CHAPTER 13 FILTERS AND TUNED AMPLIFIERS

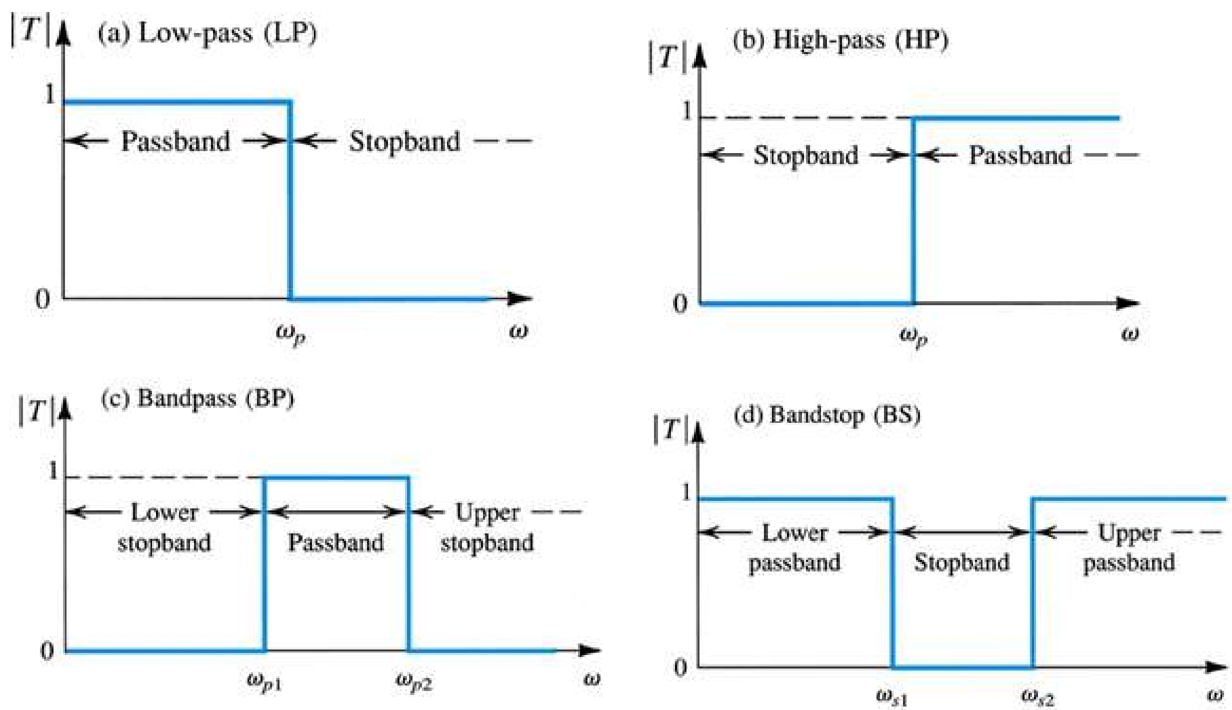
Chapter Outline

13.1 Filter Transmission, Types and Specifications

Filter Transfer Function

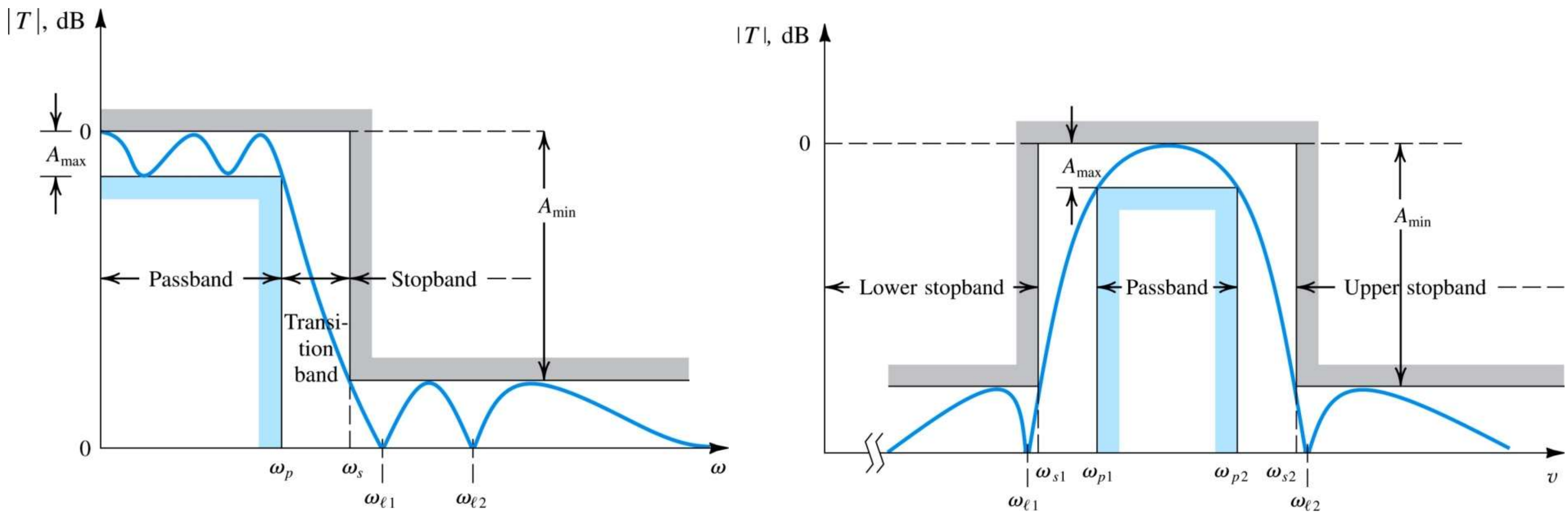
- A filter is a **linear** two-port network represented by the ratio of the output to input voltage
- Transfer function $T(s) \equiv V_o(s)/V_i(s)$
- Transmission : evaluating $T(s)$ for physical frequency $s = j\omega \rightarrow T(j\omega) = |T(j\omega)| \cdot e^{j\phi(\omega)}$
  - Gain: $20 \log|T(j\omega)|$ (dB)
  - Attenuation: $-20 \log|T(j\omega)|$ (dB)
- Output frequency spectrum : $|V_o(s)| = |T(s)|\,|V_i(s)|$

Types of Filters



Filter Specification

- Passband edge : ω_p
- Maximum allowed variation in passband transmission : A_{max}
- Stopband edge : ω_s
- Minimum required stopband attenuation : A_{min}



- The first step of filter design is to determine the filter specifications
- Then find a transfer function $T(s)$ whose magnitude $|T(j\omega)|$ meets the specifications
- The process of obtaining a transfer function that meets given specifications is called **filter approximation**

13.2 Filter Transfer Function

Transfer Function

- The filter transfer function is written as the ratio of two polynomials:

$$T(s)=\frac{a_M s^M + a_{M-1}s^{M-1}+\ldots+a_0}{s^N + b_{N-1}s^{N-1}+\ldots+b_0}$$

- The degree of the denominator → filter order
- To ensure the stability of the filter → $N \geq M$
- The coefficients a_i and b_j are real numbers
- The transfer function can be factored and expressed as:

$$T(s)=\frac{a_M(s-z_1)(s-z_2)\ldots(s-z_M)}{(s-p_1)(s-p_2)\ldots(s-p_N)}$$

- Zeros: z_1 , z_2 , … , z_M and (N–M) zeros at infinity
- Poles: p_1 , p_2 , … , p_N
- Zeros and poles can be either a real or a complex number
- Complex zeros and poles must occur in conjugate pairs
- The poles have to be on the LHP of s-plane

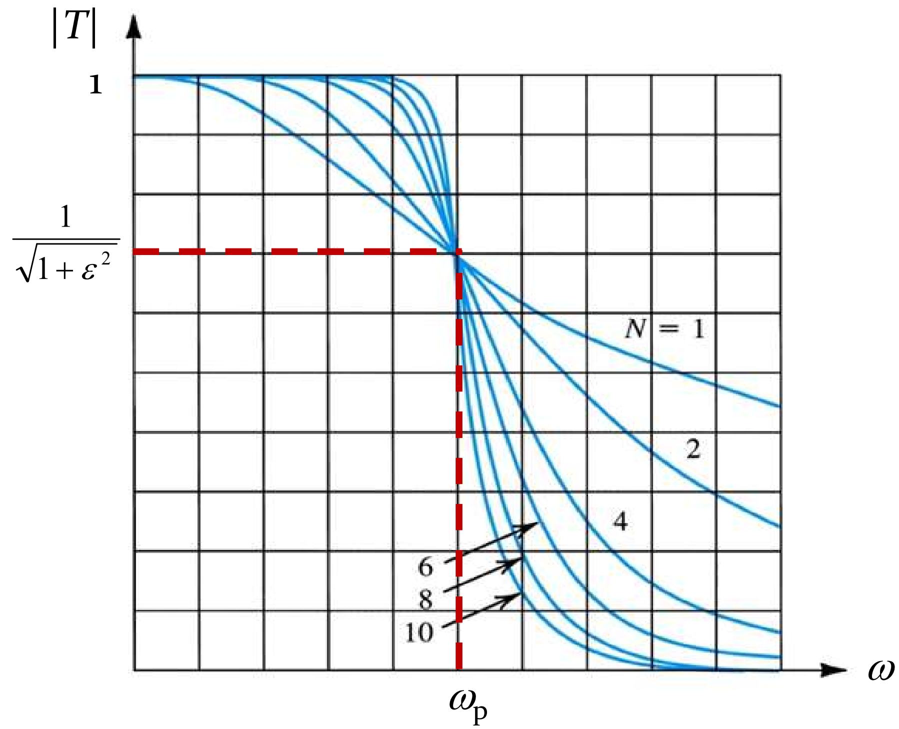
13.3 Butterworth and Chebyshev Filters

The Butterworth Filter

- Butterworth filters exhibit monotonically decreasing transmission with all zeros at $\omega = \infty$
- Maximally flat response → degree of passband flatness increases as the order N is increased
- Higher order filter has a sharp cutoff in the transition band
- The magnitude function of the Butterworth filter is:

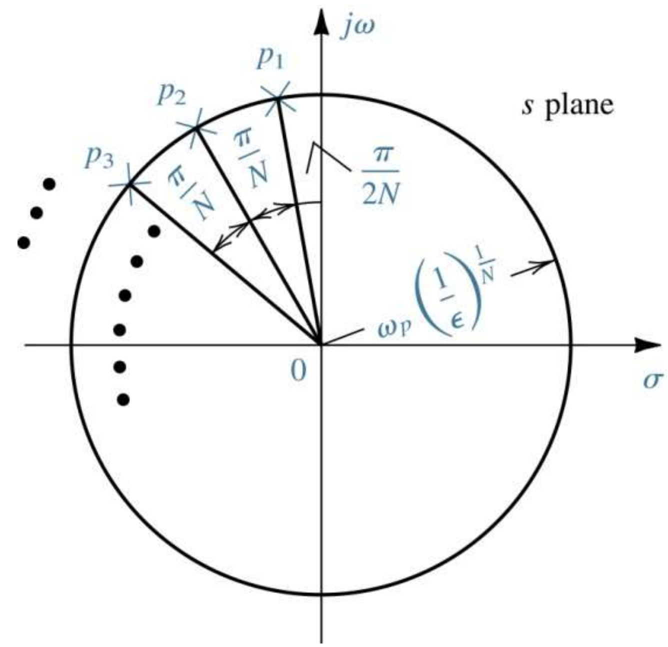
$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2(\omega/\omega_p)^{2N}}}$$

- Required transfer functions can be defined based on filter specifications (A_{max}, A_{min}, ω_p , ω_s)



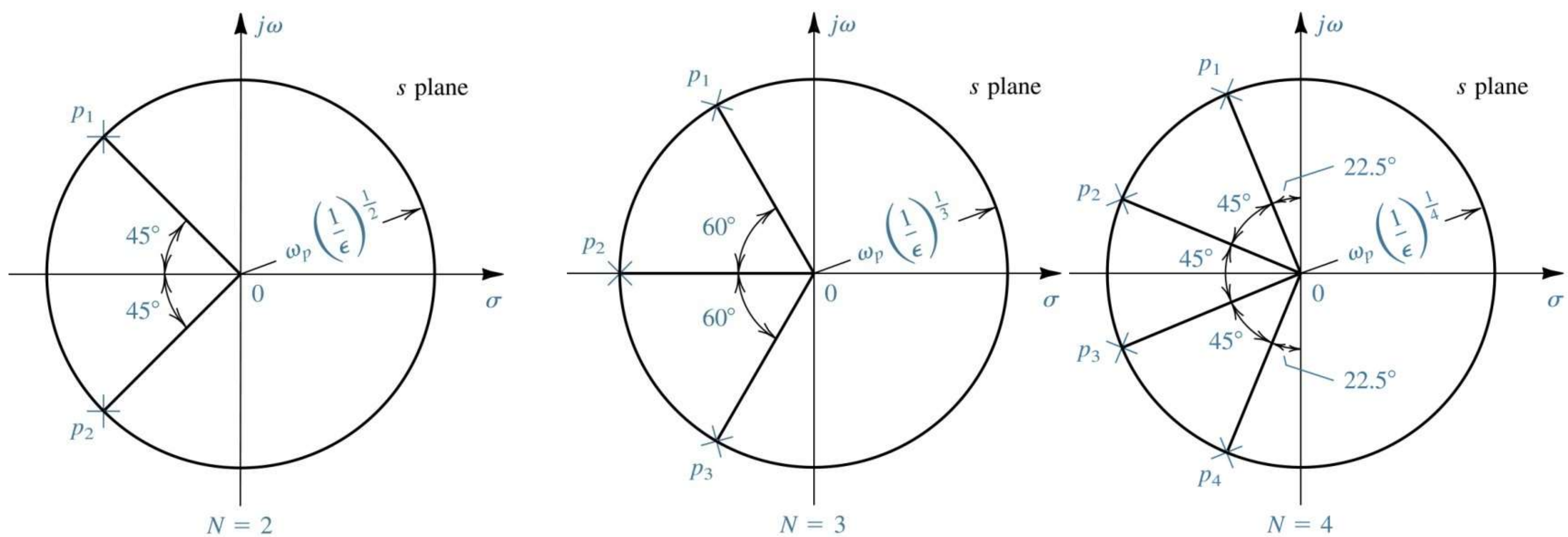
Natural Modes of the Butterworth Filter

- The natural modes (poles) locate on a circle
- The radius of the circle is $\omega_0 = \omega_p (1/\varepsilon)^{1/N}$
- Equal angle space $\dfrac{\pi}{N}$



$$T(s) = \frac{K}{\left(1-\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)\cdots\left(1-\frac{s}{p_N}\right)}$$

$$= \frac{K\omega_0^N}{(s-p_1)(s-p_2)\ldots(s-p_N)} \qquad \text{where } \omega_0 = \omega_p (1/\varepsilon)^{1/N}$$



