \[ \frac{\omega_0}{2Q} \]

(high-frequency gain = \( a_2 \))

(e) Low-Pass Notch (LPN)

\[ T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \]

\( \omega_n \to \omega_0 \)

dc gain = \( a_2 \frac{\omega_0^2}{\omega_0} \)

(high-frequency gain = \( a_2 \))

(f) High-Pass Notch (HPN)

\[ T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \]

\( \omega_n \to \omega_0 \)

dc gain = \( a_2 \frac{\omega_0^2}{\omega_0} \)

(high-frequency gain = \( a_2 \))
Second-Order Filters (Cont’d)

\(T(s) = \frac{s^2 - s \omega_0 + \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}\)

Flat gain = \(a_2\)
11.5 THE SECOND-ORDER LCR RESONATOR

The Resonator Natural Modes

- The LCR resonator can be excited by either current or voltage source.
- The excitation should be applied without change the natural structure of the circuit.
- The natural modes of the circuits are identical (will not be changed by the excitation methods).
- The similar characteristics also applies to series LCR resonator.

\[
\begin{align*}
\text{Current Excitation} & : \quad \frac{V_o}{I_i} = \frac{1}{Y} = \frac{1}{sL + sC + \frac{1}{R}} = \frac{s/C}{s^2 + (1/RC)s + 1/LC} \\
\text{Voltage Excitation} & : \quad \frac{V_o}{V_i} = \frac{(R \parallel 1/sC)}{(R \parallel 1/sC) + sL} = \frac{1/LC}{s^2 + (1/RC)s + 1/LC}
\end{align*}
\]

\[
\omega_0 = \frac{1}{\sqrt{LC}}, \quad Q = \omega_0 RC
\]
Realization of Transmission Zeros

- Values of $s$ at which $Z_2(s) = 0$ and $Z_1(s) \neq 0$
  $\rightarrow Z_2$ behaves as a short
- Values of $s$ at which $Z_1(s) = \infty$ and $Z_2(s) \neq \infty$
  $\rightarrow Z_1$ behaves as an open

Realization of Filter Functions

**Low-Pass Filter**

$$T(s) = \frac{V_o}{V_i} = \frac{1/LC}{s^2 + (1/RC)s + 1/LC}$$

**High-Pass Filter**

$$T(s) = \frac{V_o}{V_i} = \frac{s^2}{s^2 + (1/RC)s + 1/LC}$$

**Bandpass Filter**

$$T(s) = \frac{V_o}{V_i} = \frac{(1/RC)s}{s^2 + (1/RC)s + 1/LC}$$
11.6 SECOND-ORDER ACTIVE FILTERS (INDUCTOR REPLACEMENT)

Second-Order Active Filters by Op Amp-RC Circuits

- Inductors are not suitable for IC implementation
- Use op amp-RC circuits to replace the inductors
- Second-order filter functions based on RLC resonator

The Antoniou Inductance-Simulation Circuit

- Inductors are realized by op amp-RC circuits with negative feedbacks
- The equivalent inductance is given by

\[
Z_{in} \equiv \frac{V_1}{I_1} = sC_4R_1R_3R_5 / R_2 = sL_{eq}
\]

\[
L_{eq} = C_4R_1R_3R_5 / R_2
\]
The Op Amp-RC Resonator

- The inductor is replaced by the Antoniou circuit
- The pole frequency and the quality factor are given by

\[ \omega_0 = 1/\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2} \quad \text{and} \quad Q = \omega_0 C_6 R_6 = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}} \]

- The pole frequency and quality factor for a simplified case where \( R_1 = R_2 = R_3 = R_5 = R \) and \( C_4 = C \)

\[ \omega_0 = 1/RC \quad \text{and} \quad Q = R_6 / R \]
Filter Realization

Low-Pass Filter

High-Pass Filter

Bandpass Filter

Notch Filter
LPN Filter

HPN Filter

All-Pass Filter
11.7 SECOND-ORDER ACTIVE FILTERS (TWO-INTEGRATOR-LOOP)

