Audio Signal Processing I

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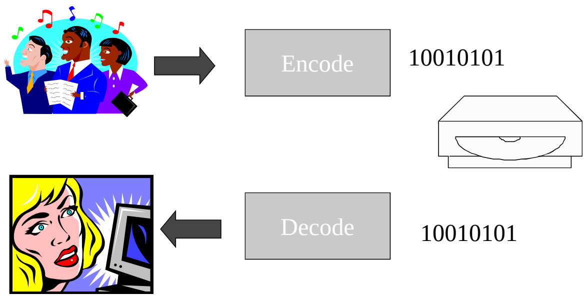
*Reference: Marina Bosi, Perceptual Audio Coding-Lecture Note of Music 422/EE367C, CCRMA, Stanford University, Spring 1999

Reference

- M. Bosi and R. E. Goldberg, Introduction to Digital Audio Coding and Standards, Kluwer Academic Publishers, 2003.

Contents

Audio Coder



Coding Goals

- Maximize the perceived quality of the sound
- Minimize the data rates and complexity
- Related parameters
  - Delay
  - Error robustness
  - Scalability
  - etc.

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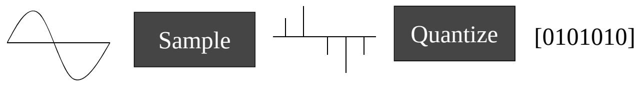
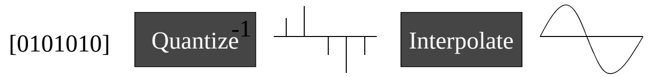
Pulse-Code Modulation

- PCM Encoder



- PCM Decoder



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PCM Example: CD Format

- Sampling frequency: Fs = 44.1 KHz (i.e. one sample every ~0.023 ms)
- Number of bits per sample: R = 16 (i.e. up to 2^{16} = 65536 levels)
- Bit rate: I = Fs*R = 706.5 kb/s per channel
- Total bit rate: I_{stereo} = 1.413 Mb/s
- Signal to noise ratio: SNR ~ 90 dB

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Fourier Transform

- Fourier transform

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

- Inverse Fourier transform

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

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Energy and Power

- Power

$$p(t) = x(t)^2$$

- Energy

$$E = \int_0^T p(t)\,dt = T\langle p(t)\rangle$$

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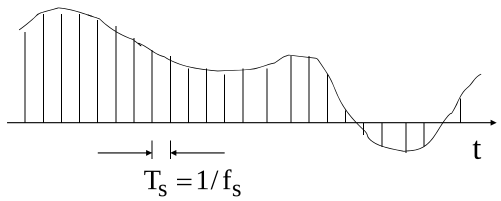
Signal Sampling



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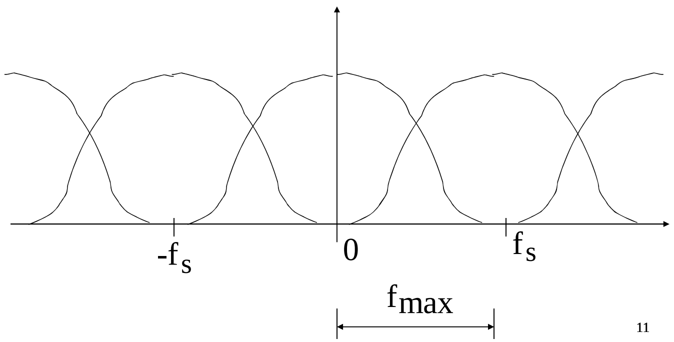
Aliasing



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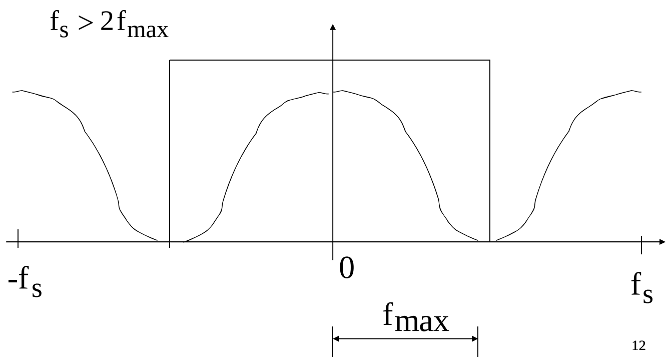
Sampling Theorem