Design Procedure of the Butterworth Filters

Design Specifications

A_{max}, A_{min}, ω_p, ω_s

Design Procedure

1. Determine ε (from A_{max})

$$|T(j\omega_p)| = \frac{1}{\sqrt{1+\varepsilon^2}} \rightarrow A_{max}[dB] = 20\log\sqrt{1+\varepsilon^2} \rightarrow \varepsilon = \sqrt{10^{A_{max}/10}-1}$$

2. Determine the required filter order N (from ω_p , ω_s , A_{min})

$$\text{Attenuation } A(\omega_s)[dB] = -20\log[1/\sqrt{1+\varepsilon^2(\omega_s/\omega_p)^{2N}}] = 10\log[1+\varepsilon^2(\omega_s/\omega_p)^{2N}] \geq A_{min}$$

3. Determine the N natural modes (poles) with $\omega_0 = \omega_p(1/\varepsilon)^{1/N}$

$p_1, p_2, \ldots\ p_N$

4. Determine $T(s)$

$$T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\ldots(s-p_N)} \quad \text{where } \omega_0 = \omega_p(1/\varepsilon)^{1/N}$$

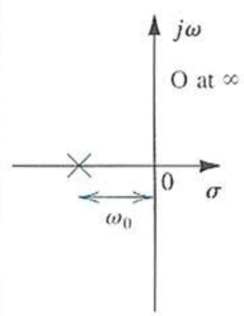
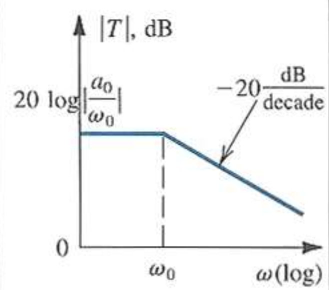
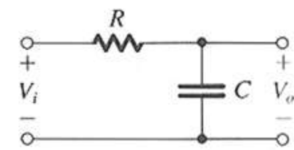
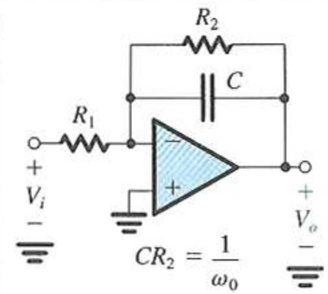
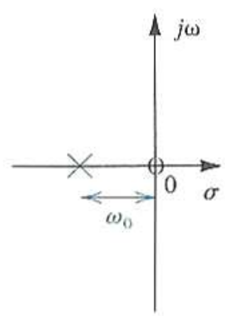
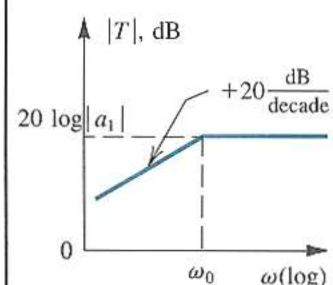
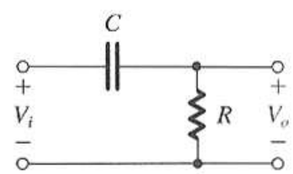
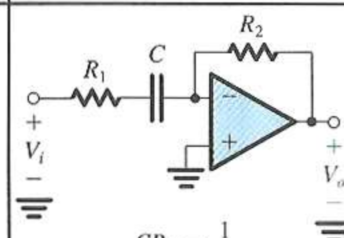
13.4 First-Order and Second-Order Filter Functions

Cascade Filter Design

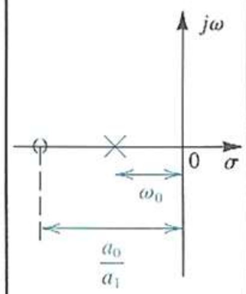
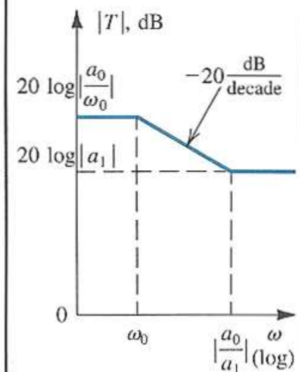
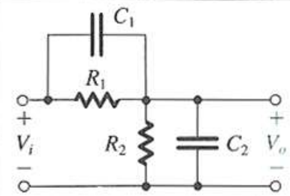
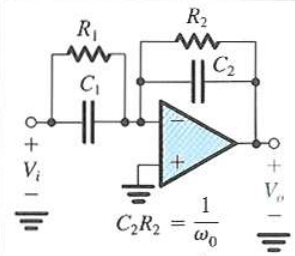
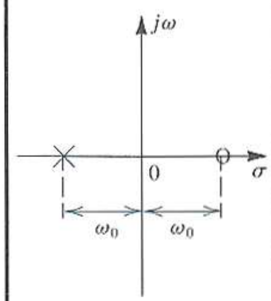
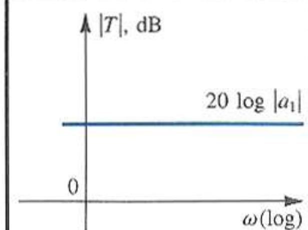
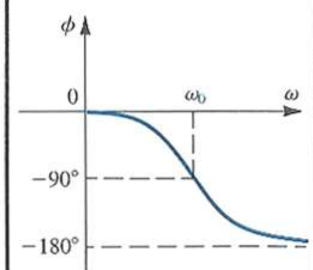
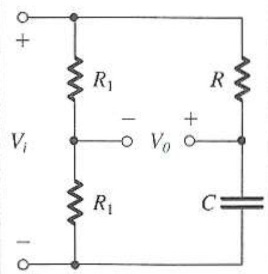
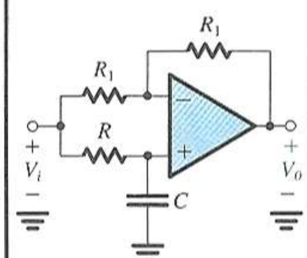
- First-order and second-order filters can be cascaded to realize high-order filters
- Cascade design is one of the most popular methods for the design of active filters
- Cascading does not change the transfer functions of individual blocks if the output resistance is low

First-Order Filters

- Bilinear transfer function $T(s)=\dfrac{a_1 s+a_0}{s+b_0}=\dfrac{a_1 s+a_0}{s+\omega_0}$

Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\lvert T\rvert$	Passive Realization	Op Amp-RC Realization
(a) Low-Pass (LP) $T(s)=\dfrac{a_0}{s+\omega_0}$			 $CR=\dfrac{1}{\omega_0}$ dc gain = 1	 $CR_2=\dfrac{1}{\omega_0}$ dc gain $=-\dfrac{R_2}{R_1}$
(b) High-Pass (HP) $T(s)=\dfrac{a_1 s}{s+\omega_0}$			 $CR=\dfrac{1}{\omega_0}$ High-frequency gain = 1	 $CR_1=\dfrac{1}{\omega_0}$ High-frequency gain $=-\dfrac{R_2}{R_1}$

First-Order Filters (Cont'd)

$T(s)$	Singularities	$\|T\|$ and ϕ	Passive Realization	Op Amp-RC Realization
(c) General $T(s) = \frac{a_1 s + a_0}{s + \omega_0}$			 $(C_1 + C_2)(R_1 // R_2) = \frac{1}{\omega_0}$ $C_1R_1 = \frac{a_0}{a_1}$ dc gain $= \frac{R_2}{R_1 + R_2}$ HF gain $= \frac{C_1}{C_1 + C_2}$	 $C_2R_2 = \frac{1}{\omega_0}$ $C_1R_1 = \frac{a_1}{a_0}$ dc gain $= -\frac{R_2}{R_1}$ HF gain $= -\frac{C_1}{C_2}$
$T(s) = -a_1\frac{s - \omega_0}{s + \omega_0}$ $a_1 > 0$			 $CR = 1/\omega_0$ Flat gain $(a_1) = 0.5$	 $CR = 1/\omega_0$ Flat gain $(a_1) = 1$

Second-Order Filters

- Biquadratic transfer function

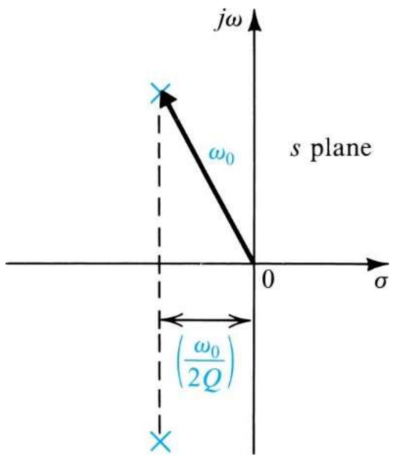
$$T(s)=\frac{a_2s^2+a_1s+a_0}{s^2+(\omega_0/Q)s+\omega_0^2}$$

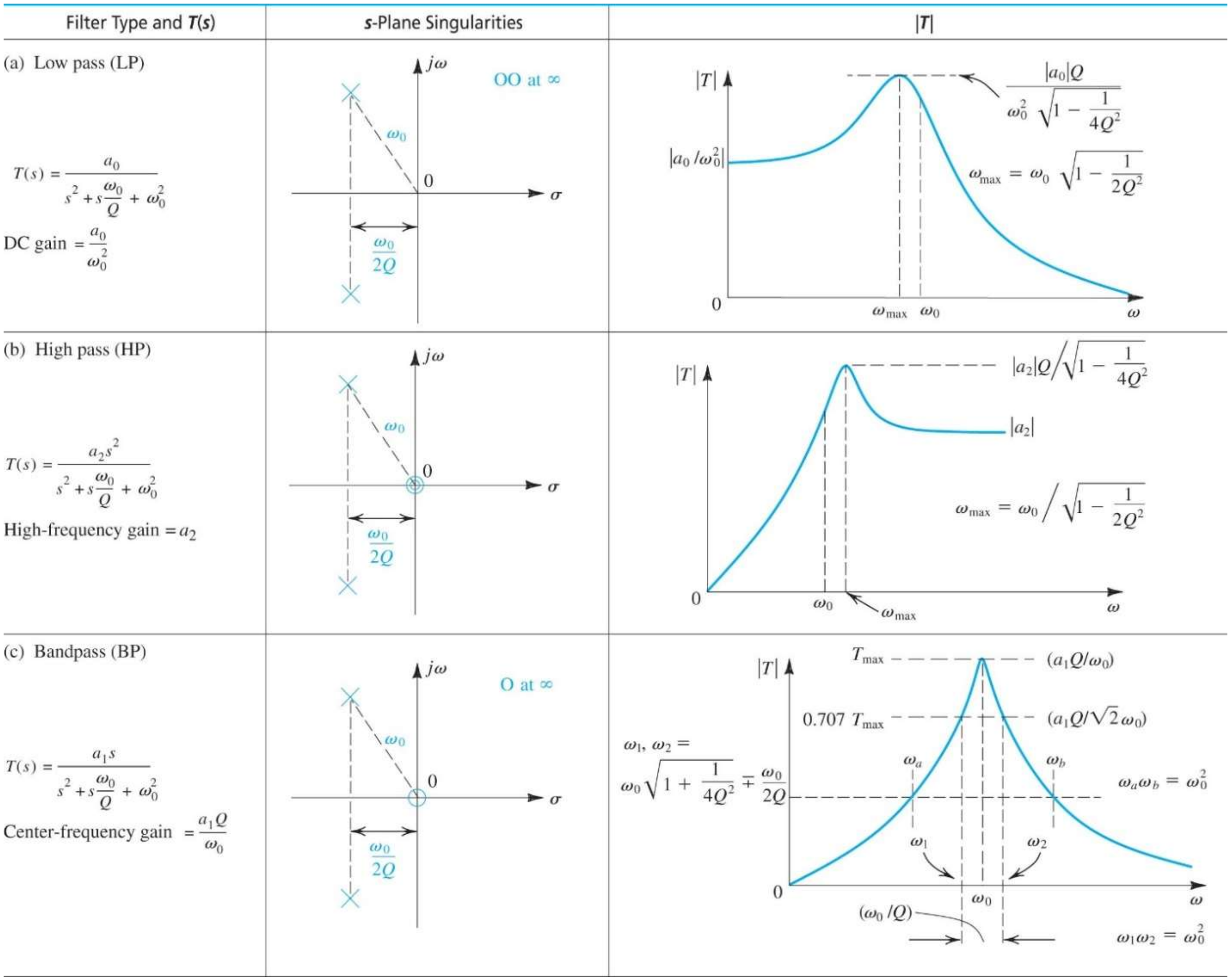
- Pole frequency: ω_o
- Pole quality factor: Q
- Poles:

