CHAPTER 13 FILTERS AND TUNED AMPLIFIERS

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13.1 Filter Transmission, Types and Specifications

Filter Transfer Function

- A filter is a **linear** two-port network represented by the ratio of the output to input voltage
- Transfer function \( T(s) \equiv \frac{V_o(s)}{V_i(s)} \)
- Transmission: evaluating \( T(s) \) for physical frequency \( s = j\omega \) → \( T(j\omega) = |T(j\omega)| \cdot e^{j\phi(\omega)} \)
  - Gain: \( 20 \log|T(j\omega)| \) (dB)
  - Attenuation: \( -20 \log|T(j\omega)| \) (dB)
- Output frequency spectrum: \( |V_o(s)| = |T(s)||V_i(s)| \)

Types of Filters

- **(a) Low-pass (LP)**
- **(b) High-pass (HP)**
- **(c) Bandpass (BP)**
- **(d) Bandstop (BS)**
Filter Specification

- Passband edge: \( \omega_p \)
- Maximum allowed variation in passband transmission: \( A_{\text{max}} \)
- Stopband edge: \( \omega_s \)
- Minimum required stopband attenuation: \( A_{\text{min}} \)

- The first step of filter design is to determine the filter specifications
- Then find a transfer function \( T(s) \) whose magnitude \( |T(j\omega)| \) meets the specifications
- The process of obtaining a transfer function that meets given specifications is called filter approximation
13.2 Filter Transfer Function

Transfer Function

- The filter transfer function is written as the ratio of two polynomials:
  \[ T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \ldots + a_0}{s^N + b_{N-1} s^{N-1} + \ldots + b_0} \]

- The degree of the denominator \( \rightarrow \) filter order
- To ensure the stability of the filter \( \rightarrow N \geq M \)
- The coefficients \( a_i \) and \( b_j \) are real numbers
- The transfer function can be factored and expressed as:
  \[ T(s) = \frac{a_M (s - z_1)(s - z_2)\ldots(s - z_M)}{(s - p_1)(s - p_2)\ldots(s - p_N)} \]

- Zeros: \( z_1, z_2, \ldots, z_M \) and \( (N-M) \) zeros at infinity
- Poles: \( p_1, p_2, \ldots, p_N \)
- Zeros and poles can be either a real or a complex number
- Complex zeros and poles must occur in conjugate pairs
- The poles have to be on the LHP of s-plane
13.3 Butterworth and Chebyshev Filters

The Butterworth Filter

- Butterworth filters exhibit monotonically decreasing transmission with all zeros at $\omega = \infty$
- Maximally flat response $\rightarrow$ degree of passband flatness increases as the order $N$ is increased
- Higher order filter has a sharp cutoff in the transition band
- The magnitude function of the Butterworth filter is:

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 (\omega / \omega_p)^{2N}}}$$

- Required transfer functions can be defined based on filter specifications ($A_{\text{max}}, A_{\text{min}}, \omega_p, \omega_s$)
Natural Modes of the Butterworth Filter

- The natural modes (poles) locate on a circle
- The radius of the circle is $\omega_0 = \omega_p (1/\varepsilon)^{1/N}$
- Equal angle space $\frac{\pi}{N}$

$$T(s) = \frac{K}{\left(1 - \frac{s}{p_1}\right) \left(1 - \frac{s}{p_2}\right) \cdots \left(1 - \frac{s}{p_N}\right)}$$

$$= \frac{K \omega_0^N}{(s - p_1)(s - p_2) \cdots (s - p_N)} \text{ where } \omega_0 = \omega_p (1/\varepsilon)^{1/N}$$
Design Procedure of the Butterworth Filters

**Design Specifications**

\[ A_{\text{max}}, A_{\text{min}}, \omega_p, \omega_s \]

**Design Procedure**

1. Determine \( \varepsilon \) (from \( A_{\text{max}} \))
   \[
   |T(j\omega_p)| = \frac{1}{\sqrt{1+\varepsilon^2}} \rightarrow A_{\text{max}}[dB] = 20 \log \sqrt{1+\varepsilon^2} \rightarrow \varepsilon = \sqrt{10^{A_{\text{max}}/10} - 1}
   \]

2. Determine the required filter order \( N \) (from \( \omega_p, \omega_s, A_{\text{min}} \))
   Attenuation \( A(\omega_s)[dB] = -20 \log[1/\sqrt{1+\varepsilon^2 (\omega_s / \omega_p)^{2N}}] = 10 \log[1 + \varepsilon^2 (\omega_s / \omega_p)^{2N}] \geq A_{\text{min}} \)

3. Determine the \( N \) natural modes (poles) with \( \omega_0 = \omega_p (1/\varepsilon)^{1/N} \)
   \( p_1, p_2, \ldots, p_N \)

4. Determine \( T(s) \)
   \[
   T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\ldots(s-p_N)} \quad \text{where} \quad \omega_0 = \omega_p (1/\varepsilon)^{1/N}
   \]
# 13.4 First-Order and Second-Order Filter Functions

## Cascade Filter Design
- First-order and second-order filters can be cascaded to realize high-order filters
- Cascade design is one of the most popular methods for the design of active filters
- Cascading does not change the transfer functions of individual blocks if the output resistance is low

## First-Order Filters
- Bilinear transfer function \( T(s) = \frac{a_1 s + a_0}{s + b_0} = \frac{a_1 s + a_0}{s + \omega_0} \)

| Filter Type and \( T(s) \) | \( s \)-Plane Singularities | Bode Plot for \(|T|\) | Passive Realization | Op Amp-RC Realization |
|-----------------------------|-----------------------------|----------------------|---------------------|---------------------|
| (a) Low-Pass (LP) \( T(s) = \frac{a_0}{s + \omega_0} \) | \( j\omega \) 0 at \( \infty \) | 20 log \( \frac{\omega_0}{\omega_0} \) -20 dB decade | \( CR = \frac{1}{\omega_0} \) dc gain = 1 | \( CR_2 = \frac{1}{\omega_0} \) dc gain = \(-\frac{R_2}{R_1}\) |
| (b) High-Pass (HP) \( T(s) = \frac{a_1 s}{s + \omega_0} \) | \( j\omega \) 0 at \( \infty \) | 20 log \( a_1 \) +20 dB decade | \( CR = \frac{1}{\omega_0} \) High-frequency gain = 1 | \( CR_1 = \frac{1}{a_0} \) High-frequency gain = \(-\frac{R_2}{R_1}\) |
First-Order Filters (Cont’d)

\[ T(s) = \frac{a_1 s + a_0}{s + \omega_0} \]

- **General**
- **Graphs**
- **Equations**
  - \( |T|, \text{ dB} \)
  - \( 20 \log \left| \frac{a_0}{\omega_0} \right| \) dB/decade
  - \( 20 \log |a_1| \)

- **Analysis**
  - \( C_1R_1 = \frac{a_0}{a_1} \)
  - dc gain = \( \frac{R_3}{R_1 + R_2} \)
  - HF gain = \( \frac{C_1}{C_1 + C_2} \)

- **Passive Realization**

| \( T(s) \) | Singularity | \(|T| \) and \( \phi \) | Passive Realization | Op Amp-RC Realization |
|------------|-------------|----------------|---------------------|----------------------|
| \( T(s) = \frac{a_1 s + a_0}{s + \omega_0} \) | \( a_1 > 0 \) | \( 20 \log |a_1| \) | \( CR = 1/\omega_0 \) | \( CR = 1/\omega_0 \) |
| \( \omega_0 \) | \( \omega_0 \) | \( \omega_0 \) | Flat gain \((a_1) = 0.5\) | Flat gain \((a_1) = 1\) |
Second-Order Filters

- Biquadratic transfer function

\[ T(s) = \frac{a_2s^2 + a_1s + a_0}{s^2 + (\omega_0/Q)s + \omega_0^2} \]

- Pole frequency: \( \omega_0 \)
- Pole quality factor: \( Q \)
- Poles:

\[ p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0 \sqrt{1 - \frac{1}{4Q^2}} \]

- Bandwidth:

\[ BW \equiv \omega_2 - \omega_1 = \frac{\omega_0}{Q} \]
Second-Order Filters (Cont’d)

| Filter Type and $T(s)$ | $s$-Plane Singularities | $|T|$ |
|------------------------|--------------------------|------|
| (d) Notch $T(s) = \frac{s^2 + \omega_n^2}{s^2 + \frac{s}{\omega_n} + \frac{\omega_n^2}{Q}}$ | ![Diagram of Notch Filter](image) | ![Frequency Response of Notch Filter](image) |
| DC gain = $\omega_n/Q$ | $|\omega_n/\sqrt{2}|$ | $\alpha_1 \omega_1 = \omega_0^2$ |
| High-frequency gain = $\alpha_2$ | | |

| (c) Low-pass notch (LPN) $T(s) = \frac{s^2 + \omega_n^2}{s^2 + \frac{s}{\omega_n} + \frac{\omega_n^2}{Q}}$ | ![Diagram of LPN Filter](image) | ![Frequency Response of LPN Filter](image) |
| DC gain = $\frac{\omega_n^2}{\omega_0^2}$ | $|\omega_n/\omega_0|$ | $\omega_{max} = \omega_0 \sqrt{\left(\frac{\omega_n^2}{\omega_0^2}\right) \left(1 - \frac{1}{2Q^2}\right) - 1}$ |
| High-frequency gain = $\alpha_2$ | | |

| (f) High-pass notch (HPN) $T(s) = \frac{s^2 + \omega_n^2}{s^2 + \frac{s}{\omega_n} + \frac{\omega_n^2}{Q}}$ | ![Diagram of HPN Filter](image) | ![Frequency Response of HPN Filter](image) |
| DC gain = $\frac{\omega_n^2}{\omega_0^2}$ | $|\omega_n/\omega_0|$ | $T_{max} = \frac{|\omega_n^2 - \omega_0^2|}{\sqrt{(\omega_0^2 - \omega_n^2)^2 + \left(\frac{\omega_n}{\omega_0}\right)^2 \omega_n^2}}$ |
| High-frequency gain = $\alpha_2$ | | |
Second-Order Filters (Cont’d)

(g) All pass (AP)

\[ T(s) = a_2 \frac{s^2 - \frac{\omega_0}{Q} s + \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \]

Flat gain = \(a_2\)
13.5 The Second-Order LCR Resonator

The Resonator Natural Modes

- The LCR resonator can be excited by either current or voltage source.
- The excitation should be applied without changing the natural structure of the circuit.
- The natural modes of the circuits are identical (will not be changed by the excitation methods).
- The similar characteristics also apply to series LCR resonator.

\[
\begin{align*}
\frac{V_o}{I_i} &= \frac{1}{Y} = \frac{1}{sL + sC + 1/R} = \frac{s/C}{s^2 + (1/RC)s + 1/LC} \\
\frac{V_o}{V_i} &= \frac{(R||1/sC)}{(R||1/sC)+sL} = \frac{1/LC}{s^2 + (1/RC)s + 1/LC} \\
\end{align*}
\]

\[
\begin{align*}
\omega_b &= 1/\sqrt{LC} \\
Q &= \omega_b RC
\end{align*}
\]
Realization of Transmission Zeros

- Values of $s$ at which $Z_2(s) = 0$ and $Z_1(s) \neq 0$
  $\rightarrow$ $Z_2$ behaves as a short
- Values of $s$ at which $Z_1(s) = \infty$ and $Z_2(s) \neq \infty$
  $\rightarrow$ $Z_1$ behaves as an open

Realization of Filter Functions

**Low-Pass Filter**

$$T(s) = \frac{V_o}{V_i} = \frac{1/LC}{s^2 + (1/RC)s + 1/LC}$$

**High-Pass Filter**

$$T(s) = \frac{V_o}{V_i} = \frac{s^2}{s^2 + (1/RC)s + 1/LC}$$

**Bandpass Filter**

$$T(s) = \frac{V_o}{V_i} = \frac{(1/RC)s}{s^2 + (1/RC)s + 1/LC}$$
Notch Filter

Low-Pass Notch Filter

High-Pass Notch Filter

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13.6 Second-Order Active Filters (Inductor Replacement)

Second-Order Active Filters by Op Amp-RC Circuits

- Inductors are not suitable for IC implementation
- Use op amp-RC circuits to replace the inductors
- Second-order filter functions based on RLC resonator

The Antoniou Inductance-Simulation Circuit

- Inductors are realized by op amp-RC circuits with negative feedbacks
- The equivalent inductance is given by

\[
Z_{in} = \frac{V_1}{I_1} = sC_4R_1R_3R_5 / R_2 = sL_{eq}
\]

\[
L_{eq} = C_4R_1R_3R_5 / R_2
\]
The Op Amp-RC Resonator

- The inductor is replaced by the Antoniou circuit
- The pole frequency and the quality factor are given by

\[ \omega_0 = \frac{1}{\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2}} \quad Q = \omega_0 C_6 R_6 = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_4 R_3 R_5}} \]

- A simplified case where \( R_1 = R_2 = R_3 = R_5 = R \) and \( C_4 = C_6 = C \)

\[ \omega_0 = \frac{1}{RC} \quad Q = \frac{R_6}{R} \]
Filter Realization

Low-Pass Filter

High-Pass Filter

Bandpass Filter

Notch Filter
13.7 Second-Order Active Filters (Two-Integrator-Loop)

Derivation of the Two-Integrator-Loop Biquad

- **High-pass implementation:**
  \[ V_{hp} = KV_i - \frac{1}{Q} \frac{\omega_0}{s} V_{hp} - \frac{\omega_0^2}{s^2} V_{hp} \]
  \[ T_{hp}(s) = \frac{Ks^2}{s^2 + (\omega_0 / Q)s + \omega_0^2} \]

- **Band-pass implementation:**
  \[ T_{bp}(s) = \left( -\frac{\omega_0}{s} \right) V_{hp}(s) = \frac{-K\omega_0}{s^2 + (\omega_0 / Q)s + \omega_0^2} \]

- **Low-pass implementation:**
  \[ T_{lp}(s) = \left( -\frac{\omega_0}{s} \right)^2 V_{hp}(s) = \frac{K\omega_0^2}{s^2 + (\omega_0 / Q)s + \omega_0^2} \]
Circuit Implementation (I)

\[ V_{hp} = \frac{R_3}{R_2 + R_3} \left( 1 + \frac{R_f}{R_1} \right) V_i + \frac{R_2}{R_2 + R_3} \left( 1 + \frac{R_f}{R_1} \right) \left( -\frac{\omega_0}{s} V_{hp} \right) - \frac{R_f}{R_1} \left( \frac{\omega_0^2}{s^2} V_{hp} \right) \]

\[ \frac{R_f}{R_1} = 1 \quad \frac{R_3}{R_2} = 2Q - 1 \quad K = 2 - \frac{1}{Q} \]

- High-pass transfer function:
  \[ T_{hp}(s) = \frac{V_{hp}}{V_i} = \frac{s^2[2R_3/(R_2 + R_3)]}{s^2 + s[2R_2/(R_2 + R_3)]\omega_0 + \omega_0^2} \]

- Band-pass transfer function:
  \[ T_{bp}(s) = \frac{V_{hp}}{V_i} = \frac{s[2R_3/(R_2 + R_3)]\omega_0}{s^2 + s[2R_2/(R_2 + R_3)]\omega_0 + \omega_0^2} \]

- Low-pass transfer function:
  \[ T_{lp}(s) = \frac{V_{lp}}{V_i} = \frac{[2R_3/(R_2 + R_3)]\omega_0^2}{s^2 + s[2R_2/(R_2 + R_3)]\omega_0 + \omega_0^2} \]

- Notch and all-pass transfer function:
  \[ T(s) = \frac{V_o}{V_i} = -\frac{2R_3}{R_2 + R_3} \frac{(R_F / R_H) s^2 + (R_F / R_B) \omega_0 s + (R_F / R_L) \omega_0^2}{s^2 + s[2R_2/(R_2 + R_3)]\omega_0 + \omega_0^2} \]
Circuit Implementation (II) – Tow-Thomas Biquad

- Use an additional inverter to make all the coefficients of the summer the same sign
- All op amps are in single-ended mode
- The high-pass function is no longer available
- It is known as the Tow-Thomas biquad
- An economical feedforward scheme can be employed with the Tow-Thomas circuit

\[ T(s) = \frac{V_o}{V_i} = -\frac{s^2 \frac{C_1}{C} + s \frac{1}{C} \left( \frac{1}{R_1} - \frac{r}{RR_2} \right) + \frac{1}{C^2 RR_2}}{s^2 + s \frac{1}{QCR} + \frac{1}{C^2 R^2}} \]
13.8 Single-Amplifier Biquadratic Active Filters

**Characteristics of the SAB Circuits**
- Only one op amp is required to implement biquad circuit
- Exhibit greater dependence on the limited gain and bandwidth of the op amp
- More sensitive to the unavoidable tolerances in the values of resistors and capacitors
- Limited to less stringent filter specifications with pole $Q$ factors less than 10

**Synthesis of the SAB Circuits**
- Use feedback to move the poles of an RC circuit from the negative real axis to the complex conjugate locations to provide selective filter response
- Steps of SAB synthesis:
  - Synthesis of a feedback loop that realizes a pair of complex conjugate poles characterized by $\omega_0$ and $Q$
  - Injecting the input signal in a way that realizes the desired transmission zeros
- Natural modes of the filter:
  \[
  t(s) = \frac{N(s)}{D(s)} \\
  L(s) = At(s) = \frac{AN(s)}{D(s)}
  \]

The closed-loop characteristics equation:

\[1 + L(s) = 0 \rightarrow t(s_p) = -1/A \approx 0\]

⇒ The poles of the closed-loop system are identical to the zeros of the RC network
RC Networks with complex transmission zeros

\[
t(s) = \frac{s^2 + s \left( \frac{1}{C_1} + \frac{1}{C_2} \right) + \frac{1}{C_1C_2R_3R_4}}{s^2 + s \left( \frac{1}{C_1R_3} + \frac{1}{C_2R_3} + \frac{1}{C_1R_4} \right) + \frac{1}{C_1C_2R_3R_4}}
\]

Characteristics Equation of the Filter

\[
s^2 + s \frac{\omega_0}{Q} + \omega_0^2 = s^2 + s \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}
\]

\[
\rightarrow \omega_0 = \frac{1}{\sqrt{C_1C_2R_3R_4}}
\]

\[
\rightarrow Q = \left[ \frac{\sqrt{C_1C_2R_3R_4}}{R_3} \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \right]^{-1}
\]

Let \( C_1 = C_2 = C \), \( R_3 = R \), \( R_4 = R/m \) \( \rightarrow m = 4Q^2 \)

\[
\rightarrow CR = 2Q/\omega_0
\]
Injection the Input Signal

- The method of injection the input signal into the feedback loop through the grounded nodes
- A component with a ground node can be connected to the input source
- The filter transmission zeros depends on where the input signal is injected

\[
\begin{align*}
\frac{V_o}{V_i} &= \frac{-s(\alpha / C_1 R_4)}{s^2 + s \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{R_3} + \frac{1}{C_1 C_2 R_3 R_4} \right)} \\
V_x &= -\frac{V_o}{s C_2 R_3}
\end{align*}
\]
Generation of Equivalent Feedback Loops

Characteristics Equation:
\[ 1 + L(s) = 0 \rightarrow 1 + At(s) = 0 \]

Characteristics Equation:
\[ 1 - \frac{A}{A+1}(1-t) = 0 \rightarrow 1 + At(s) = 0 \]
Generation of Equivalent Feedback Loops (Cont'd)
13.9 Sensitivity

Filter Sensitivity

- Deviation in filter response due to the tolerances in component values
- Especially for RC component values and amplifier gain

Classical Sensitivity Function

- Definition:

\[ S_x^y = \lim_{{\Delta x \to 0}} \frac{{\Delta y / y}}{{\Delta x / x}} \]

\[ S_x^y = \frac{{\partial y}}{{\partial x}} \cdot \frac{x}{y} \]

- For small changes:

\[ S_x^y \approx \frac{{\Delta y / y}}{{\Delta x / x}} \]
13.10 Transconductance-C Filters

Limitations of Op Amp-RC Circuits
- Suitable for audio-frequency filters using discrete op amps, resistors and capacitors
- High-frequency applications limited by the relatively low bandwidth of general-purpose op amps
- Impractical for IC implementations due to:
  - The need for large capacitors and resistors increases the IC cost
  - The need for very precise values of RC time constant requires expensive trimming/tuning
  - The need for op amps that can drive resistive and large capacitive loads

Methods for IC Filter Implementations
- Transconductance-C filters:
  - Utilize transconductance amplifiers or transconductors together with capacitors for filters
  - High-quality and high-frequency transconductors can be easily realized in CMOS technology
  - Has been widely used for medium/high-frequency applications (up to hundreds of MHz)
- MOSFET-C filters:
  - Replace resistors with MOSFETs in linear region
  - Techniques have been evolved to obtain linear operation with large input signals
- Switched-capacitor filters:
  - Replace the required resistor by switching a capacitor at a relatively high frequency
  - The resulting filters are discrete-time circuits as opposed to the continuous-time ones
  - Is ideally suited for implementation low-frequency filters in IC form using CMOS technology
Transconductors

- An ideal transconductor has infinite input resistance and infinite output resistance
- The output can be positive or negative depending on the current direction
- Transconductor can be single-ended or fully differential
Basic Building Blocks

- Negative transconductor used to realize a resistance
- Transconductor loaded with a capacitor as an integrator

First-Order Gm-C Low-Pass Filter

\[
\frac{V_o}{V_i} = \frac{G_{m1}}{sC + G_{m2}} = \frac{G_{m1}}{1 + sC / G_{m2}}
\]
Second-Order Gm-C Low-Pass Filter

\[
V_2 = \frac{G_{m2}}{sC_2} V_1
\]

\[
V_1 = -\frac{sG_{m4} / C_1}{s^2 + sG_{m3} / C_1 + G_{m1}G_{m2} / C_1C_2}
\]

\[
V_2 = -\frac{G_{m2}G_{m4} / C_1C_2}{s^2 + sG_{m3} / C_1 + G_{m1}G_{m2} / C_1C_2}
\]

BP center - freq gain = \( \frac{G_{m4}}{G_{m3}} \)

\[\omega_0 = \sqrt{\frac{G_{m1}G_{m2}}{C_1C_2}}\]

LP dc gain = \( \frac{G_{m4}}{G_{m1}} \)

\[Q = \sqrt{\frac{G_{m1}G_{m2}}{G_{m3}}} \sqrt{\frac{C_1}{C_2}}\]
Simplified Circuit

- \( G_{m1} = G_{m2} = G_m \)
- \( C_1 = C_2 = C \)

\[ \omega_0 = \frac{G_m}{C} \]

\[ Q = \frac{G_m}{G_{m3}} \]

Fully Differential Circuit
13.11 Switched-Capacitor Filters

Basic Principle

- A capacitor switched between two nodes at a sufficiently high rate is equivalent to a resistor
- The resistor in the active-RC integrator can be replaced by the capacitor and the switches
- Equivalent resistor:
  \[ i_{av} = \frac{C_1 v_i}{T_c} \rightarrow R_{eq} \equiv \frac{v_i}{i_{av}} = \frac{T_c}{C_1} \]
- Equivalent time constant for the integrator = \( T_c(C_2/C_1) \)
Practical Circuits

- Can realize both inverting and non-inverting integrator
- Insensitive to stray capacitances
- Noninverting switched-capacitor (SC) integrator

- Inverting switched-capacitor (SC) integrator
Filter Implementation

- Circuit parameters for the two integrators with the same time constant

\[ T_c \frac{C_2}{C_3} = T_c \frac{C_1}{C_4} \rightarrow C_1 = C_2 = C \rightarrow C_3 = C_4 = KC \]

\[ \omega_0 = \frac{1}{\sqrt{C_1 C_2 R_3 R_4}} = \frac{1}{T_c} \sqrt{\frac{C_3 C_4}{C_2 C_1}} \Rightarrow Q = \frac{R_5}{R_4} = \frac{C_4}{C_5} \rightarrow C_5 = \omega_0 T_c \frac{C}{Q} \]