Chapter 11 Frequency response and filter

11.1 Frequency response

<u>Basics</u>

1. Complex-frequency waveform representation of a real signal

$$x(t) = X_m e^{\sigma t}\cos(wt+\phi_x) = \mathrm{Re}[X_m e^{j\phi_x} e^{(\sigma+jw)t}] = \mathrm{Re}[\underline{X}e^{st}],\ \underline{X} = X_m e^{j\phi_x}$$

$$\longrightarrow \boxed{H(s) = \frac{\underline{Y}}{\underline{X}},\quad s = \sigma + jw} \longrightarrow$$

$$x(t) = X_m\cos(wt+\phi_x) = \mathrm{Re}[X_m e^{j\phi_x} e^{jwt}] = \mathrm{Re}[\underline{X}e^{jwt}]$$

2. σ =0, complex-frequency waveform $\rightarrow$ sinusoidal ac waveform

$$x(t) = X_m\cos(wt+\phi_x) = \mathrm{Re}[X_m e^{j\phi_x} e^{jwt}] = \mathrm{Re}[\underline{X}e^{jwt}]$$

$$\longrightarrow \boxed{H(jw) = \frac{\underline{Y}}{\underline{X}}} \longrightarrow$$

$$y(t) = Y_m\cos(wt+\phi_y) = \mathrm{Re}[Y_m e^{j\phi_y} e^{jwt}] = \mathrm{Re}[\underline{Y}e^{jwt}]$$

$$a(w) \equiv |H(jw)| = \frac{Y_m}{X_m} : \text{amplitude ratio},\ \theta(\mathrm{w}) \equiv \angle H(jw) = \phi_x - \phi_y : \text{phase shift}$$

3. Frequency-response curves: plots of *a(w)* and $\theta(w)$

$$H(jw) = K\frac{(jw-z_1)(jw-z_2)\ldots}{(jw-p_1)(jw-p_2)\ldots}$$

$$a(w) = |K|\frac{|jw-z_1||jw-z_2|\ldots}{|jw-p_1||jw-p_2|\ldots}$$

$$\theta(\text{w}) \equiv \angle K + [\angle(jw-z_1)+\angle(jw-z_2)+\ldots] - [\angle(jw-p_1)+\angle(jw-p_2)+\ldots]$$

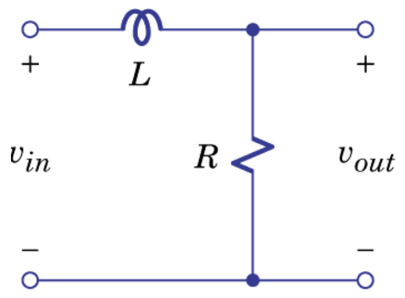
4. $w \rightarrow \infty$ $\quad a(w) \rightarrow \begin{cases} |K|, m=n \\ 0, m<n \end{cases}, \theta(w) \rightarrow \angle K + (m-n)\times 90^\circ$

5. $w \rightarrow 0$, *a(w)* and $\theta(w)$ depend on zeros and poles

Discussion

1. Ex. 11.1 L=0.2H, R=8Ω find steady-state response v_{out}(t)

$$v_{in}(t)=10\cos 20t+10\cos 300t$$



$$\underline{V}_{out}=\underline{V}_{in}\frac{R}{sL+R}=\underline{V}_{in}\frac{R/L}{s+R/L}$$

$$H(jw)=\frac{\underline{V}_{out}}{\underline{V}_{in}}=\frac{40}{jw+40}$$

$$a(w)=|H(jw)|=\frac{40}{\sqrt{w^2+1600}},\ \theta(w)=\angle H(jw)=-\tan^{-1}\frac{w}{40}$$

$$a(20)=0.894,\ \theta(20)=-26.6^\circ,\ a(300)=0.132,\ \theta(300)=-82.4^\circ$$

$$\underline{V}_{out}=\begin{cases}10\times H(20),\ \text{for } w=20\\ 10\times H(300),\ \text{for } w=300\end{cases}$$

$$v_{out}(t)=8.94\cos(20t-26.6^\circ)+1.32\cos(300t-82.4^\circ)$$

frequency selective response, lowpass filter

Frequency response approach is much easier than time-domain approach or phasor approach.

2. Ex.11.2 all-pass circuit

$$\underline{V}_{out}=\underline{V}_C-\underline{V}_x$$

$$=\underline{V}_{in}\left(\frac{1/sC}{R+1/sC}-\frac{R_x}{2R_x}\right)$$

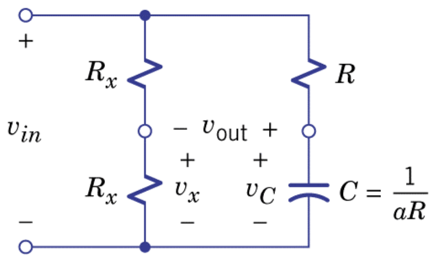
$$=\underline{V}_{in}\left(\frac{1}{sRC+1}-\frac{1}{2}\right)$$

$$=\underline{V}_{in}\left(-\frac{1}{2}\right)\frac{sRC-1}{sRC+1}$$

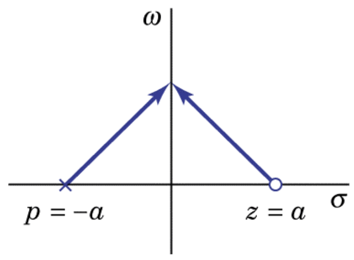
$$\rightarrow H(jw)=\frac{\underline{V}_{out}}{\underline{V}_{in}}=-\frac{1}{2}\,\frac{jw-1/RC}{jw+1/RC}$$

$$a(w)=-\frac{1}{2}$$

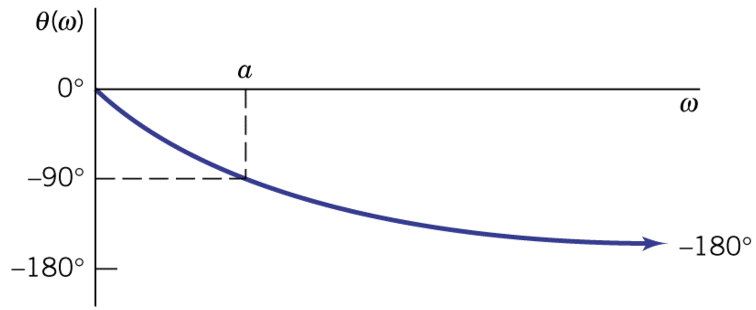
$$\theta(w)=\tan^{-1}\frac{-w}{1/RC}-\tan^{-1}\frac{w}{1/RC}=-2\tan^{-1}\frac{w}{1/RC}$$



(a) All–pass network

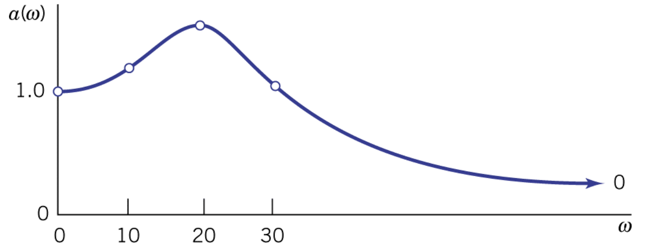


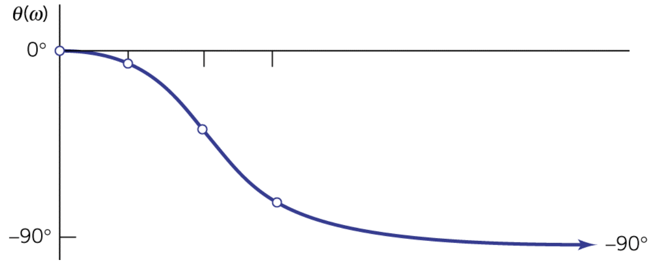
(b) Pole–zero pattern



(c) Phase shift versus ω

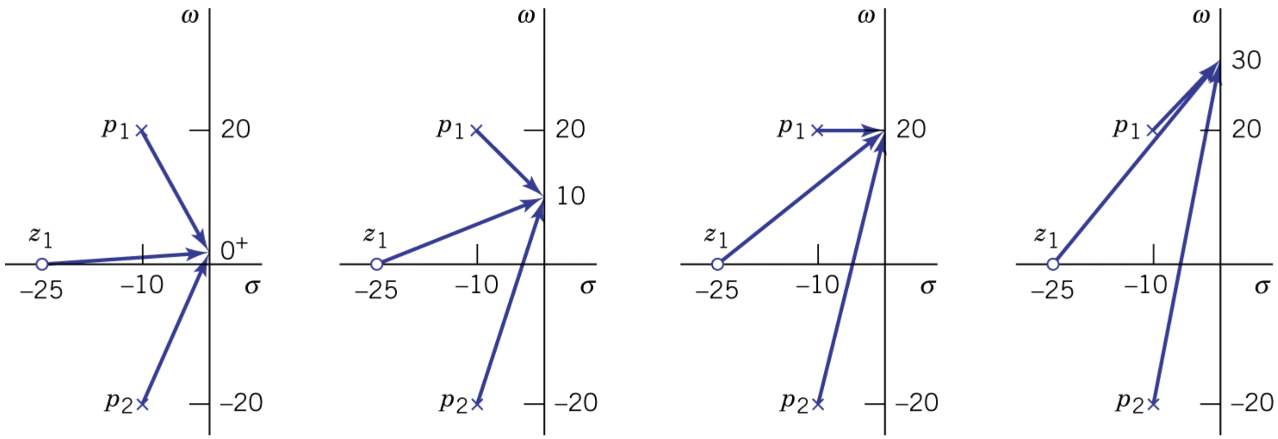
3. Ex. 11.3 plot frequency response





$$H(s)=\frac{20(s+25)}{s^2+20s+500}=\frac{20[s-(-25)]}{(s-p_1)(s-p_2)}$$

$$p_1, p_2=-10\pm j20$$

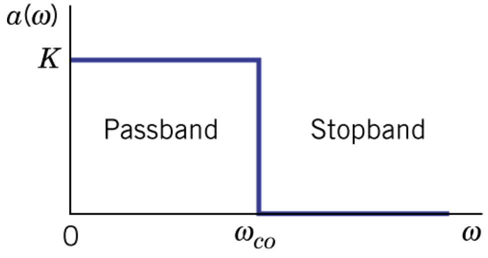


(a) (b) (c) (d)