$$p_1, p_2=-\frac{\omega_0}{2Q}\pm j\omega_0\sqrt{1-\frac{1}{4Q^2}}$$

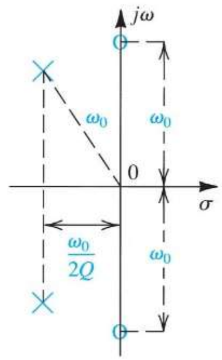
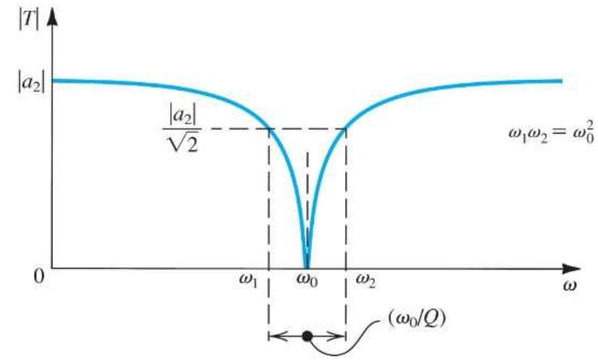
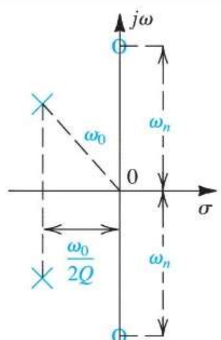
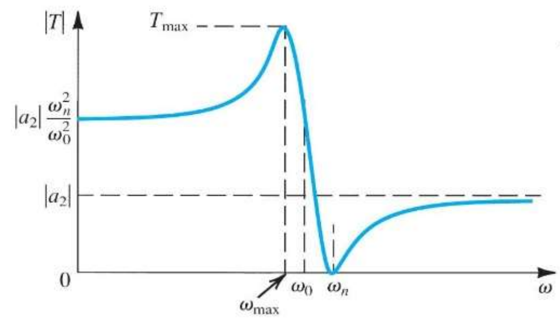
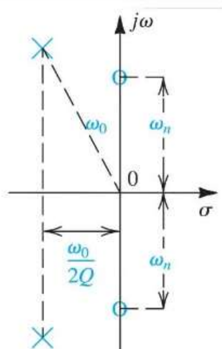
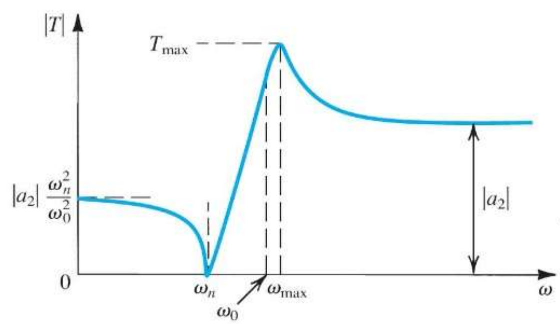
- Bandwidth:

$$BW\equiv\omega_2-\omega_1=\frac{\omega_0}{Q}$$





Second-Order Filters (Cont'd)

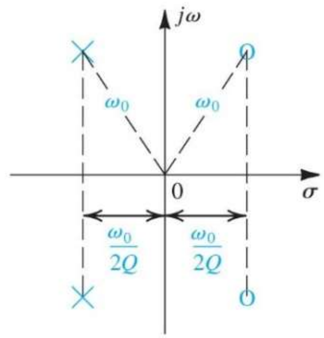
Filter Type and $T(s)$	s-Plane Singularities	$\lvert T \rvert$
(d) Notch $T(s) = a_2 \dfrac{s^2 + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ DC gain = High-frequency gain $= a_2$		
(e) Low-pass notch (LPN) $T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \geq \omega_0$ DC gain $= a_2 \dfrac{\omega_n^2}{\omega_0^2}$ High-frequency gain $= a_2$		 $\omega_{max} = \omega_0 \sqrt{\dfrac{\left(\dfrac{\omega_n^2}{\omega_0^2}\right)\left(1 - \dfrac{1}{2Q^2}\right) - 1}{\dfrac{\omega_n^2}{\omega_0^2} + \dfrac{1}{2Q^2} - 1}}$ $T_{max} = \dfrac{\lvert a_2 \rvert \quad \lvert \omega_n^2 - \omega_{max}^2 \rvert}{\sqrt{(\omega_0^2 - \omega_{max}^2)^2 + \left(\dfrac{\omega_0}{Q}\right)^2 \omega_{max}^2}}$
(f) High-pass notch (HPN) $T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \leq \omega_0$ DC gain $= a_2 \dfrac{\omega_n^2}{\omega_0^2}$ High-frequency gain $= a_2$		

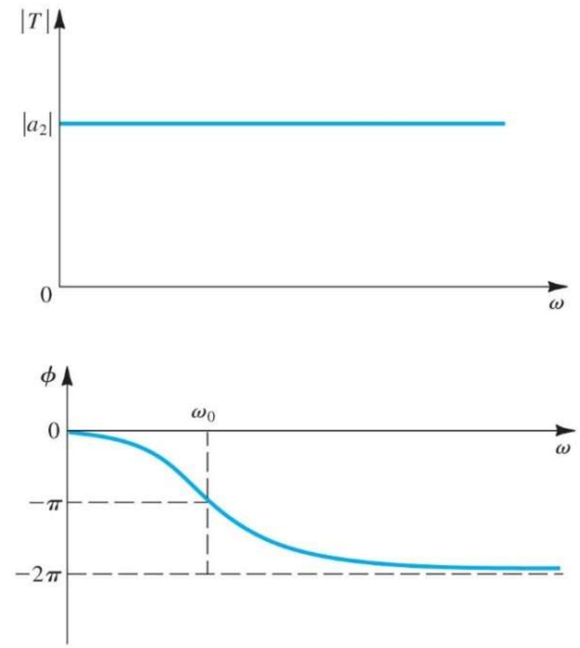
Second-Order Filters (Cont'd)

(g) All pass (AP)

$$T(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Flat gain $= a_2$

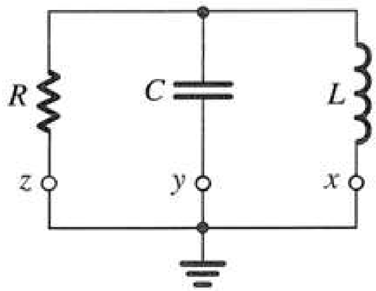




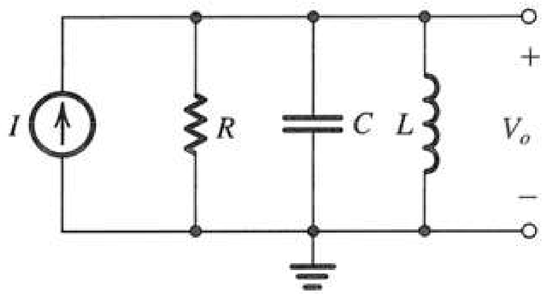
13.5 The Second-Order LCR Resonator

The Resonator Natural Modes

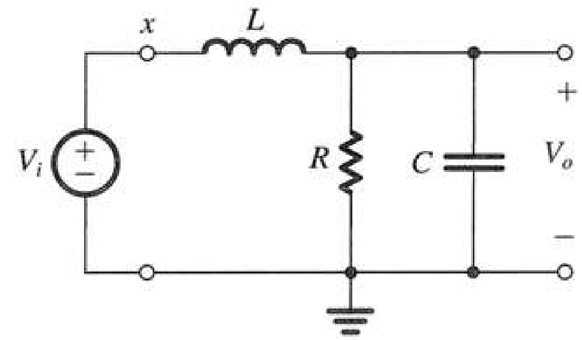
Parallel Resonator



Current Excitation



Voltage Excitation

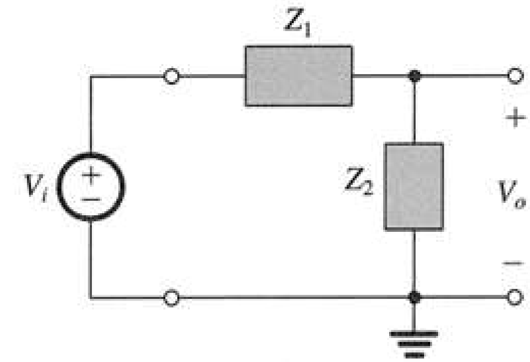


$$\begin{cases} \text{Current Excitation} & \dfrac{V_o}{I_i} = \dfrac{1}{Y} = \dfrac{1}{1/sL + sC + 1/R} = \dfrac{s/C}{s^2 + (1/RC)s + 1/LC} \\[2ex] \text{Voltage Excitation} & \dfrac{V_o}{V_i} = \dfrac{(R \,\|\, 1/sC)}{(R \,\|\, 1/sC) + sL} = \dfrac{1/LC}{s^2 + (1/RC)s + 1/LC} \end{cases} \Rightarrow \begin{cases} \omega_0 = 1/\sqrt{LC} \\[2ex] Q = \omega_0 RC \end{cases}$$

- ❑ The *LCR* resonator can be excited by either current or voltage source
- ❑ The excitation should be applied without change the natural structure of the circuit
- ❑ The natural modes of the circuits are identical (will not be changed by the excitation methods)
- ❑ The similar characteristics also applies to series *LCR* resonator

Realization of Transmission Zeros

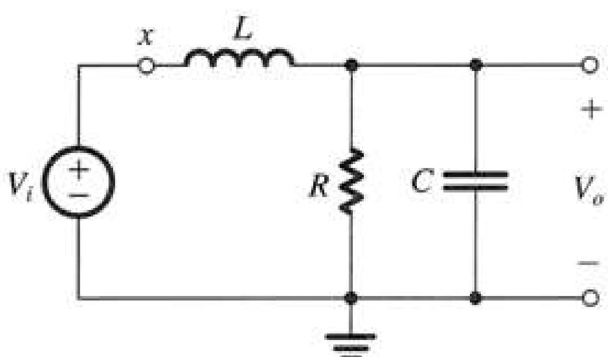
- Values of s at which $Z_2(s) = 0$ and $Z_1(s) \neq 0$
  $\rightarrow Z_2$ behaves as a short
- Values of s at which $Z_1(s) = \infty$ and $Z_2(s) \neq \infty$
  $\rightarrow Z_1$ behaves as an open



Realization of Filter Functions

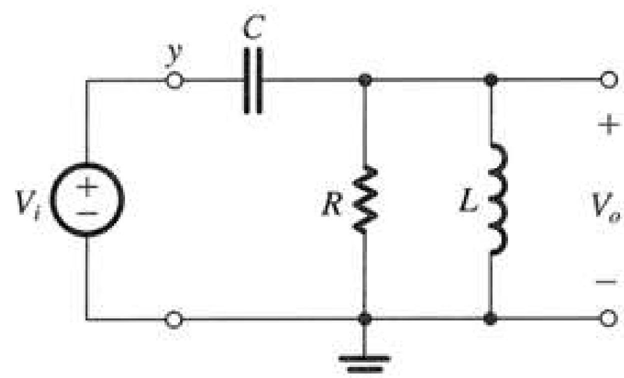
Low-Pass Filter

$$T(s) = \frac{V_o}{V_i} = \frac{1/LC}{s^2 + (1/RC)s + 1/LC}$$



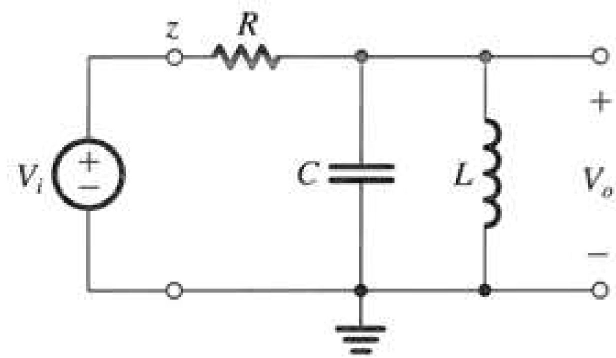
High-Pass Filter

$$T(s) = \frac{V_o}{V_i} = \frac{s^2}{s^2 + (1/RC)s + 1/LC}$$

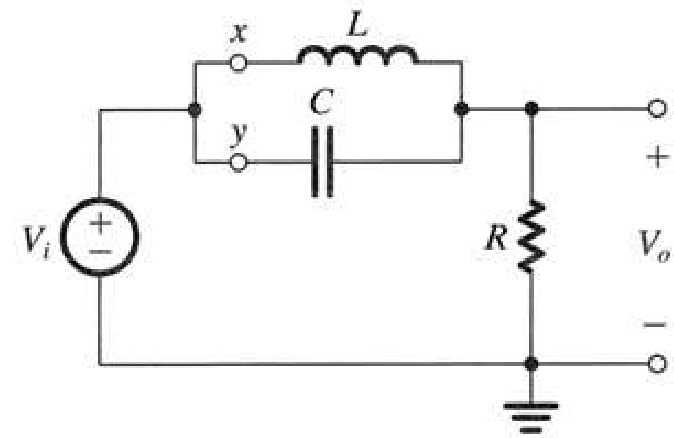


Bandpass Filter

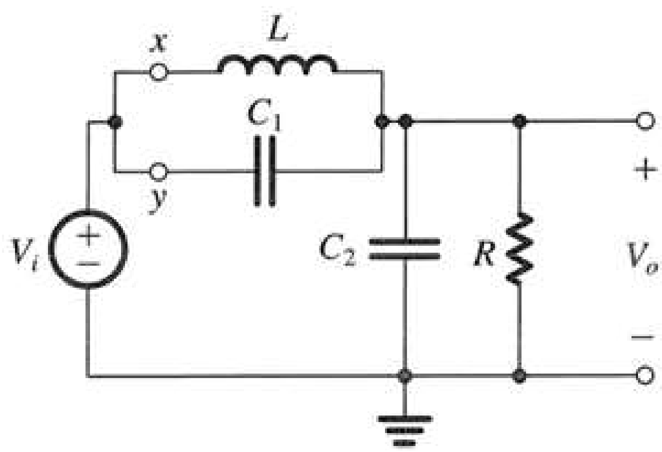
$$T(s) = \frac{V_o}{V_i} = \frac{(1/RC)s}{s^2 + (1/RC)s + 1/LC}$$



