CHAPTER 14 SIGNAL GENERATORS AND WAVEFORM-SHAPING CIRCUITS

Chapter Outline

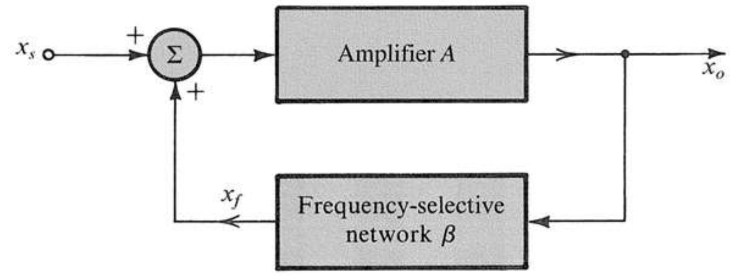
14.1 Basic Principles of Sinusoidal Oscillators

Types of Oscillators

- Linear oscillator:
  - Employs a positive feedback loop consisting of an amplifier and a frequency-selective network
  - Some form of nonlinearity has to be employed to provide control of the amplitude of the output
- Nonlinear oscillator:
  - Generates square, triangular, pulse waveforms
  - Employs multivibrators: bistable, astable and monostable

The Oscillator Feedback Loop and Oscillation Criterion

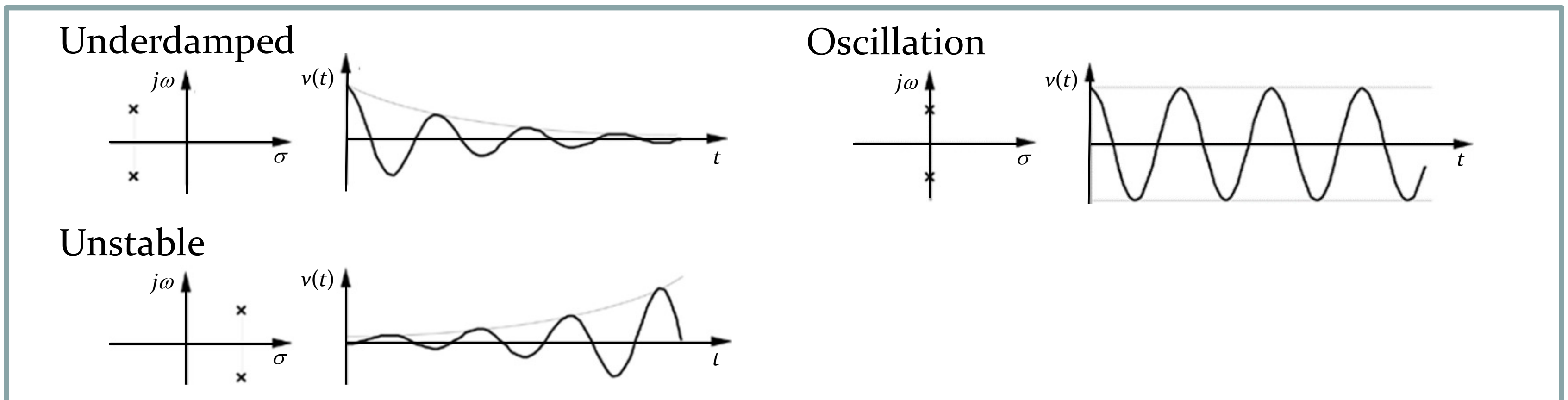
- Positive feedback loop analysis:



$$A_f(s) \equiv \frac{x_o}{x_i} = \frac{A(s)}{1-A(s)\beta(s)} = \frac{A(s)}{1-L(s)}, \text{ where } L(s) \text{ is the loop gain}$$

- Characteristic equation: 1-$L(s)$ = 0

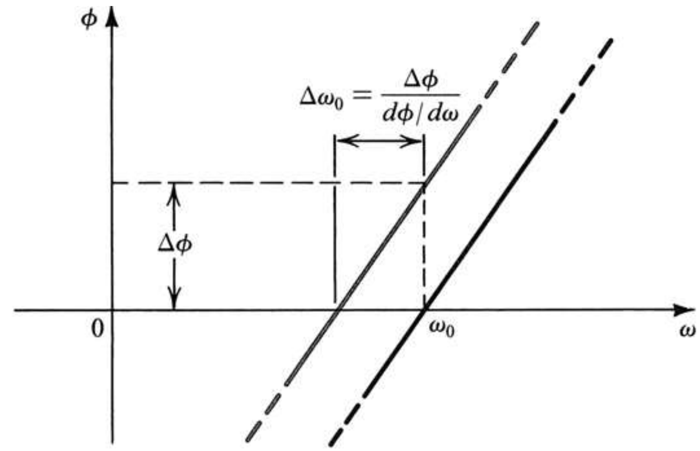
$\rightarrow$ Find poles of the closed-loop system by solving $L(s)$ = 1



❑ Barkhausen criterion:

- The phase of loop gain should be zero at ω_o
- The magnitude of the loop gain should be unity at ω_o
- The characteristic equation has roots at $s = \pm j\omega_o$

$$L(j\omega_0) = A(j\omega_0)\beta(j\omega_0) = 1$$
$$\rightarrow |L(j\omega_0)| = 1$$
$$\rightarrow \angle L(j\omega_0) = 0°$$



❑ Stability of oscillation frequency:

- ω_o is determined solely by the phase characteristics
- A steep function $f(\omega)$ results in a more stable frequency

❑ Stability of oscillation frequency:

- Oscillation: loop gain $A\beta = 1$
- Growing output: loop gain $A\beta > 1$
- Decaying output: loop gain $A\beta < 1$

Nonlinear Amplitude Control

❑ Oscillation mechanism:

- Initiating oscillation: loop gain slightly larger than unity (poles in RHP)
- Gain control: nonlinear network reduces loop gain to unity (poles on $j\omega$-axis)

Limiter Circuits for Amplitude Control

- For small output amplitude (D_1 off, D_2 off)

  $\rightarrow$ incremental gain (slope) = $-R_f/R_1$

- For large negative output swing (D_1 on, D_2 off)

$$v_A = \frac{R_3 V + R_2 v_O}{R_2 + R_3} = -V_D \rightarrow v_O = -\left(\frac{R_3}{R_2} V + \frac{R_2 + R_3}{R_2} V_D\right) = L_-$$

$$\begin{cases} v_I / R_1 + v_O / R_f = I_{D1} \\ (V + V_D)/R_2 + (v_O + V_D)/R_3 + I_{D1} = 0 \end{cases}$$

$$v_O = -\frac{R_f \parallel R_3}{R_1} v_I - \frac{R_f \parallel R_3}{R_2} V - \frac{R_f \parallel R_3}{R_2 \parallel R_3} V_D$$

  $\rightarrow$ incremental gain (slope) = $-(R_f||R_4)/R_1$

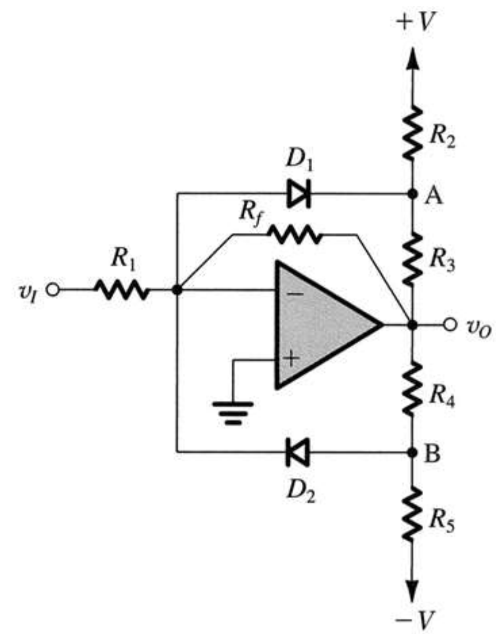
- For large positive output swing (D_1 off, D_2 on)

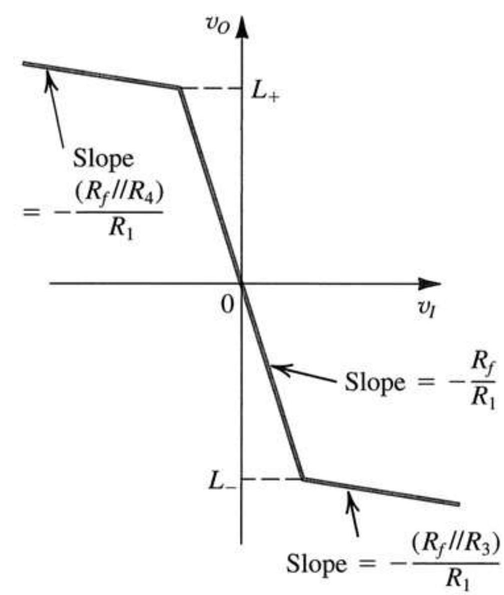
$$v_B = \frac{-R_4 V + R_5 v_O}{R_4 + R_5} = V_D \rightarrow v_O = \frac{R_4}{R_5} V + \frac{R_4 + R_5}{R_5} V_D = L_+$$

$$\begin{cases} v_I / R_1 + v_O / R_f + I_{D2} = 0 \\ (v_O - V_D)/R_4 + (-V - V_D)/R_5 = I_{D2} \end{cases}$$

$$v_O = -\frac{R_f \parallel R_4}{R_1} v_I + \frac{R_f \parallel R_4}{R_5} V + \frac{R_f \parallel R_4}{R_4 \parallel R_5} V_D$$