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Eliminating Aliasing

- If your application requires a sample rate below the highest frequencies in the signal, you will need to low pass filter the signal before sampling
- Example: The telephone sample rate is 8 KHz and a 4 KHz low pass filter is employed. (speech: ~100 Hz to ~7 KHz, you really do sound different on the phone)

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Coder Implications

- We can only hear up to ~20 KHz so we should filter out higher frequencies and sample at ~40 KHz to get high quality reproductions of broadband sound
- For example, CDs sample at 44.1 KHz and provide much greater sound quality than telephone system

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Binary Numbers

- Decimal notation
  - Symbols: 0, 1, 2, 3, 4, …, 9
  - e.g., $1999 = 1*10^3 + 9*10^2 + 9*10^1 + 9*10^0$
- Binary notation
  - Symbols: 0, 1
  - e.g.,

$$[01100100] = 0*2^7 + 1*2^6 + 1*2^5 + 0*2^4 + 0*2^3 + 1*2^2 + 0*2^1 + 0*2^0 = 100$$

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Negative Numbers

- Folded binary
  - Use the highest order bit as an indicator of sign
- Two's complement
  - Follows the highest positive number with the lowest negative
  - e.g., 3 bits, $3 \equiv [011], -4 \equiv [100] = 2^4 - 4$
- We use folded binary notation when we need to represent negative numbers

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Two Quantization Methods

- Uniform quantization
  - Constant limit on absolute round-off error $\Delta/2$
  - Poor performance on SNR at low input power
- Floating point quantization
  - Some bits for an exponent
  - the rest for an mantissa
  - SNR is determined by the number of mantissa bits and remain roughly constant
  - Gives up accuracy for high signals but gains much greater accuracy for low signals

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Quantization Error

- Main source of coder error
- Characterized by $\langle q^2 \rangle$
- A better measure
  $\mathrm{SNR} = 10\log_{10}(\langle x^2 \rangle / \langle q^2 \rangle)$
- Does not reflect auditory perception
- Can not describe how perceivable the errors are
- Satisfactory objective error measure that reflects auditory perception does not exist

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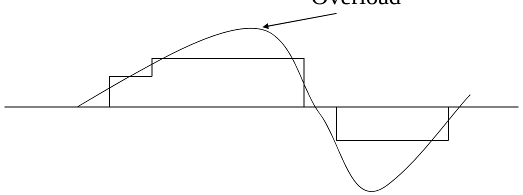
Quantization Error (cont.)

- Round-off error
- Overload error



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Round-Off Error

- Comes from mapping ranges of input amplitudes onto single codes
- Worse when the range of input amplitude onto a code is wider
- Assume that the error follows a uniform distribution
- Average error power $\langle q^2 \rangle = \int_{-\Delta/2}^{\Delta/2} q^2 \frac{1}{\Delta} dq = \Delta^2/12$
- For a uniform quantizer $\langle q^2 \rangle = x_{\max}^2 / (3 * 2^{2R})$

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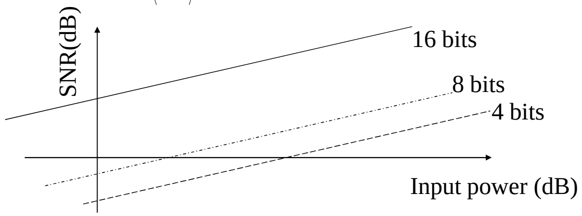
Round-Off Error (cont.)

