Chapter 14 Two-port networks

14.1 Two-ports and impedance parameters

two-port concept, impedance parameters, reciprocal networks

14.2 Admittance, hybrid, and transmission parameters

admittance parameters, hybrid parameters, transmission parameters, parameter conversion

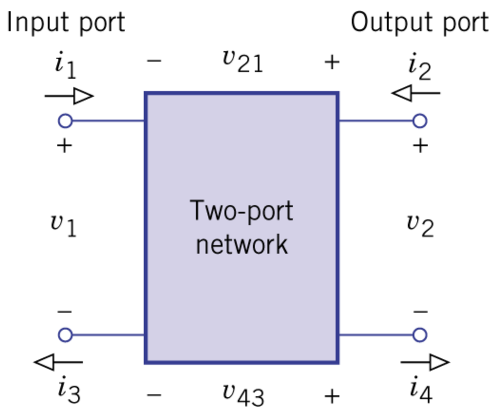
14.3 Circuit analysis with two-ports

terminated two-ports, two-ports in cascade, two-ports in series, two-ports in parallel

14.1 Two-ports and impedance parameters

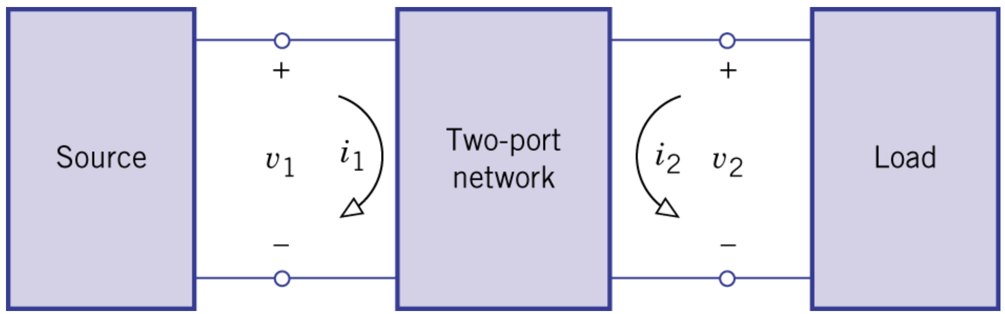
Basics

1. Two-port network
   - a four-terminal network with input port and output port
   - the network characteristics is completely described by v_1, i_1, v_2, i_2
   - a useful method to analyze filter, amplifier,….
   - can be extended to multi-port networks



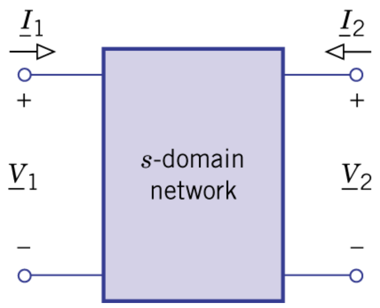
$$i_1 = i_3, i_2 = i_4$$

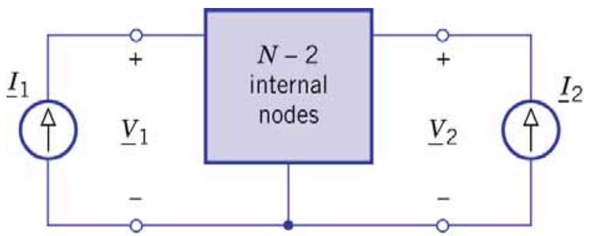
v_{21} and v_{43} are not concerned



No independent sources are in the two-port network and load.

2. O.C. impedance parameters





sources: i_1, i_2, responses: v_1, v_2

in s – domain: $\begin{cases} \underline{V}_1 = z_{11}\underline{I}_1 + z_{12}\underline{I}_2 \\ \underline{V}_2 = z_{21}\underline{I}_1 + z_{22}\underline{I}_2 \end{cases} \rightarrow \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \end{bmatrix} = [z]\begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix}$

$[z] \equiv \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$: o.c. impedance parameter matrix

$z_{11} = \left.\frac{\underline{V}_1}{\underline{I}_1}\right|_{\underline{I}_2=0}$: input impedance at port 1 with port 2 o.c.

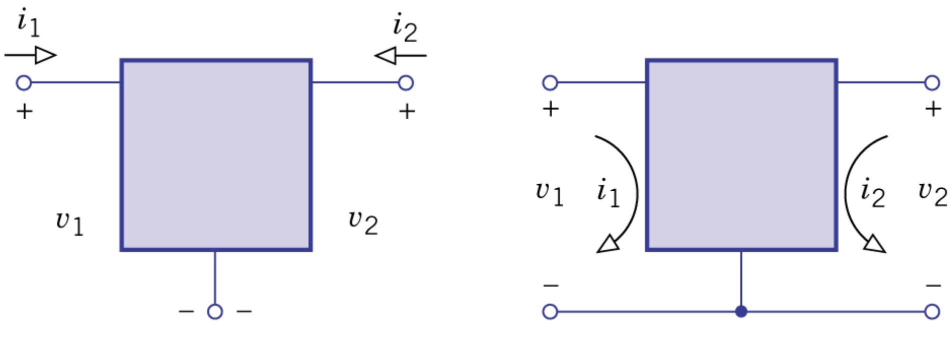
$z_{22} = \left.\frac{\underline{V}_2}{\underline{I}_2}\right|_{\underline{I}_1=0}$: input impedance at port 2 with port 1 o.c.

$z_{12} = \left.\frac{\underline{V}_1}{\underline{I}_2}\right|_{\underline{I}_1=0}$: reverse transfer impedance with port 1 o.c.

$z_{21} = \left.\frac{\underline{V}_2}{\underline{I}_1}\right|_{\underline{I}_2=0}$: forward transfer impedance with port 2 o.c.

Discussion

1. Most two-port networks are three-terminal networks.

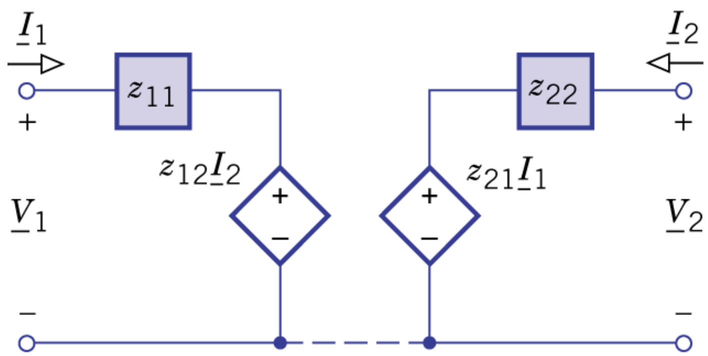


(a) Three-terminal network *(b)* Two-port with common ground

2. Equivalent circuit expressed in z-parameters

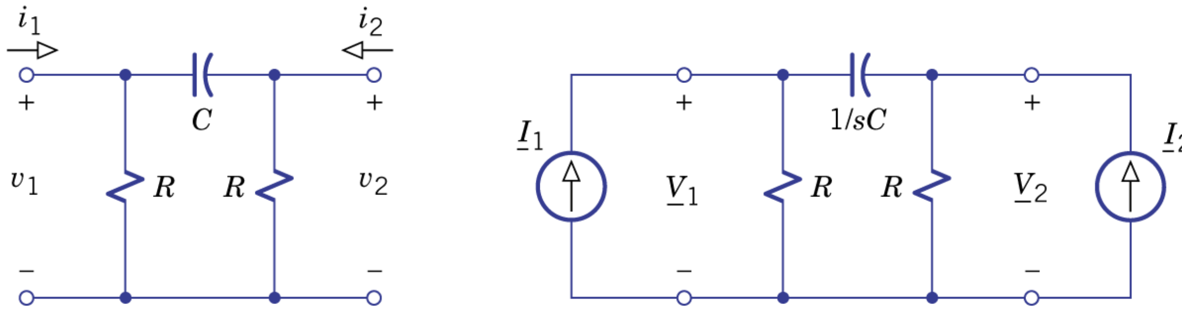
$$\begin{cases} \underline{V}_1 = z_{11}\underline{I}_1 + z_{12}\underline{I}_2 \\ \underline{V}_2 = z_{21}\underline{I}_1 + z_{22}\underline{I}_2 \end{cases}$$