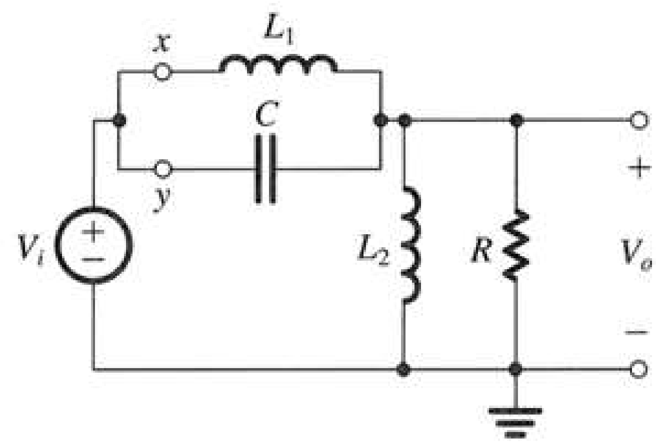
Notch Filter



Low-Pass Notch Filter



High-Pass Notch Filter



13.6 Second-Order Active Filters (Inductor Replacement)

Second-Order Active Filters by Op Amp-RC Circuits

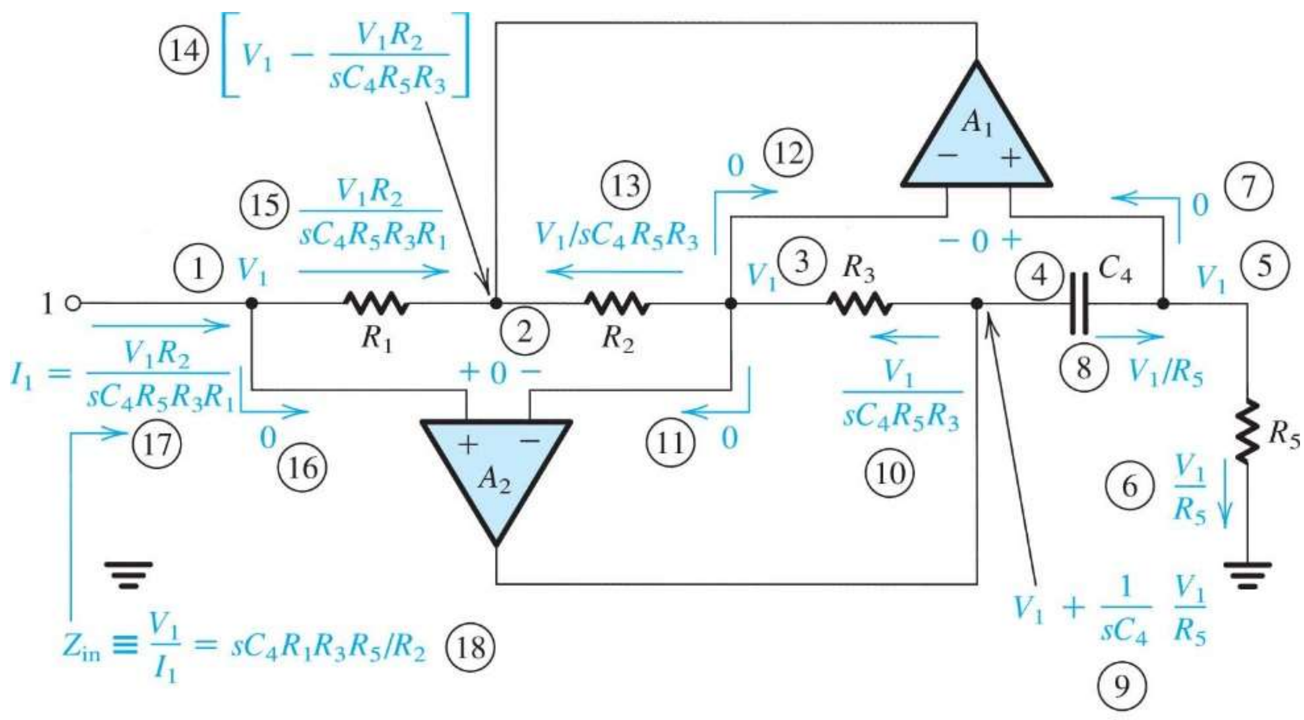
- Inductors are not suitable for IC implementation
- Use op amp-RC circuits to replace the inductors
- Second-order filter functions based on RLC resonator

The Antoniou Inductance-Simulation Circuit

- Inductors are realized by op amp-RC circuits with negative feedbacks
- The equivalent inductance is given by

$$Z_{in} \equiv \frac{V_1}{I_1} = sC_4R_1R_3R_5 / R_2 = sL_{eq}$$

$$L_{eq} = C_4R_1R_3R_5 / R_2$$



(b)

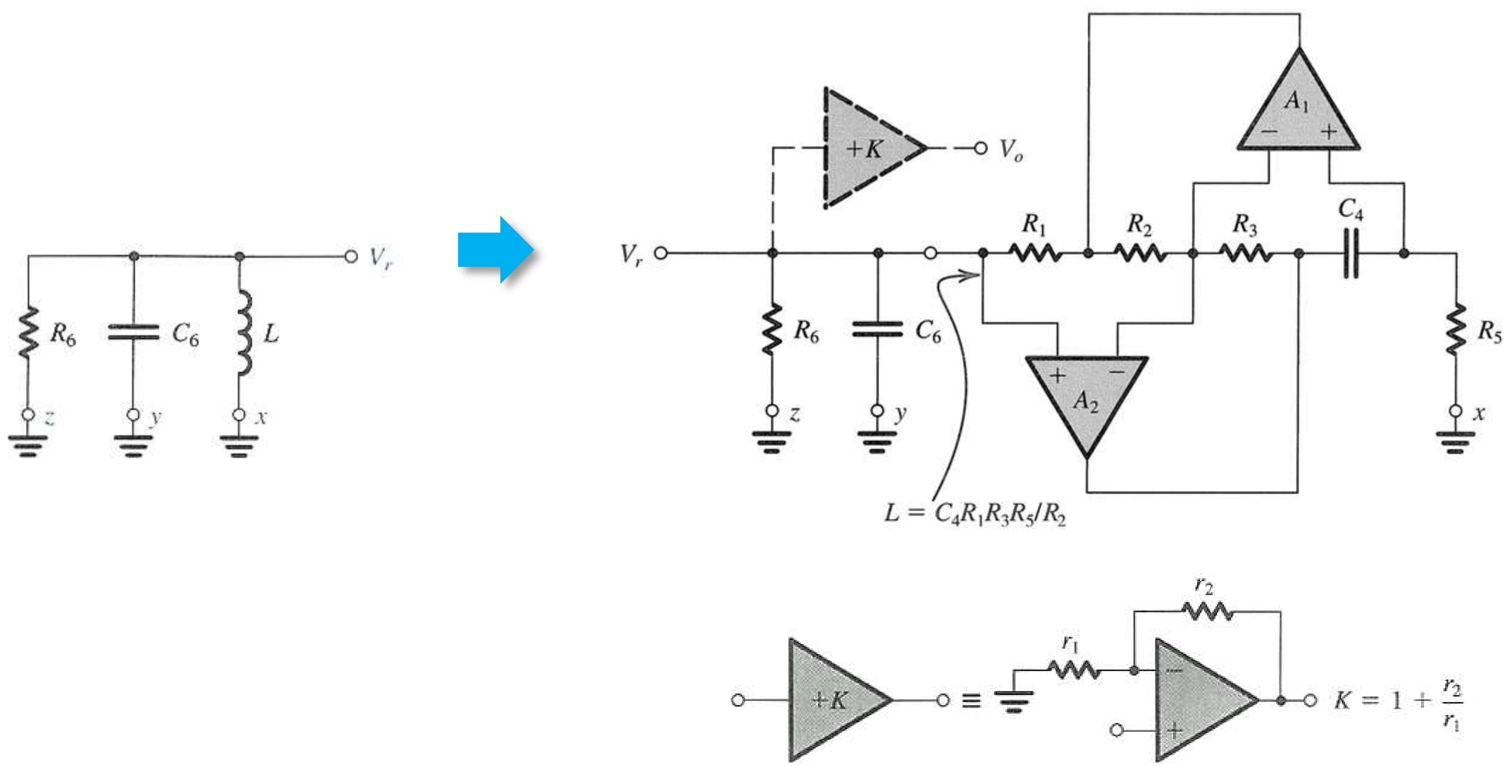
The Op Amp-RC Resonator

- The inductor is replaced by the Antoniou circuit
- The pole frequency and the quality factor are given by

$$\omega_0 = 1/\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2} \qquad Q = \omega_0 C_6 R_6 = R_6 \sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1 R_3 R_5}}$$

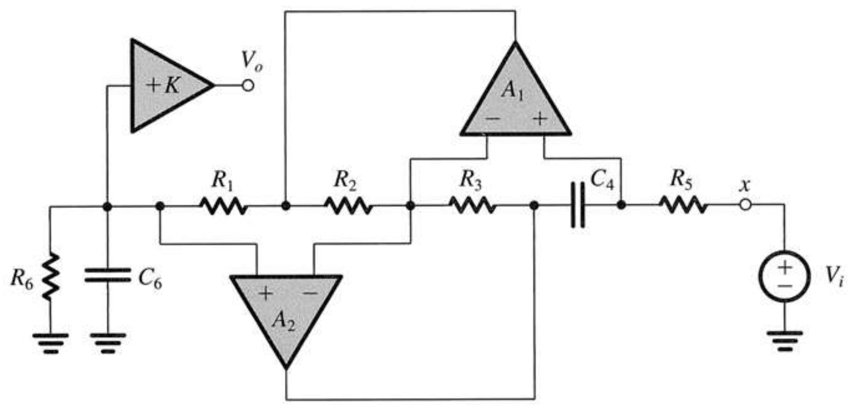
- A simplified case where $R_1 = R_2 = R_3 = R_5 = R$ and $C_4 = C_6 = C$