Derivation of the Two-Integrator-Loop Biquad

\[
\frac{V_{hp}}{V_i} = \frac{Ks^2}{s^2 + (\omega_0/Q)s + \omega_0^2}
\]

\[
V_{hp} + \frac{1}{Q} \left( \frac{\omega_0}{s} V_{hp} \right) + \left( \frac{\omega_0^2}{s^2} V_{hp} \right) = KV_i
\]

- **High-pass implementation:**

\[
V_{hp} = KV_i - \frac{\omega_0}{Q} V_{hp} - \frac{\omega_0^2}{s^2} V_{hp}
\]

- **Bandpass implementation:**

\[
\frac{(-\omega_0/s)V_{hp}}{V_i} = \frac{-K\omega_0 s}{s^2 + (\omega_0/Q)s + \omega_0^2} = T_{hp}(s)
\]

- **Low-pass implementation:**

\[
\frac{(\omega_0^2/s^2)V_{hp}}{V_i} = \frac{K\omega_0^2}{s^2 + (\omega_0/Q)s + \omega_0^2} = T_{lp}(s)
\]
Circuit Implementation (I)

\[ V_{hp} = \frac{R_3}{R_2 + R_3} (1 + \frac{R_f}{R_1}) V_i + \frac{R_2}{R_2 + R_3} (1 + \frac{R_f}{R_1}) (-\frac{\omega_0}{s} V_{hp}) - \frac{R_f}{R_1} (\frac{\omega_0^2}{s^2} V_{hp}) \]

\[ \rightarrow R_f / R_1 = 1 \quad R_3 / R_2 = 2Q - 1 \quad K = 2 - 1/Q \]

- **High-pass transfer function:**
  \[ T_{hp}(s) = \frac{V_{hp}}{V_i} = \frac{s^2 [2R_3/(R_2 + R_3)]}{s^2 + s[2R_2/(R_2 + R_3)]\omega_0 + \omega_0^2} \]

- **Bandpass transfer function:**
  \[ T_{bp}(s) = \frac{V_{bp}}{V_i} = \frac{s[2R_3/(R_2 + R_3)]\omega_0}{s^2 + s[2R_2/(R_2 + R_3)]\omega_0 + \omega_0^2} \]

- **Low-pass transfer function:**
  \[ T_{lp}(s) = \frac{V_{lp}}{V_i} = \frac{2R_3/[2R_2/(R_2 + R_3)]\omega_0^2}{s^2 + s[2R_2/(R_2 + R_3)]\omega_0 + \omega_0^2} \]

- **Notch and all-pass transfer function:**
  \[ T(s) = \frac{V_o}{V_i} = -\frac{2R_3}{R_2 + R_3} (\frac{R_f}{R_l}) s^2 + (\frac{R_f}{R_l} \omega_0^2 s + (\frac{R_f}{R_l} \omega_0^2) \omega_0 s + (\frac{R_f}{R_l} \omega_0^2) \omega_0^2 \]

\[ R_f \quad R_f \quad R_f \]

\[ V_i \quad V_i \quad V_i \]

\[ R_1 \quad R_2 \quad R_3 \]

\[ C \quad C \]

\[ V_{hp} \quad V_{hp} \quad V_{hp} \]

\[ V_{bp} \quad V_{bp} \quad V_{bp} \]

\[ V_{lp} \quad V_{lp} \quad V_{lp} \]

\[ V_o \quad V_o \quad V_o \]
Circuit Implementation (II) – Tow-Thomas Biquad

- Use an additional inverter to make all the coefficients of the summer the same sign.
- All op amps are in single-ended mode.
- The high-pass function is no longer available.
- It is known as the Tow-Thomas biquad.
- An economical feedforward scheme can be employed with the Tow-Thomas circuit.