$$\begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix}$$



3. Use definition to determine the z-parameter of a two-port network.
4. Ex. 14.1 find z-parameters of a "symmetrical" network

$$z_{11} = z_{22}$$
$$z_{12} = z_{21}$$



(a) Network for Example 14.1 (b) s-domain diagram with current sources

sources: $\underline{I}_1, \underline{I}_2$

$$\begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix}$$

$$z_{11} = R//(\frac{1}{sC} + R) = \frac{1}{\frac{1}{R} + \frac{1}{R + \frac{1}{sC}}} = \frac{1}{\frac{1}{R} + \frac{sC}{1 + sRC}} = \frac{R + sR^2C}{1 + 2sRC} = z_{22}$$

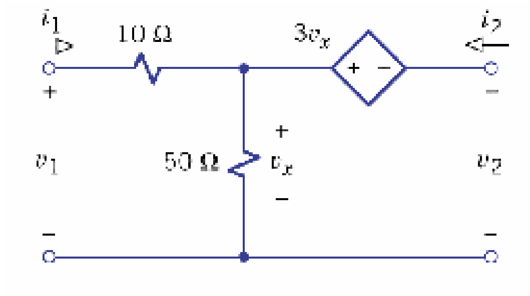
$$z_{11} = \left.\frac{\underline{V}_1}{\underline{I}_1}\right|_{\underline{I}_2=0}, z_{12} = \left.\frac{\underline{V}_1}{\underline{I}_2}\right|_{\underline{I}_1=0}$$

$$z_{21} = \left.\frac{\underline{V}_2}{\underline{I}_1}\right|_{\underline{I}_2=0}, z_{22} = \left.\frac{\underline{V}_2}{\underline{I}_2}\right|_{\underline{I}_1=0}$$

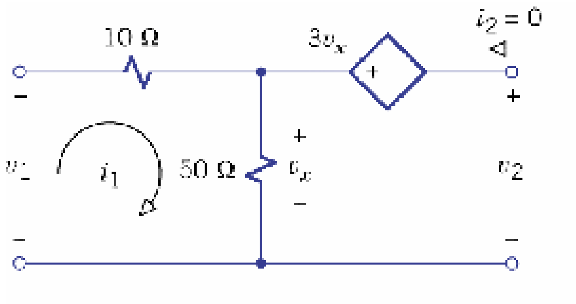
$$z_{12} = \left.\frac{\underline{V}_1}{\underline{I}_2}\right|_{\underline{I}_1=0}, \underline{V}_1\Big|_{\underline{I}_1=0} = \frac{R}{R + \frac{1}{sC}} \underline{V}_2\Big|_{\underline{I}_1=0} = \frac{sRC}{1 + sRC} \underline{I}_2 z_{22}$$

$$\rightarrow z_{12} = \frac{sRC}{1 + sRC} z_{22} = \frac{sRC}{1 + sRC} \frac{R + sR^2C}{1 + 2sRC} = \frac{sR^2C}{1 + 2sRC} = z_{21}$$

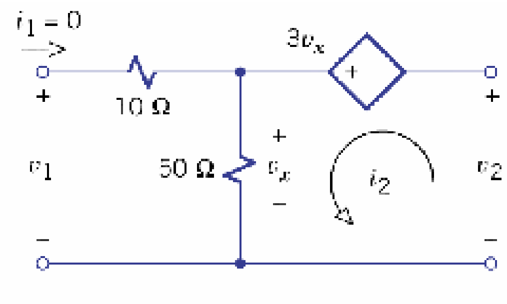
5. Ex. 14.2 find z-parameters of an "active" network



(a) Network for Example 14.2



(b) Open-output diagram



(c) Open-input diagram

sources: i_1, i_2

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0}, \quad z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0}$$

$$z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0}, \quad z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0}$$