$$\omega_0 = 1/RC \qquad Q = R_6 / R$$

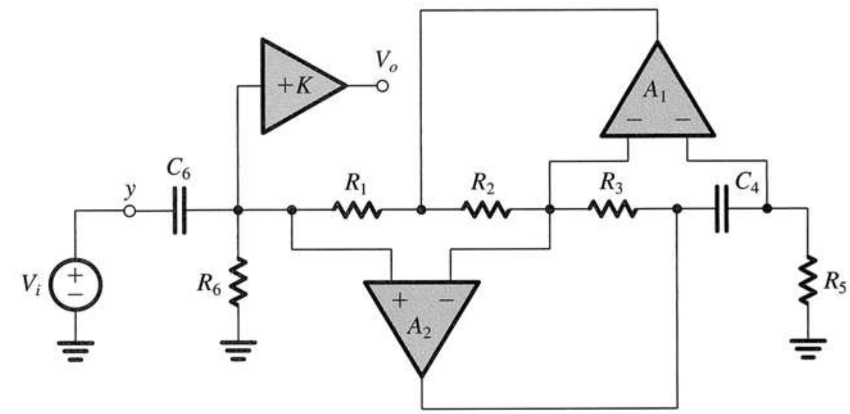


Filter Realization

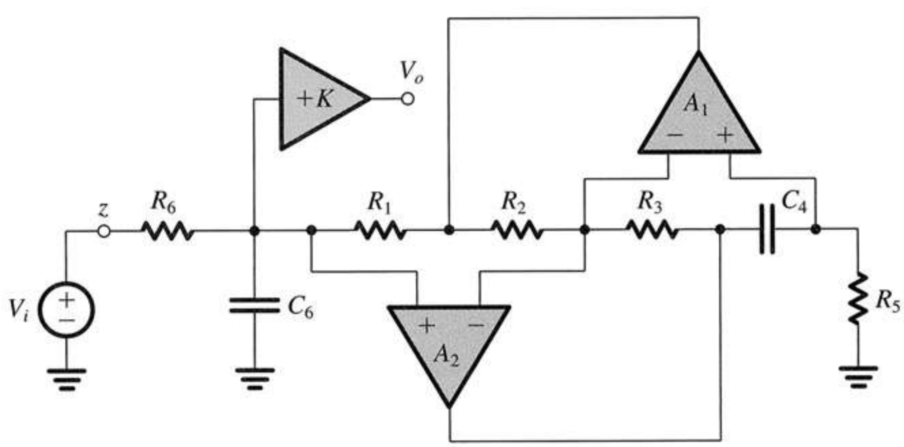
Low-Pass Filter



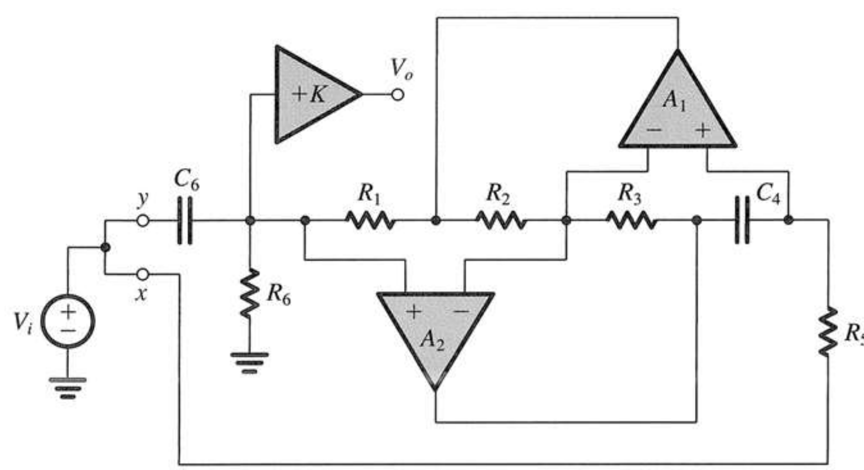
High-Pass Filter



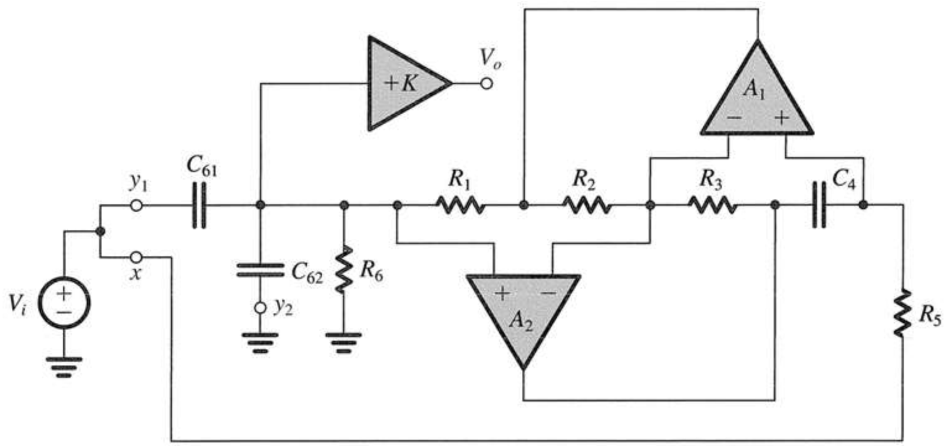
Bandpass Filter



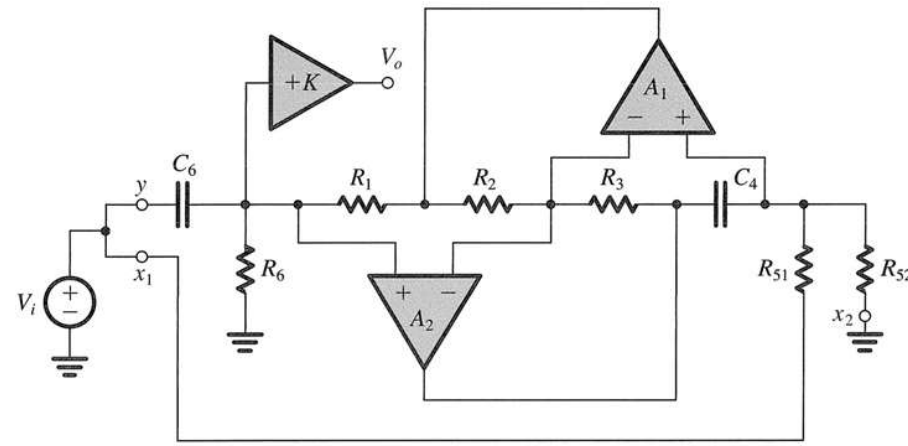
Notch Filter



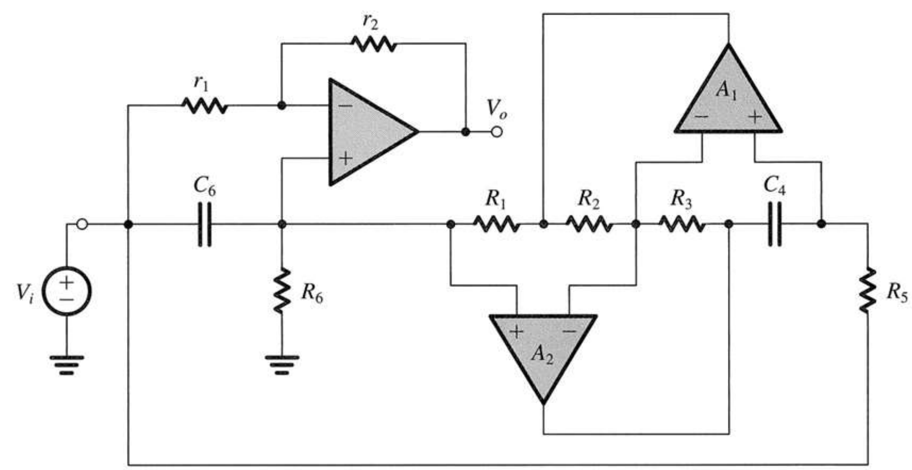
LPN Filter



HPN Filter



All-Pass Filter



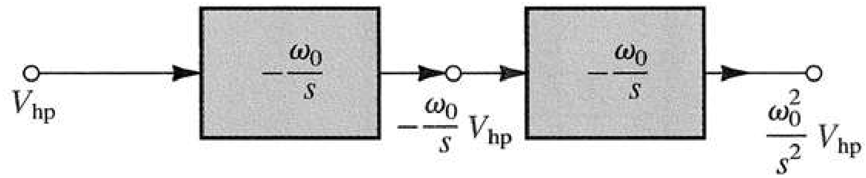
13.7 Second-Order Active Filters (Two-Integrator-Loop)

Derivation of the Two-Integrator-Loop Biquad

- High-pass implementation:

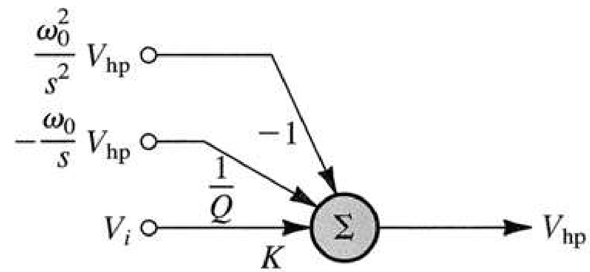
$$V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_0}{s}V_{hp} - \frac{\omega_0^2}{s^2}V_{hp}$$

$$T_{hp}(s) = \frac{Ks^2}{s^2 + (\omega_0/Q)s + \omega_0^2}$$



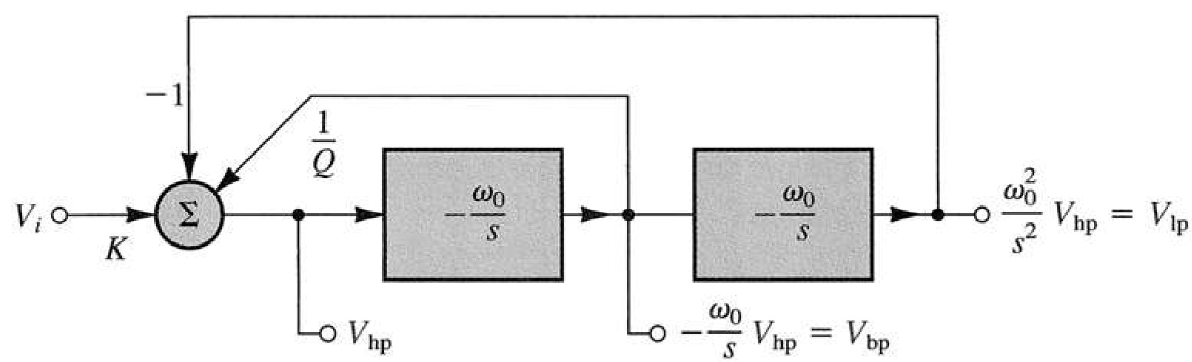
- Band-pass implementation:

$$T_{bp}(s) = \left(-\frac{\omega_0}{s}\right)V_{hp}(s) = \frac{-K\omega_0 s}{s^2 + (\omega_0/Q)s + \omega_0^2}$$



- Low-pass implementation:

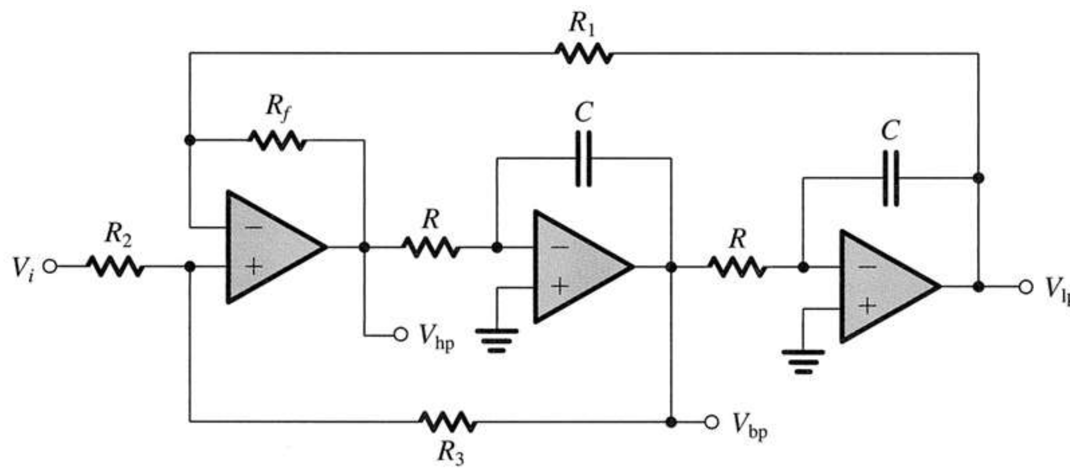
$$T_{lp}(s) = \left(-\frac{\omega_0}{s}\right)^2 V_{hp}(s) = \frac{K\omega_0^2}{s^2 + (\omega_0/Q)s + \omega_0^2}$$



Circuit Implementation (I)

$$V_{hp} = \frac{R_3}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)V_i + \frac{R_2}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right)\left(-\frac{\omega_0}{s}V_{hp}\right) - \frac{R_f}{R_1}\left(\frac{\omega_0^2}{s^2}V_{hp}\right)$$

$$\rightarrow \frac{R_f}{R_1} = 1 \quad \frac{R_3}{R_2} = 2Q-1 \quad K = 2-\frac{1}{Q}$$



- ❑ High-pass transfer function:

$$T_{hp}(s) = \frac{V_{hp}}{V_i} = \frac{s^2[2R_3/(R_2+R_3)]}{s^2+s[2R_2/(R_2+R_3)]\omega_0+\omega_0^2}$$

- ❑ Band-pass transfer function:

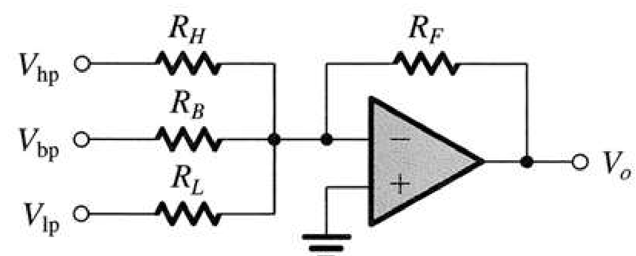
$$T_{bp}(s) = \frac{V_{bp}}{V_i} = \frac{s[2R_3/(R_2+R_3)]\omega_0}{s^2+s[2R_2/(R_2+R_3)]\omega_0+\omega_0^2}$$

- ❑ Low-pass transfer function:

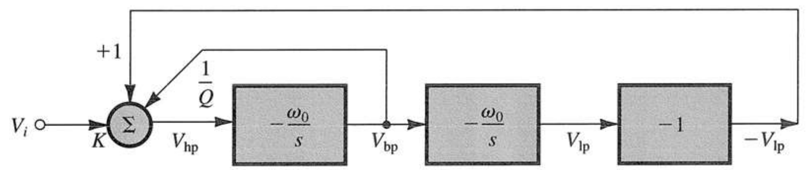
$$T_{bp}(s) = \frac{V_{bp}}{V_i} = \frac{[2R_3/(R_2+R_3)]\omega_0^2}{s^2+s[2R_2/(R_2+R_3)]\omega_0+\omega_0^2}$$

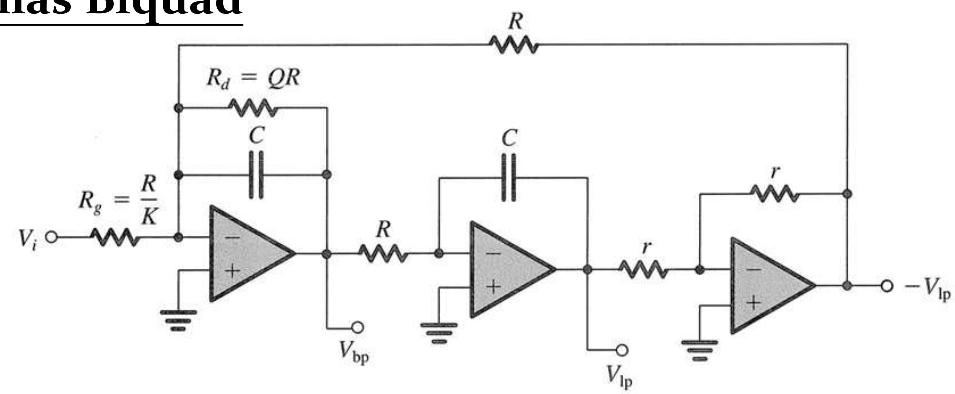
- ❑ Notch and all-pass transfer function:

$$T(s) = \frac{V_o}{V_i} = -\frac{2R_3}{R_2+R_3}\frac{(R_F/R_H)s^2+(R_F/R_B)\omega_0 s+(R_F/R_L)\omega_0^2}{s^2+s[2R_2/(R_2+R_3)]\omega_0+\omega_0^2}$$