$$\mathrm{SNR} = 10\log_{10}(\langle x^2 \rangle / \langle q^2 \rangle)$$
$$= 10\log_{10}(\langle x^2 \rangle / x_{\max}^2) + 20\,\mathrm{R}\log_{10} 2 + 10\log_{10} 3$$
$$= 10\log_{10}(\langle x^2 \rangle / x_{\max}^2) + 6.021\mathrm{R} + 4.771$$



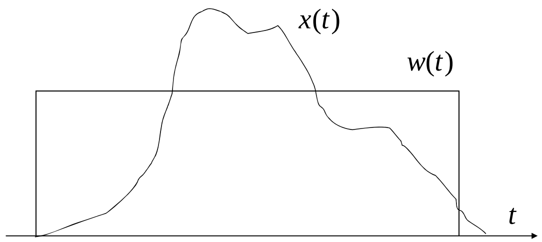
Overload Error

- Comes from signals where $x(t) > x_{\max}$
- Depends on the probability distribution of signal values
- Reduced for high $x_{\max}$
- High $x_{\max}$ implies wide levels and therefore high round-off error
- Requires a balance between the need to reduce both errors

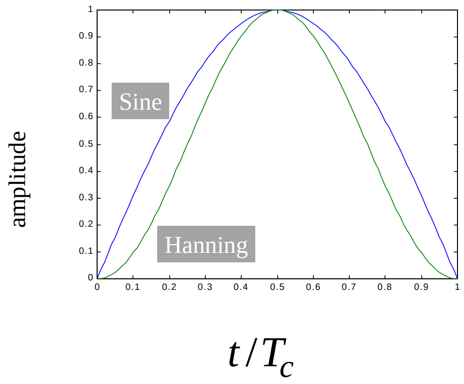
Frequency Domain Coding

- Subdivide the input signal into a number of frequency components and quantize these components separately
- Subdivision into frequency components removes redundancy in the input signal
- Number of bits to encode each frequency component can be variable, so that encoding accuracy can be placed in frequencies where is most needed

Window Function



Windows



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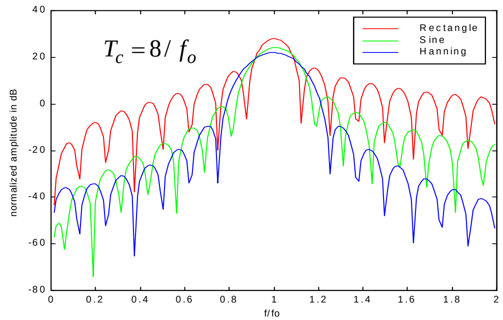
Fourier Transform of a Sine Wave with Various Windows



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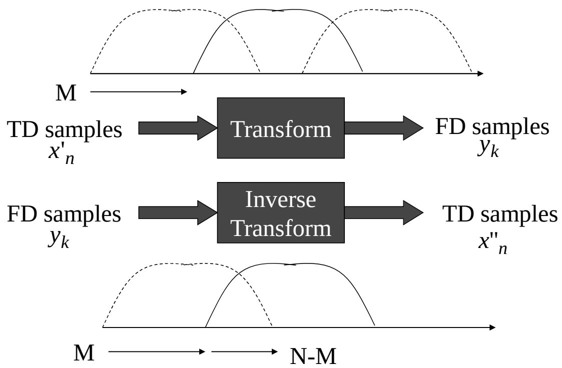
Overlap-Add Scheme



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Reconstruction

- Window input signal with analysis window

$$x_n \rightarrow x'_n = x_n \cdot w_n^a$$

- Apply transform to the windowed signal

$$x'_n \rightarrow y_k = F(x'_n)$$

- Apply the inverse transform

$$y_k \rightarrow x''_n = F^{-1}(y_k) = x'_n$$

- Window with the synthesis window

$$x'_n \rightarrow z_n = x'_n \cdot w_n^s = x_n \cdot w_n^a \cdot w_n^s$$

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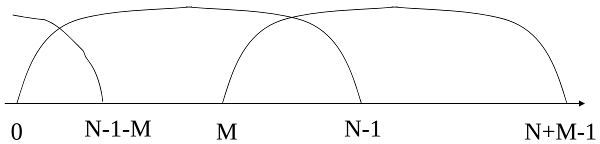
Window Constraints