(1)port − 2 o.c., $i_2 = 0$

$$v_1 = i_1(10+50)$$

$$\rightarrow z_{11} = \frac{v_1}{i_1} = 60$$

$$v_x = 50 i_1, v_2 = -3v_x + v_x$$

$$= -2v_x = -100 i_1$$

$$\rightarrow z_{21} = \frac{v_2}{i_1} = -100$$

(2)port − 1 o.c., $i_1 = 0$

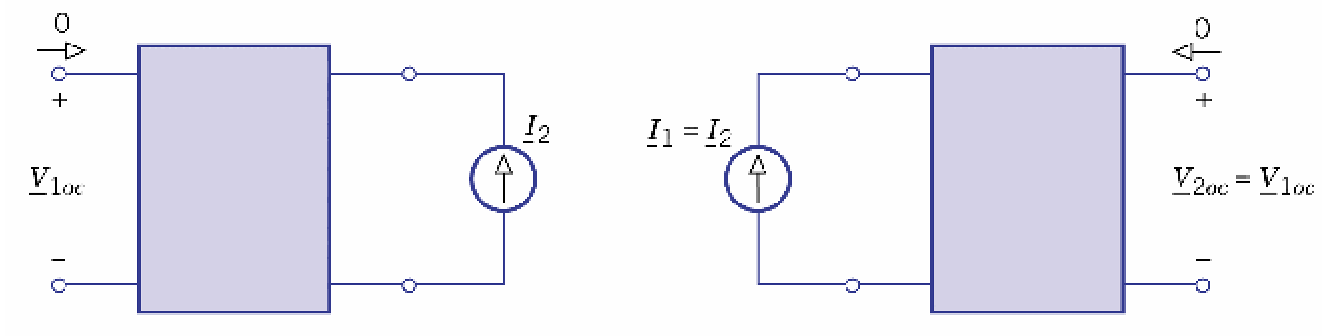
$$v_2 = -2v_x, v_x = 50 i_2$$

$$\rightarrow z_{22} = \frac{v_2}{i_2} = -100 \neq z_{11}$$

$$v_1 = v_x = 50 i_2$$

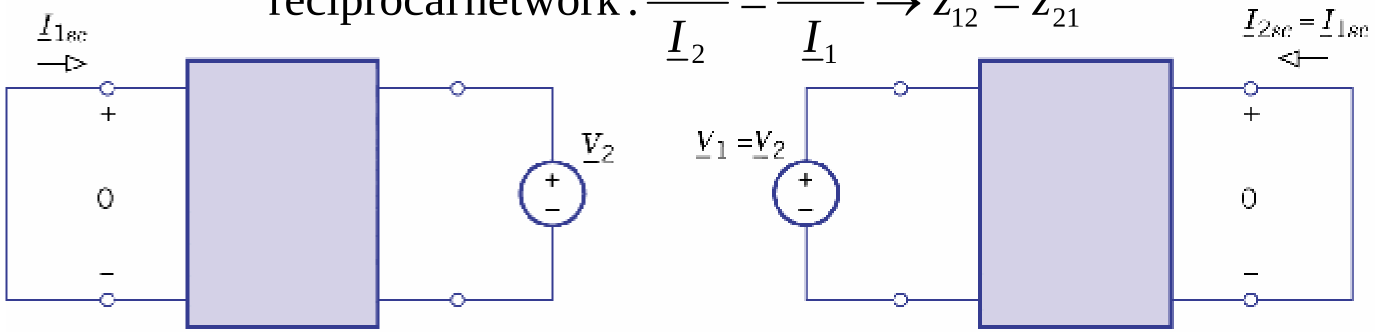
$$\rightarrow z_{12} = \frac{v_1}{i_2} = 50 \neq z_{12}$$

6. Reciprocal circuit



(a) Interchanging open circuit and current source

$$\text{reciprocal network}: \frac{V_{1oc}}{I_2} = \frac{V_{2oc}}{I_1} \rightarrow z_{12} = z_{21}$$



(b) Interchanging short circuit and voltage source

$$\text{reciprocal network}: \frac{I_{1sc}}{V_2} = \frac{I_{2sc}}{V_1} \rightarrow y_{12} = y_{21}$$

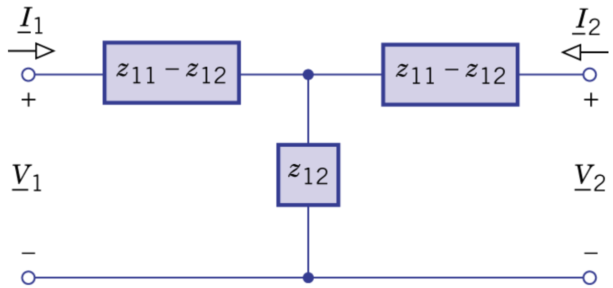
7. Any linear network containing no controlled sources is a reciprocal network.

∵node equation and mesh equation have symmetrical forms

8. Ex. 14.3 T-network

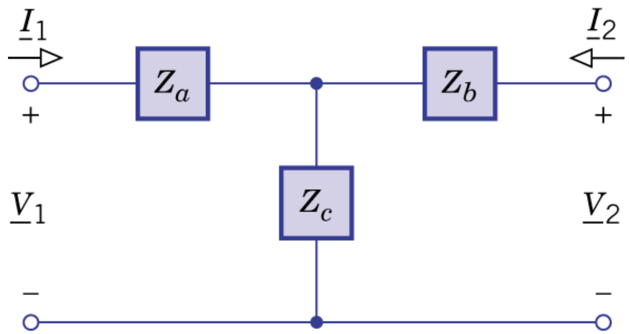
$$\text{sources}: I_1, I_2 \quad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}, z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}, z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}, z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



$$(1) I_2 = 0, z_{11} = \frac{V_1}{I_1} = z_{11} - z_{12} + z_{12}, z_{21} = \frac{V_2}{I_1} = z_{12}$$

$$(2) I_1 = 0, z_{22} = \frac{V_2}{I_2} = z_{11} - z_{12} + z_{12}, z_{12} = \frac{V_1}{I_2}$$



$$Z_a = z_{11} - z_{12}, Z_c = z_{12}$$

$$Z_b = z_{22} - z_{12}$$

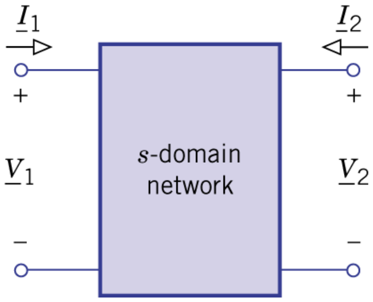
14.2 Admittance, hybrid, and transmission parameters

<u>Basics</u>

1. Not all two-ports posses meaningful or measurable z-parameters.

$\rightarrow$ other parameters

2. Admittance parameter



sources : $\underline{V}_1, \underline{V}_2$, responses : $\underline{I}_1, \underline{I}_2$

$$\begin{cases} \underline{I}_1 = y_{11}\underline{V}_1 + y_{12}\underline{V}_2 \\ \underline{I}_2 = y_{21}\underline{V}_1 + y_{22}\underline{V}_2 \end{cases}$$

$$\rightarrow \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = [y] \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \end{bmatrix}$$

$$[y] \equiv \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \text{: s.c. admittance parameter matrix}$$

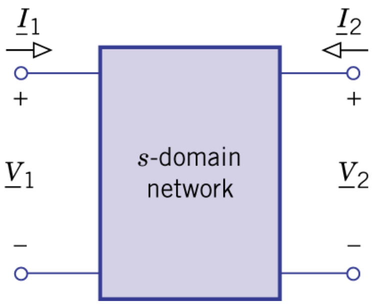
$$y_{11} = \left.\frac{\underline{I}_1}{\underline{V}_1}\right|_{\underline{V}_2=0} \text{: input admittance at port 1 with port 2 s.c.}$$

$$y_{22} = \left.\frac{\underline{I}_2}{\underline{V}_2}\right|_{\underline{V}_1=0} \text{: input admittance at port 2 with port 1 s.c.}$$

$$y_{12} = \left.\frac{\underline{I}_1}{\underline{V}_2}\right|_{\underline{V}_1=0} \text{: reverse transfer admittance with port 1 s.c.}$$

$$y_{21} = \left.\frac{\underline{I}_2}{\underline{V}_1}\right|_{\underline{V}_2=0} \text{: forward transfer admittance with port 2 s.c.}$$

3. Hybrid parameter



sources : $\underline{I}_1, \underline{V}_2$, responses : $\underline{V}_1, \underline{I}_2$

$$\begin{cases} \underline{V}_1 = h_{11}\underline{I}_1 + h_{12}\underline{V}_2 \\ \underline{I}_2 = h_{21}\underline{I}_1 + h_{22}\underline{V}_2 \end{cases}$$

$$\rightarrow \begin{bmatrix} \underline{V}_1 \\ \underline{I}_2 \end{bmatrix} = [h] \begin{bmatrix} \underline{I}_1 \\ \underline{V}_2 \end{bmatrix}$$

$$[h] \equiv \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \text{: hybrid parameter matrix}$$