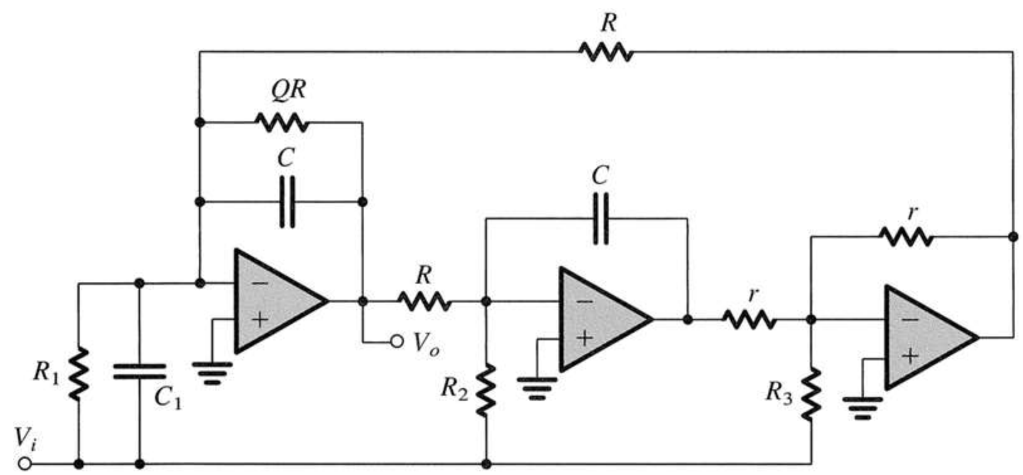
Circuit Implementation (II) – Tow-Thomas Biquad





- Use an additional inverter to make all the coefficients of the summer the same sign
- All op amps are in single-ended mode
- The high-pass function is no longer available
- It is known as the Tow-Thomas biquad
- An economical feedforward scheme can be employed with the Tow-Thomas circuit

$$T(s) = \frac{V_o}{V_i} = -\frac{s^2\frac{C_1}{C} + s\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2RR_2}}{s^2 + s\frac{1}{QCR} + \frac{1}{C^2R^2}}$$



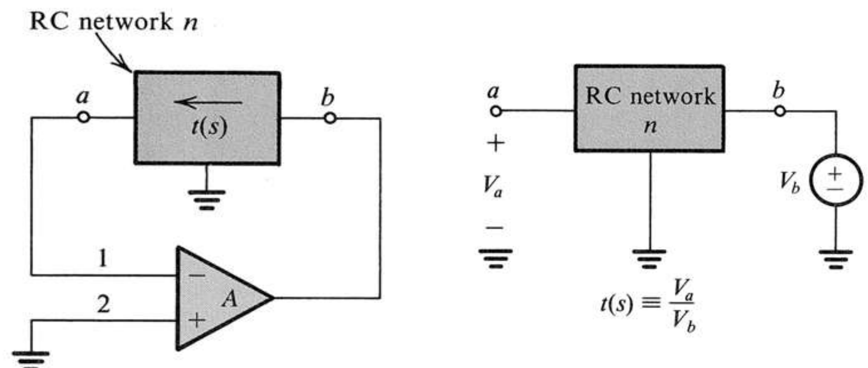
13.8 Single-Amplifier Biquadratic Active Filters

Characteristics of the SAB Circuits

- ❑ Only one op amp is required to implement biquad circuit
- ❑ Exhibit greater dependence on the limited gain and bandwidth of the op amp
- ❑ More sensitive to the unavoidable tolerances in the values of resistors and capacitors
- ❑ Limited to less stringent filter specifications with pole Q factors less than 10

Synthesis of the SAB Circuits

- ❑ Use feedback to move the poles of an RC circuit from the negative real axis to the complex conjugate locations to provide selective filter response
- ❑ Steps of SAB synthesis:
  - ■ Synthesis of a feedback loop that realizes a pair of complex conjugate poles characterized by ω_o and Q
  - ■ Injecting the input signal in a way that realizes the desired transmission zeros
- ❑ Natural modes of the filter:



$$t(s) = N(s)/D(s)$$

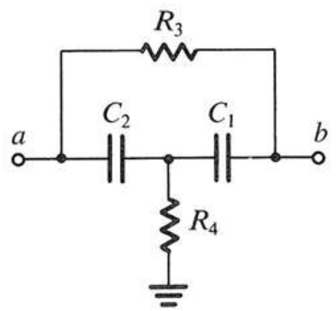
$$L(s) = At(s) = AN(s)/D(s)$$

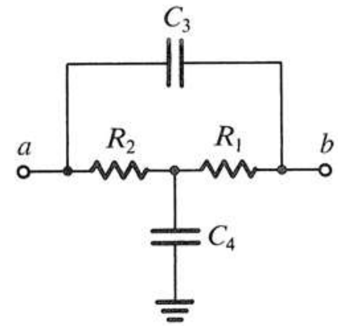
The closed-loop characteristics equation:

$$1 + L(s) = 0 \rightarrow t(s_P) = -1/A \approx 0$$

$\Rightarrow$ The poles of the closed-loop system are identical to the zeros of the *RC* network

RC Networks with complex transmission zeros





$$t(s)=\frac{s^2+s\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3}+\frac{1}{C_1C_2R_3R_4}}{s^2+s\left(\frac{1}{C_1R_3}+\frac{1}{C_2R_3}+\frac{1}{C_1R_4}\right)+\frac{1}{C_1C_2R_3R_4}}$$

$$t(s)=\frac{s^2+s\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\frac{1}{C_4}+\frac{1}{R_1R_2C_3C_4}}{s^2+s\left(\frac{1}{R_2C_4}+\frac{1}{R_2C_3}+\frac{1}{R_1C_4}\right)+\frac{1}{R_1R_2C_3C_4}}$$

Characteristics Equation of the Filter

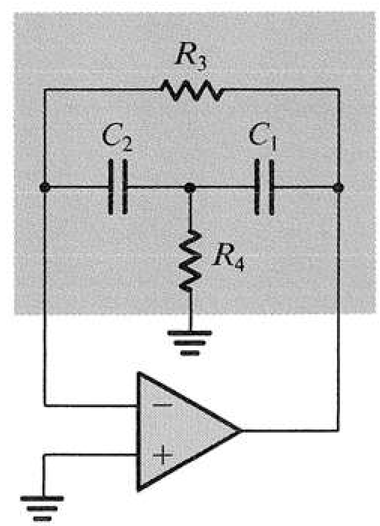
$$s^2+s\frac{\omega_0}{Q}+\omega_0^2=s^2+s\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\frac{1}{R_3}+\frac{1}{C_1C_2R_3R_4}$$

$$\rightarrow \omega_0=\frac{1}{\sqrt{C_1C_2R_3R_4}}$$

$$\rightarrow Q=\left[\frac{\sqrt{C_1C_2R_3R_4}}{R_3}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right]^{-1}$$

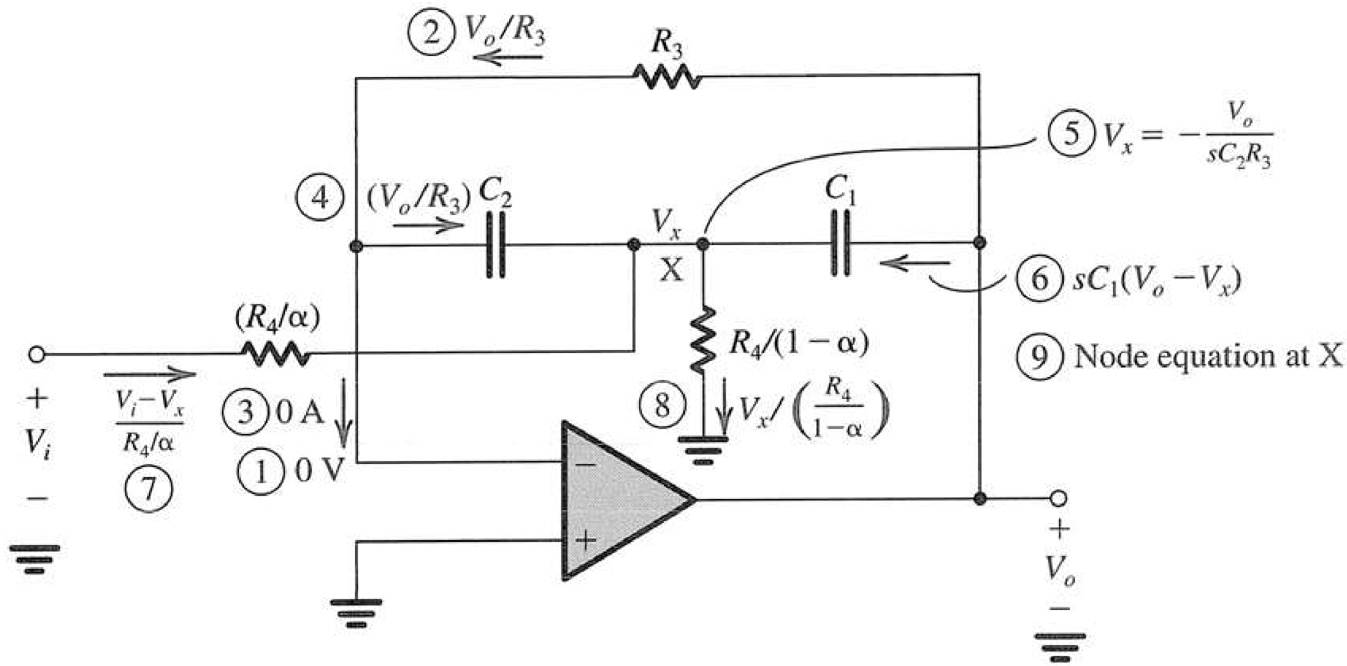
Let $C_1 = C_2 = C$, $R_3 = R$, $R_4 = R/m$ $\rightarrow m=4Q^2$

$\rightarrow CR=2Q/\omega_0$



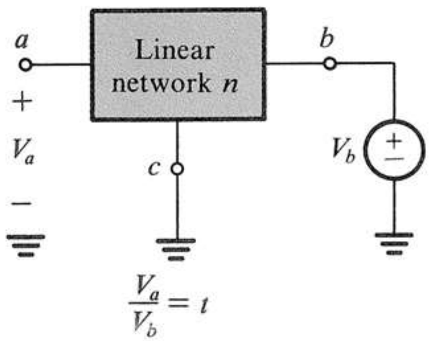
Injection the Input Signal

- The method of injection the input signal into the feedback loop through the grounded nodes
- A component with a ground node can be connected to the input source
- The filter transmission zeros depends on where the input signal is injected



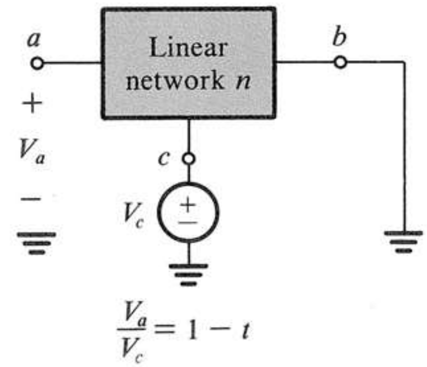
$$\frac{V_o}{V_i} = \frac{-s(\alpha / C_1R_4)}{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}$$