$$w_n^a \cdot w_n^s + w_{n+M}^a \cdot w_{n+M}^s = 1$$

$$w_n^a \cdot w_n^s = 1 \qquad w_n^a \cdot w_n^s = 1$$

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Perfect Reconstruction

- Assume that the analysis window is the same as the synthesis window
- Assume that the window is symmetrical
- Assume no quantization

$$w(n)^2 + w(n+M)^2 = 1 \quad n = 0, \cdots, N-M-1$$

- A possible window

$$w(n) = \begin{cases} \sin(\pi \cdot ((n+1/2)/(N-M))/2) & n = 0, \cdots, N-M-1 \\ 1 & n = N-M, \cdots, M-1 \\ \sin(\pi \cdot ((N-n-1/2)/(N-M))/2) & n = M, \cdots, N-1 \end{cases}$$

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Overlapping and Required System Rate

- Overlap N-M samples
  - Slide the window by M samples
  - Perform an N-point transform to obtain N frequency samples
  - Transmit N frequency samples every M time samples
  - If there is no overlap, we need only to transmit N frequency samples every N time samples
  - Thus the required system rate is higher than that of the no-overlapping case, because M<N

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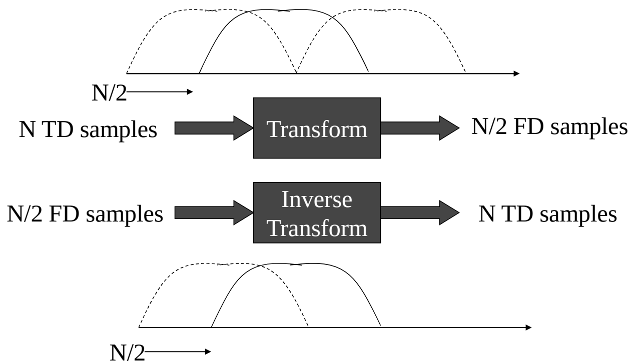
Perfect Reconstruction TDAC Transform



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Oddly Stacked TDAC (OTDAC)

- Modified discrete cosine transform (MDCT)

$$X(k)=\sum_{n=0}^{N-1} x'(n)\cos\left(\frac{2\pi}{N}(n+n_o)(k+\frac{1}{2})\right) \quad k=0,\cdots,\frac{N}{2}-1$$

$$n_o=(\frac{N}{2}+1)/2 \qquad X(k)=-X(N-k-1)$$

- Inverse modified discrete cosine transform (IMDCT)

$$x''(n)=\frac{2}{N}\sum_{k=0}^{N-1} X(k)\cos\left(\frac{2\pi}{N}(n+n_o)(k+\frac{1}{2})\right) \quad n=0,\cdots,N-1$$

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Perfect Reconstruction TDAC Transform

- Symmetric analysis and synthesis windows
- Identical analysis and synthesis windows

$$[w(n)]^2+[w(N/2+n)]^2=1 \quad n=0,\cdots,N/2-1$$

- Sine window

$$w(n)=\sin(\pi*((n+1/2)/(N/2))/2) \quad n=0,\cdots,N/2-1$$

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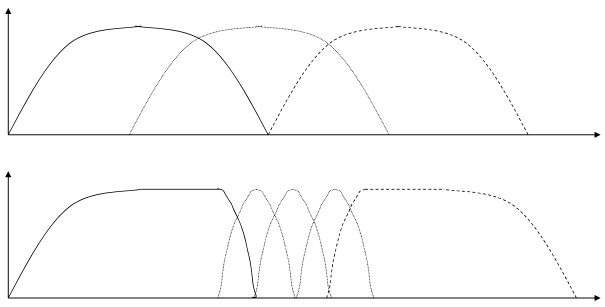
Steady-State vs. Transient Block Selection



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