CHAPTER 14 SIGNAL GENERATORS AND WAVEFORM-SHAPING CIRCUITS

Chapter Outline
14.1 Basic Principles of Sinusoidal Oscillators
14.2 Op Amp-RC Oscillators
14.3 LC and Crystal Oscillators
14.4 Bistable Multivibrators
14.5 Generation of Square and Triangular Waveforms using Astable Multivibrators
14.6 Generation of a Standardized Pulse-The Monostable Multivibrators
14.7 Integrated-Circuit Timers
14.8 Nonlinear Waveform-Shaping Circuits
14.1 Basic Principles of Sinusoidal Oscillators

Types of Oscillators

- Linear oscillator:
  - Employs a positive feedback loop consisting of an amplifier and a frequency-selective network
  - Some form of nonlinearity has to be employed to provide control of the amplitude of the output

- Nonlinear oscillator:
  - Generates square, triangular, pulse waveforms
  - Employs multivibrators: bistable, astable and monostable

The Oscillator Feedback Loop and Oscillation Criterion

- Positive feedback loop analysis:
  \[ A_f(s) = \frac{x_o}{x_i} = \frac{A(s)}{1 - A(s)\beta(s)} = \frac{A(s)}{1 - L(s)} \]
  where \( L(s) \) is the loop gain

- Characteristic equation: \( 1 - L(s) = 0 \)
  \( \rightarrow \) Find poles of the closed-loop system by solving \( L(s) = 1 \)

- Underdamped
- Oscillation
- Unstable
Barkhausen criterion:
- The phase of loop gain should be zero at $\omega_0$
- The magnitude of the loop gain should be unity at $\omega_0$
- The characteristic equation has roots at $s = \pm j\omega_0$

$$L(j\omega_0) = A(j\omega_0)\beta(j\omega_0) = 1$$
$$\rightarrow |L(j\omega_0)| = 1$$
$$\rightarrow \angle L(j\omega_0) = 0^\circ$$

Stability of oscillation frequency:
- $\omega_0$ is determined solely by the phase characteristics
- A steep function $f(\omega)$ results in a more stable frequency

Stability of oscillation frequency:
- Oscillation: loop gain $A\beta = 1$
- Growing output: loop gain $A\beta > 1$
- Decaying output: loop gain $A\beta < 1$

Nonlinear Amplitude Control

Oscillation mechanism:
- Initiating oscillation: loop gain slightly larger than unity (poles in RHP)
- Gain control: nonlinear network reduces loop gain to unity (poles on $j\omega$-axis)
Limiter Circuits for Amplitude Control

- For small output amplitude ($D_1$ off, $D_2$ off)
  \[ \text{incremental gain (slope)} = -\frac{R_f}{R_1} \]

- For large negative output swing ($D_1$ on, $D_2$ off)
  \[ v_A = \frac{R_3V + R_2V_o}{R_2 + R_3} = V_D \quad v_O = \left( \frac{R_3}{R_2}V + \frac{R_2 + R_3}{R_2}V_D \right) = L_-
  \]
  \[ \frac{v_I}{R_1} + \frac{v_O}{R_f} = I_{D_1} \]
  \[ (V + V_D)/R_2 + (v_O + V_D)/R_3 + I_{D_1} = 0 \]
  \[ v_O = -\frac{R_f}{R_1} - \frac{R_f}{R_2}V - \frac{R_f}{R_2 || R_3}V_D \]
  \[ \text{incremental gain (slope)} = -(R_f || R_4)/R_1 \]

- For large positive output swing ($D_1$ off, $D_2$ on)
  \[ v_B = \frac{-R_3V + R_2V_o}{R_4 + R_5} = V_D \quad v_O = \frac{R_4}{R_5}V + \frac{R_4 + R_5}{R_5}V_D = L_+ \]
  \[ \frac{v_I}{R_1} + \frac{v_O}{R_f} + I_{D_2} = 0 \]
  \[ (v_O - V_D)/R_4 + (-V - V_D)/R_5 = I_{D_2} \]
  \[ v_O = -\frac{R_f}{R_1} - \frac{R_f}{R_2}V - \frac{R_f}{R_2 || R_3}V_D \]
  \[ \text{incremental gain (slope)} = -(R_f || R_3)/R_1 \]
14.2 OP Amp-RC Oscillator Circuits

Wien-Bridge Oscillator

- Define the loop gain
  \[ A(s) = 1 + \frac{R_2}{R_1} \text{ and } \beta(s) = \frac{Z_p}{Z_p + Z_s} \]
  \[ L(s) = \left[ 1 + \frac{R_2}{R_1} \right] \frac{Z_p}{Z_p + Z_s} \rightarrow L(s) = \frac{1 + \frac{R_2}{R_1}}{3 + sRC + 1/sRC} \]

- Pole locations by solving the characteristic equation
  \[ L(s) = \frac{1 + \frac{R_2}{R_1}}{3 + sRC + 1/sRC} = 1 \rightarrow s^2R^2C^2 + \left( 2 - \frac{R_2}{R_1} \right) RC + 1 = 0 \]
  - Oscillation condition: \( 2 - \frac{R_2}{R_1} = 0 \) and \( s = \pm j\omega_0 = \pm j/RC \)
  - Start-up condition: \( 2 - \frac{R_2}{R_1} < 0 \) (poles at RHP)

- Barkhausen criterion:
  \[ L(j\omega) = \frac{1 + \frac{R_2}{R_1}}{3 + j\omega_0 RC + 1/j\omega_0 RC} = 1 \rightarrow (1 - \omega_0^2R^2C^2) + j\omega_0\left( 2 - \frac{R_2}{R_1} \right) RC = 0 \rightarrow 1 - \omega_0^2R^2C^2 = 0 \text{ and } 2 - \frac{R_2}{R_1} = 0 \]
  - Oscillation condition: \( R_2/R_1 = 2 \) and \( \omega_0 = 1/RC \)
  - Start-up condition: \( \omega_0 = RC \) and \( R_2/R_1 = 2 + \Delta \)

- Limiter is used for amplitude control
Wien-Bridge Oscillator with Amplitude Control

- Diodes are used to limit the amplitude of the output swing
- Diodes are off with small-signal operation and can be neglected for analysis of the oscillation condition
Phase-Shift Oscillator

- The circuit oscillates at the frequency for which the phase shift of the RC network is $180^\circ$.
- Only at this frequency will the total phase shift around the loop be $0^\circ$ or $360^\circ$.
- The minimum number of RC sections is three.
- $K$ should be equal to the inverse of the magnitude of the RC network at oscillation frequency.
- Slightly higher $K$ is used to ensure that the oscillation starts.
- Limiter is used for amplitude control.
Quadrature Oscillator

- Based on the two-integrator loop without damping
- Loop gain:

\[ L(s) = \frac{v_{O2}}{v_i} \times \frac{v_{O2}}{v_{O1}} = \left( -\frac{1}{sRC} \right) \frac{1}{sRC + \left( \frac{1}{2} - \frac{1}{2R_f} \right)} \]

- Oscillation condition:

\[ L(s) = -\frac{1}{s^2R^2C^2 + sRC \left( \frac{1}{2} - \frac{1}{2R_f} \right)} \rightarrow s = \pm j \omega_0 = \pm j/R \]

\[ \rightarrow R_f = 2R \text{ and } s = \pm j \omega_0 = \pm j/RC \]

- Poles are initially located in RHP (for \( R_f < 2R \)) to ensure that oscillation starts
- Too much positive feedback results in higher output distortion
- \( v_{O2} \) is purer than \( v_{O1} \) because of the filtering action provided by the second integrator on the peak-limited output of the first integrator
- \( v_{O2} \) and \( v_{O1} \) have a phase difference of 90° due to the integrator function

Norton equivalent of the 1st stage

\[ v = \frac{v_{O2}}{2} \]

\[ \frac{v_{O2}}{v_{O1}} = \frac{1}{sRC + \left( \frac{1}{2} - \frac{R}{R_f} \right)} = \frac{1}{sRC} \]
Active-Filter Tuned Oscillator

- The circuit consists of a high-Q bandpass filter connected in a positive-feedback loop with a hard limiter.
- Any filter circuit with positive gain can be used to implement the bandpass filter.
- Can generate high-quality output sine waves.
- Have independent control of frequency, amplitude and distortion of the output sinusoid.

Final Remark

- Op amp-RF oscillators ~ 10 to 100kHz.
- Lower limit: passive components.
- Upper limit: frequency response and slew rate of op amp.
14.3 LC and Crystal Oscillators

**LC Tuned Oscillators**

- Colpitts oscillator: capacitive divider
- Hartley oscillator: inductive divider
- A parallel LC circuit between base and collector
- R models the overall losses

**Analysis of Colpitts Oscillators**

\[
\omega_0 = \frac{1}{\sqrt{L(1/C_1 + 1/C_2)}}
\]

\[
\omega_0 = \frac{1}{\sqrt{(L_1 + L_2)C}}
\]

- Utilize the transistor’s nonlinear I-V characteristics for amplitude control (self-limiting)
- Collector (drain) current waveforms are distorted due to the nonlinear characteristics
- Output voltage is a sinusoid with high purity because of the filtering action of the LC tuned circuit
Complete Circuit for a Colpitts Oscillator

DC Analysis

AC Analysis

NTUEE Electronics – L. H. Lu
The Cross-Coupled LC Oscillator

- Popular LC oscillator circuit suitable for IC implementation
- Capable of operating at high frequencies (up to hundreds of GHz)
- The oscillation frequency is defined by the LC tank
- The cross-couple pair is to start up the oscillation
- Differential oscillation output available

Loop gain:

\[ L(s) = \left( \frac{g_m}{1/R + 1/sL + sC} \right)^2 \]

Barkhausen criterion:

\[ L(j\omega_0) = \left( \frac{g_m}{1/R + 1/j\omega_0L + j\omega_0C} \right)^2 = 1 \rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \text{ and } g_m = \frac{1}{R} \]
Crystal Oscillators

- Crystal impedance:

\[
Z(s) = \frac{1}{sC_p + \frac{1}{sL + \frac{1}{sC_s}}} \\
Z(s) = \frac{1}{sC_p} \frac{s^2 + 1/LC_s}{s^2 + [(C_p + C_s)/LC_p C_s]}
\]

\[
\omega_s = \frac{1}{\sqrt{LC_s}} \\
\omega_p = \frac{1}{\sqrt{L(1/C_s + 1/C_p)^{-1}}}
\]

- Crystal reactance is inductive over very narrow frequency (\(\omega_s\) to \(\omega_p\))
- The frequency band is well defined for a given crystal
- Use the crystal to replace the inductor of the Colpitts oscillators
- Oscillation frequency is dominated by \(C_s\) (much smaller than other \(C\)'s)

\[
\omega_0 \approx \frac{1}{\sqrt{LC_s}} = \omega_s
\]

- Crystals are available with resonance frequencies KHz ~ hundred MHz
- The oscillation frequency is fixed (tuning is not possible)
14.4 Bistable Multivibrators

**Bistable Characteristics**
- Positive feedback for bistable multivibrator
- Stable states:
  1. $v_O = L_+ \text{ and } v_+ = L_+ R_1/(R_1+R_2)$
  2. $v_O = L_- \text{ and } v_+ = L_- R_1/(R_1+R_2)$
- Metastable state: $v_O = 0 \text{ and } v_+ = 0$

**Transfer Characteristics of the Inverting Bistable Circuit**
- Initially $v_1 = L_-$, the bistable is in the state of $v_O = L_+ \text{ and } v_+ = L_+ R_1/(R_1+R_2)$
  $\rightarrow v_O \text{ change state to } L_- \text{ when } v_1 \text{ increases to a value of } L_+ R_1/(R_1+R_2)$
- Initially $v_1 = L_+$, the bistable is in the state of $v_O = L_- \text{ and } v_+ = L_- R_1/(R_1+R_2)$
  $\rightarrow v_O \text{ change state to } L_+ \text{ when } v_1 \text{ decreases to a value of } L_- R_1/(R_1+R_2)$
- The circuit exhibits hysteresis with a width of $(V_{TH} - V_{TL})$
- Input $v_1$ is referred to as a trigger signal which merely initiates or triggers regeneration
Transfer Characteristics of the Noninverting Bistable Circuit

- Initially $v_I = L_-$, the bistable is in the state of $v_O = L_-$ and $v_+ = v_I R_2/(R_1+R_2)+L_- R_1/(R_1+R_2) < 0$
  - $v_O$ change state to $L_+$ when $v_I$ increases to a value ($V_{TH}$) that causes $v_+ = 0 \rightarrow V_{TH} = -L_-(R_1/R_2)>0$

- Initially $v_I = L_+$, the bistable is in the state of $v_O = L_+$ and $v_+ = v_I R_2/(R_1+R_2)+L_+ R_1/(R_1+R_2) > 0$
  - $v_O$ change state to $L_-$ when $v_I$ decreases to a value ($V_{TL}$) that causes $v_+ = 0 \rightarrow V_{TL} = -L_+(R_1/R_2)<0$

Application of the Bistable Circuit as a Comparator
Limiter Circuits for Precise Output Levels

\[
L_+ = V_{Z_1} + V_D \\
L_- = -(V_{Z_1} + V_D)
\]

\[
L_+ = V_Z + V_{D_1} + V_{D_2} \\
L_- = -(V_Z + V_{D_3} + V_{D_4})
\]
14.5 Generation of Square and Triangular Waveforms using Astable Multivibrators

Operation of the Astable Multivibrator

- **RC charge/discharge:** \( V = V_{\infty} - (V_{\infty} - V_0) e^{-t/RC} \) \( \Rightarrow \Delta t = RC \ln \left( \frac{V_{\infty} - V_0}{V_{\infty} - V} \right) \)

- For \( V_O = L_+ \) and \( V_+ = V_O R_1/(R_1+R_2) > 0 \)
  - \( v_\text{\textendash} \) is charged toward \( L_+ \) through \( RC \)
  - \( v_O \) change stage to \( L_- \) when \( v_\text{\textendash} = v_+ \)

- For \( V_O = L_- \) and \( V_+ = V_O R_1/(R_1+R_2) < 0 \)
  - \( v_\text{\textendash} \) is discharged toward \( L_- \) through \( RC \)
  - \( v_O \) change stage to \( L_+ \) when \( v_\text{\textendash} = v_+ \)

- For \( L_- = -L_+ \): \( T \approx 2RC \ln \frac{1+\beta}{1-\beta} \)
Generation of Triangular Waveforms

- Triangular can be obtained by replacing the low-pass $RC$ circuit with an integrator
- The bistable circuit required is of the noninverting type

\[
\frac{V_{TH} - V_{TL}}{T_1} = \frac{L_+}{RC} \Rightarrow T_1 = RC \frac{V_{TH} - V_{TL}}{L_+}
\]

\[
\frac{V_{TH} - V_{TL}}{T_2} = \frac{-L_-}{RC} \Rightarrow T_2 = RC \frac{V_{TH} - V_{TL}}{-L_-}
\]

Slope $= \frac{-L_+}{RC}$

Slope $= \frac{-L_-}{RC}$
14.6 Generation of a Standardized Pulse
- The Monostable Multivibrators

Op-Amp Monostable Multivibrators

- Circuit components:
  - Trigger: \( C_2, R_4 \) and \( D_2 \)
  - Clamping diode: \( D_1 \)
  - \( R_4 \gg R_1 \) \( \Rightarrow \) \( i_{D4} \approx 0 \)

- The circuit has one stable state:
  - \( v_O = L_+ \)
  - \( v_B = V_{D1} \approx 0 \)
  - \( D_1 \) and \( D_2 \) on

- Operation of monostable multivibrator
  - Negative step as the trigger input
  - \( D_2 \) conducts heavily
  - \( v_C \) is pulled below \( v_B \) for effective trigger
  - \( v_O \) changes state to \( L_- \) and \( v_C \) becomes negative
  - \( D_1 \) and \( D_2 \) off and \( C_1 \) is discharged toward \( L_- \)
  - \( v_O \) changes state to \( L_+ \) as \( v_B = v_C = \beta L_- \)
  - \( C_1 \) is charged toward \( L_+ \)
  - \( v_B \) is clamped to \( V_{D1} \approx 0 \) and the circuit is back to its stable state
  - Positive trigger step turns off \( D_2 \) (invalid trigger)

\[
v_b(t) = L_+ - (L_- - V_{D1}) e^{-t/R_0C_1}
\]

\[
T \approx C_1R_3 \ln \left( \frac{L_- - V_{D1}}{L_- - \beta \cdot L_-} \right) \approx C_1R_3 \ln \left( \frac{1}{1 - \beta} \right)
\]
14.7 Integrated-Circuit Timers

Monostable Multivibrator using 555 Timer Circuit

- **Stable state:** $S = R = 0$ and $Q = 0$  
  $\rightarrow Q_1$ on and $v_C = 0$

- **Trigger ($v_{\text{trigger}} < V_{TL}$):** $S = 1$ and $Q = 1$  
  $\rightarrow Q_1$ off and $v_C$ is charged toward $V_{CC}$

- **Trigger pulse removal ($v_{\text{trigger}} > V_{TL}$):** $S = R = 0$ and $Q = 1$  
  $\rightarrow Q_1$ off and $v_C$ is charged toward $V_{CC}$

- **End of recovery period ($v_C = V_{TH}$):** $R = 1$ and $Q = 0$  
  $\rightarrow Q_1$ on and $v_C$ is discharged toward GND

- **Stable state:** $v_C$ drops to 0 and $S = R = 0$ and $Q = 0$

\[
v_C(t) = V_{CC}(1 - e^{-t/RC})
\]

\[
T = RC \ln 3 \approx 1.1RC
\]
Astable Multivibrator using 555 Timer Circuit

Operation of astable multivibrator

- Initially $v_C = 0$: $S/R = 1/0$ and $Q = 1 \rightarrow Q_1$ off and $v_C$ is charged toward $V_{CC}$ thru $R_A$ and $R_B$
- $v_C$ reaches $V_{TH}$: $S/R = 0/1$ and $Q = 0 \rightarrow Q_1$ on and $v_C$ is discharged toward GND thru $R_B$
- $v_C$ reaches $V_{TL}$: $S/R = 1/0$ and $Q = 1 \rightarrow Q_1$ off and $v_C$ is charged toward $V_{CC}$ thru $R_A$ and $R_B$

$T_H = C(R_A + R_B) \ln 2 \approx 0.69C(R_A + R_B)$

$T_L = CR_B \ln 3 \approx 0.69CR_B$

$T = T_H + T_L = 0.69CR_B$

Duty cycle $= \frac{T_H}{T_H + T_L} = \frac{R_A + R_B}{R_A + 2R_B}$
14.8 Nonlinear Waveform-Shaping Circuits

Nonlinear Amplification Method

- Use amplifiers with nonlinear transfer characteristics
to convert triangular wave to sine wave
- Differential pair with an emitter degeneration resistance can be used as sine-wave shaper

Breakpoint Method

- \( R_4, R_5 \gg R_1, R_2 \) and \( R_3 \) to avoid loading effect
  - \(-V_1 < v_{\text{IN}} < V_1:\)
  \[ v_O = v_{\text{IN}} \]
  - \(-V_2 < v_{\text{IN}} < -V_1 \) or \( V_1 < v_{\text{IN}} < V_2 \)
  \[ v_O = V_1 + \frac{(v_{\text{IN}} - V_1) R_5}{(R_4 + R_5)} \]
  - \( v_{\text{IN}} < -V_2 \) or \( V_2 < v_{\text{IN}} \)
  \[ v_O = V_2 \]