  $\rightarrow$ incremental gain (slope) = $-(R_f||R_3)/R_1$





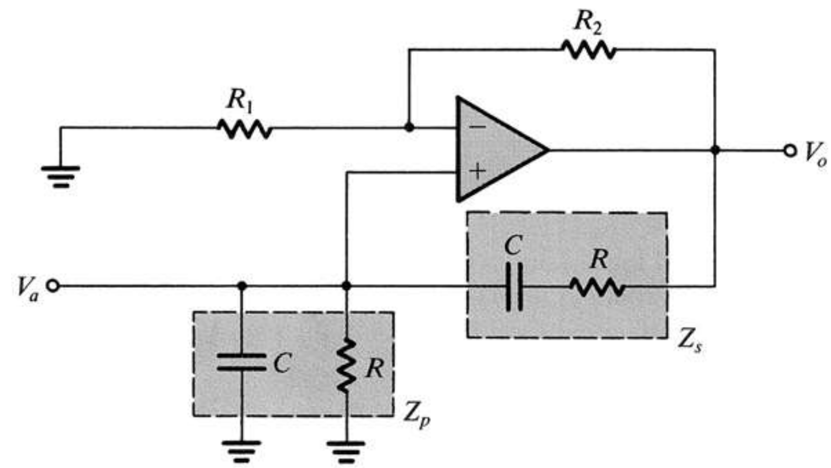
14.2 OP Amp-RC Oscillator Circuits

Wien-Bridge Oscillator

❑ Define the loop gain

$$A(s) = 1 + \frac{R_2}{R_1} \text{ and } \beta(s) = \frac{Z_p}{Z_p + Z_s}$$

$$L(s) = \left[1 + \frac{R_2}{R_1}\right] \frac{Z_p}{Z_p + Z_s} \rightarrow L(s) = \frac{1 + R_2 / R_1}{3 + sRC + 1/sRC}$$



❑ Pole locations by solving the characteristic equation

$$L(s) = \frac{1 + R_2 / R_1}{3 + sRC + 1/sRC} = 1 \rightarrow s^2 R^2 C^2 + s\left(2 - \frac{R_2}{R_1}\right) RC + 1 = 0$$

- Oscillation condition: $2 - R_2/R_1 = 0$ and $s = \pm j\omega_0 = \pm j/RC$
- Start-up condition: $2 - R_2/R_1 < 0$ (poles at RHP)

❑ Barkhausen criterion:

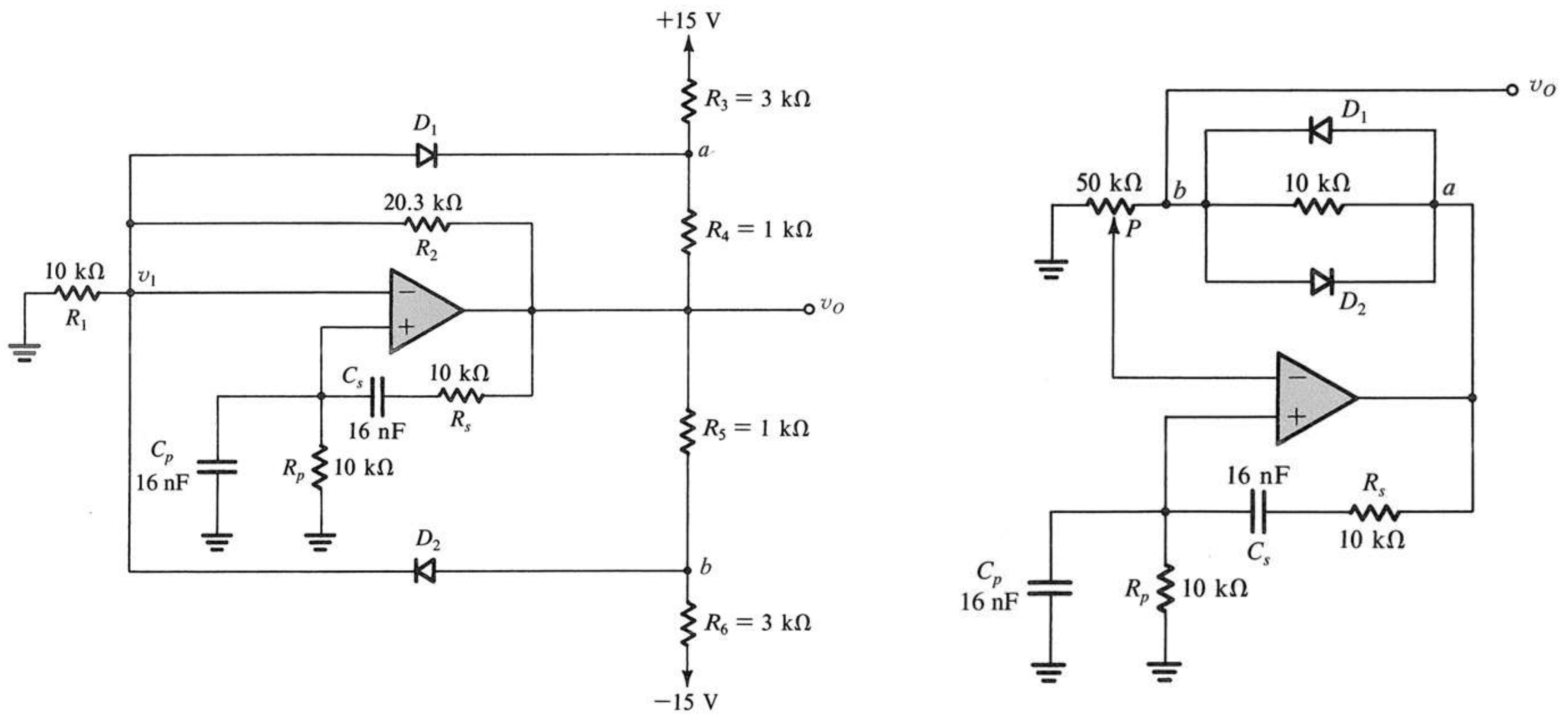
$$L(j\omega) = \frac{1 + R_2 / R_1}{3 + j\omega_0 RC + 1/j\omega_0 RC} = 1 \rightarrow (1 - \omega_0^2 R^2 C^2) + j\omega_0\left(2 - \frac{R_2}{R_1}\right) RC = 0 \rightarrow 1 - \omega_0^2 R^2 C^2 = 0 \text{ and } 2 - \frac{R_2}{R_1} = 0$$

- Oscillation condition: $R_2/R_1 = 2$ and $\omega_0 = 1/RC$
- Start-up condition: $\omega_0 = RC$ and $R_2/R_1 = 2 + \Delta$

❑ Limiter is used for amplitude control

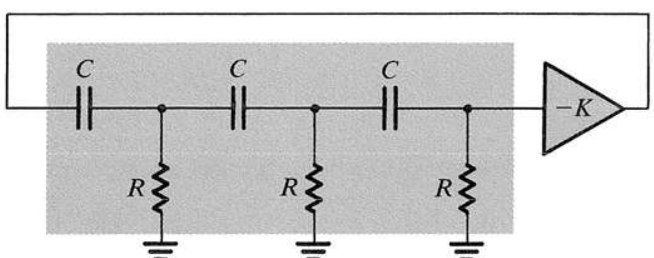
Wien-Bridge Oscillator with Amplitude Control

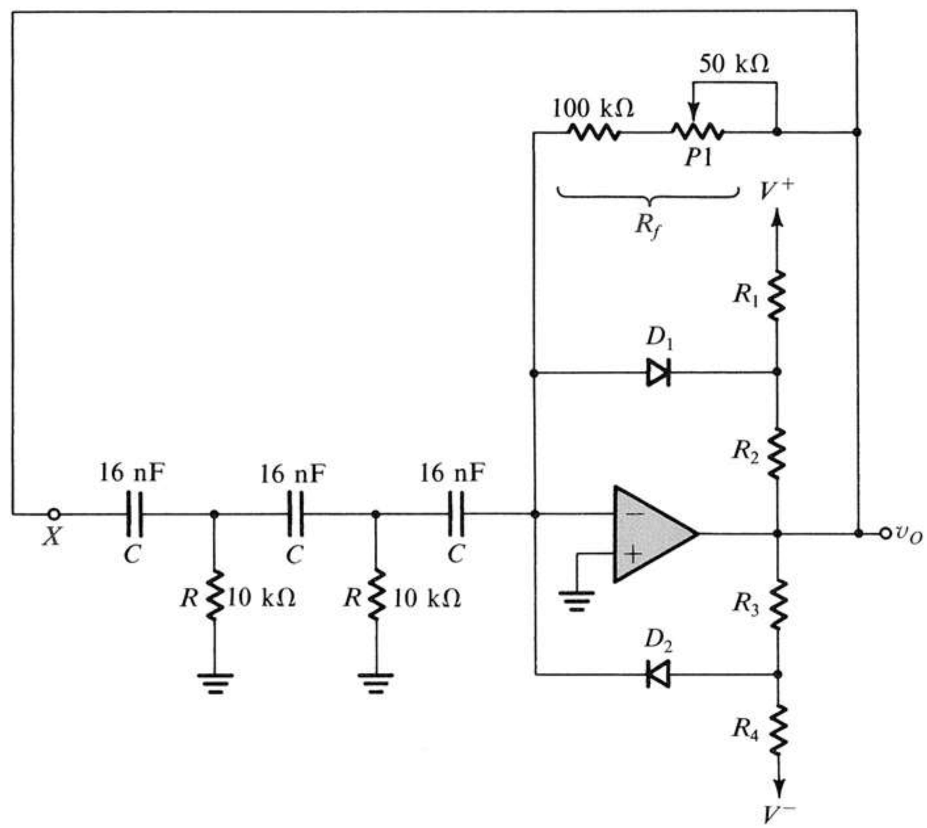
- ❑ Diodes are used to limit the amplitude of the output swing
- ❑ Diodes are off with small-signal operation and can be neglected for analysis of the oscillation condition