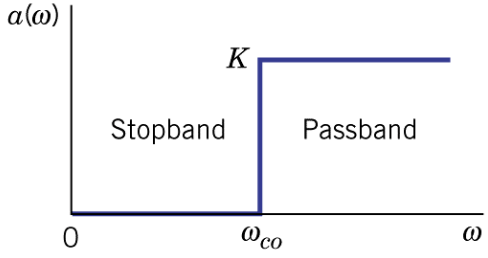
11.2 Filters

Basics

1. Ideal lowpass (LPF) and highpass (HPF) filters, w_{co}: cutoff frequency

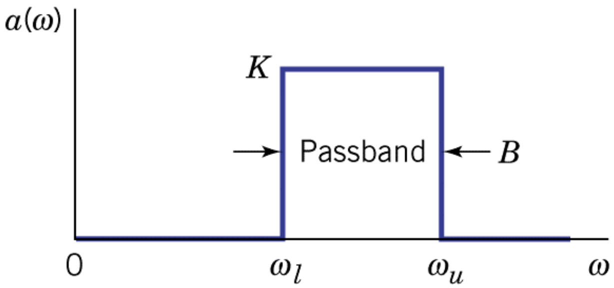


(a) Ideal lowpass filter

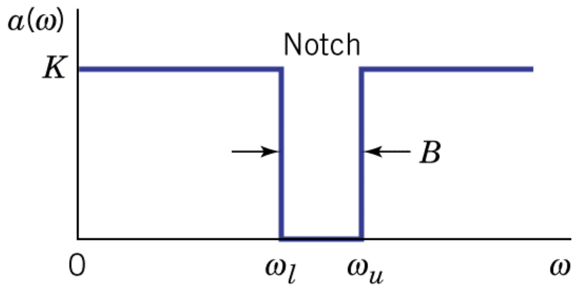


(b) Ideal highpass filter

2. Ideal banspass (BPF) and notch (BSF) filters: w_l: lower cutoff frequency, w_u: upper cutoff frequency, B=w_u-w_l: bandwidth



(a) Ideal bandpass filter



(b) Ideal notch filter

Discussion

1. First-order LPF

$$H_{lp,1}(s)=\frac{V_{out}}{V_{in}}=\frac{1/sC}{R+1/sC}=\frac{1/RC}{s+1/RC}$$

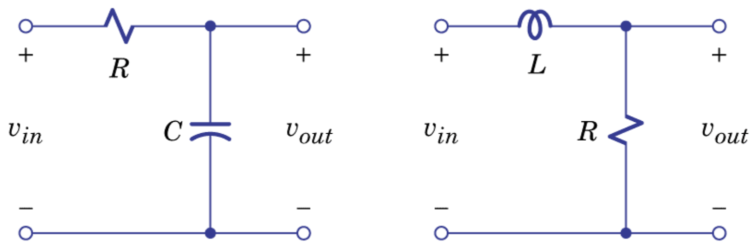
$$H_{lp,2}(s)=\frac{V_{out}}{V_{in}}=\frac{R}{sL+R}=\frac{R/L}{s+R/L}$$

$$\boxed{H_{lp}(s)=\frac{Kw_{co}}{s+w_{co}},\ K=1,\ w_{co}=\frac{1}{\tau},\ \tau=RC}$$

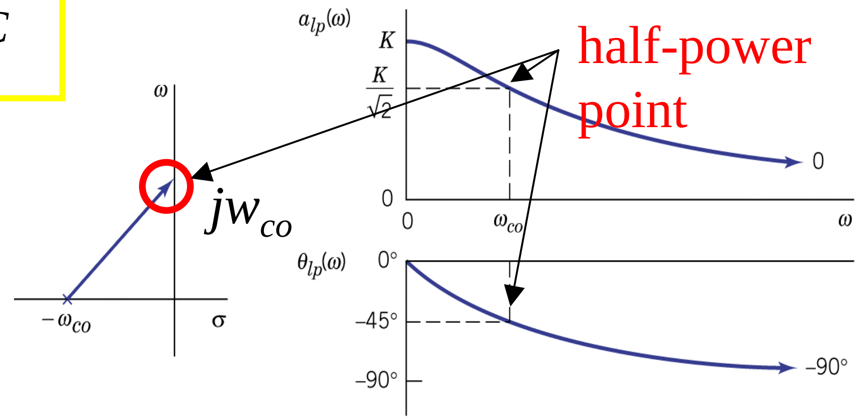
$$H_{lp}(jw)=\frac{Kw_{co}}{jw+w_{co}}=\frac{K}{1+j\dfrac{w}{w_{co}}}$$

$$a_{lp}(w)=\frac{K}{\sqrt{1+(w/w_{co})^2}}$$

$$\theta_{lp}(w)=-\tan^{-1}\frac{w}{w_{co}}$$



(a) Lowpass filters



(a) s–plane diagram (b) Frequency response curves

2. First-order highpass filter

$$H_{hp,1}(s)=\frac{V_{out}}{V_{in}}=\frac{R}{R+1/sC}=\frac{s}{s+1/RC}$$

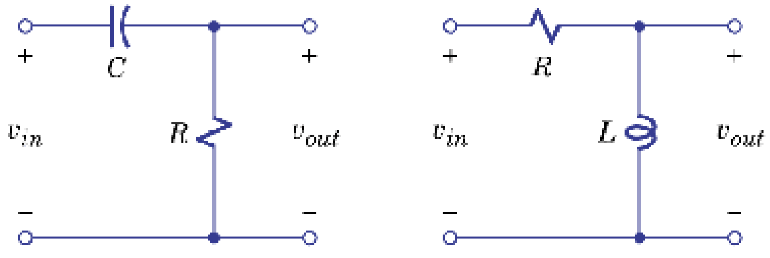
$$H_{hp,2}(s)=\frac{V_{out}}{V_{in}}=\frac{sL}{sL+R}=\frac{s}{s+R/L}$$

$$\boxed{H_{hp}(s)=\frac{Ks}{s+w_{co}},\ K=1,\ w_{co}=\frac{1}{\tau},\ \tau=\frac{L}{R}}$$

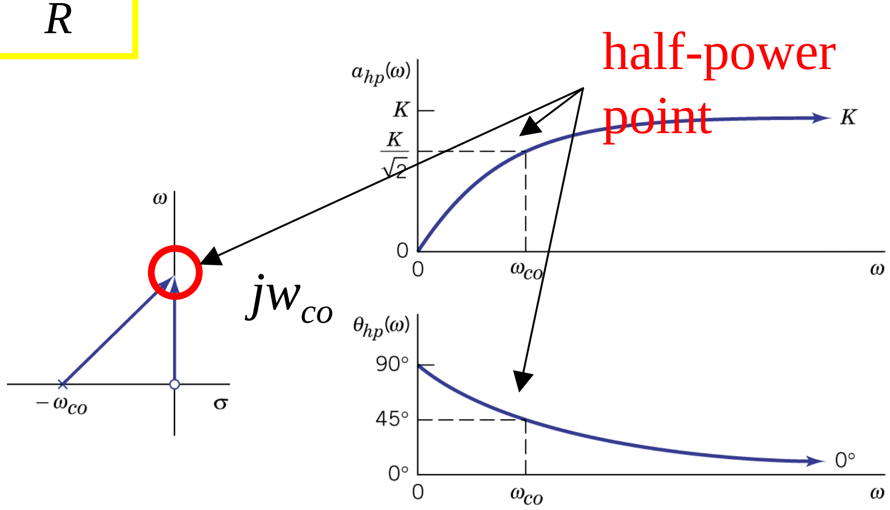
$$H_{hp}(jw)=\frac{Kjw}{jw+w_{co}}=\frac{K}{1-j\dfrac{w_{co}}{w}}$$

$$a_{hp}(w)=\frac{K}{\sqrt{1+(w_{co}/w)^2}}$$

$$\theta_{hp}(w)=\tan^{-1}\frac{w_{co}}{w}$$



(b) Highpass filters

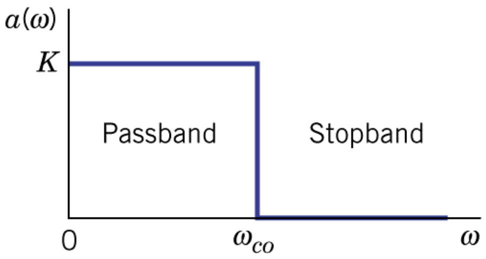


(a) s–plane diagram (b) Frequency response curves

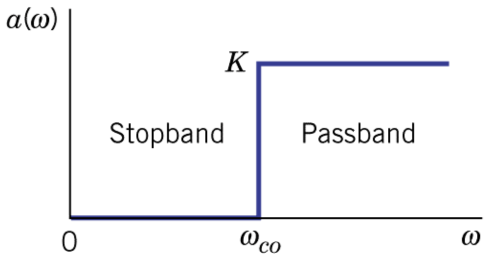
3. LPF to HPF transformation

$$H_{lp}(s)=\frac{Kw_{co}}{s+w_{co}},H_{hp}(s)=\frac{Ks}{s+w_{co}}$$

$$s\to\frac{w_{co}^2}{s}\Rightarrow LPF\to HPF,H_{lp}(\frac{w_{co}^2}{s})=\frac{Kw_{co}}{\frac{w_{co}^2}{s}+w_{co}}=\frac{Ks}{s+w_{co}}=H_{hp}(s)$$



