Multivariate Statistical Analysis Final Exam
January 19, 2007
2 pages, 10 problems, 100 points

1. Let $\mathbf{X}_{1j}$ .and $\mathbf{X}_{2j}$ denote the responses to treatments 1 and 2, respectively, with $j = 1, 2, \cdots, n$ in a paired comparison experiment. Assume that the n independent observed differences $\mathbf{D}_j = \mathbf{X}_{1j} - \mathbf{X}_{2j}$ can be regarded as a random sample from a multivariate normal population. From a certain experiment of 30 observations, we found that the sample average and the sample covariance matrix of the observed differences are $\bar{\mathbf{d}} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\mathbf{S}_d = \begin{bmatrix} 8 & 2 \\ 2 & 5 \end{bmatrix}$, respectively. Test the hypothesis that there is no treatment difference at 90% significance level and compute the 90% Bonferroni simultaneous confidence interval for the individual mean difference. (10%)

2. Two samples were taken from an experiment where two characteristics X_1 and X_2 were measured. Assume that samples 1 and 2 are taken from $N_2(\boldsymbol{\mu}_1, \boldsymbol{\Sigma})$ and $N_2(\boldsymbol{\mu}_2, \boldsymbol{\Sigma})$, respectively. The summary statistics of the two samples for the observations are $\bar{\mathbf{x}}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\mathbf{S}_1 = \begin{bmatrix} 6 & 1 \\ 1 & 4 \end{bmatrix}$, $n_1 = 16$ and $\bar{\mathbf{x}}_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $\mathbf{S}_2 = \begin{bmatrix} 10 & 3 \\ 3 & 6 \end{bmatrix}$, $n_2 = 16$, respectively. Test the hypothesis $H_0 : \boldsymbol{\mu}_1 = \boldsymbol{\mu}_2$ .versus $H_1 : \boldsymbol{\mu}_1 \neq \boldsymbol{\mu}_2$ at 90% significance level and find the 90% Bonferroni simultaneous confidence interval for the individual mean difference. (10%)

3. Consider the following independent samples. Population 1: $\begin{bmatrix} 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix}$; Population 2: $\begin{bmatrix} 9 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix}$; Population 3: $\begin{bmatrix} 3 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \end{bmatrix} \begin{bmatrix} 2 \\ 7 \end{bmatrix}$. Assume that all three samples are from multivariate normal distributions with identical population covariance matrices. Are the mean vectors of those three populations the same at 90% significance level? (10%)

4. Calculate the least square estimates $\hat{\boldsymbol{\beta}}$, the residuals $\hat{\varepsilon}$, and the residual sum of squares for a straight line model $Y = \beta_0 + \beta_1 z$ fit to the data

z	0	1	2	3
y	2	1	3	4

(10%)

5. Use the results of Problem 4 to construct a 90% confidence interval for $E(Y \mid z = 1.5)$ and a 90% prediction interval for a new observation Y when $z = 1.5$. (10%)

6. Suppose the random variables X_1 , X_2 , X_3 .have the covariance matrix $\mathbf{\Sigma} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 8 & 2 \\ 0 & 2 & 5 \end{bmatrix}$. Find the first principal component Y_1 and its proportion of total population variance. (10%)

7. Assume an $m = 1$ orthogonal factor model. Calculate the loading matrix $\mathbf{L}$ and matrix of specific variance $\mathbf{\Psi}$ for the covariance matrix of Problem 6 using the principal component solution method. Compute also the residual matrix. (10%)

8. Establish the inequality (9-19) in p. 486 of the text book. (See the hint for Exercise 9.5 in p. 533 of the textbook). (10%)

9. The (2×1) .random vectors $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ have the joint covariance matrix $\mathrm{Cov}\left(\begin{bmatrix} X_1^{(1)} \\ X_2^{(1)} \\ -- \\ X_1^{(2)} \\ X_2^{(2)} \end{bmatrix}\right) = \mathbf{\Sigma} = \begin{bmatrix} 64 & 0 & | & 0 & 0 \\ 0 & 1 & | & 0.6 & 0 \\ -- & -- & + & -- & -- \\ 0 & 0.6 & | & 1 & 0 \\ 0 & 0 & | & 0 & 64 \end{bmatrix}$. Calculate the first canonical correlation ρ_1^* and the canonical variate pair (U_1, V_1). Note that if $\mathbf{A} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$, $\mathbf{A}^{1/2} = \begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{bmatrix}$ and $\mathbf{A}^{-1} = \begin{bmatrix} 1/\lambda_1 & 0 \\ 0 & 1/\lambda_2 \end{bmatrix}$ (10%)

10. Which part of this course could be most useful/interesting to you? Why? Give your suggestions to improve this course. (10%)