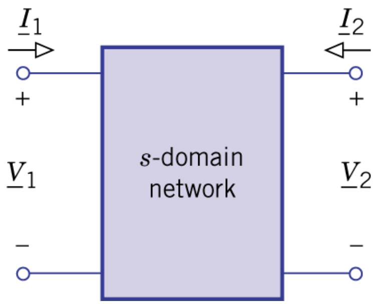
$$h_{11} = \left.\frac{\underline{V}_1}{\underline{I}_1}\right|_{\underline{V}_2=0} = \frac{1}{y_{11}} \text{: input admittance at port 1 with port 2 s.c.}$$

$$h_{22} = \left.\frac{\underline{I}_2}{\underline{V}_2}\right|_{\underline{I}_1=0} = \frac{1}{z_{22}} \text{: input impedance at port 2 with port 1 o.c.}$$

$$h_{12} = \left.\frac{\underline{V}_1}{\underline{V}_2}\right|_{\underline{I}_1=0} \text{: reverse voltage ratio with port 1 o.c.}$$

$$h_{21} = \left.\frac{\underline{I}_2}{\underline{I}_1}\right|_{\underline{V}_2=0} \text{: forward current ratio with port 2 s.c.}$$

4. Transmission parameter



sources : $\underline{V}_2, \underline{I}_2$, responses : $\underline{V}_1, \underline{I}_1$

$$\begin{cases} \underline{V}_1 = A\underline{V}_2 - B\underline{I}_2 \\ \underline{I}_1 = C\underline{V}_2 - D\underline{I}_2 \end{cases}$$

$$\rightarrow \begin{bmatrix} \underline{V}_1 \\ \underline{I}_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \underline{V}_2 \\ -\underline{I}_2 \end{bmatrix} = [T] \begin{bmatrix} \underline{V}_2 \\ -\underline{I}_2 \end{bmatrix}$$

$[T] \equiv \begin{bmatrix} A & B \\ C & D \end{bmatrix}$: transmission parameter matrix

$A = \left. \dfrac{\underline{V}_1}{\underline{V}_2} \right|_{\underline{I}_2 = 0}$: reverse voltage ratio with port 2 o.c.

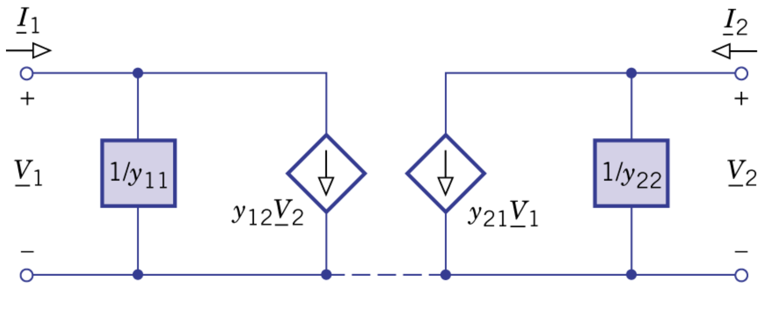
$B = \left. \dfrac{\underline{V}_1}{-\underline{I}_2} \right|_{\underline{V}_2 = 0}$: reverse transfer impedance with port 2 s.c.

$C = \left. \dfrac{\underline{I}_1}{\underline{V}_2} \right|_{\underline{I}_2 = 0}$: reverse transfer admittance with port 2 o.c.

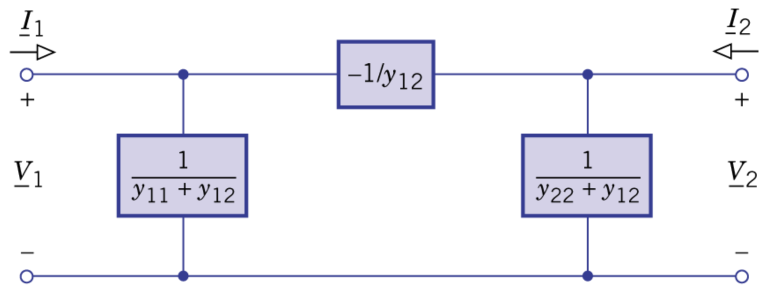
$D = \left. \dfrac{\underline{I}_1}{-\underline{I}_2} \right|_{\underline{V}_2 = 0}$: forward current ratio with port 2 s.c.

Discussion

1. Equivalent circuit expressed in y-parameters



(a) *y*-parameter model



(b) Tee model for reciprocal two-port

sources: $\underline{V}_1, \underline{V}_2$

$$\begin{cases} \underline{I}_1 = y_{11}\underline{V}_1 + y_{12}\underline{V}_2 \\ \underline{I}_2 = y_{21}\underline{V}_1 + y_{22}\underline{V}_2 \end{cases}$$

$$\begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \end{bmatrix}$$

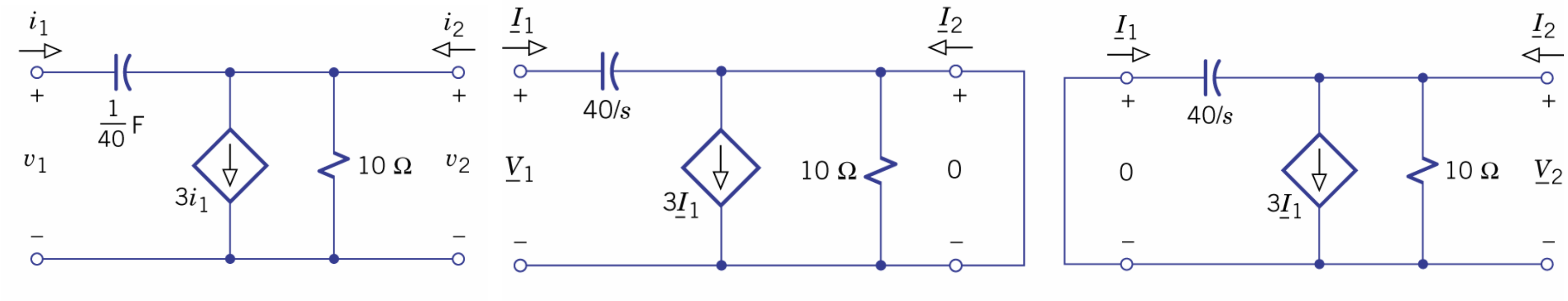
(1) $\underline{V}_2 = 0$, $y_{11} = \frac{\underline{I}_1}{\underline{V}_1} = y_{11} + y_{12} - y_{12}$

$y_{21} = \frac{\underline{I}_2}{\underline{V}_1} = -\frac{1}{-1/y_{12}} = y_{12}$

(2) $\underline{V}_1 = 0$, $y_{22} = \frac{\underline{I}_2}{\underline{V}_2} = y_{22} + y_{12} - y_{12}$

$y_{12} = \frac{\underline{I}_1}{\underline{V}_2} = -\frac{1}{-1/y_{12}} = y_{12}$

2. Ex. 14.5 find y-parameters of an "active" network



(a) Network for Example 14.5 (b) Shorted-output diagram (c) Shorted-input diagram

sources: $\underline{V}_1, \underline{V}_2$

$$\begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \end{bmatrix}$$

(1) port − 2 s.c., $\underline{V}_2 = 0$

$$\underline{I}_1 = \underline{V}_1 \frac{s}{40} \rightarrow y_{11} = \frac{\underline{I}_1}{\underline{V}_1} = \frac{s}{40}$$

$$\underline{I}_2 = 3\underline{I}_1 - \underline{I}_1 = 2\underline{I}_1 = 2\frac{s}{40}\underline{V}_1$$

$$\rightarrow y_{21} = \frac{\underline{I}_2}{\underline{V}_1} = \frac{s}{20}$$

(2) port − 1 s.c., $\underline{V}_1 = 0$

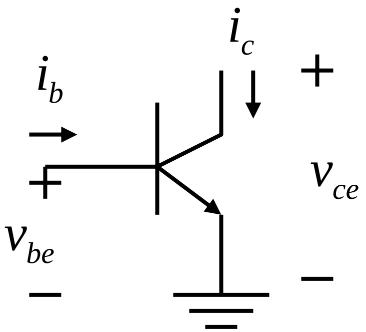
$$\underline{I}_1 = -\underline{V}_2 \frac{s}{40} \rightarrow y_{12} = \frac{\underline{I}_1}{\underline{V}_2} = -\frac{s}{40} \neq y_{21}$$

$$\underline{I}_2 = 3\underline{I}_1 - \underline{I}_1 + \frac{\underline{V}_2}{10} = 2\underline{I}_1 + \frac{\underline{V}_2}{10}$$

$$= -\frac{s\underline{V}_2}{20} + \frac{\underline{V}_2}{10} = \frac{2-s}{20}\underline{V}_2$$

$$\rightarrow y_{22} = \frac{\underline{I}_2}{\underline{V}_2} = \frac{2-s}{20}$$

3. H-parameters are applied to transistor because they are measured physical quantities.

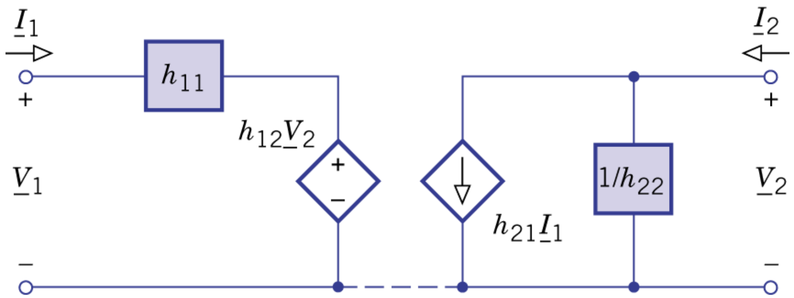


sources : i_1, v_2, responses : v_1, i_2

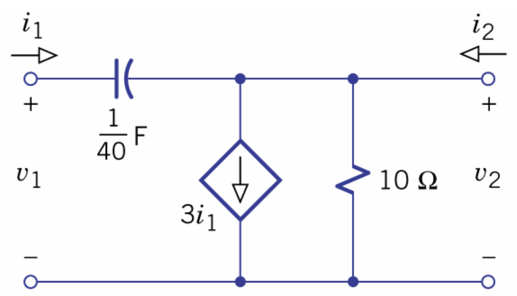
$$\begin{cases} v_1 = h_{11} i_1 + h_{12} v_2 \\ i_2 = h_{21} i_1 + h_{22} v_2 \end{cases} \rightarrow \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

$$CE \rightarrow \begin{bmatrix} v_{be} \\ i_c \end{bmatrix} = \begin{bmatrix} h_{ie} & h_{re} \\ h_{fe} & h_{oe} \end{bmatrix} \begin{bmatrix} i_b \\ v_{ce} \end{bmatrix}$$

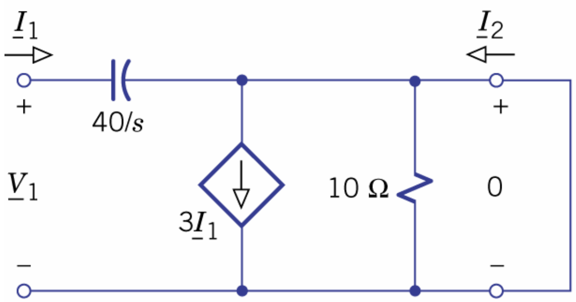
4. Equivalent circuit expressed in h-parameters



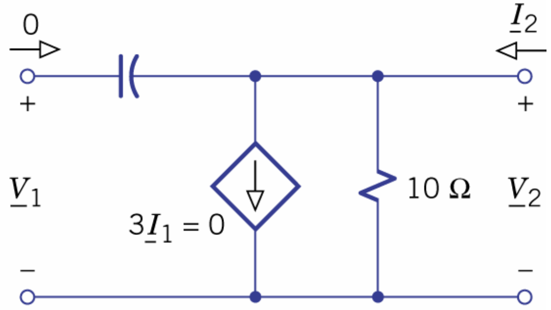
5. Ex. 14.6 find h-parameters of an "active" network



(a) Network for Example 14.5



(b) Shorted-output diagram



(b) Open input

sources: $\underline{I}_1, \underline{V}_2$

responses: $\underline{V}_1, \underline{I}_2$

$$\begin{cases} \underline{V}_1 = h_{11}\underline{I}_1 + h_{12}\underline{V}_2 \\ \underline{I}_2 = h_{21}\underline{I}_1 + h_{22}\underline{V}_2 \end{cases}$$

$$\rightarrow \begin{bmatrix} \underline{V}_1 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \underline{I}_1 \\ \underline{V}_2 \end{bmatrix}$$

(1)port – 2 s.c., $\underline{V}_2 = 0$

$$\underline{I}_1 = \underline{V}_1 \frac{s}{40}$$

$$\rightarrow h_{11} = \frac{\underline{V}_1}{\underline{I}_1} = \frac{40}{s}$$

$$\underline{I}_2 = 3\underline{I}_1 - \underline{I}_1 = 2\underline{I}_1$$

$$\rightarrow h_{21} = \frac{\underline{I}_2}{\underline{I}_1} = 2$$

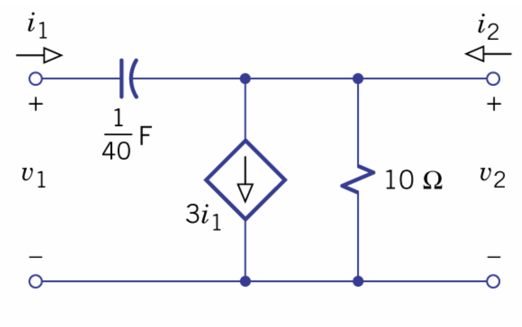
(2)port – 1 o.c., $\underline{I}_1 = 0$

$$\underline{V}_1 = \underline{V}_2 \rightarrow h_{12} = \frac{\underline{V}_1}{\underline{V}_2} = 1$$

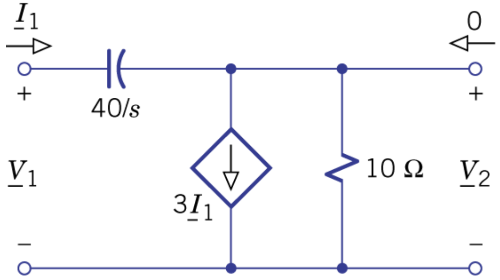
$$\underline{V}_2 = 10\underline{I}_2$$

$$\rightarrow h_{22} = \frac{\underline{I}_2}{\underline{V}_2} = 0.1$$

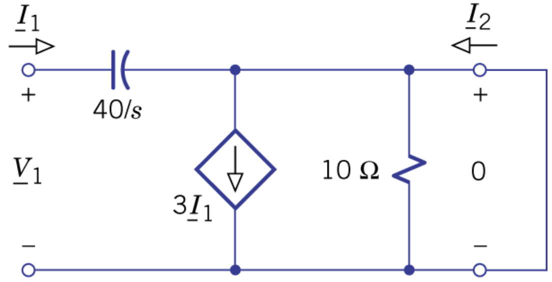
6. Ex. 14.7 find ABCD-parameters of an "active" network



(a) Network for Example 14.5



(a) Open output



(b) Shorted output

sources: $\underline{V}_2, \underline{I}_2$

responses: $\underline{V}_1, \underline{I}_1$

$$\begin{cases} \underline{V}_1 = A\underline{V}_2 - B\underline{I}_2 \\ \underline{I}_1 = C\underline{V}_2 - D\underline{I}_2 \end{cases}$$

$$\rightarrow \begin{bmatrix} \underline{V}_1 \\ \underline{I}_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \underline{V}_2 \\ -\underline{I}_2 \end{bmatrix}$$

(1)port – 2 o.c., $\underline{I}_2 = 0$

$$\underline{I}_1 = 3\underline{I}_1 + \frac{\underline{V}_2}{10}, \underline{I}_1 = -\frac{\underline{V}_2}{20}$$

$$\rightarrow C = \frac{\underline{I}_1}{\underline{V}_2} = -\frac{1}{20}$$

$$\underline{V}_1 = \underline{V}_2 + \frac{40}{s}\underline{I}_1 = (1 - \frac{2}{s})\underline{V}_2$$

$$\rightarrow A = \frac{\underline{V}_1}{\underline{V}_2} = 1 - \frac{2}{s}$$

(2)port – 2 s.c., $\underline{V}_2 = 0$

$$\underline{I}_1 = 3\underline{I}_1 - \underline{I}_2, 2\underline{I}_1 = \underline{I}_2$$

$$\rightarrow D = \frac{\underline{I}_1}{-\underline{I}_2} = -\frac{1}{2}$$

$$\underline{V}_1 = \frac{40}{s}\underline{I}_1 = \frac{20}{s}\underline{I}_2$$

$$\rightarrow B = \frac{\underline{V}_1}{-\underline{I}_2} = -\frac{20}{s}$$

7. All the 2-port parameters are related as given in Table 14.2.

8. Conversion between z-parameters and y-parameters

$$z-\text{parameter}\begin{bmatrix}\underline{V}_1\\ \underline{V}_2\end{bmatrix}=\begin{bmatrix}z_{11} & z_{12}\\ z_{21} & z_{22}\end{bmatrix}\begin{bmatrix}\underline{I}_1\\ \underline{I}_2\end{bmatrix}=[z]\begin{bmatrix}\underline{I}_1\\ \underline{I}_2\end{bmatrix}\rightarrow\begin{bmatrix}\underline{I}_1\\ \underline{I}_2\end{bmatrix}=[z]^{-1}\begin{bmatrix}\underline{V}_1\\ \underline{V}_2\end{bmatrix}=[y]\begin{bmatrix}\underline{V}_1\\ \underline{V}_2\end{bmatrix}$$

$$\Rightarrow[y]=[z]^{-1}=\begin{bmatrix}\frac{z_{22}}{\Delta_z} & -\frac{z_{12}}{\Delta_z}\\ -\frac{z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z}\end{bmatrix},\Delta_z=\left|[z]\right|=z_{11}z_{22}-z_{12}z_{21}$$

$$[z]=[y]^{-1}=\begin{bmatrix}\frac{y_{22}}{\Delta_y} & -\frac{y_{12}}{\Delta_y}\\ -\frac{y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y}\end{bmatrix},\Delta_y=\left|[y]\right|=y_{11}y_{22}-y_{12}y_{21}$$

9. Derivation of h-parameters from z-parameters

$$z-\text{parameter}\begin{bmatrix}\underline{V}_1\\ \underline{V}_2\end{bmatrix}=\begin{bmatrix}z_{11} & z_{12}\\ z_{21} & z_{22}\end{bmatrix}\begin{bmatrix}\underline{I}_1\\ \underline{I}_2\end{bmatrix},\ h-\text{parameter}\begin{bmatrix}\underline{V}_1\\ \underline{I}_2\end{bmatrix}=\begin{bmatrix}h_{11} & h_{12}\\ h_{21} & h_{22}\end{bmatrix}\begin{bmatrix}\underline{I}_1\\ \underline{V}_2\end{bmatrix}$$

$$\begin{cases}\underline{V}_1 = z_{11}\underline{I}_1 + z_{12}\underline{I}_2 \ldots(1)\\ \underline{V}_2 = z_{21}\underline{I}_1 + z_{22}\underline{I}_2 \ldots(2)\end{cases}\qquad (2)\rightarrow \underline{I}_2 = -\frac{z_{21}}{z_{22}}\underline{I}_1 + \frac{1}{z_{22}}\underline{V}_2 \ldots(3)$$

$$(1)\overset{(3)}{\rightarrow}\underline{V}_1 = z_{11}\underline{I}_1 + z_{12}\left(-\frac{z_{21}}{z_{22}}\underline{I}_1 + \frac{1}{z_{22}}\underline{V}_2\right) = \frac{\Delta_z}{z_{22}}\underline{I}_1 + \frac{z_{12}}{z_{22}}\underline{V}_2,\ \Delta_z = z_{11}z_{22} - z_{12}z_{21}$$

$$[h]=\begin{bmatrix}\dfrac{\Delta_z}{z_{22}} & \dfrac{z_{12}}{z_{22}}\\ -\dfrac{z_{21}}{z_{22}} & \dfrac{1}{z_{22}}\end{bmatrix}$$

10. Derivation of z-parameters from T-parameters

$$T-\text{parameter}\begin{bmatrix}\underline{V}_1\\ \underline{I}_1\end{bmatrix}=\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\underline{V}_2\\ -\underline{I}_2\end{bmatrix}, z-\text{parameter}\begin{bmatrix}\underline{V}_1\\ \underline{V}_2\end{bmatrix}=\begin{bmatrix}z_{11} & z_{12}\\ z_{21} & z_{22}\end{bmatrix}\begin{bmatrix}\underline{I}_1\\ \underline{I}_2\end{bmatrix}$$

$$\begin{cases}\underline{V}_1 = A\underline{V}_2 - B\underline{I}_2 \ldots(1)\\ \underline{I}_1 = C\underline{V}_2 - D\underline{I}_2 \ldots(2)\end{cases} \quad (2)\rightarrow \underline{V}_2 = \frac{1}{C}\underline{I}_1 + \frac{D}{C}\underline{I}_2 \ldots(3)$$

$$(1)\overset{(3)}{\rightarrow}\underline{V}_1 = A\left(\frac{1}{C}\underline{I}_1 + \frac{D}{C}\underline{I}_2\right) - B\underline{I}_2 = \frac{A}{C}\underline{I}_1 + \frac{\Delta_T}{C}\underline{I}_2, \Delta_T = AD - BC$$

$$[z]=\begin{bmatrix}\frac{A}{C} & \frac{\Delta_T}{C}\\ \frac{1}{C} & \frac{D}{C}\end{bmatrix}$$

11. For a reciprocal 2-port network,

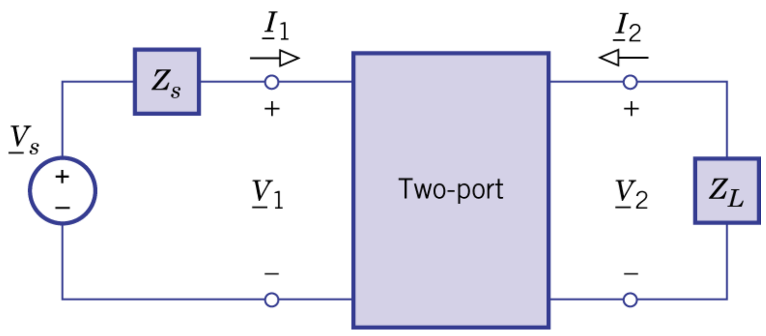
$$z_{12} = z_{21}, y_{12} = y_{21}, h_{12} = -h_{21}, \Delta_T = AD - BC = 1$$

14.3 Circuit analysis with two-ports

Basics

1. Terminated two-ports using z-parameters

$$\begin{cases} \underline{V}_1 = z_{11}\underline{I}_1 + z_{12}\underline{I}_2 \ldots(1) \\ \underline{V}_2 = z_{21}\underline{I}_1 + z_{22}\underline{I}_2 \ldots(2) \end{cases}, \begin{cases} \underline{V}_s = Z_S\underline{I}_1 + \underline{V}_1 \ldots(3) \\ \underline{V}_2 = -Z_L\underline{I}_2 \ldots\ldots(4) \end{cases}$$



$$(2)\xrightarrow{(4)} -Z_L\underline{I}_2 = z_{21}\underline{I}_1 + z_{22}\underline{I}_2, z_{21}\underline{I}_1 = -(Z_L + z_{22})\underline{I}_2$$

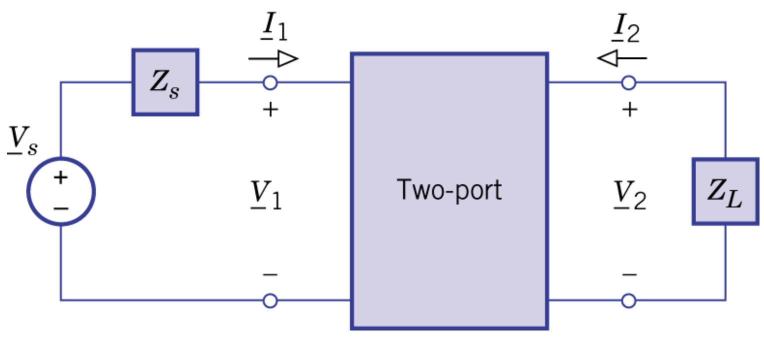
$$\Rightarrow \text{current transfer function } H_i \equiv \frac{\underline{I}_2}{\underline{I}_1} = \frac{-z_{21}}{Z_L + z_{22}}$$

$$(1)\xrightarrow{(4)} \underline{V}_1 = z_{11}\underline{I}_1 - \frac{z_{12}}{Z_L}\underline{V}_2, \underline{I}_1 = \frac{1}{z_{11}}\underline{V}_1 + \frac{z_{12}}{z_{11}Z_L}\underline{V}_2 \ldots(5)$$

$$(2)\xrightarrow{(5)} \underline{V}_2 = z_{21}\left(\frac{1}{z_{11}}\underline{V}_1 + \frac{z_{12}}{z_{11}Z_L}\underline{V}_2\right) - \frac{z_{22}}{Z_L}\underline{V}_2$$

$$\Rightarrow \text{voltage transfer function } H_v \equiv \frac{\underline{V}_2}{\underline{V}_1} = \frac{\frac{z_{21}}{z_{11}}}{1 - \frac{z_{12}z_{21}}{z_{11}Z_L} + \frac{z_{22}}{Z_L}} = \frac{z_{21}Z_L}{z_{11}Z_L - z_{12}z_{21} + z_{11}z_{22}} = \frac{z_{21}Z_L}{\Delta_z + z_{11}Z_L}$$

$$\begin{cases} \underline{V}_1 = z_{11}\underline{I}_1 + z_{12}\underline{I}_2 \ldots(1) \\ \underline{V}_2 = z_{21}\underline{I}_1 + z_{22}\underline{I}_2 \ldots(2) \end{cases}, \begin{cases} \underline{V}_s = Z_S\underline{I}_1 + \underline{V}_1 \ldots(3) \\ \underline{V}_2 = -Z_L\underline{I}_2 \ldots\ldots(4) \end{cases}$$



$$H_i \equiv \frac{\underline{I}_2}{\underline{I}_1} = \frac{-z_{21}}{Z_L + z_{22}} \ldots(6)$$

$$(1) \xrightarrow{(6)} \underline{V}_1 = z_{11}\underline{I}_1 - z_{12}\frac{-z_{21}}{Z_L + z_{22}}\underline{I}_1$$

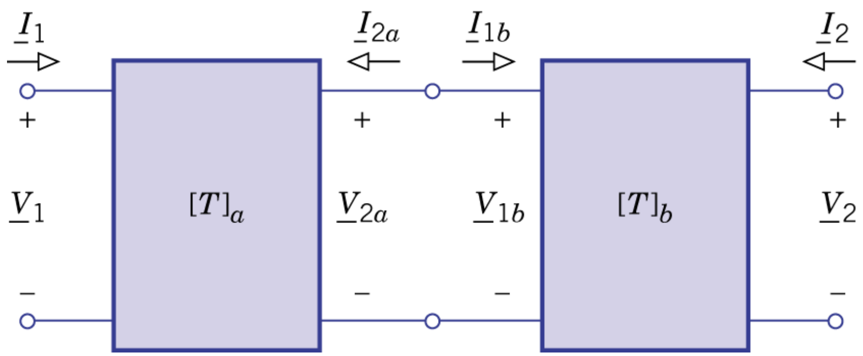
$$\Rightarrow \text{equivalent input impedance } Z_i \equiv \frac{\underline{V}_1}{\underline{I}_1} = \frac{z_{11}Z_L + z_{11}z_{22} - z_{12}z_{21}}{Z_L + z_{22}} = \frac{\Delta_z + z_{11}Z_L}{Z_L + z_{22}}$$

$$\underline{V}_s = 0, (3) \to \underline{V}_1 = -Z_S\underline{I}_1 \ldots(7), (1) \xrightarrow{(7)} -Z_S\underline{I}_1 = z_{11}\underline{I}_1 + z_{12}\underline{I}_2 \to \underline{I}_1 = -\frac{z_{12}}{Z_S + z_{11}}\underline{I}_2 \ldots(8)$$

$$(2) \xrightarrow{(8)} \underline{V}_2 = z_{21}\left(-\frac{z_{12}}{Z_S + z_{11}}\underline{I}_2\right) + z_{22}\underline{I}_2$$

$$\Rightarrow \text{equivalent output impedance } Z_o \equiv \left.\frac{\underline{V}_2}{\underline{I}_2}\right|_{\underline{V}_s = 0} = \frac{z_{22}Z_S + z_{11}z_{22} - z_{12}z_{21}}{Z_S + z_{11}} = \frac{\Delta_z + z_{22}Z_S}{Z_S + z_{11}}$$