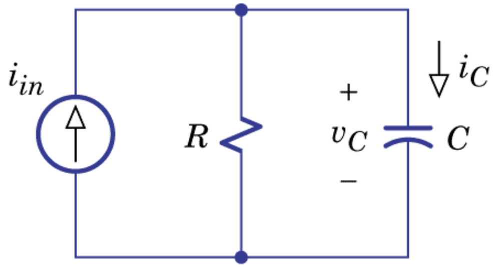
(a) Ideal lowpass filter



(b) Ideal highpass filter

$$0\to\infty,w_{co}\to w_{co}$$

4. Ex.11.4 find H(jw)

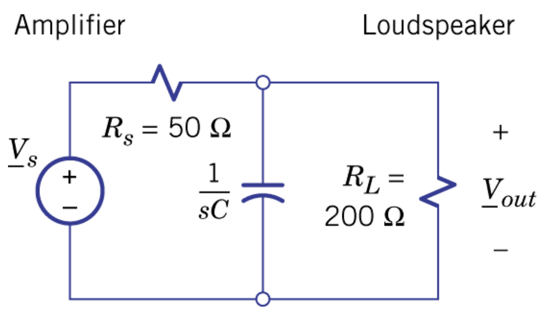


$$\underline{I}_C = \underline{I}_{in}\frac{R}{R+1/sC}$$

$$H(s) = \frac{\underline{I}_C}{\underline{I}_{in}} = \frac{R}{R+1/sC} = \frac{s}{s+1/RC}$$

$$H(jw) = \frac{jw}{jw+1/RC} = \frac{1}{1-j\frac{w_{co}}{w}}, w_{co} = \frac{1}{RC}, HPF$$

5. Ex.11.5 find H(jw) to have f_{co}~4kHz



$$\underline{V}_{out} = \underline{V}_s \frac{\frac{1}{G_L+sC}}{\frac{1}{G_s}+\frac{1}{G_L+sC}} = \underline{V}_s \frac{G_s}{G_s+G_L+sC}$$

$$H(s) = \frac{\underline{V}_{out}}{\underline{V}_s} = \frac{G_s}{G_s+G_L+sC} = \frac{G_s/C}{s+(G_s+G_L)/C} = \frac{Kw_{co}}{s+w_{co}},$$

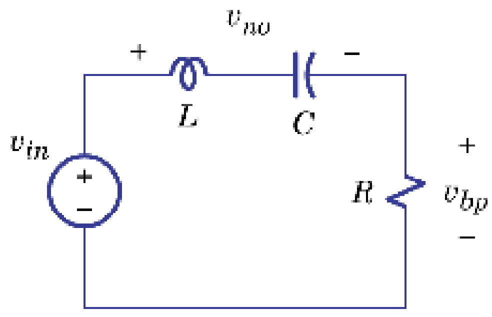
$$w_{co} = 2\pi f_{co} = \frac{G_s+G_L}{C} = \frac{1}{40C} \to C \approx 1uF, K = \frac{G_s}{G_s+G_L} = 0.8$$

if $v_s = 5\cos w_1 t + 0.5\cos w_2 t,\ f_1 = 3kHz,\ f_2 = 16kHz$

$H(jw_1) = 0.64\angle -37°, H(jw_2) = 0.19\angle -76°$

$\to v_s = 3.2\cos(w_1 t - 37°) + 0.095\cos(w_2 t - 76°)$

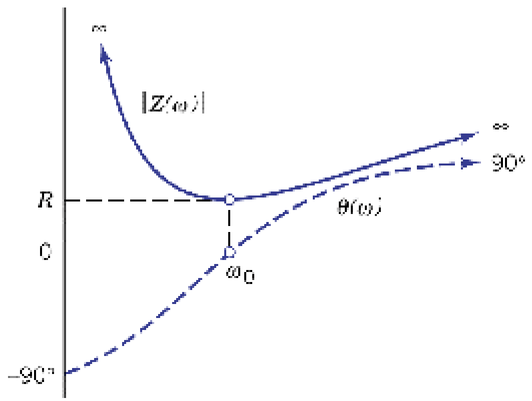
6. Series RLC resonator



$$Z(jw) = R + jwL + \frac{1}{jwC}$$

$$= R + \frac{-w^2LC+1}{jwC}, \; w_o \equiv \frac{1}{\sqrt{LC}}$$

$$X(w_o) = 0, Z(w_o) = R$$



$$H_{bp}(s) \equiv \frac{V_{bp}}{V_{in}} = \frac{R}{sL + 1/sC + R}$$

$$= \frac{sRC}{s^2LC + sRC + 1} = \frac{s\frac{R}{L}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

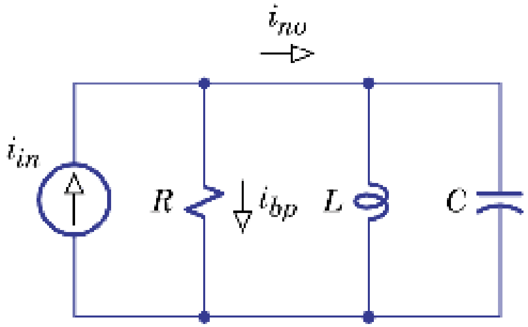
$$\boxed{= \frac{Ksw_o/Q}{s^2 + 2\alpha s + w_o^2} = \frac{Ksw_o/Q}{(s-p_1)(s-p_2)}}$$

$$w_o^2 \equiv \frac{1}{LC}, \alpha \equiv \frac{R}{2L}, K = 1$$

$$Q_{ser} \equiv w_o \frac{W_L}{P_{loss}} = w_o \frac{L}{R} = \frac{w_o}{2\alpha}$$

$$p_1, p_2 = -\frac{w_o}{2Q} \pm jw_o\sqrt{1 - \frac{1}{4Q^2}}$$

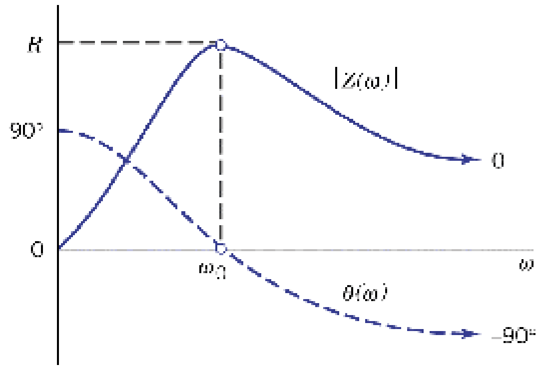
7. Parallel RLC resonator



$$Y(jw) = \frac{1}{R} + jwC + \frac{1}{jwL}$$

$$= \frac{1}{R} + \frac{-w^2LC+1}{jwL}$$

$$B(w_o) = 0, Y(w_o) = \frac{1}{R}$$



$$H_{bp}(s) = \frac{\underline{I}_{bp}}{\underline{I}_{in}} = \frac{\frac{1}{sC+1/sL}}{R+\frac{1}{sC+1/sL}}$$

$$= \frac{\frac{sL}{s^2LC+1}}{R+\frac{sL}{s^2LC+1}} = \frac{sL}{s^2RLC+sL+R}$$

$$= \frac{s\frac{1}{RC}}{s^2+s\frac{1}{RC}+\frac{1}{LC}} = \boxed{\frac{Ksw_o/Q}{s^2+2\alpha s+w_o^2}}$$

$$w_o^2 = \frac{1}{LC}, \alpha \equiv \frac{1}{2RC}, K = 1$$

$$Q_{par} \equiv w_o \frac{W_C}{P_{loss}} = w_o RC = \frac{w_o}{2\alpha}$$

8. Second-order BPF

$$H_{bp}(s)=\frac{K(w_o/Q)s}{s^2+(w_o/Q)s+w_o^2}$$

$$H_{bp}(jw)=\frac{K(w_o/Q)\,jw}{-w^2+(w_o/Q)\,jw+w_o^2}=\frac{K}{1+jQ(\frac{w}{w_o}-\frac{w_o}{w})}$$

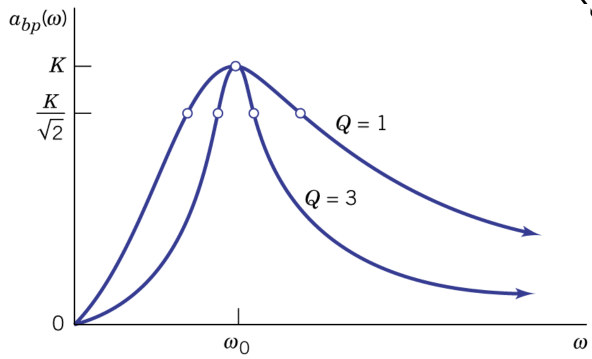
$$a_{bp}(w)=\frac{K}{\sqrt{1+Q^2(\frac{w}{w_o}-\frac{w_o}{w})^2}},\ \theta_{bp}=-\tan^{-1}Q(\frac{w}{w_o}-\frac{w_o}{w})$$