\[ T(s) = \frac{V_o}{V_i} = \frac{s^2 C_1 + s \left( \frac{1}{R_1} - \frac{r}{R_1 R_2} \right) + \frac{1}{C^2 R^2}}{s^2 + s \left( \frac{1}{QCR} \right) + \frac{1}{C^2 R^2}} \]
11.8 SINGLE-AMPLIFIER BIQUADRATIC ACTIVE FILTERS

Characteristics of the SAB Circuits
- Only one op amp is required to implement biquad circuit.
- Exhibit a greater dependence on the limited gain and bandwidth of the op amp.
- More sensitive to the unavoidable tolerances in the values of resistors and capacitors.
- Limited to less stringent filter specifications with pole $Q$ factors less than 10.

Synthesis of the SAB Circuits
- Use feedback to move the poles of an RC circuit from the negative real axis to the complex conjugate locations to provide selective filter response.
- Steps of SAB synthesis:
  - Synthesis of a feedback loop that realizes a pair of complex conjugate poles characterized by $\omega_0$ and $Q$.
  - Injecting the input signal in a way that realizes the desired transmission zeros.
- Natural modes of the filter:

\[
t(s) = \frac{N(s)}{D(s)}
\]

\[
L(s) = At(s) = \frac{AN(s)}{D(s)}
\]

The closed-loop characteristics equation:

\[
1 + L(s) = 0 \rightarrow t(s_p) = -1 / A \approx 0
\]

$\Rightarrow$ The poles of the closed-loop system are identical to the zeros of the $RC$ network.
RC Networks with complex transmission zeros

\[ t(s) = \frac{s^2 + s \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \frac{1}{R_3} + \frac{1}{C_1 C_2 R_3 R_4}}{s^2 + s \left( \frac{1}{C_1 R_3} + \frac{1}{C_2 R_3} + \frac{1}{C_1 R_4} \right) + \frac{1}{C_1 C_2 R_3 R_4}} \]

Characteristics Equation of the Filter

\[ s^2 + s \frac{\omega_0}{Q} + \omega_0^2 = s^2 + s \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \frac{1}{R_3} + \frac{1}{C_1 C_2 R_3 R_4} \]

\[ \rightarrow \omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}} \]

\[ \rightarrow Q = \left[ \frac{\sqrt{C_1 C_2 R_3 R_4}}{R_3} \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \right]^{-1} \]

Let \( C_1 = C_2 = C \), \( R_3 = R \), \( R_4 = R/m \) \( \rightarrow m = 4Q^2 \)

\[ \rightarrow CR = \frac{2Q}{\omega_0} \]
**Injection the Input Signal**

- The method of injection the input signal into the feedback loop through the grounded nodes.
- A component with a ground node can be disconnected from the ground and connected to the input source.
- The filter transmission zeros depends on the components through which the input signal is injected.

\[ V_o = \frac{-s(\alpha/C_1 R_4)}{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1 C_2 R_3 R_4}} \]
Generation of Equivalent Feedback Loops

Characteristics Equation:
\[ 1 + L(s) = 0 \rightarrow 1 + At(s) = 0 \]

Characteristics Equation:
\[ 1 - \frac{A}{A+1}(1-t) = 0 \rightarrow 1 + At(s) = 0 \]
Generation of Equivalent Feedback Loops (Cont’d)
11.9 SENSITIVITY

Filter Sensitivity
- Deviation in filter response due to the tolerances in component values
- Especially for RC component values and amplifier gain

Classical Sensitivity Function
- Definition:
  \[
  S_y^x = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \frac{y}{x}
  \]
  \[
  S_y^x = \frac{\partial y}{\partial x} \frac{x}{y}
  \]
- For small changes:
  \[
  S_y^x \approx \frac{\Delta y}{\Delta x} \frac{y}{x}
  \]
11.10 SWITCHED-CAPACITOR FILTERS

