Multivariate Statistical Analysis Mid Term
November 17, 2006
2 pages, 16 problems, 100 points

1. What is your name in Chinese? (6%)

2. Verify that $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ is the inverse matrix of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. (6%)

3. Verify that $\lambda_1 = 9$, $\lambda_2 = 4$ and $\mathbf{e}_1' = [2 \quad 1]$, $\mathbf{e}_2' = [-1 \quad 2]$ are the eigenvalues and eigenvectors of $\begin{bmatrix} 8 & 2 \\ 2 & 5 \end{bmatrix}$. (6%)

4. Expand the quadratic form $(\mathbf{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})$, where $\mathbf{x}' = [x_1 \quad x_2]$, $\boldsymbol{\mu}' = [-1 \quad 1]$, and $\boldsymbol{\Sigma} = \begin{bmatrix} 8 & 2 \\ 2 & 5 \end{bmatrix}$. (6%)

5. Compute the major axes and directions formed by the ellipse $(\mathbf{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu}) = c^2$, where c is a real constant and $\mathbf{x}$, $\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$ are defined in Problem 4. (6%)

6. Find the major axes and directions of the solid ellipse which contains 90% of the probability for bivariate normal distribution $N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$ have been defined in Problem 4. (6%)

7. Find the major axes and directions of the solid ellipse which contains 90% of the probability for sample mean distribution of a sample of size 100. Assume that the probability density distribution of the bivariate population is unknown except that the population mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ is the vector and matrix given in Problem 4. Assume also that 100 can be treated as a large sample size. (6%)

8. Set up a null hypothesis $H_0 : \boldsymbol{\mu} = \boldsymbol{\mu}_0$ and an alternative hypothesis $H_1 : \boldsymbol{\mu} \neq \boldsymbol{\mu}_0$, with $\boldsymbol{\mu}_0' = [-1 \quad 1]$ for bivariate normal distribution $N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Show that the Hotelling's T^2 test rejects the null hypothesis at 90% significance level. Suppose that the sample size is 30, and the sample mean $\bar{\mathbf{x}}' = [-0.5 \quad -0.5]$, the

sample covariance matrix $\mathbf{S} = \begin{bmatrix} 8 & 2 \\ 2 & 5 \end{bmatrix}$. (6%)

9. Find the major axes and directions of the Hotelling's 90% T^2 confidence region for Problem 8. (6%)

10. Determine the 90% simultaneous T^2 confidence intervals for Problem 8. (6%)

11. Determine the 90% Bonferroni simultaneous confidence intervals for Problem 8. (6%)

12. Which component of $\boldsymbol{\mu}_0$ is not a plausible value of the corresponding component of $\boldsymbol{\mu}$ in Problem 8? (6%)

13. Determine the lower control limit (LCL) and the 90% upper control limit (LCL) of the T^2 control chart for future observation which is based on a sample of size n = 30 from a bivariate normal population. Assume that the sample mean is $\bar{\mathbf{x}}' = \begin{bmatrix} -1 & 1 \end{bmatrix}$ and the sample covariance matrix $\mathbf{S} = \begin{bmatrix} 8 & 2 \\ 2 & 5 \end{bmatrix}$ (6%)

14. Show that $\mathbf{x} = \begin{bmatrix} -0.5 & -0.5 \end{bmatrix}$ is under control for the T^2 control chart constructed in Problem 13. (6%)

15. Prove that the function $\eta^b e^{-\eta/2}$ has a maximum, with respect to η, of $(2b)^b e^{-b}$, occurring at $\eta = 2b, \quad b > 0$. This result appeared in p. 171 of the Textbook. (6%)

16. Suppose that we want to measure the weight of a diamond ring. Why and when can we estimate the unknown exact weight by the sample mean of the data of measurements repeated under almost-identical conditions? (10%)