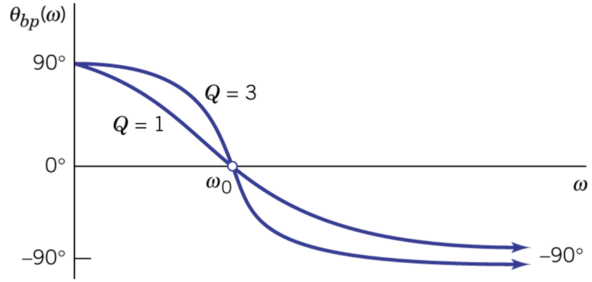
K : midband gain, w_o : center frequency

$$a_{bp}(w_l)=a_{bp}(w_u)\equiv\frac{K}{\sqrt{2}}\rightarrow w_u,w_l=w_o\sqrt{1+\frac{1}{4Q^2}}\pm\frac{w_o}{2Q}$$

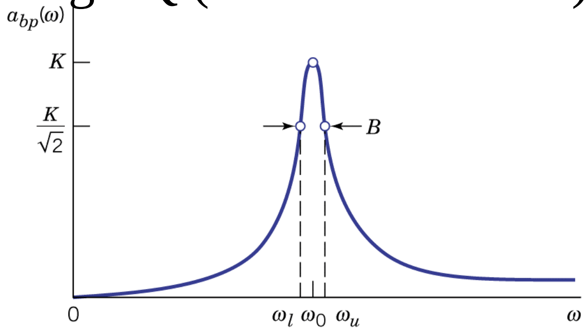
$$\rightarrow \boxed{B\equiv w_u-w_l=\frac{w_o}{Q},\ w_uw_l=w_o^2}$$

$$\xrightarrow{Q\geq 10} w_u,w_l\approx w_o\pm\frac{w_o}{2Q}$$





high-Q (narrowband BPF)



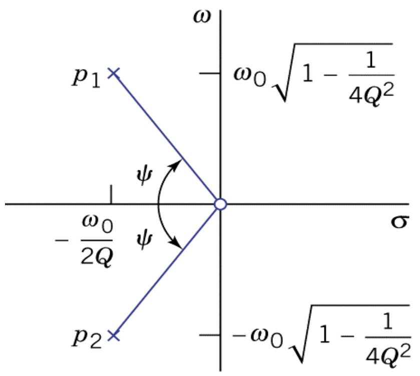
$$H_{bp}(s)=\frac{K(w_o/Q)s}{s^2+(w_o/Q)s+w_o^2}=\frac{K(w_o/Q)s}{(s-p_1)(s-p_2)},Q=\frac{w_o}{2\alpha}$$

(1)underdampe d case : $Q>\frac{1}{2}$, or $\alpha<w_o$

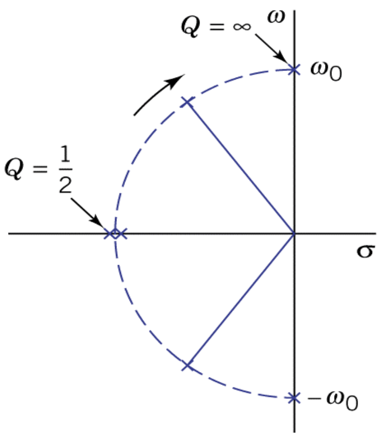
$$p_1,p_2=-\frac{w_o}{2Q}\pm jw_o\sqrt{1-\frac{1}{4Q^2}},|p_1|=|p|_2=Q,$$

$$\angle p_1=180°-\varphi,\angle p_2=180°+\varphi,\varphi=\cos^{-1}\frac{1}{2Q}$$

(2)critical damped case : $Q=\frac{1}{2}$



(a) Pole–zero pattern for second–order bandpass filter



(b) Pole movement as Q changes

9. Second-order notch filter

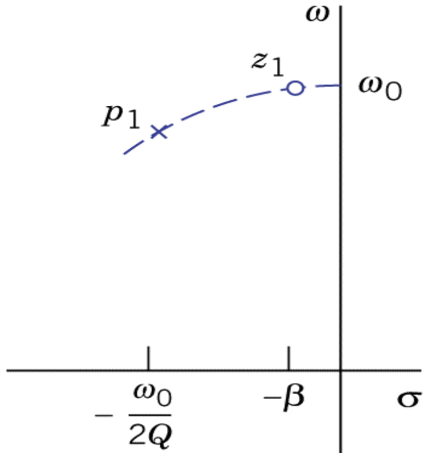
$$H_{no}(s)=\frac{K(s^2+2\beta s+w_o^2)}{s^2+2\alpha s+w_o^2}=\frac{K(s^2+2\beta s+w_o^2)}{s^2+(w_o/Q)s+w_o^2}=\frac{K(s-z_1)(s-z_2)s}{(s-p_1)(s-p_2)},\beta<<\alpha$$

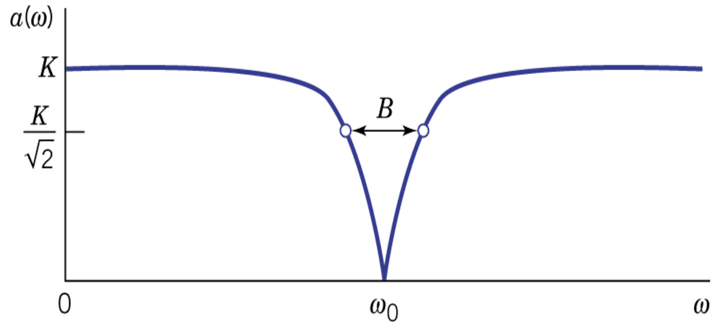
$$\text{underdamped case}: Q(Q=\frac{w_o}{2\alpha})>\frac{1}{2},\text{ or }\alpha<w_o$$

$$a_{no}(w_o)=\frac{2KQ\beta}{w_o}<<K$$

$$p_1,p_2=-\alpha\pm j\sqrt{w_o^2-\alpha^2}=-\frac{w_o}{2Q}\pm jw_o\sqrt{1-\frac{1}{4Q^2}}$$

$$z_1,z_2=-\beta\pm j\sqrt{w_o^2-\beta^2}\approx-\beta\pm jw_o$$





10. Ex.11.6 design a parallel BPF to give w_o=20kHz, B=500Hz

for a low-loss inductor $R_w << wL$

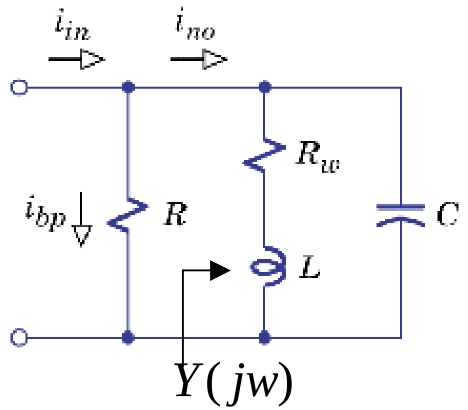
$$Y(jw) = jwC + \frac{1}{R_w + jwL} = \frac{jwCR_w - w^2LC + 1}{R_w + jwL}$$

$$\approx \frac{CR_w}{L} + j(wC - \frac{1}{wL}) = \frac{1}{R_{par}} + jwC + \frac{1}{jwL}$$

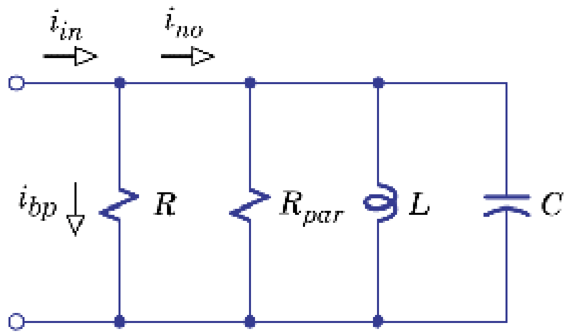
$$w_o = \frac{1}{\sqrt{LC}} \rightarrow L = 63.3nF$$

$$Q = \frac{w_o}{B} = 40 = w_o(R // R_{par})C \rightarrow R // R_{par} = 5.03k$$

$$R_{par} = \frac{L}{CR_w} \rightarrow R_{par} = 13.2k, R = 8.13k$$



$$R_w = 1.2\Omega, L = 1mH$$

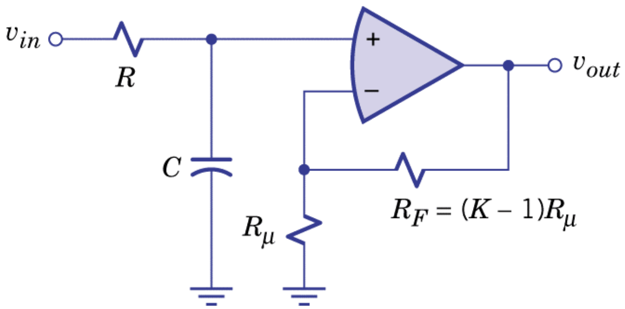


(b) Equivalent network for $\omega \approx \omega_0$

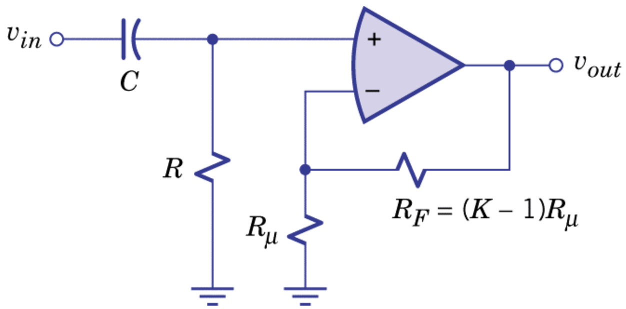
11.3 Op-amp filter circuits

Option

1. Advantages: active filter
2. Noninverting LPF and HPF
   - no output loading effects
   - $$V_{in}=\frac{R_\mu}{R_\mu+(K-1)R_\mu}V_{out}\rightarrow V_{out}=KV_{in}$$



(a) Lowpass

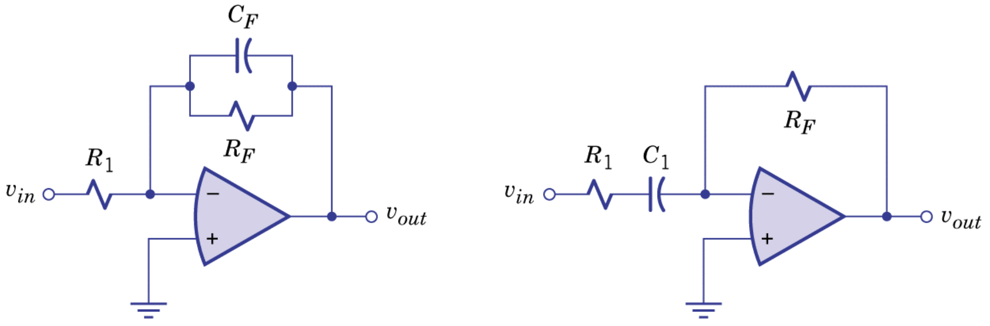


(b) Highpass

3. Inverting LPF and HPF

- A voltage follower could be inserted as a buffer at the input.
- Network functions

$$H(s)=\frac{\underline{V}_{out}}{\underline{V}_{in}}=-\frac{Z_F(s)}{Z_1(s)}\begin{cases}\text{LPF}: Z_F(s)=\dfrac{1}{\dfrac{1}{R_F}+sC_F}=\dfrac{R_F}{1+sR_FC_F}=\dfrac{R_F w_{co}}{s+w_{co}},\, w_{co}=\dfrac{1}{R_FC_F}\\ \rightarrow H(s)=-\dfrac{R_F}{R_1}\dfrac{w_{co}}{s+w_{co}}=\dfrac{Kw_{co}}{s+w_{co}},\, K=-\dfrac{R_F}{R_1}\\ \text{HPF}: Z_1(s)=R_1+\dfrac{1}{sC_1}=\dfrac{1+sR_1C_1}{sC_1}=R_1\dfrac{s+w_{co}}{s},\, w_{co}=\dfrac{1}{R_1C_1}\\ \rightarrow H(s)=-\dfrac{R_F}{R_1}\dfrac{s}{s+w_{co}}=\dfrac{Ks}{s+w_{co}},\, K=-\dfrac{R_F}{R_1}\end{cases}$$

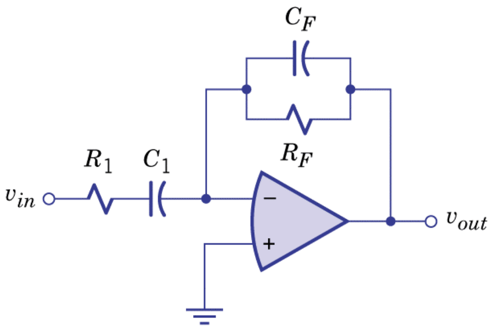


(a) Lowpass (b) Highpass

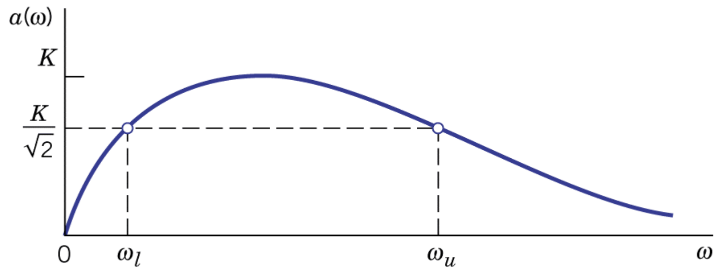
4. Wideband BPF

- Combined LPF and HPF
- Network functions

$$H(s) = -\frac{R_F}{R_1}\frac{s}{s+w_l}\frac{w_u}{s+w_u}, w_l = \frac{1}{R_1C_1}, w_u = \frac{1}{R_F C_F}, w_l << w_u$$



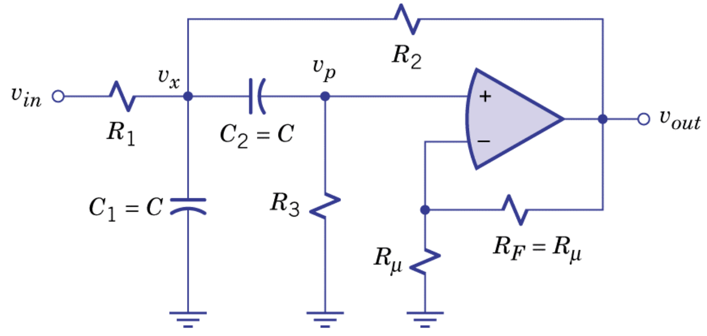
(a) Circuit



(b) Amplitude ratio

5. Narrowband BPF

- Resonant type BPF to give high Q
- Network functions



$$KCL:\begin{cases}\dfrac{V_{in}-V_x}{R_1}+\dfrac{V_{out}-V_x}{R_2}+\dfrac{V_p-V_x}{1/sC}-\dfrac{V_x}{1/sC}=0\\[2ex] \dfrac{V_x-V_p}{1/sC}-\dfrac{V_p}{R_3}=0\\[2ex] \dfrac{V_x-V_p}{1/sC}\end{cases}$$

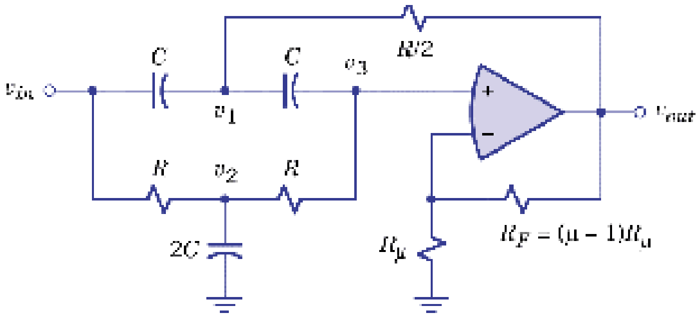
$$\Rightarrow H(s)=\frac{V_{out}}{V_{in}}=\begin{cases}K=\dfrac{2Q}{R_1Cw_o}\\[2ex] w_o=\sqrt{\dfrac{R_1+R_2}{R_1R_2R_3C^2}}\\[2ex] Q=\dfrac{R_1R_2R_3Cw_o}{2R_1R_2+R_2R_3-R_1R_3}\end{cases}\rightarrow\begin{cases}R_1=\dfrac{2Q}{Kw_oC}\\[2ex] R_2=\dfrac{KR_1}{\sqrt{(K-1)^2+8Q^2}-1}\\[2ex] R_3=\dfrac{K^2R_1(R_1+R_2)}{4Q^2R_2}\end{cases}$$

- Q<0 for small R_2→oscillation

$$Q=\frac{Cw_o}{2G_3+G_1-G_2}$$

6. Notch filter

- Resonant type BSF to give high Q
- Network functions



$$H(s) = \frac{V_{out}}{V_{in}} = \mu \frac{s^2 + a^2}{s^2 + (4 - 2\mu)as + a^2}, a = \frac{1}{RC}$$

$$= K \frac{s^2 + 2\beta s + w_o^2}{s^2 + (w_o / Q)s + w_o^2}$$

$$K = \mu, \beta = 0, w_o = \frac{1}{RC}, Q = \frac{1}{4 - 2\mu}$$

11.4 Bode plots

Basics

1.dB gain $$g(w)dB \equiv 20\log|H(jw)| = 20\log a(w), a(w) = 10^{\frac{g(w)}{20}}$$

$$dB \equiv 20\log\frac{V_2}{V_1} = 10\log\frac{P_2}{P_1}$$

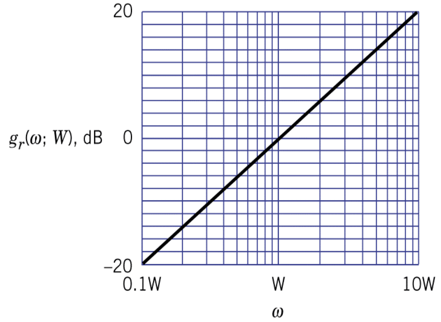
2.Bode plot: plot of *a(w)* in a log frequency scale

Discussion

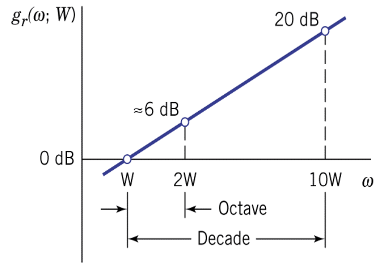
1. Ramp function

$$H_r(s;W) \equiv \frac{s}{W}, H_r(jw;W) \equiv \frac{jw}{W} = \frac{w}{W}\angle 90°$$

$$g_r(w;W) = 20\log\frac{w}{W}, \theta(w;W) = 90°$$



(a)



(b)

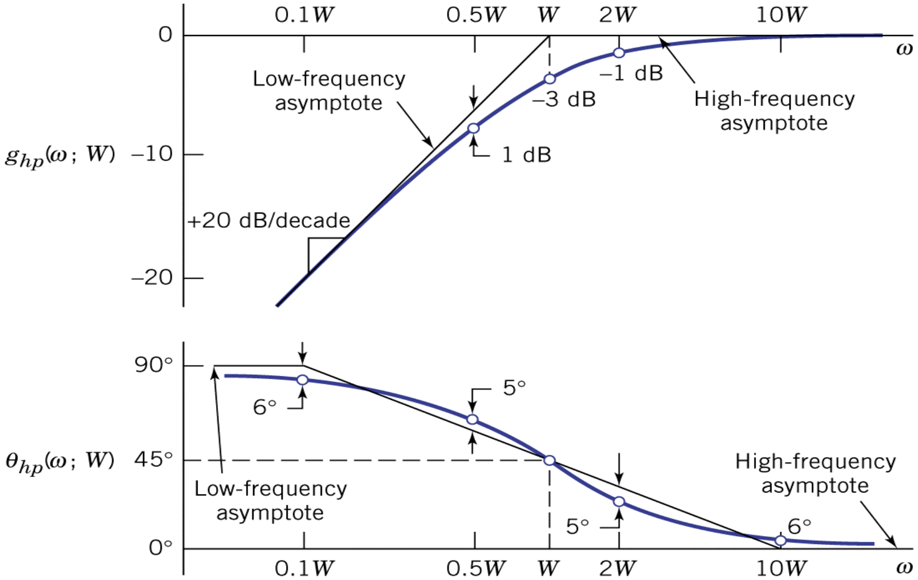
2. Highpass function

$$H_{hp}(s;W) \equiv \frac{s}{s+W}, H_{hp}(jw;W) = \frac{j(w/W)}{1+j(w/W)}$$

$$\text{low-frequency}: w < 0.1W, g_{hp} \approx 20\log\frac{w}{W}, \theta_{hp} \approx 90°$$

$$\text{high-frequency}: w > 10W, g_{hp} \approx 0dB, \theta_{hp} \approx 0°$$

$$\text{corner frequency}: w = W, g_{hp} \approx -3dB, \theta_{hp} = 45°$$



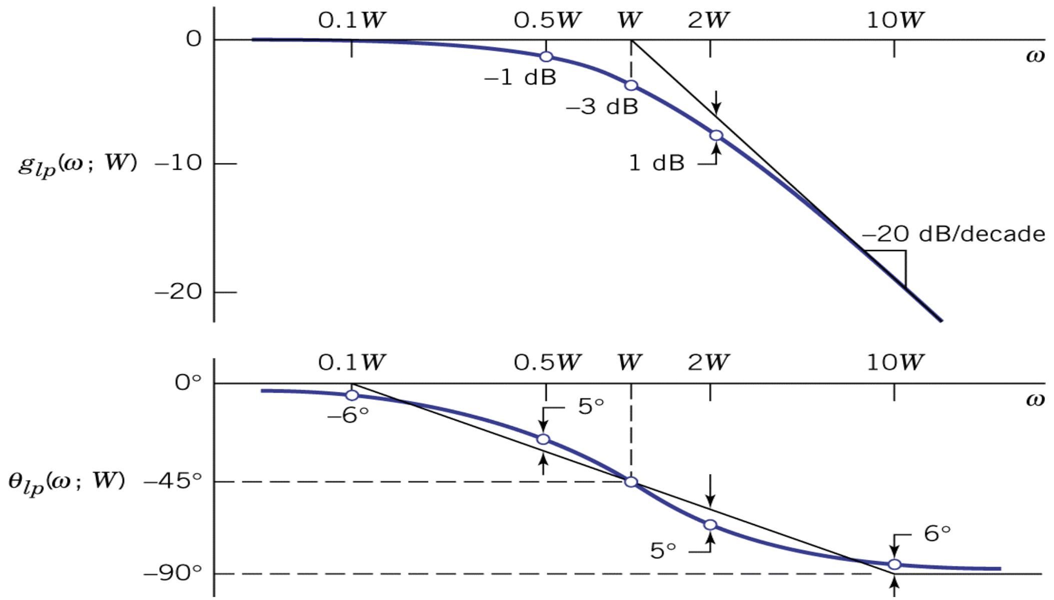
3. Lowpass function

$$H_{lp}(s;W) \equiv \frac{W}{s+W}, H_{lp}(jw;W) = \frac{1}{1+j(w/W)}$$

$$\text{low-frequency}: w < 0.1W, g_{lp} \approx 0dB, \theta_{hp} \approx 0°$$

$$\text{high-frequency}: w > 10W, g_{hp} \approx -20\log\frac{w}{W}, \theta_{hp} \approx -90°$$

$$\text{corner frequency}: w = W, g_{hp} \approx -3dB, \theta_{hp} = -45°$$



4. Ex.11.8 plot the Bode plot

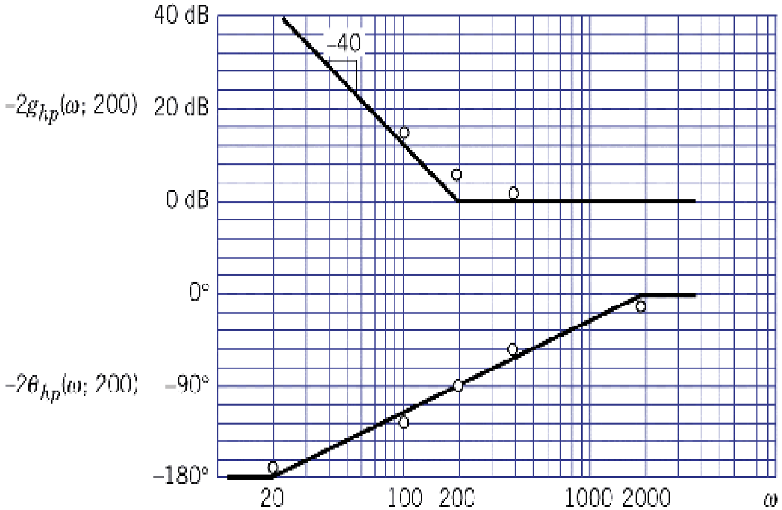
$$H(s) = -\frac{(s+200)^2}{10s^2} = -\frac{1}{10}\left(\frac{s}{s+200}\right)^{-1} = -0.1H_{hp}^{-2}(s;200)$$

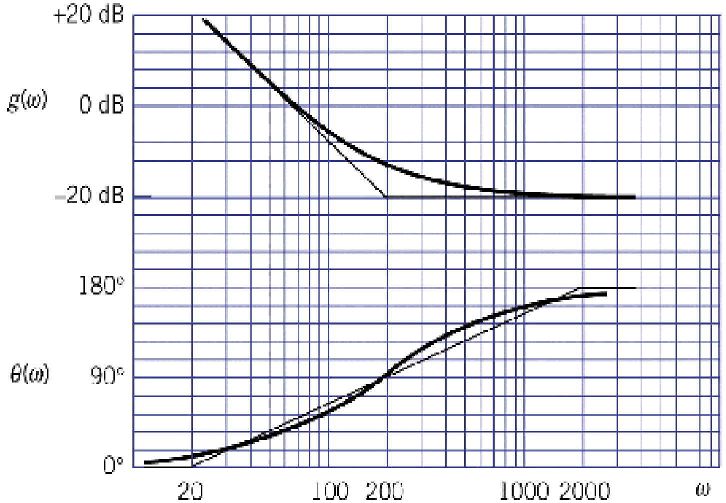
$$g(w) = -20dB - 2g_{hp}(w;200), \theta(w) = 180° - 2\theta_{hp}(w;200)$$

$$\text{low-frequency}: w < 20, g(w) \approx -20dB - 40\log\frac{w}{W}, \theta(w) \approx 180° - 2\times 90°$$

$$\text{high-frequency}: w > 2000, g_{hp} \approx -20dB, \theta_{hp} \approx 180°$$

$$\text{corner frequency}: w = W, g_{hp} \approx -20dB - (-6dB), \theta_{hp} = 180° - 2\times 45°$$



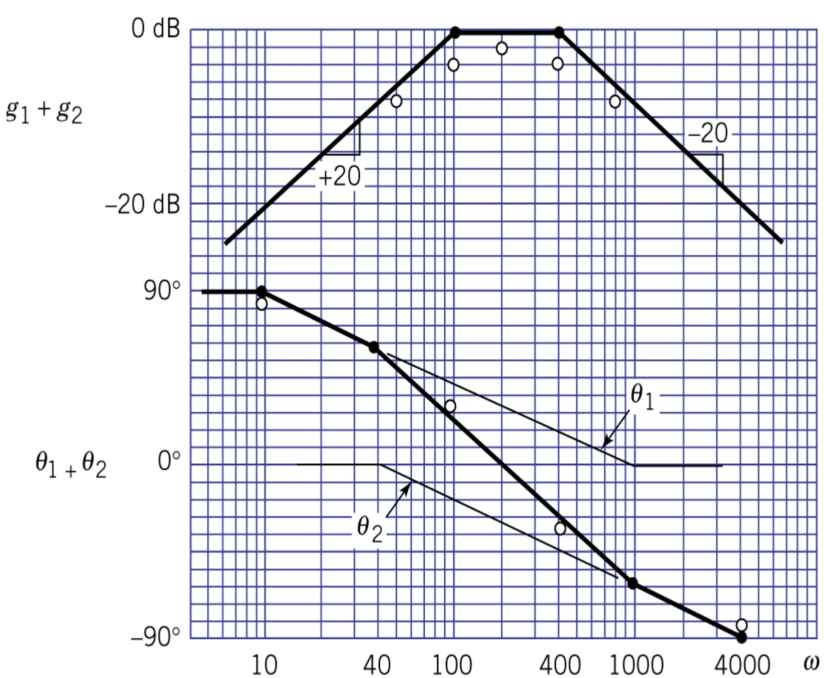


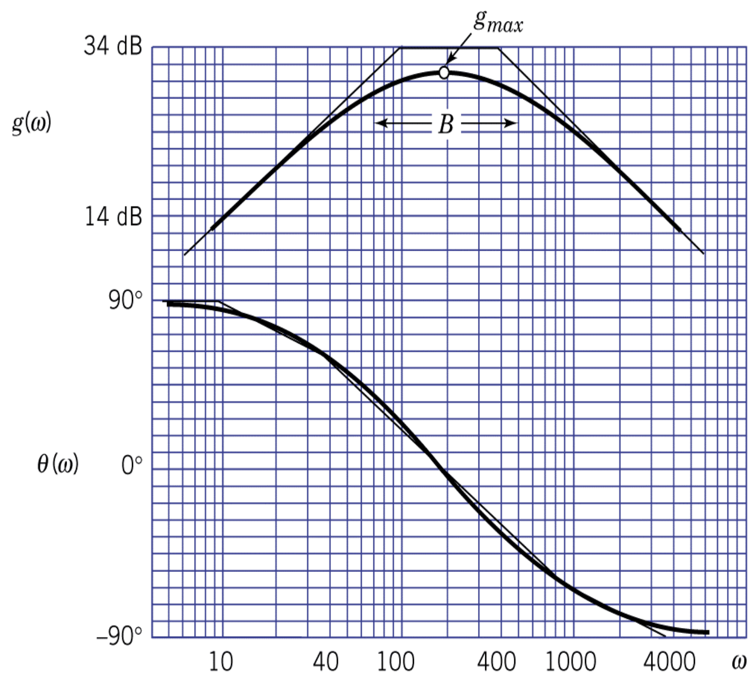
5. Ex.11.9 plot the Bode plot

$$H(s)=\frac{20000s}{(s+100)(s+400)}=50\frac{s}{s+100}\frac{400}{s+400}=50H_{1,hp}(s;100)H_{2,lp}(s;400)$$

$$g(w)=34dB+g_{1,hp}(w;100)+g_{2,lp}(w;400),\theta(w)=\theta_{1,lp}(w;100)+\theta_{2,hp}(w;400)$$

$$w\geq 100, g_{1,hp}(w)\approx 0dB; w\leq 400, g_{2,lp}(w)\approx 0dB$$

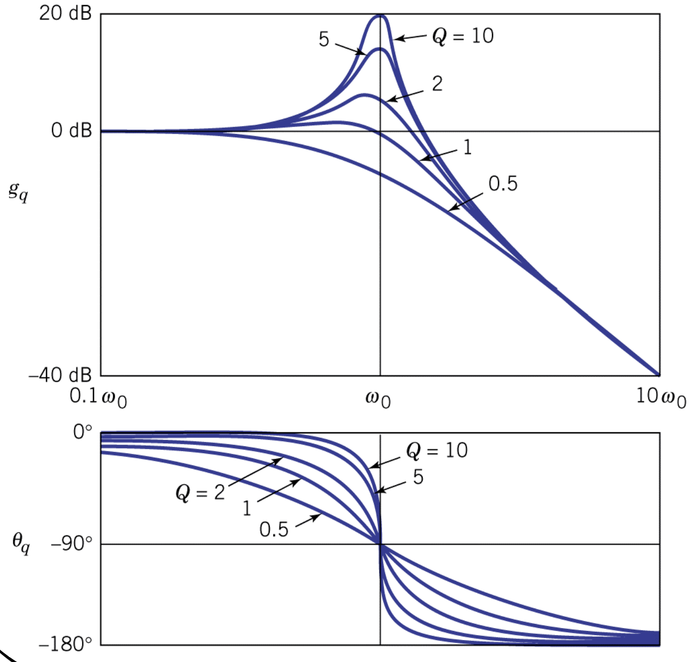


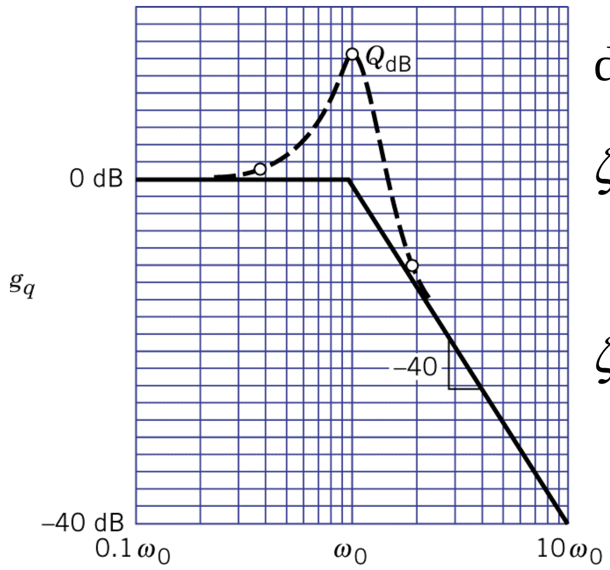


6. Quadratic function

$$H_q(s;w_o,Q)\equiv\frac{w_o^2}{s^2+(w_o/Q)s+w_o^2},\ H_q(jw;w_o,Q)=\frac{1}{1-(w/w_o)^2+j(w/Qw_o)}$$

$$H_q(jw;w_o,Q)\approx\begin{cases}1,w<<w_o\\-(\frac{w_o}{w})^2,w>>w_o\\-jQ,w=w_o\end{cases}\rightarrow g_q\approx\begin{cases}0dB,w<0.1w_o\\-40\log\frac{w}{w_o},w>10w_o\\20\log Q=Q_{dB},w=w_o\end{cases}$$





damping ratio

$$\zeta\equiv\frac{1}{2Q}=\frac{\alpha}{w_o}$$

$$\zeta\begin{cases}<1,\text{underdamp}\\=1,\text{critical damp}\\>1,\text{overdamp}\end{cases}$$

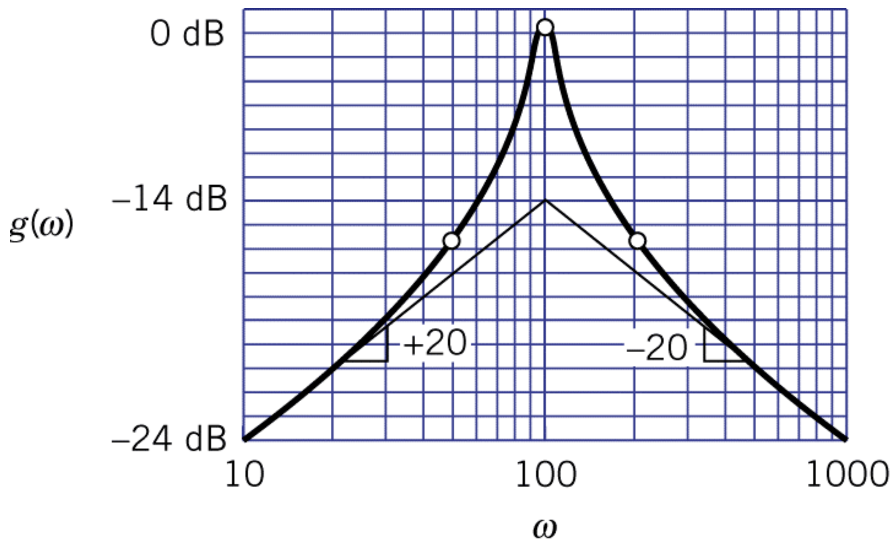
7. Ex.11.10

$$H(s)=\frac{20s}{s^2+20s+10^4}=0.2\frac{s}{100}\frac{10^4}{s^2+20s+10^4}=0.2H_r(s;100)H_q(s;100,5)$$

$$w_o=100,\frac{w_o}{Q}=20\rightarrow Q=5$$

$$g(w)=-14dB+g_r(w;100)+g_q(w;100,5)$$

$$w=100, g(w)=-14+0+20\log 5=0dB$$



11.5 Frequency-response design

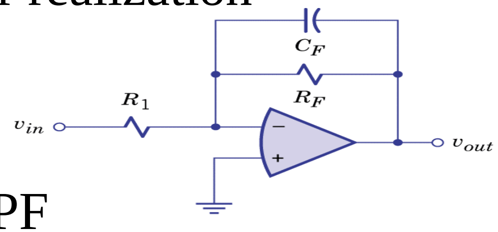
Optional

1. Design the filter from frequency-response and Bode plot

- line slope $m(\pm 20dB/decade)$
- corner frequencies W_1, W_2, number of decades $= \log \dfrac{W_2}{W_1}$
- factored frequency-response

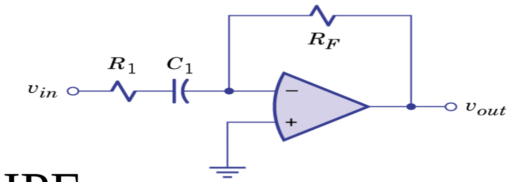
$$g(w) = K_{dB} + g_1(w) + g_2(w) + \ldots \rightarrow H(s) = \pm|K| H_1(s) H_2(s)\ldots$$