**Basic Principle**

- A capacitor switched between two nodes at a sufficiently high rate is equivalent to a resistor.
- The resistor in the active-RC integrator can be replaced by the capacitor and the switches.
- Equivalent resistor:
  \[
  i_{av} = \frac{C_i v_i}{T_c} \rightarrow R_{eq} \equiv \frac{v_i}{i_{av}} = C_1 T_c
  \]
- Equivalent time constant for the integrator = \( T_c \left( \frac{C_2}{C_1} \right) \)
Practical Circuits

- Can realize both inverting and non-inverting integrator
- Insensitive to stray capacitances
- Noninverting switched-capacitor (SC) integrator

- Inverting switched-capacitor (SC) integrator
Filter Implementation

- Circuit parameters for the two integrators with the same time constant

\[ T_c \frac{C_2}{C_3} = T_c \frac{C_1}{C_4} \rightarrow C_1 = C_2 = C \rightarrow C_3 = C_4 = KC \]

\[ \omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}} = \frac{1}{T_c} \sqrt{\frac{C_3 C_4}{C_2 C_1}} = \frac{K}{T_c} \]

\[ Q = \frac{R_3}{R_4} = \frac{C_4}{C_5} \rightarrow C_5 = \omega_0 T_c \frac{C}{Q} \]
11.11 TUNED AMPLIFIERS

Characteristics of a Tuned Amplifier

- Center frequency: $\omega_0$
- 3-dB bandwidth: $B$
- Skirt selectivity: $S$

(3-dB bandwidth is less than 5% of $\omega_0$ in many applications → narrow-band characteristics.

The circuits studied are small-signal voltage amplifiers in which transistors operate in class A mode.

The Basic Principle for Single-Tuned Amplifiers

- Parallel $RLC$ circuit is used as the load for an amplifier.
- The amplifier exhibits a second-order bandpass characteristic.

$$
\begin{align*}
V_o &= -g_m V_i = \frac{-g_m V_i}{sC + 1/R + 1/sL} \\
V_o/V_i &= -g_m \frac{s}{s^2 + s(1/RC) + 1/LC} \\
\omega_0 &= 1/\sqrt{LC} \\
B &= 1/RC \\
Q &= \omega_0/B = \omega_0 RC \\
\frac{V_o(j\omega_0)}{V_i(j\omega_0)} &= -g_m R
\end{align*}
$$
Inductor Losses

\[ Q \equiv \frac{\omega L}{r_s} \rightarrow Q_0 \equiv \frac{\omega_0 L}{r_s} \]

\[ Y(j\omega_0) = \frac{1}{r_s + j\omega_0 L} = \frac{1}{j\omega_0 L 1 - j(1/Q_0)} = \frac{1}{1 + j(1/Q_0)} \]

For \( Q_0 \gg 1 \rightarrow Y(j\omega_0) \approx \frac{1}{j\omega_0 L \left(1 + j \frac{1}{Q_0}\right)} = \frac{1}{j\omega_0 L} + \frac{1}{\omega_0 L Q_0} \]

\[ R_p \approx \omega_0 L Q_0 = (\omega_0 L)^2 / r_s \]

- Power loss in a practical inductor is represented by a series resistance \( r_s \).
- Rather than specifying the value of \( r_s \), a inductor is specified by the \( Q \) factor at the frequency at interest.
- It is desirable to replace the series connection of \( L \) and \( r_s \) with an equivalent parallel connection of \( L \) and \( R_p \).
- \( R_p \) is expressed in terms of \( r_s, L \) and the frequency of interest \( \omega_0 \) under the assumption that \( Q_0 \gg 1 \).
- In fact, the value of resulting \( R_p \) varies with frequency, however, it is approximated as a constant resistance at frequencies around \( \omega_0 \) to simplify the analysis.
- High inductor \( Q \) factor means low inductor loss
  - \( r_s \) as small as possible for a given \( L \).
  - \( R_p \) as large as possible for a given \( L \).
The Use of Transformers