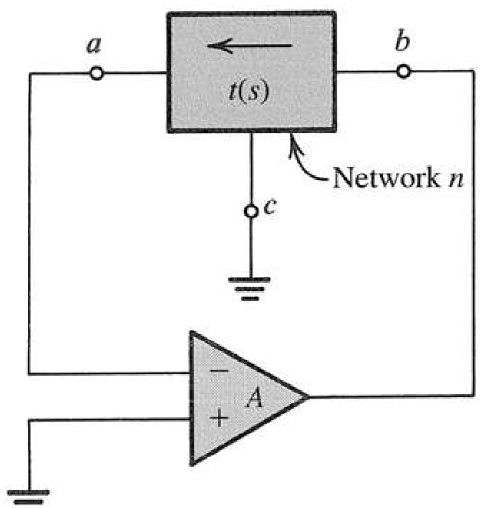
Generation of Equivalent Feedback Loops



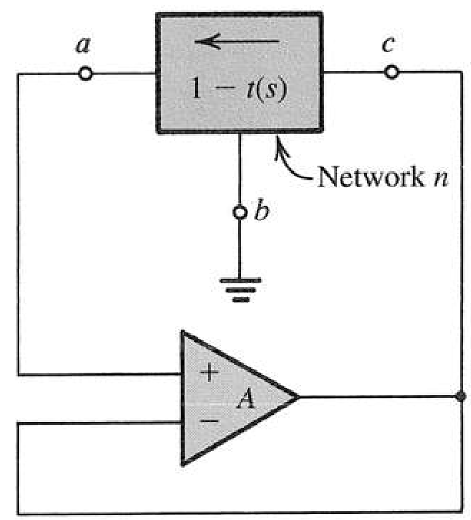
$$\frac{V_a}{V_b} = t$$

$$\frac{V_a}{V_c} = 1 - t$$





Equivalent Loop



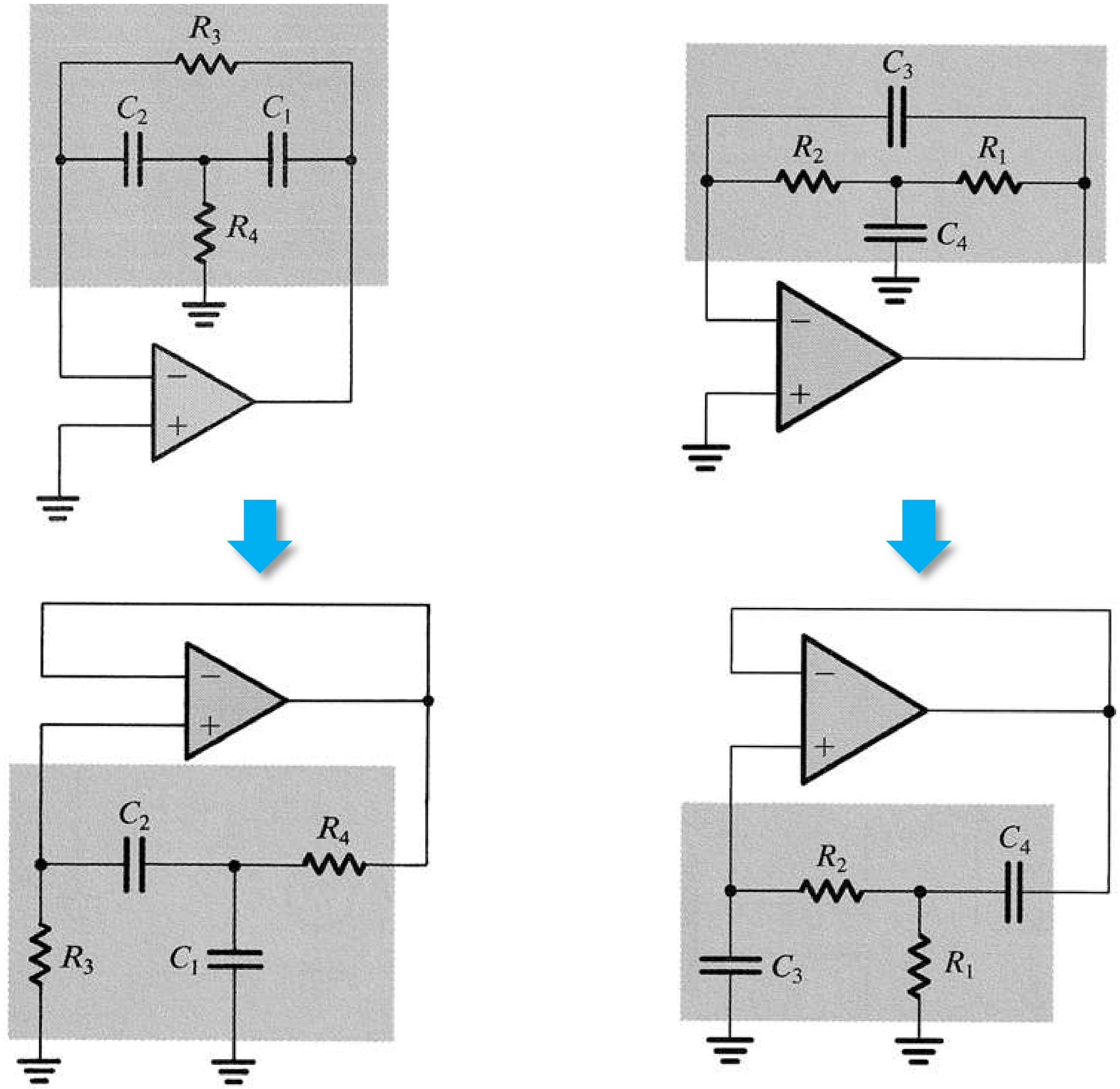
Characteristics Equation:

$$1 + L(s) = 0 \rightarrow 1 + At(s) = 0$$

Characteristics Equation:

$$1 - \frac{A}{A+1}(1-t) = 0 \rightarrow 1 + At(s) = 0$$

Generation of Equivalent Feedback Loops (Cont'd)



13.9 Sensitivity

Filter Sensitivity

- Deviation in filter response due to the tolerances in component values
- Especially for RC component values and amplifier gain

Classical Sensitivity Function

- Definition:

$$S_x^y \equiv \lim_{\Delta x \to 0} \frac{\Delta y / y}{\Delta x / x}$$

$$S_x^y = \frac{\partial y}{\partial x} \frac{x}{y}$$

- For small changes:

$$S_x^y \approx \frac{\Delta y / y}{\Delta x / x}$$

13.10 Transconductance-C Filters

Limitations of Op Amp-RC Circuits

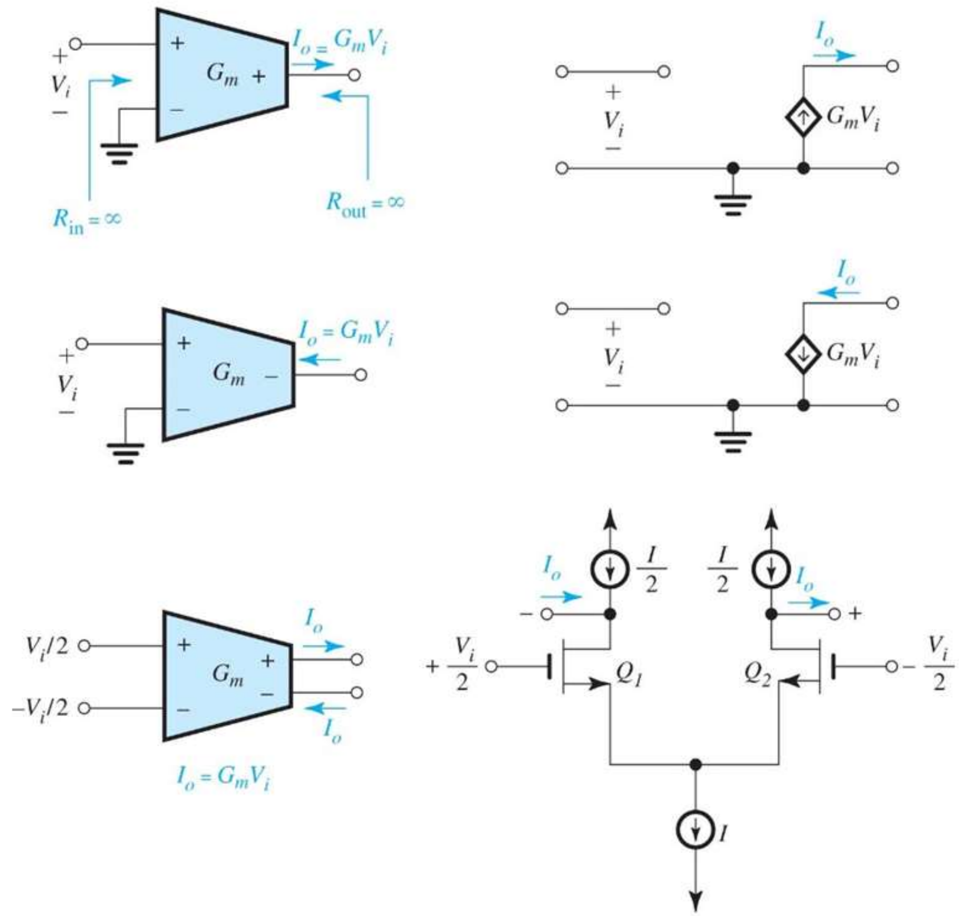
- Suitable for audio-frequency filters using discrete op amps, resistors and capacitors
- High-frequency applications limited by the relatively low bandwidth of general-purpose op amps
- Impractical for IC implementations due to:
  - The need for large capacitors and resistors increases the IC cost
  - The need for very precise values of RC time constant requires expensive trimming/tuning
  - The need for op amps that can drive resistive and large capacitive loads

Methods for IC Filter Implementations

- Transconductance-C filters:
  - Utilize transconductance amplifiers or transconductors together with capacitors for filters
  - High-quality and high-frequency transconductors can be easily realized in CMOS technology
  - Has been widely used for medium/high-frequency applications (up to hundreds of MHz)
- MOSFET-C filters:
  - Replace resistors with MOSFETs in linear region
  - Techniques have been evolved to obtain linear operation with large input signals
- Switched-capacitor filters:
  - Replace the required resistor by switching a capacitor at a relatively high frequency
  - The resulting filters are discrete-time circuits as opposed to the continuous-time ones
  - Is ideally suited for implementation low-frequency filters in IC form using CMOS technology

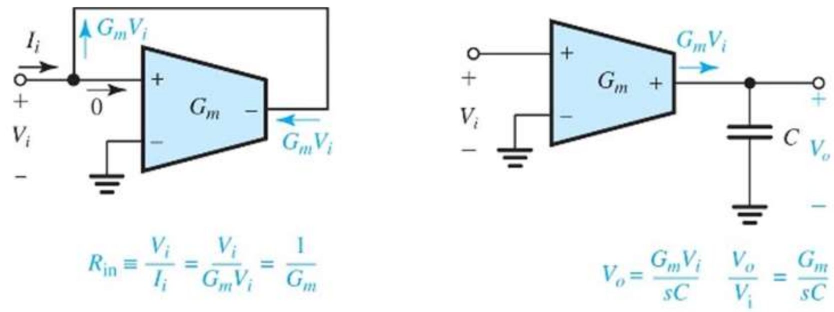
Transconductors

- An ideal transconductor has infinite input resistance and infinite output resistance
- The output can be positive or negative depending the current direction
- Transconductor can be single-ended or fully differential

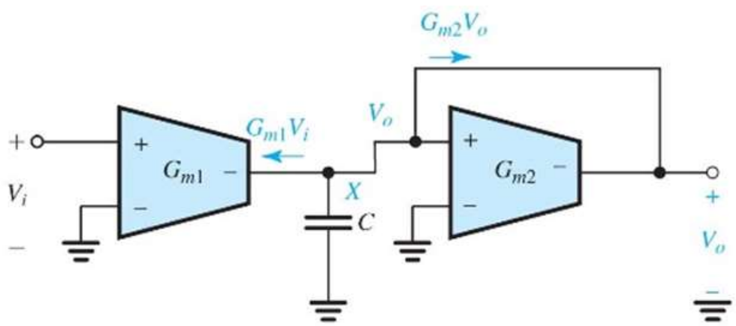


Basic Building Blocks

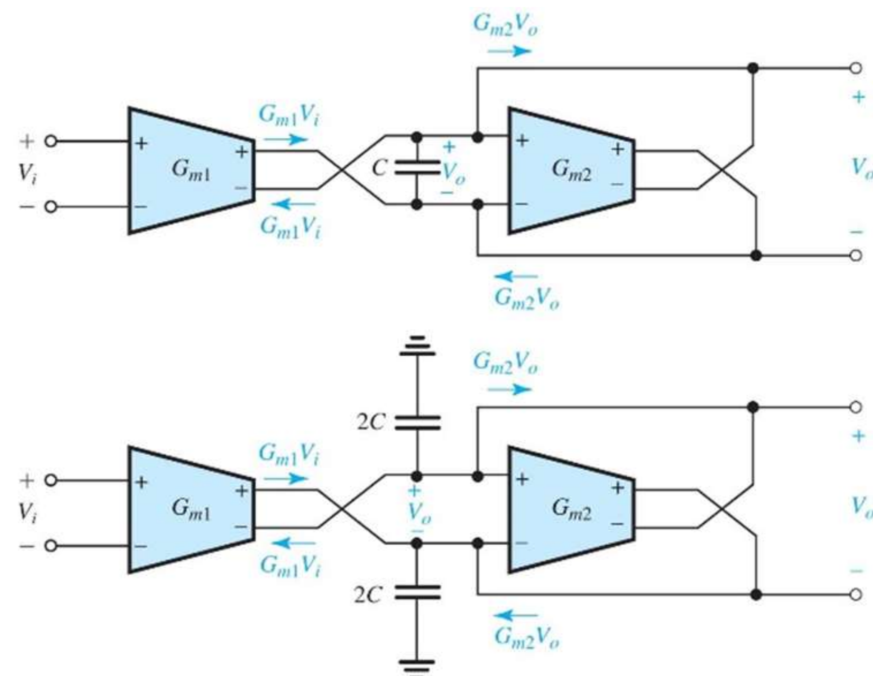
- ❑ Negative transconductor used to realize a resistance
- ❑ Transconductor loaded with a capacitor as an integrator



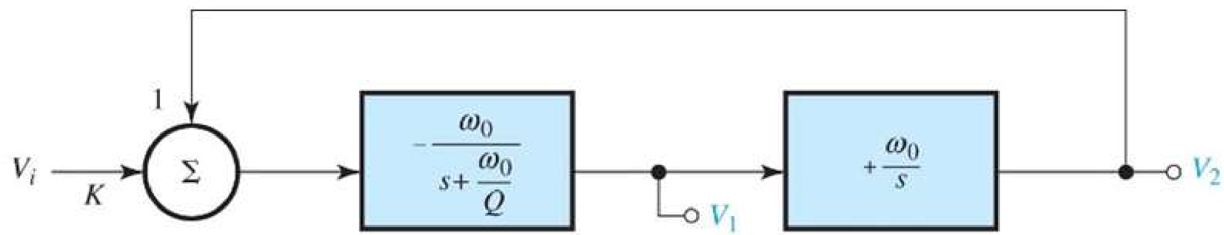
First-Order Gm-C Low-Pass Filter

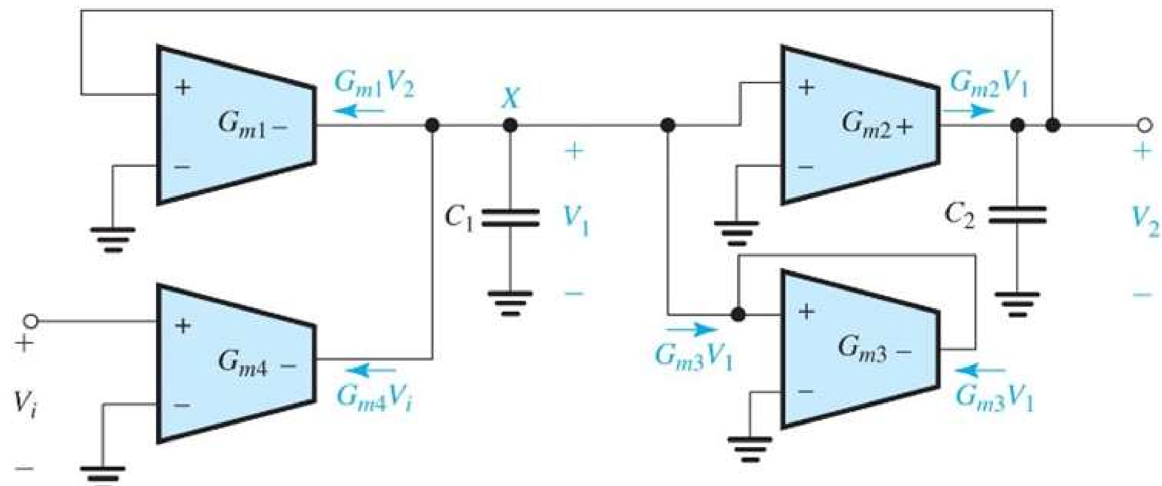


$$\frac{V_o}{V_i} = -\frac{G_{m1}}{sC + G_{m2}} = -\frac{G_{m1}/G_{m2}}{1 + sC/G_{m2}}$$



Second-Order Gm-C Low-Pass Filter





$$V_2 = \frac{G_{m2}}{sC_2} V_1$$

$$\frac{V_1}{V_i} = -\frac{sG_{m4}/C_1}{s^2 + sG_{m3}/C_1 + G_{m1}G_{m2}/C_1C_2} \quad \Rightarrow \quad \text{BP center-freq gain} = \frac{G_{m4}}{G_{m3}} \qquad \omega_0 = \sqrt{\frac{G_{m1}G_{m2}}{C_1C_2}}$$

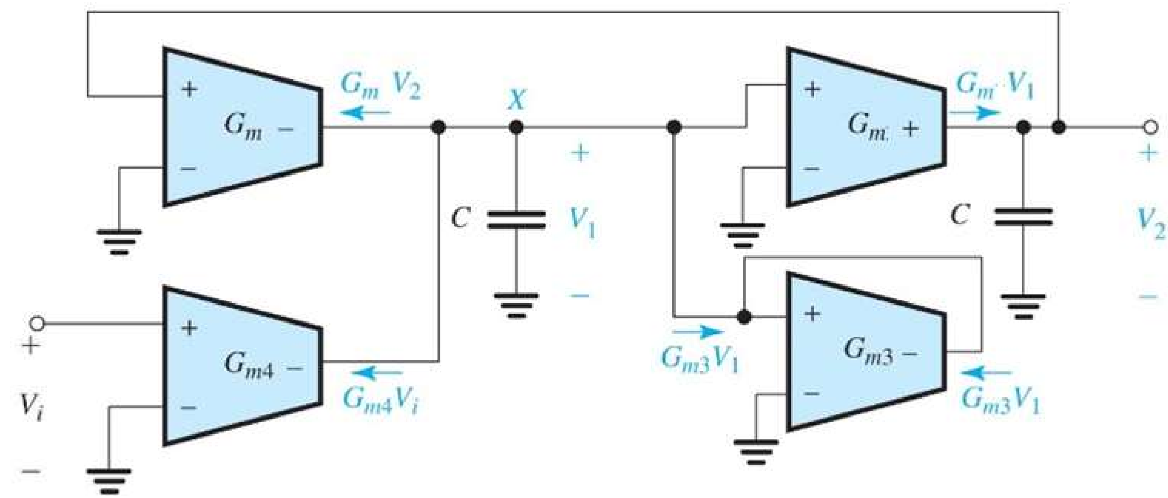
$$\frac{V_2}{V_i} = -\frac{G_{m2}G_{m4}/C_1C_2}{s^2 + sG_{m3}/C_1 + G_{m1}G_{m2}/C_1C_2} \quad \Rightarrow \quad \text{LP dc gain} = -\frac{G_{m4}}{G_{m1}} \qquad Q = \frac{\sqrt{G_{m1}G_{m2}}}{G_{m3}}\sqrt{\frac{C_1}{C_2}}$$

Simplified Circuit

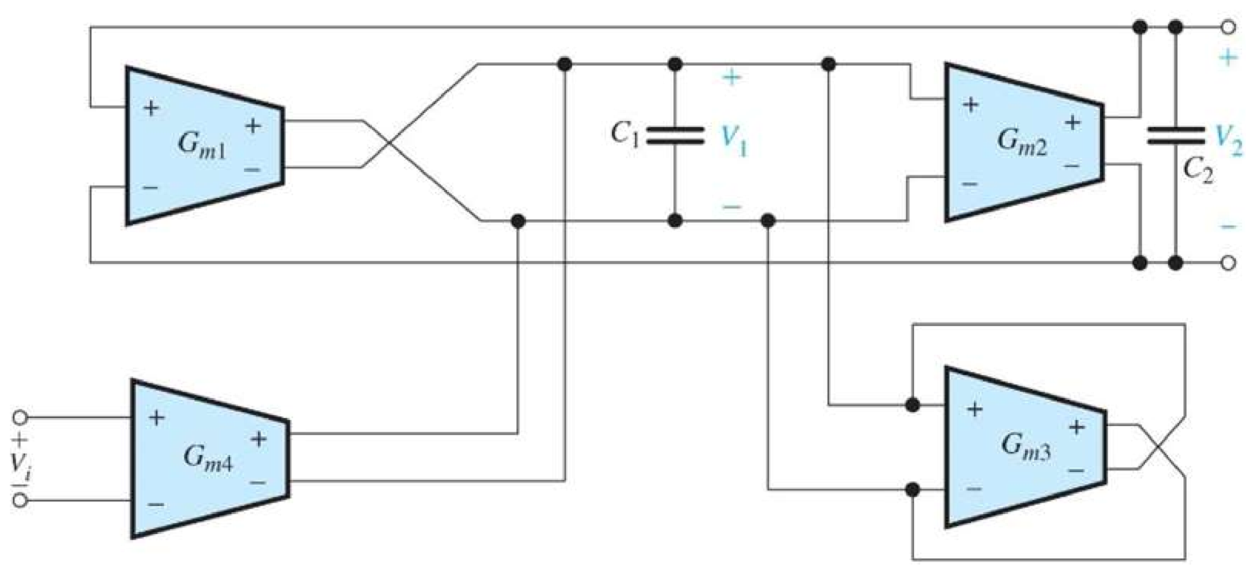
- $G_{m1} = G_{m2} = G_m$
- $C_1 = C_2 = C$

$$\omega_0 = \frac{G_m}{C}$$

$$Q = \frac{G_m}{G_{m3}}$$



Fully Differential Circuit



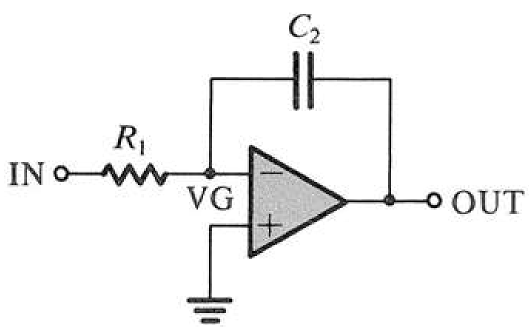
13.11 Switched-Capacitor Filters

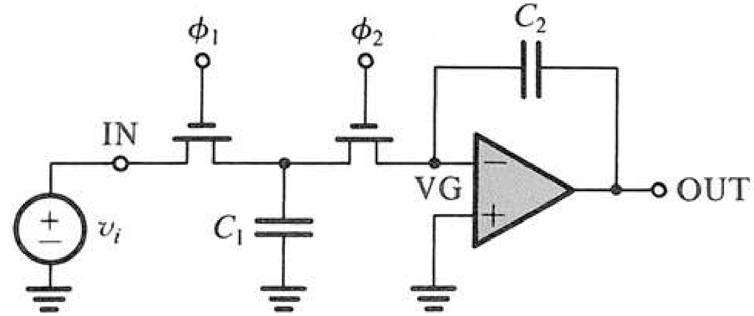
Basic Principle

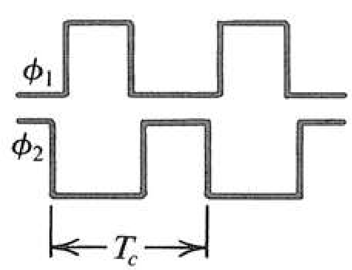
- ❑ A capacitor switched between two nodes at a sufficiently high rate is equivalent to a resistor
- ❑ The resistor in the active-*RC* integrator can be replaced by the capacitor and the switches
- ❑ Equivalent resistor:

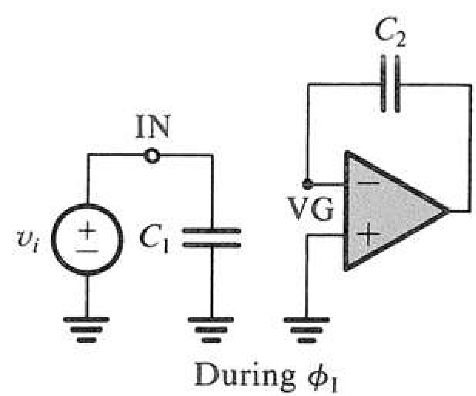
$$i_{av} = \frac{C_1 v_i}{T_c} \rightarrow R_{eq} \equiv \frac{v_i}{i_{av}} = \frac{T_c}{C_1}$$

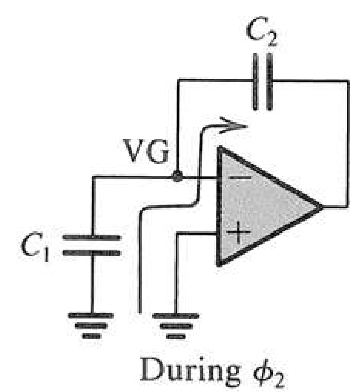
- ❑ Equivalent time constant for the integrator = $T_c(C_2/C_1)$





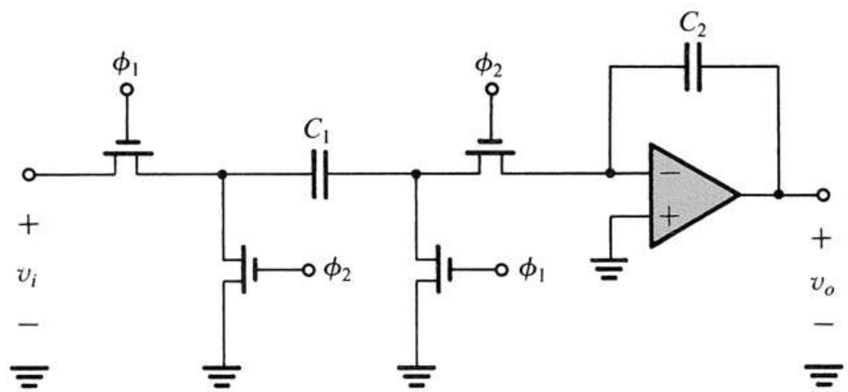




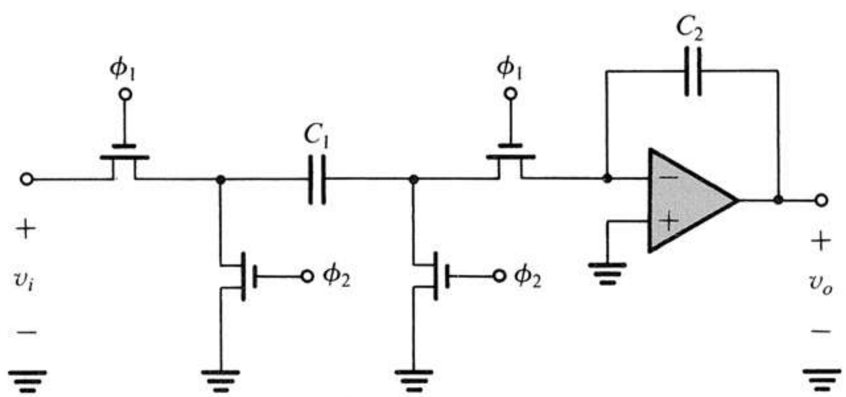


Practical Circuits

- Can realize both inverting and non-inverting integrator
- Insensitive to stray capacitances
- Noninverting switched-capacitor (SC) integrator



- Inverting switched-capacitor (SC) integrator



Filter Implementation

- Circuit parameters for the two integrators with the same time constant

$$T_c \frac{C_2}{C_3} = T_c \frac{C_1}{C_4} \rightarrow C_1 = C_2 = C \rightarrow C_3 = C_4 = KC$$

$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}} = \frac{1}{T_c}\sqrt{\frac{C_3}{C_2}\frac{C_4}{C_1}} = \frac{K}{T_c} \qquad Q = \frac{R_5}{R_4} = \frac{C_4}{C_5} \rightarrow C_5 = \omega_0 T_c \frac{C}{Q}$$