Phase-Shift Oscillator

- The circuit oscillates at the frequency for which the phase shift of the *RC* network is 180°
- Only at this frequency will the total phase shift around the loop be 0° or 360°
- The minimum number of *RC* sections is three
- *K* should be equal to the inverse of the magnitude of the *RC* network at oscillation frequency
- Slightly higher *K* is used to ensure that the oscillation starts
- Limiter is used for amplitude control





Quadrature Oscillator

- Based on the two-integrator loop without damping
- Loop gain:

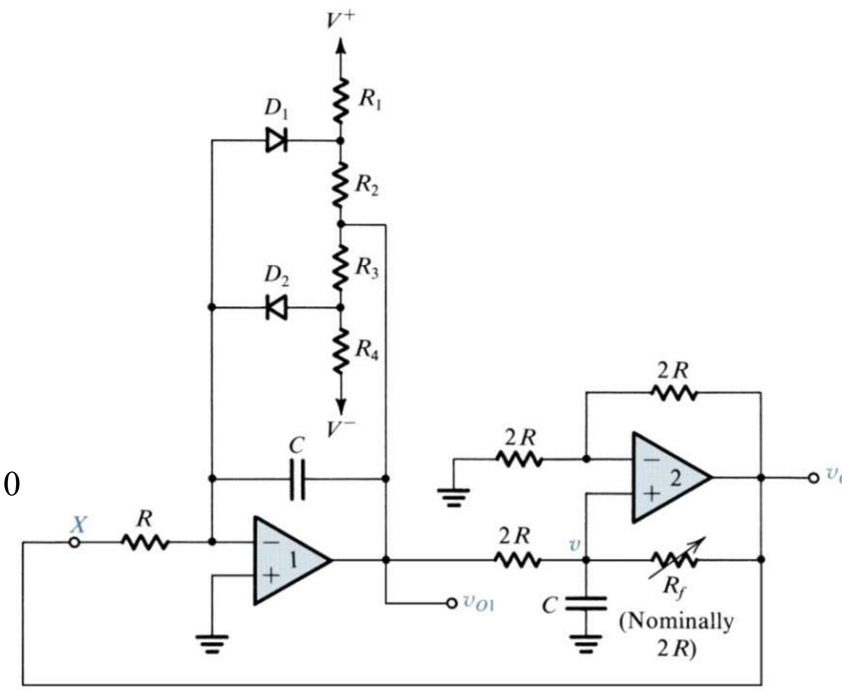
$$L(s)=\frac{v_{O1}}{v_i}\times\frac{v_{O2}}{v_{O1}}=\left(-\frac{1}{sRC}\right)\frac{1}{sRC+\left(\frac{1}{2}-\frac{1}{2}\frac{2R}{R_f}\right)}$$

- Oscillation condition:

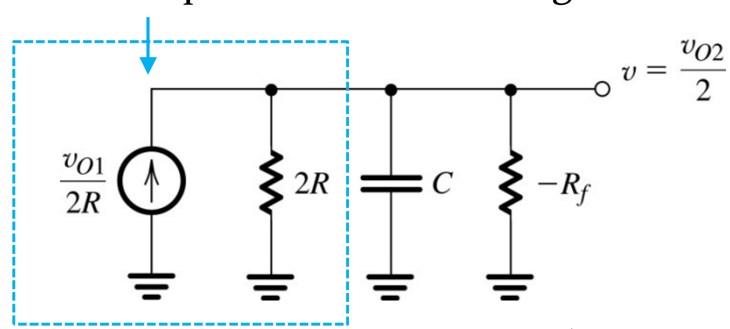
$$L(s)=-\frac{1}{s^2R^2C^2+sRC\left(\frac{1}{2}-\frac{1}{2}\frac{2R}{R_f}\right)}=1\rightarrow s^2R^2C^2+sRC\left(\frac{1}{2}-\frac{R}{R_f}\right)+1=0$$

$$\rightarrow R_f=2R \text{ and } s=\pm j\omega_0=\pm j/RC$$

- Poles are initially located in RHP (for $R_f < 2R$) to ensure that oscillation starts
- Too much positive feedback results in higher output distortion
- v_{O2} is purer than v_{O1} because of the filtering action provided by the second integrator on the peak-limited output of the first integrator
- v_{O2} and v_{O1} have a phase difference of 90° due to the integrator function



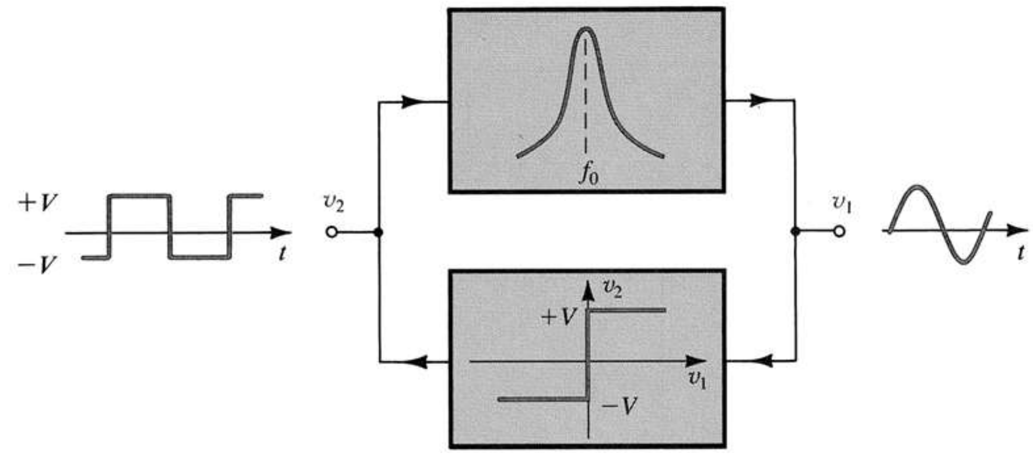
Norton equivalent of the 1st stage

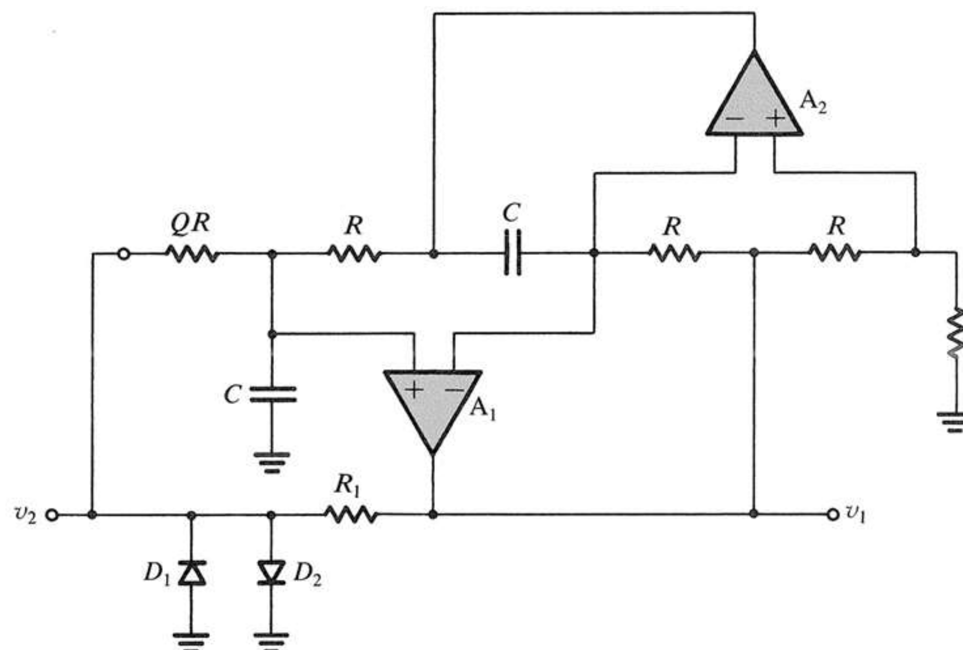


$$\frac{v_{O2}}{v_{O1}}=\frac{1}{sRC+\left(\frac{1}{2}-\frac{R}{R_f}\right)}=\frac{1}{sRC}$$

Active-Filter Tuned Oscillator

- ❑ The circuit consists of a high-Q bandpass filter connected in a positive-feedback loop with a hard limiter
- ❑ Any filter circuit with positive gain can be used to implement the bandpass filter
- ❑ Can generate high-quality output sine waves
- ❑ Have independent control of frequency, amplitude and distortion of the output sinusoid





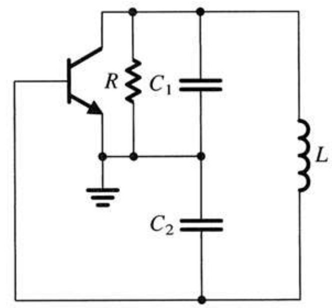
Final Remark

- ❑ Op amp-RF oscillators ~ 10 to 100kHz
- ❑ Lower limit: passive components
- ❑ Upper limit: frequency response and slew rate of op amp

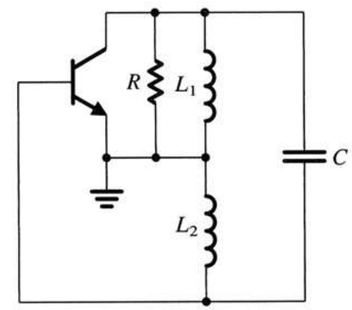
14.3 LC and Crystal Oscillators

LC Tuned Oscillators

- ❑ Colpitts oscillator: capacitive divider
- ❑ Hartley oscillator: inductive divider
- ❑ A parallel *LC* circuit between base and collector
- ❑ *R* models the overall losses

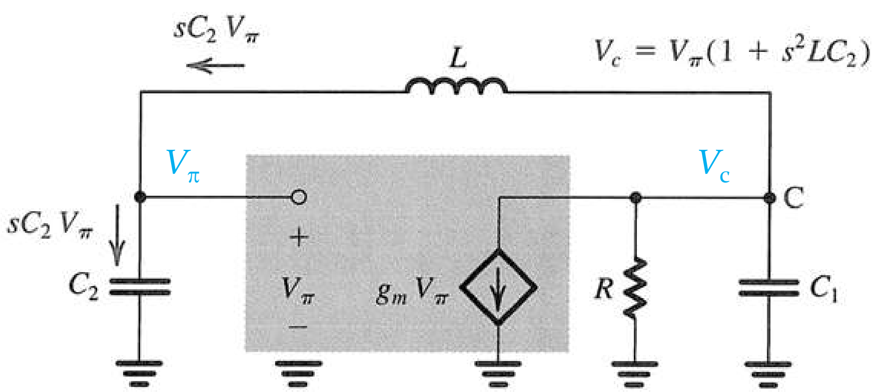


$\omega_0 = 1/\sqrt{L(1/C_1 + 1/C_2)^{-1}}$



$\omega_0 = 1/\sqrt{(L_1 + L_2)C}$

Analysis of Colpitts Oscillators



$$\begin{cases} sC_2 v_\pi = sL(v_c - v_\pi) \\ sL(v_c - v_\pi) + g_m v_\pi + v_c / R + sC_1 v_c = 0 \end{cases}$$

$$\rightarrow s^3 LC_1C_2 + s^2 \frac{LC_2}{R} + s(C_1 + C_2) + (g_m + \frac{1}{R}) = 0$$

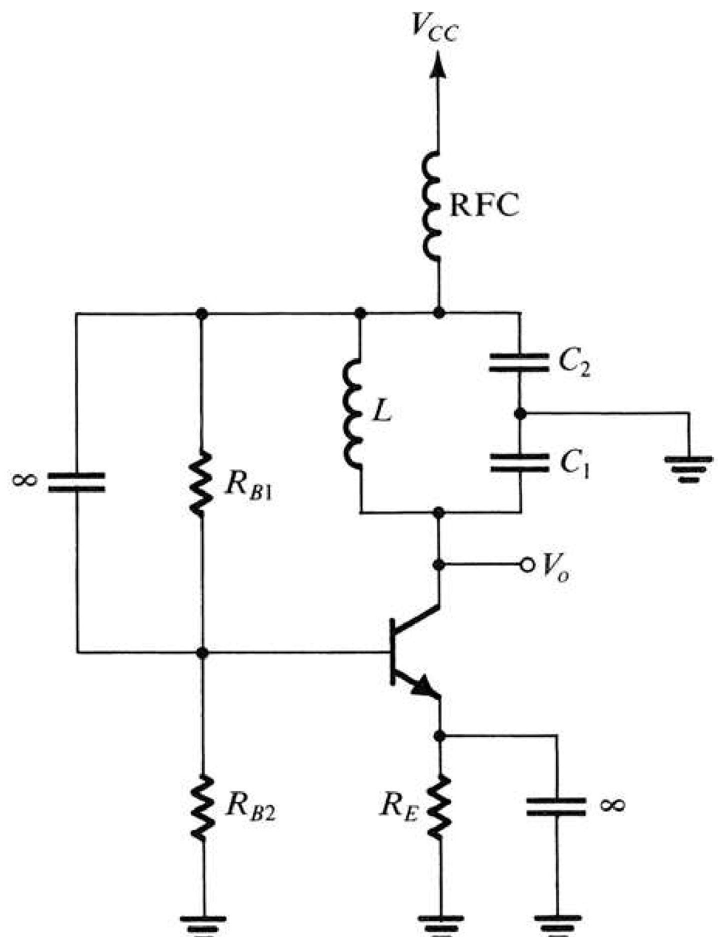
Barkhausen criterion:

$$s = \pm j\omega_0 \rightarrow \left(g_m + \frac{1}{R} - \frac{\omega_0^2 LC_2}{R} \right) + j\left[\omega_0 (C_1 + C_2) - \omega_0^3 LC_1C_2\right] = 0$$

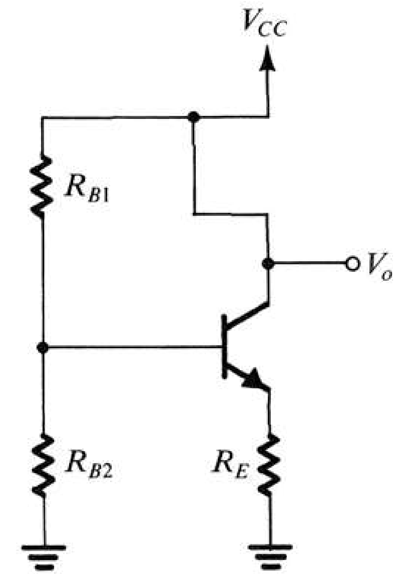
$$\rightarrow \omega_0 = 1/\sqrt{LC_1C_2/(C_1 + C_2)} \text{ and } g_m = \frac{1}{R}\frac{C_2}{C_1}$$

- ❑ Utilize the transistor's nonlinear *I-V* characteristics for amplitude control (self-limiting)
- ❑ Collector (drain) current waveforms are distorted due to the nonlinear characteristics
- ❑ Output voltage is a sinusoid with high purity because of the filtering action of the *LC* tuned circuit

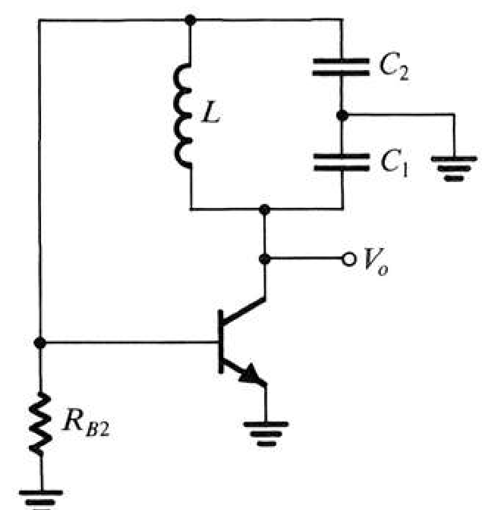
Complete Circuit for a Colpitts Oscillator



DC Analysis

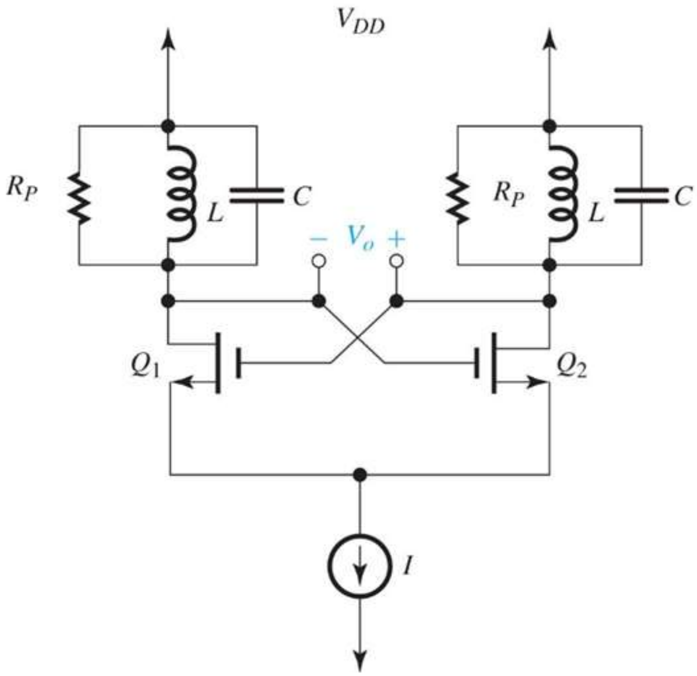


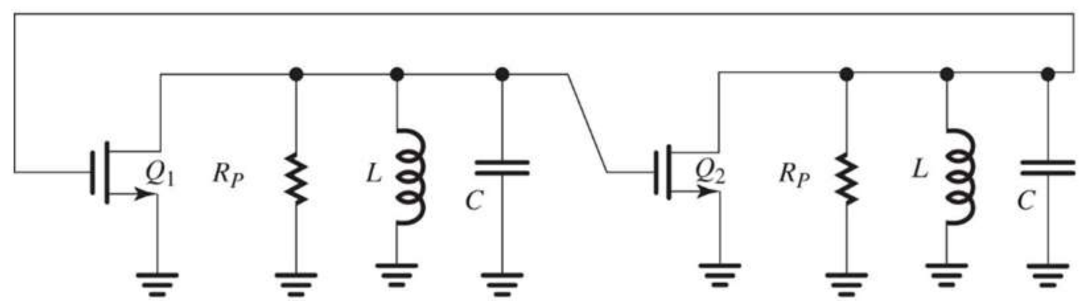
AC Analysis



The Cross-Coupled LC Oscillator

- ❑ Popular LC oscillator circuit suitable for IC implementation
- ❑ Capable of operating at high frequencies (up to hundreds of GHz)
- ❑ The oscillation frequency is defined by the LC tank
- ❑ The cross-couple pair is to start up the oscillation
- ❑ Differential oscillation output available





Loop gain:

$$L(s) = \left(\frac{g_m}{1/R + 1/sL + sC} \right)^2$$

Barkhausen criterion:

$$L(j\omega_0) = \left(\frac{g_m}{1/R + 1/j\omega_0 L + j\omega_0 C} \right)^2 = 1 \rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \text{ and } g_m = \frac{1}{R}$$

Crystal Oscillators

- Crystal impedance:

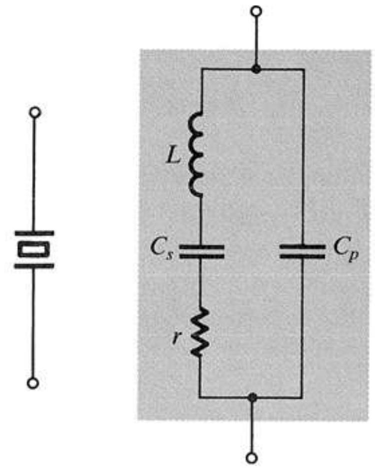
$$Z(s)=1/\left[sC_p+\frac{1}{sL+1/sC_s}\right]$$

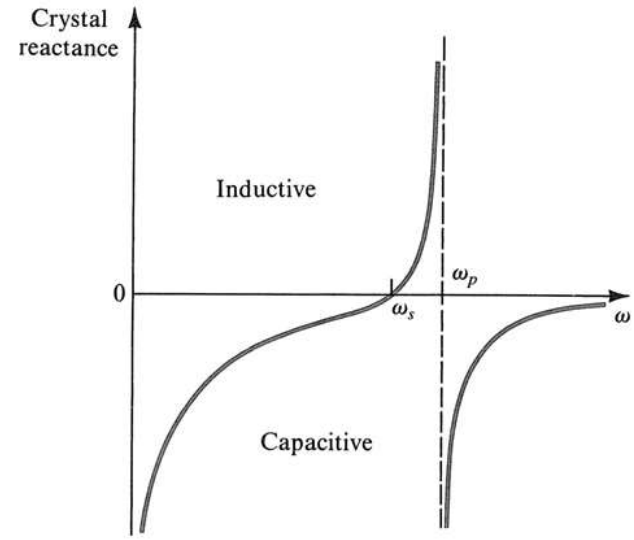
$$Z(s)=\frac{1}{sC_p}\frac{s^2+1/LC_s}{s^2+[(C_p+C_s)/LC_pC_s]}$$

$$\omega_s=1/\sqrt{LC_s}$$

$$\omega_p=1/\sqrt{L(1/C_s+1/C_p)^{-1}}$$

$$Z(j\omega)=-j\frac{1}{\omega C_p}\left(\frac{\omega^2-\omega_s^2}{\omega^2-\omega_p^2}\right)$$

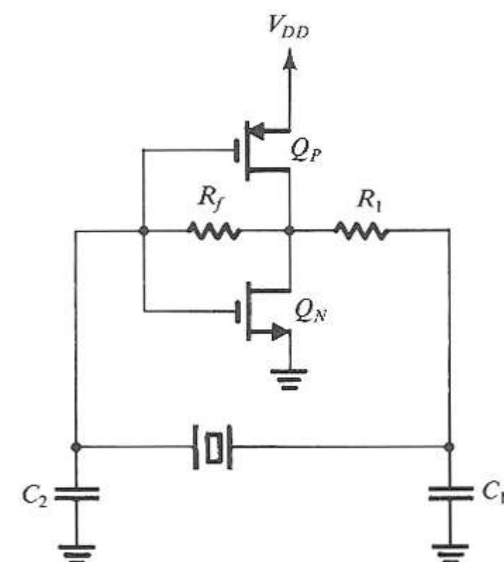




- Crystal reactance is inductive over very narrow frequency (ω_s to ω_p)
- The frequency band is well defined for a given crystal
- Use the crystal to replace the inductor of the Colpitts oscillators
- Oscillation frequency is dominated by C_s (much smaller than other C's)

$$\omega_0\approx 1/\sqrt{LC_s}=\omega_s$$

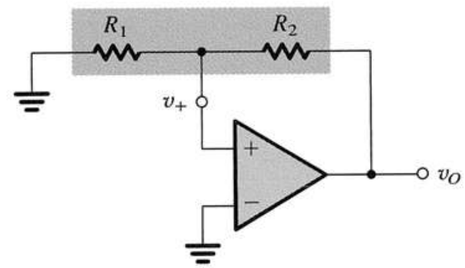
- Crystals are available with resonance frequencies KHz ~ hundred MHz
- The oscillation frequency is fixed (tuning is not possible)

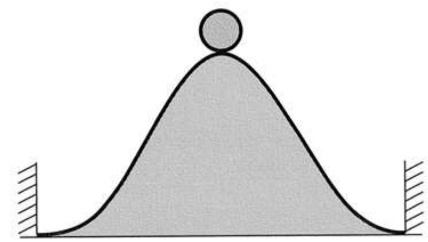


14.4 Bistable Multivibrators

Bistable Characteristics

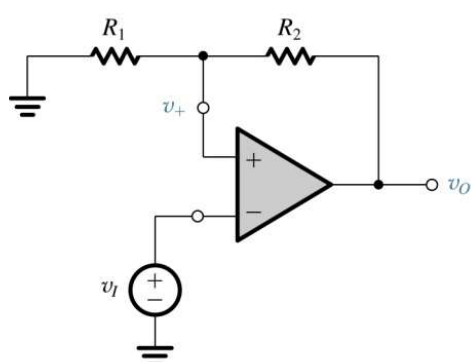
- ❑ Positive feedback for bistable multivibrator
- ❑ Stable states:
  - (1) $v_O = L_+$ and $v_+ = L_+R_1/(R_1+R_2)$
  - (2) $v_O = L_-$ and $v_+ = L_-R_1/(R_1+R_2)$
- ❑ Metastable state: $v_O = 0$ and $v_+ = 0$

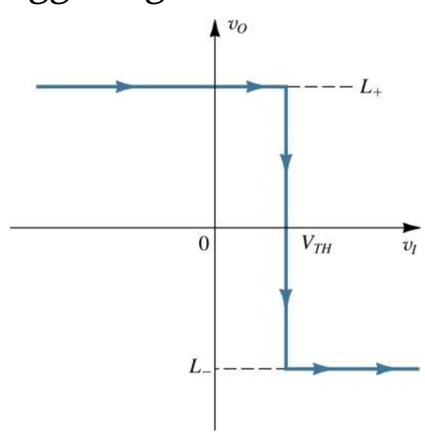


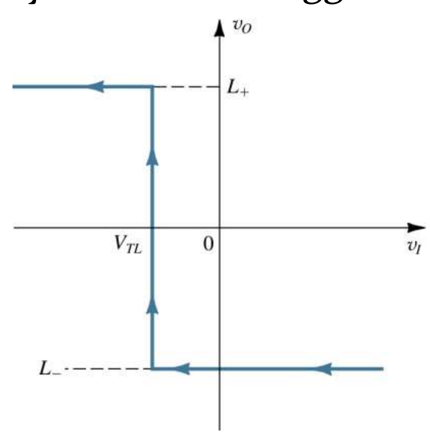


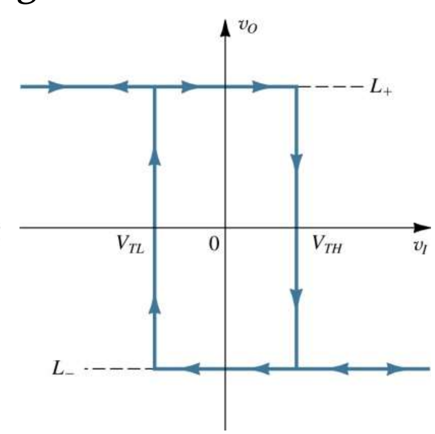
Transfer Characteristics of the Inverting Bistable Circuit

- ❑ Initially $v_I = L_-$, the bistable is in the state of $v_O = L_+$ and $v_+ = L_+R_1/(R_1+R_2)$
  - $\rightarrow$ v_O change state to L_- when v_I increases to a value of $L_+R_1/(R_1+R_2)$
- ❑ Initially $v_I = L_+$, the bistable is in the state of $v_O = L_-$ and $v_+ = L_-R_1/(R_1+R_2)$
  - $\rightarrow$ v_O change state to L_+ when v_I decreases to a value of $L_-R_1/(R_1+R_2)$
- ❑ The circuit exhibits hysteresis with a width of $(V_{TH}-V_{TL})$
- ❑ Input v_I is referred to as a trigger signal which merely initiates or triggers regeneration







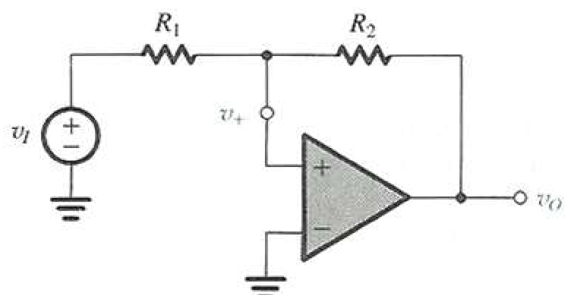


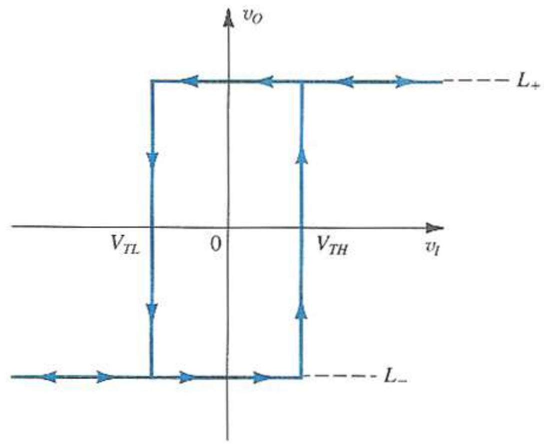
Transfer Characteristics of the Noninverting Bistable Circuit

- Initially $v_I = L_-$, the bistable is in the state of $v_O = L_-$ and $v_+ = v_I R_2/(R_1+R_2)+L\_R_1/(R_1+R_2) < 0$

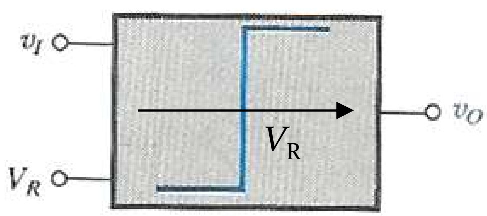
  $\rightarrow v_O$ change state to L_+ when v_I increases to a value (V_{TH}) that causes $v_+ = 0 \rightarrow V_{TH} = -L\_(R_1/R_2)>0$
- Initially $v_I = L_+$, the bistable is in the state of $v_O = L_+$ and $v_+ = v_I R_2/(R_1+R_2)+L_+ R_1/(R_1+R_2) > 0$

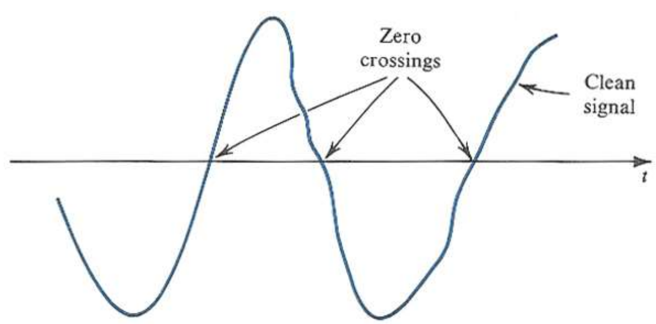
  $\rightarrow v_O$ change state to L_- when v_I decreases to a value (V_{TL}) that causes $v_+ = 0 \rightarrow V_{TL} = -L_+(R_1/R_2)<0$

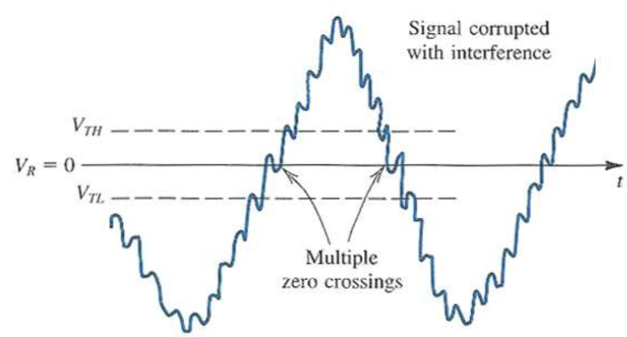




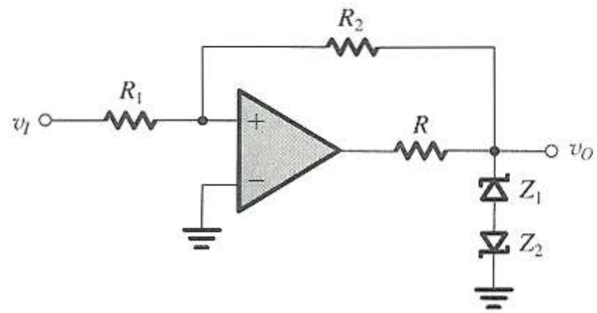
Application of the Bistable Circuit as a Comparator



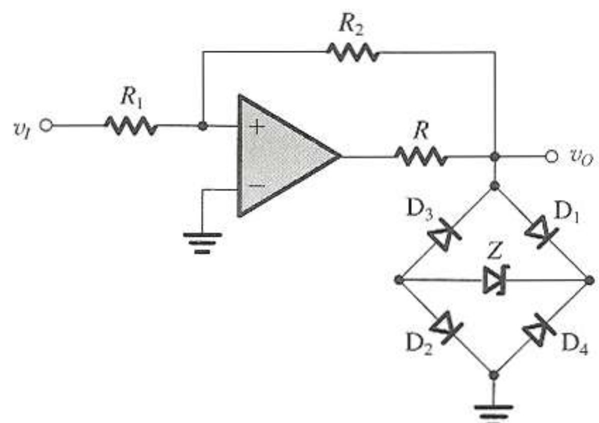




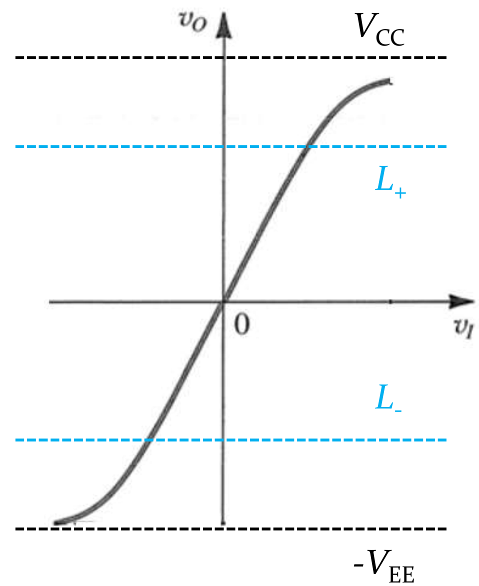
Limiter Circuits for Precise Output Levels



$$L_+ = V_{Z1} + V_D$$
$$L_- = -(V_{Z1} + V_D)$$

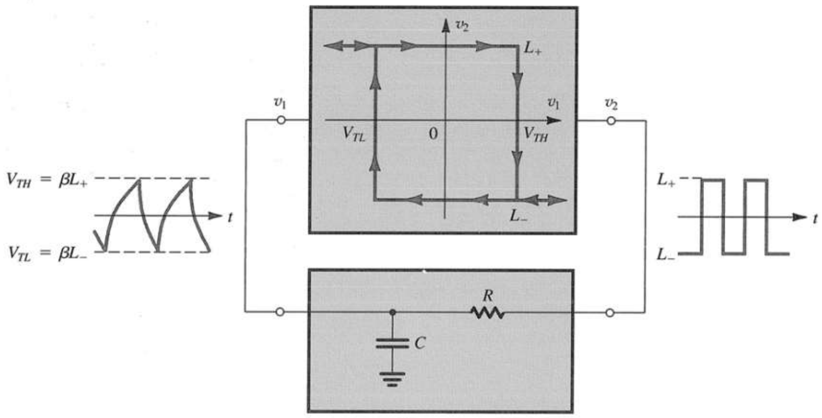


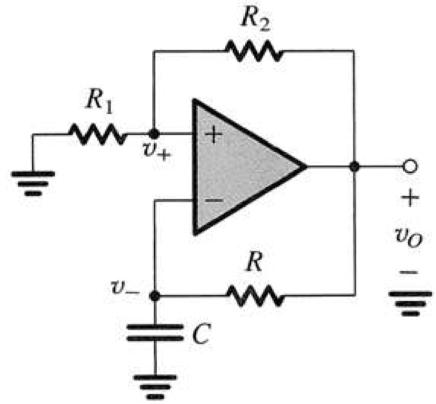
$$L_+ = V_Z + V_{D1} + V_{D2}$$
$$L_- = -(V_Z + V_{D3} + V_{D4})$$

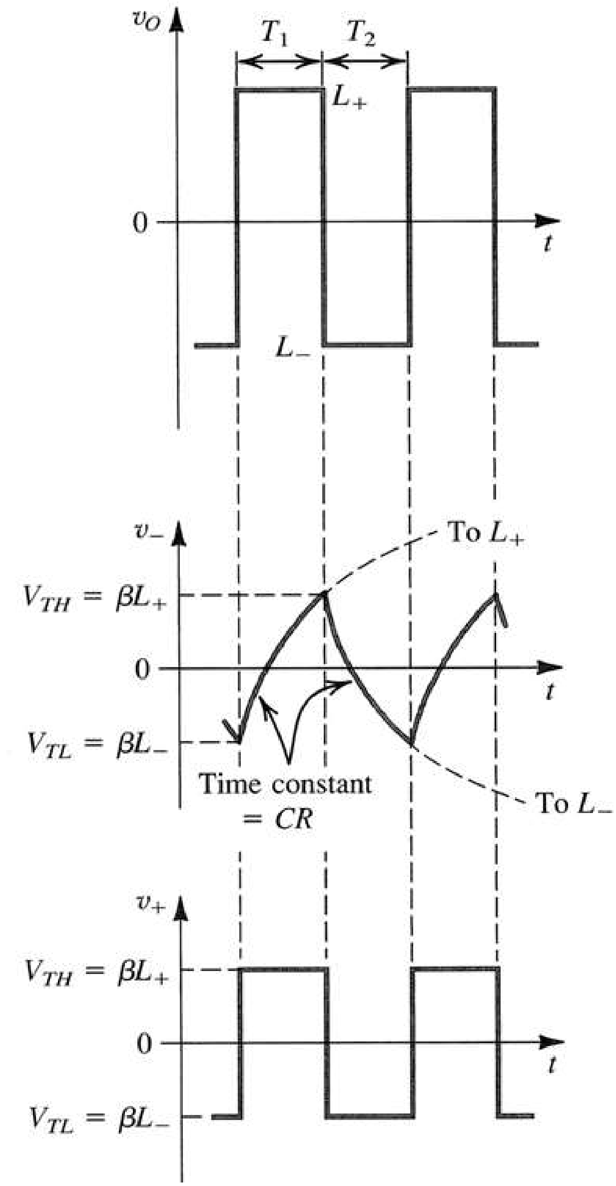


14.5 Generation of Square and Triangular Waveforms using Astable Multivibrators

Operation of the Astable Multivibrator







- RC charge/discharge: $V = V_\infty - (V_\infty - V_0)e^{-t/RC} \rightarrow \Delta t = RC\ln\left(\frac{V_\infty - V_0}{V_\infty - V}\right)$
- For $v_O = L_+$ and $v_+ = v_O R_1/(R_1+R_2) > 0$
  - $\rightarrow v_-$ is charged toward L_+ through RC
  - $\rightarrow v_O$ change stage to L_- when $v_- = v_+$
- For $v_O = L_-$ and $v_+ = v_O R_1/(R_1+R_2) < 0$
  - $\rightarrow v_-$ is discharged toward L_- through RC
  - $\rightarrow v_O$ change stage to L_+ when $v_- = v_+$

$$v_- = L_+ - (L_+ - \beta \cdot L_-)e^{-t/RC} \rightarrow T_1 = RC\ln\frac{1-\beta(L_-/L_+)}{1-\beta}$$