- In many cases, the required values of inductance and capacitance are not practical
  - use a transformer to effect an impedance change.
  - use a tapped coil (autotransformer).

\[
\begin{align*}
R & \quad L & \quad C \\
\end{align*}
\]

\[
\begin{align*}
R & \quad L & \quad C' = \frac{C}{n^2} \\
L' &= n^2L
\end{align*}
\]
Amplifiers with Multiple Tuned Circuits

- Multiple tuned circuits are used if high selectivity is required → tuned circuit at both input and output.
- **Radio-frequency choke** (RFC) is frequently used for bias circuit
  - RFC behaves as a short-circuit at dc → provide dc biasing.
  - RFC exhibit high impedance at frequency of interest → eliminate the loading effect of dc bias circuit.
- The Miller capacitance $C_{\mu}$ may results in many problems in the multiple tuned circuit
  - The Miller impedance at the input is complex since the load is not simply resistive.
  - The reflected impedance will cause detuning of the input circuit.
  - Skewing of the response of the input circuit.
  - May results in circuit oscillation.
- Using additional feedback circuits to provide current cancellation can neutralize the effect of $C_{\mu}$.
- An alternative approach is to change the circuit configuration to avoid Miller effect.
The Cascode and CC-CB Cascade

- Two amplifier configuration do not suffer the Miller Effect and are suitable for multiple tuned circuits:
  - Cascode configuration
  - CC-CB cascade configuration
- The CC-CB cascade is usually preferred in IC implementations because of its differential structure.
Synchronous Tuning

- Assume the stages do not interact → the overall response is the product of the individual responses.
- Synchronous tuning → cascading $N$ identical resonant circuits.
- The 3-dB bandwidth $B$ of the overall amplifier is related to that of the individual tuned circuit ($\omega_0/Q$):

$$B = \frac{\omega_0}{Q} \sqrt{2^{1/N} - 1}$$

- Bandwidth-shrinkage factor: $\sqrt{2^{1/N} - 1}$
  - Bandwidth decreased by the factor of $\sqrt{2^{1/N} - 1}$
  - $Q$ factor increased by the factor of $\sqrt{2^{1/N} - 1}$

Given $B$ and $N$, we can determine the bandwidth required of the individual stages.
Stagger-Tuning

- Stagger-tuned amplifiers exhibit overall response with maximal flatness around the center frequency $f_0$.
- Such a response can be obtained by transforming the response of a Butterworth low-pass filter to $\omega_0$.
- The transfer function of a second-order bandpass filter can be expressed in terms of its poles as:

$$T(s) = \frac{a_ts}{\left(s + \omega_0 - j\omega_0\sqrt{1 - \frac{1}{4Q^2}}\right)\left(s + \omega_0 + j\omega_0\sqrt{1 - \frac{1}{4Q^2}}\right)}$$

- For a narrow-band filter $Q >> 1$, and for values of $s$ in the neighborhood of $+j\omega_0$ the transfer function is approximated as (narrow-band approximation):

$$T(s) \approx \frac{a_t}{(s + \omega_0 / 2Q - j\omega_0)(2j\omega_0)} = \frac{a_t / 2}{s + \omega_0 / 2Q - j\omega_0} = \frac{a_t / 2}{(s - j\omega_0) + \omega_0 / 2Q}$$

- The response of the second-order bandpass filter in the neighborhood of its center frequency $s = j\omega_0$ is identical to a first-order low-pass filter with a pole at $(-\omega_0 / 2Q)$ in the neighborhood of $p = 0$.
- The transformation $p = s - j\omega_0$ can be applied to low-pass filters of order greater than one.
- The second-order narrow-band bandpass filter:

- The fourth-order stagger-tuned narrow-band bandpass amplifier: