Multivariate Statistical Analysis, Exercise 2, Fall 2011, Prof. S.K. Jeng

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(Transformation of Multivariate Gaussian Distribution)

The population in *Exercise 1* is indeed a multivariate Gaussian distribution. From the statistics derived in *Exercise 1*, we can guess the true parameters of the distribution assuming the estimation error is sufficiently small. Now according to the random sample from *Exercise 1*, please answer the following questions.

(a) Calculate the generalized sample variance and the total sample variance of the random sample in *Exercise 1*.

(b) The true covariance matrix of the population in *Exercise 1* is

$$\Sigma = \begin{pmatrix} 0.8000 & 0.7505 \\ 0.7505 & 1.1000 \end{pmatrix}.$$

Use the computational program to derive the eigenvalues of the covariance matrix. Calculate the product, sum of the eigenvalues and compare it to the generalized sample variance and the total sample variance.

(c) Linearly translate the original random sample according to the matrix

$$A = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}.$$

Plot the transformed random sample and carefully observe the difference between the original random sample. Calculate the new generalized sample variance and the total sample variance.

(d) Linearly transform the original random sample according to the matrix

$$B = \begin{pmatrix} 1000.8012 & 0.7485 \\ 0.7485 & 1001.0858 \end{pmatrix}.$$

Calculate the new generalized sample variance and the total sample variance. (Hint: The *Cayley-Hamilton* says that, for each matrix A with eigenvalues $\{\lambda_i\}_1^n$, then the transformed matrix $f(A)$ has eigenvalues $\{f(\lambda_i)\}_1^n$, where $f(x)$ is an analytic function.)