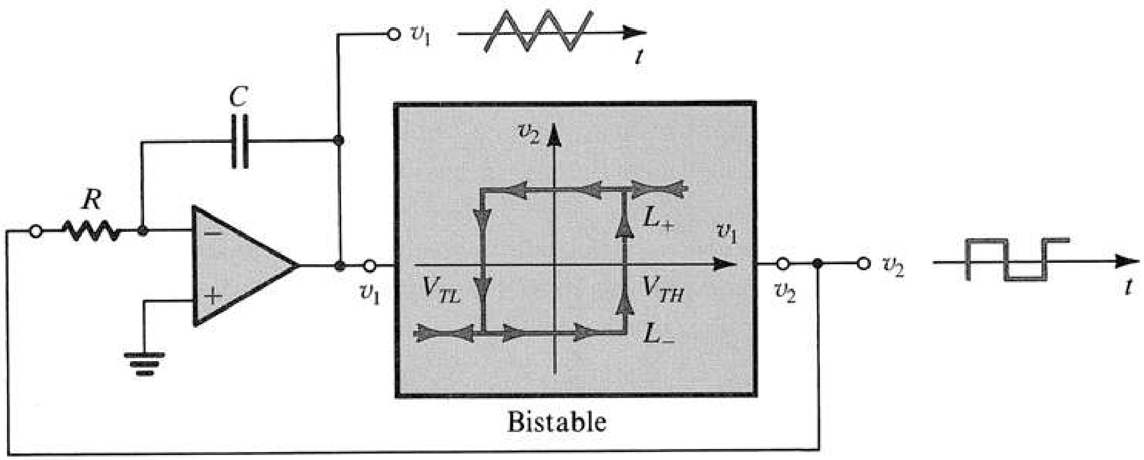
$$v_- = L_- - (L_- - \beta \cdot L_+)e^{-t/RC} \rightarrow T_2 = RC\ln\frac{1-\beta(L_+/L_-)}{1-\beta}$$

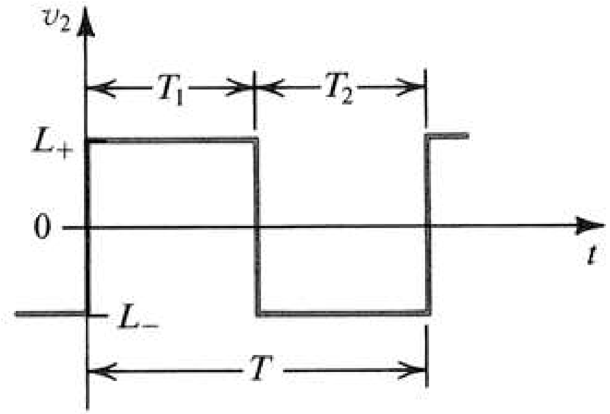
- For $L_- = -L_+$: $T \approx 2RC\ln\frac{1+\beta}{1-\beta}$

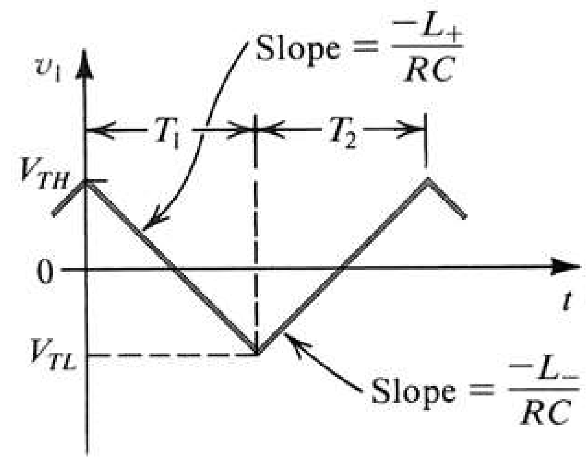
Generation of Triangular Waveforms

❑ Triangular can be obtained by replacing the low-pass *RC* circuit with an integrator

❑ The bistable circuit required is of the noninverting type







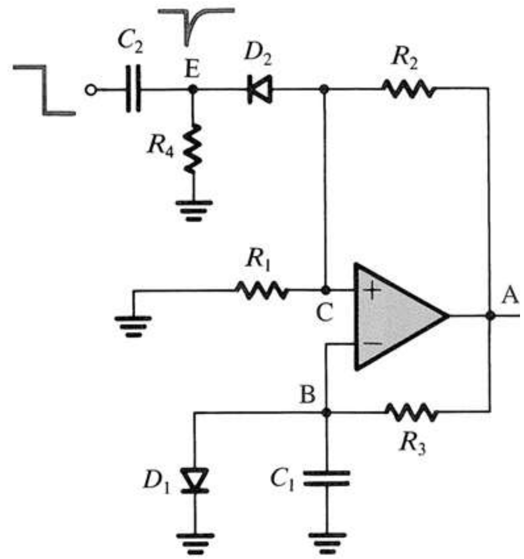
$$\frac{V_{TH}-V_{TL}}{T_1}=\frac{L_+}{RC}\rightarrow T_1=RC\frac{V_{TH}-V_{TL}}{L_+}$$

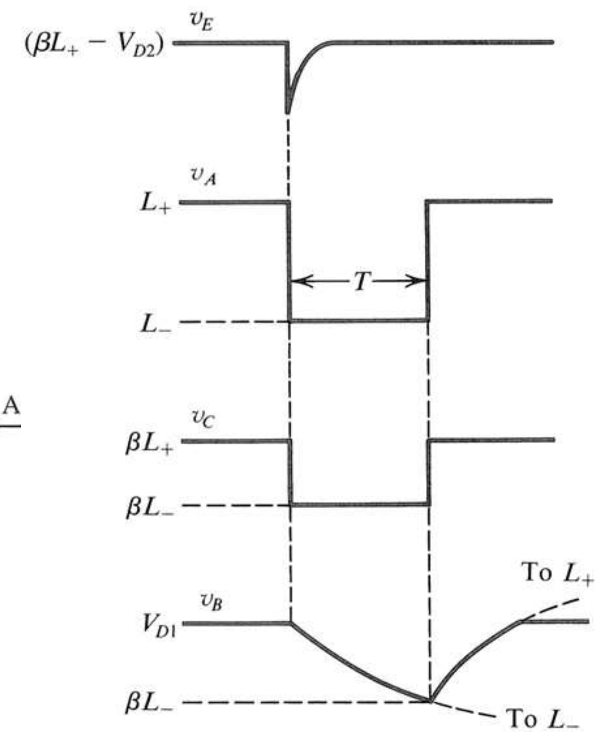
$$\frac{V_{TH}-V_{TL}}{T_2}=\frac{-L_-}{RC}\rightarrow T_2=RC\frac{V_{TH}-V_{TL}}{-L_-}$$

14.6 Generation of a Standardized Pulse – The Monostable Multivibrators

Op-Amp Monostable Multivibrators

- Circuit components:
  - Trigger: C_2, R_4 and D_2
  - Clamping diode: D_1
  - $R_4 >> R_1 \rightarrow i_{D4} \approx 0$
- The circuit has one stable state:
  - $v_O = L_+$
  - $v_B = V_{D1} \approx 0$
  - D_1 and D_2 on
- Operation of monostable multivibrator
  - Negative step as the trigger input
  - D_2 conducts heavily
  - v_C is pulled below v_B for effective trigger
  - v_O changes state to L_- and v_C becomes negative
  - D_1 and D_2 off and C_1 is discharged toward L_-
  - v_O changes state to L_+ as $v_B = v_C = \beta L_-$
  - C_1 is charged toward L_+
  - v_B is clamped to $V_{D1} \approx 0$ and the circuit is back to its stable state
  - Positive trigger step turns off D_2 (invalid trigger)



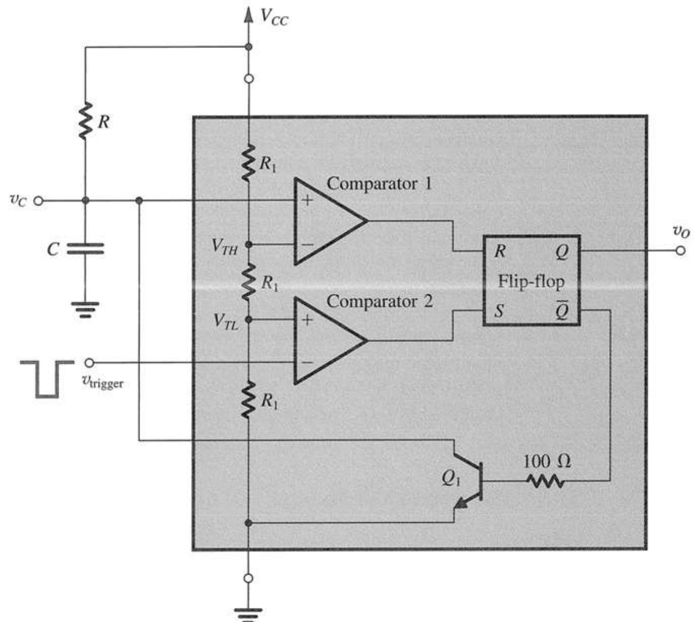


$$v_B(t) = L_- - (L_- - V_{D1})e^{-t/R_3C_1}$$

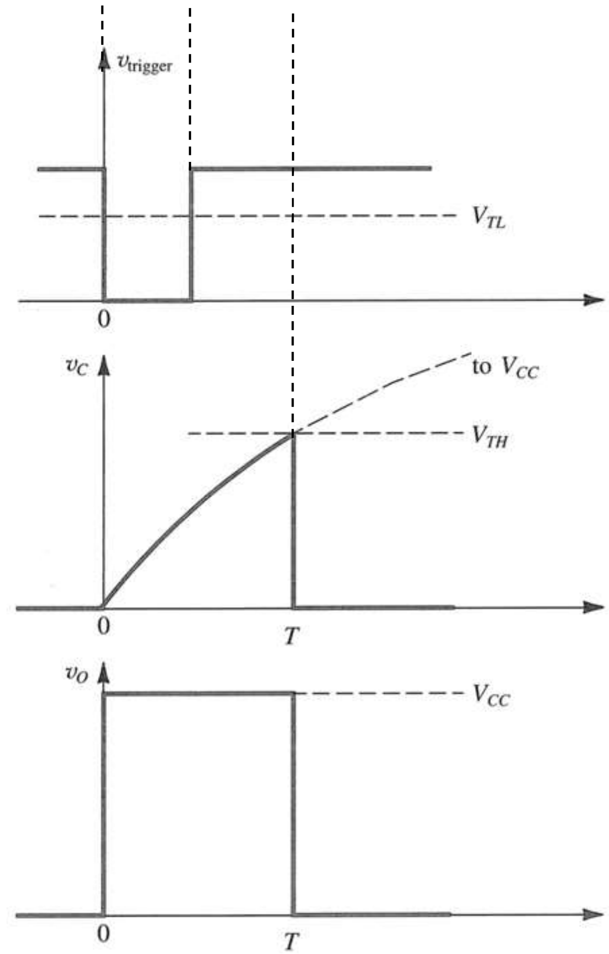
$$\rightarrow T \approx C_1R_3 \ln\left(\frac{L_- - V_{D1}}{L_- - \beta \cdot L_-}\right) \approx C_1R_3 \ln\left(\frac{1}{1-\beta}\right)$$

14.7 Integrated-Circuit Timers

Monostable Multivibrator using 555 Timer Circuit



S	0	1	0	$0 \to 0$
R	0	0	0	$1 \to 0$

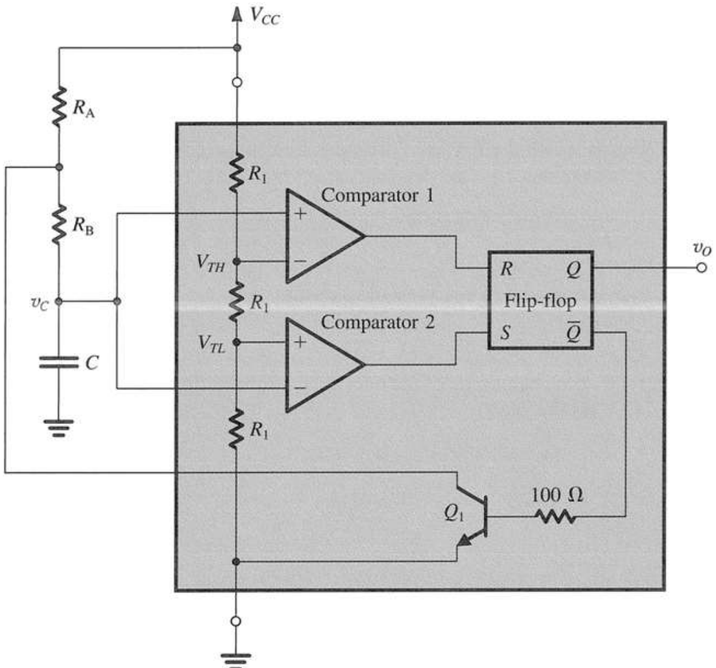


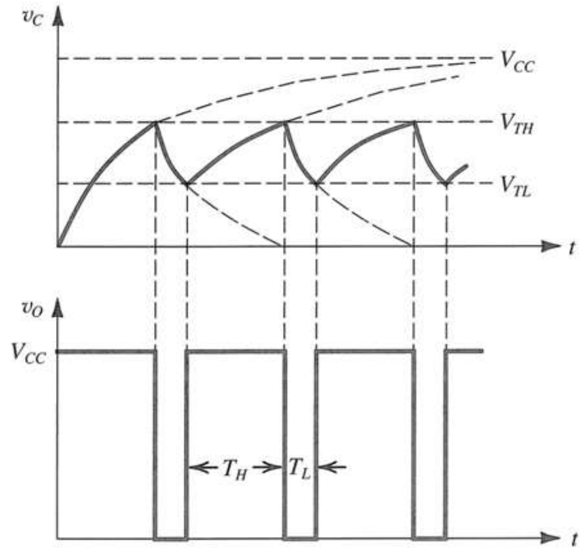
- Stable state: $S = R = 0$ and $Q = 0$
  $\rightarrow Q_1$ on and $v_C = 0$
- Trigger ($v_{trigger} < V_{TL}$): $S = 1$ and $Q = 1$
  $\rightarrow Q_1$ off and v_C is charged toward V_{CC}
- Trigger pulse removal ($v_{trigger} > V_{TL}$): $S = R = 0$ and $Q = 1$
  $\rightarrow Q_1$ off and v_C is charged toward V_{CC}
- End of recovery period ($v_C = V_{TH}$): $R = 1$ and $Q = 0$
  $\rightarrow Q_1$ on and v_C is discharged toward GND
- Stable state: v_C drops to 0 and $S = R = 0$ and $Q = 0$

$$v_C(t) = V_{CC}(1 - e^{-t/RC})$$

$$T = RC \ln 3 \approx 1.1RC$$

Astable Multivibrator using 555 Timer Circuit





$$v_C(t) = V_{CC} - (V_{CC} - V_{TL})e^{-t/C(R_A+R_B)}$$

$$T_H = C(R_A + R_B)\ln 2 \approx 0.69C(R_A + R_B)$$

$$v_C = V_{TH}e^{-t/CR_B}$$

$$T_L = CR_B \ln 3 \approx 0.69CR_B$$

$$T = T_H + T_L = 0.69CR_B$$

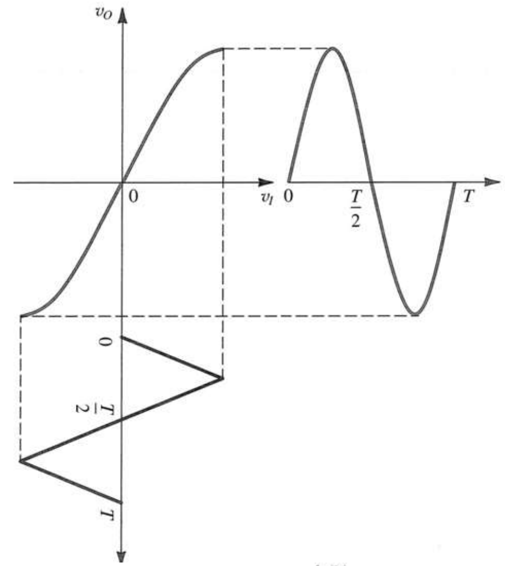
$$\text{Duty cycle} \equiv \frac{T_H}{T_H + T_L} = \frac{R_A + R_B}{R_A + 2R_B}$$

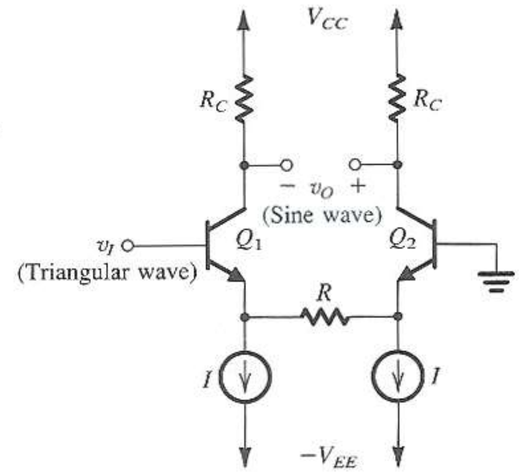
- Operation of astable multivibrator
  - Initially v_C = 0: S/R = 1/0 and Q = 1 → Q_1 off and v_C is charged toward V_{CC} thru R_A and R_B
  - v_C reaches V_{TH}: S/R = 0/1 and Q = 0 → Q_1 on and v_C is discharged toward GND thru R_B
  - v_C reaches V_{TL}: S/R = 1/0 and Q = 1 → Q_1 off and v_C is charged toward V_{CC} thru R_A and R_B

14.8 Nonlinear Waveform-Shaping Circuits

Nonlinear Amplification Method

- ❑ Use amplifiers with nonlinear transfer characteristics
  to convert triangular wave to sine wave
- ❑ Differential pair with an emitter degeneration resistance can be used as sine-wave shaper





Breakpoint Method

- ❑ R_4, R_5 >> R_1, R_2 and R_3 to avoid loading effect
  - ■ $-V_1 < v_{IN} < V_1$:

    $\rightarrow v_O = v_{IN}$
  - ■ $-V_2 < v_{IN} < -V_1$ or $V_1 < v_{IN} < V_2$

    $\rightarrow v_O = V_1 + (v_{IN} - V_1)\, R_5 / (R_4 + R_5)$
  - ■ $v_{IN} < -V_2$ or $V_2 < v_{IN}$

    $\rightarrow v_O = V_2$