- filter realization



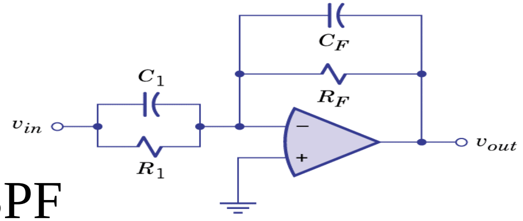
LPF

$$(a)\ H_a(s) = -\frac{1}{R_1 C_F}\,\frac{1}{s + 1/R_F C_F}$$



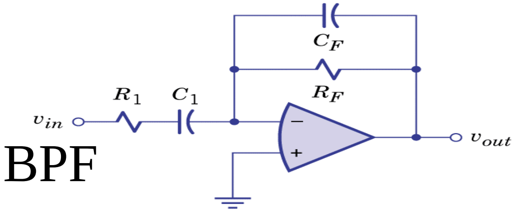
HPF

$$(b)\ H_b(s) = -\frac{R_F}{R_1}\,\frac{s}{s + 1/R_1 C_1}$$



BPF

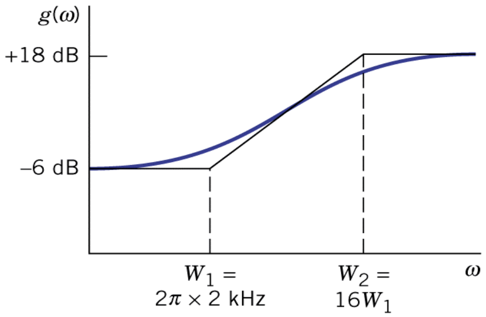
$$(c)\ H_c(s) = -\frac{C_1}{C_F}\,\frac{s + 1/R_1 C_1}{s + 1/R_F C_F}$$



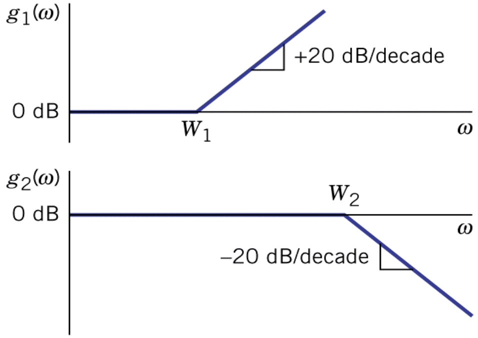
BPF

$$(d)\ H_d(s) = -\frac{1}{R_1 C_F}\,\frac{s}{(s + 1/R_1 C_1)(s + 1/R_F C_F)}$$

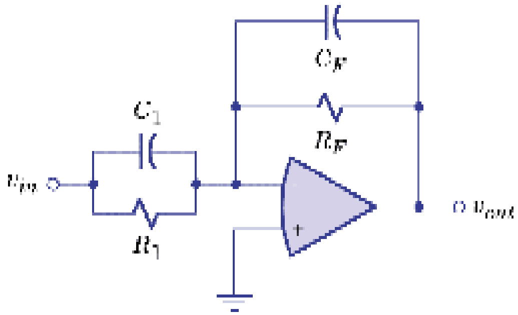
2. Design a FM pre-emphasis circuit



(a) Gain curve for FM premphasis



(b) Asymptotic plots



(c) $H_f(s) = -\frac{C_1}{C_F}\frac{s+1/R_1C_1}{s+1/R_FC_F}$

$$\text{slope} = \frac{24dB}{\log\frac{16W_1}{W_1}} = \frac{24dB}{1.2\text{decades}} = 20dB/decade$$

$$g(w) = K_{dB} + g_1(w) + g_2(w), K_{dB} = -6dB, g_1(w) = g_{lp}^{-1}(w;W_1), g_2(w) = g_{lp}(w;W_2)$$

$$H(s) = 0.5\frac{s+12600}{12600}\frac{20100}{s+20100} = 8\frac{s+12600}{s+20100}$$

$$\rightarrow \frac{C_1}{C_F} = 8, \frac{1}{R_1C_1} = 12600, \frac{1}{R_FC_F} = 20100$$

take $R_F = 10k\Omega \rightarrow C_F = 0.5nF, C_1 = 4nF, R_1 = 20k\Omega$

11.6 Butterworth filters

Optional

1. Butterworth polynomial

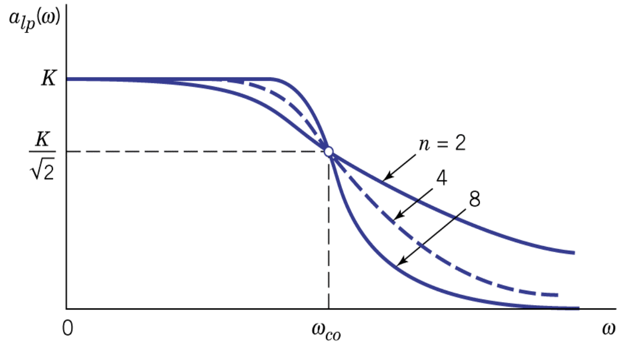
$$\left|B_n(jw)\right| = \sqrt{w^{2n} + w_{co}^{2n}}, \frac{d^m}{dw^m}\left|B_n(jw)\right|_{w=0} = 0, m = 1,2,\ldots 2n-1$$

$$\left|B_n(jw)\right| \approx \begin{cases} w_{co}^n, w < w_{co} \\ w^n, w > w_{co} \end{cases}$$

2. n-th order LPF

$$H_{lp}(s) = \frac{Kw_{co}^n}{\sqrt{s^{2n} + w_{co}^{2n}}}, H_{lp}(jw) = \frac{Kw_{co}^n}{\sqrt{w^{2n} + w_{co}^{2n}}} = \frac{K}{\sqrt{1 + (\frac{w}{w_{co}})^{2n}}}$$

$$a_{lp}(w) = \frac{K}{\sqrt{1 + (\frac{w}{w_{co}})^{2n}}} \begin{cases} \approx K, w < w_{co} \\ \approx (\frac{w_{co}}{w})^n K << K, w > w_{co} \\ = \frac{K}{\sqrt{2}}, w = w_{co} \end{cases}$$

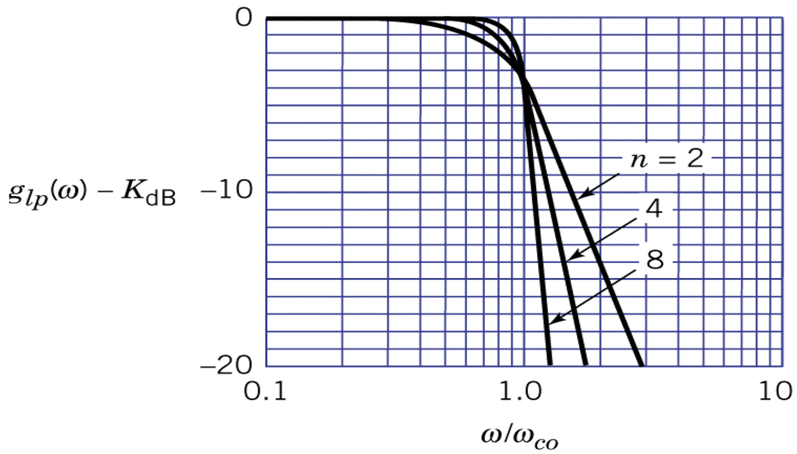


3. Bode plot

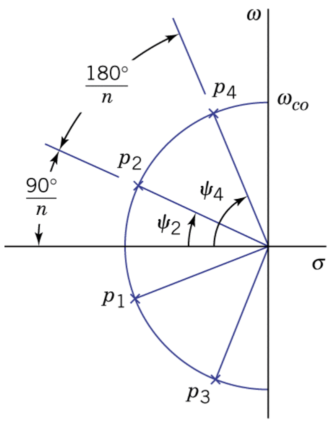
$$g_{lp}(w) = 20\log a_{lp}(w)$$

maximal flatness response

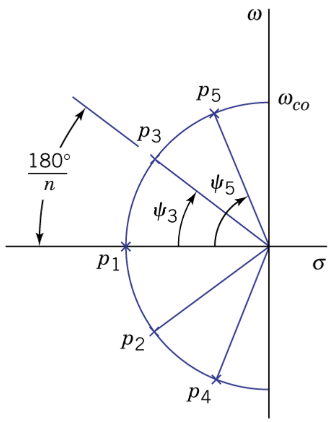
slope : $20ndB / decade$



4. Poles are uniformly spaced by angles 180°/n around a semicircle of radius w_{co} in the left half of s-plane.



(a) Even n



(b) Odd n

5. Ex.11.12 Design a Butterworth LPF to give f_{co}=15kHz, $a_{lp}(w)$ ≦K/8 for f≧23kHz

$$a_{lp}^2(w)=\frac{K^2}{1+(\frac{w}{w_{co}})^{2n}}\le\frac{K^2}{8^2},w\ge 2\pi\times 23k\rightarrow\frac{K^2}{1+(\frac{23}{15})^{2n}}\le\frac{K^2}{8^2}\rightarrow n\ge 4.85,n=5$$

6. n-th order HPF

$$H_{hp}(s)=\frac{Ks^n}{\sqrt{s^{2n}+w_{co}^{2n}}},a_{lp}(w)=\frac{K}{\sqrt{1+(\frac{w_{co}}{w})^{2n}}}$$