2. Cascade connection using T-parameters

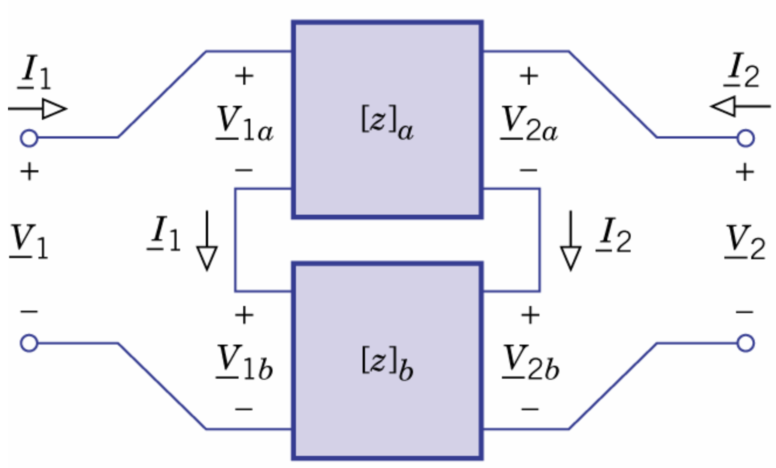


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T]_a \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} = [T]_a \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix}$$

$$= [T]_a [T]_b \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\Rightarrow [T]_{cas} = [T]_a [T]_b$$

3. Series connection using z-parameters



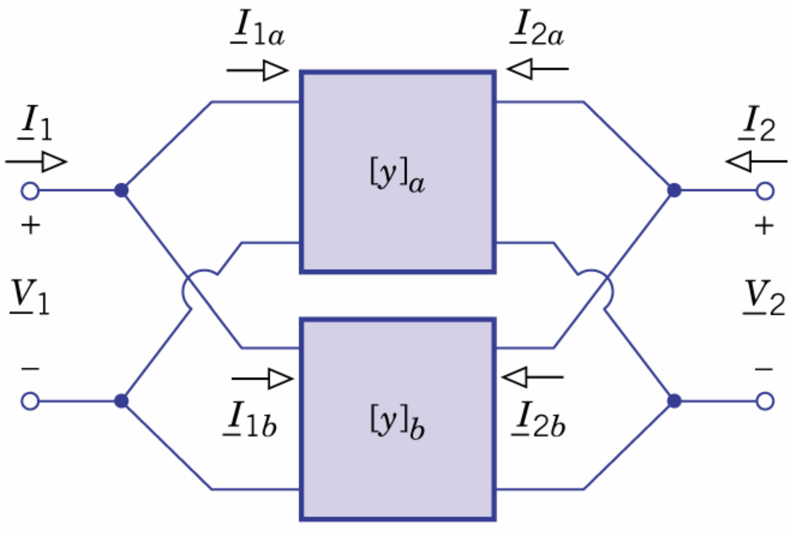
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} + \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$= [z]_a \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + [z]_b \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= ([z]_a + [z]_b) \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow [z]_{ser} = [z]_a + [z]_b$$

4. Parallel connection using y-parameters



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} + \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

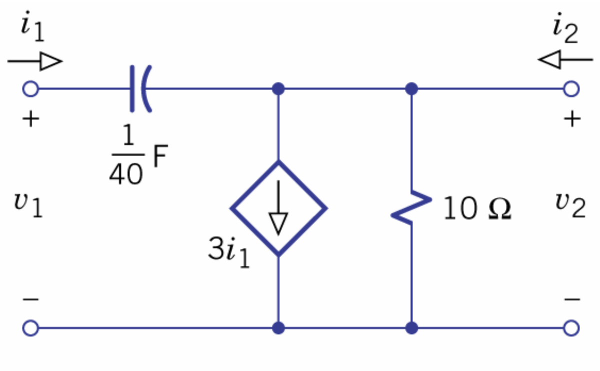
$$= [y]_a \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + [y]_b \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= ([y]_a + [y]_b) \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow [y]_{par} = [y]_a + [y]_b$$

Discussion

1. Relations of terminated two-ports in terms of z- y- h- and T-parameters are given in Table 14.3. They are useful in network analysis.
2. Ex. 14.9 given load be a 2.5H inductor, find I_2/V_1 from T-parameters



(a) Network for Example 14.5

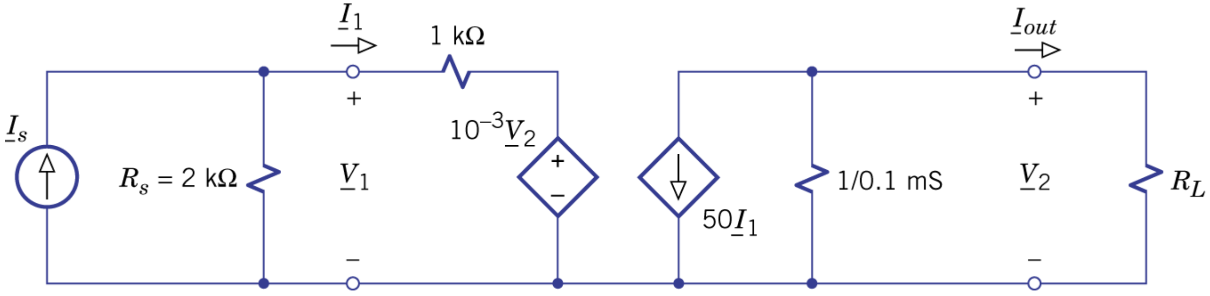
$$\text{From ex.14.7}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1-\frac{2}{s} & -\frac{20}{s} \\ -\frac{1}{20} & -\frac{1}{2} \end{bmatrix}$$

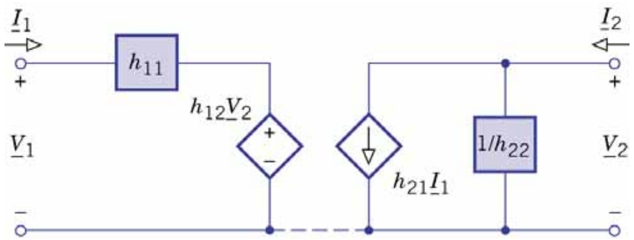
$$H(s) = \frac{\underline{I}_2}{\underline{V}_1} = \frac{\underline{I}_2}{Z_i \underline{I}_1} = \frac{H_i}{Z_i} \overset{Table14.2}{=} \frac{-1}{CZ_L + D} \frac{CZ_L + D}{AZ_L + B}$$

$$= \frac{-1}{AZ_L + B} = \frac{-1}{(1-\frac{2}{s})2.5s - \frac{20}{s}} = \frac{-0.4s}{s^2 - 2s - 8}$$

$$= \frac{-0.4s}{(s-4)(s+2)}$$

3. Ex. 14.10 find RL to give Ai=Iout/Is=-25 from transistor h-parameters





$$[h]=\begin{bmatrix}1000 & 10^{-3}\\ 50 & 0.1\times10^{-3}\end{bmatrix}$$

$$\Delta_h = 0.05$$

$$Z_i(s)=\frac{V_1}{I_1}=\frac{\Delta_h + h_{11}Y_L}{h_{22}+Y_L}$$

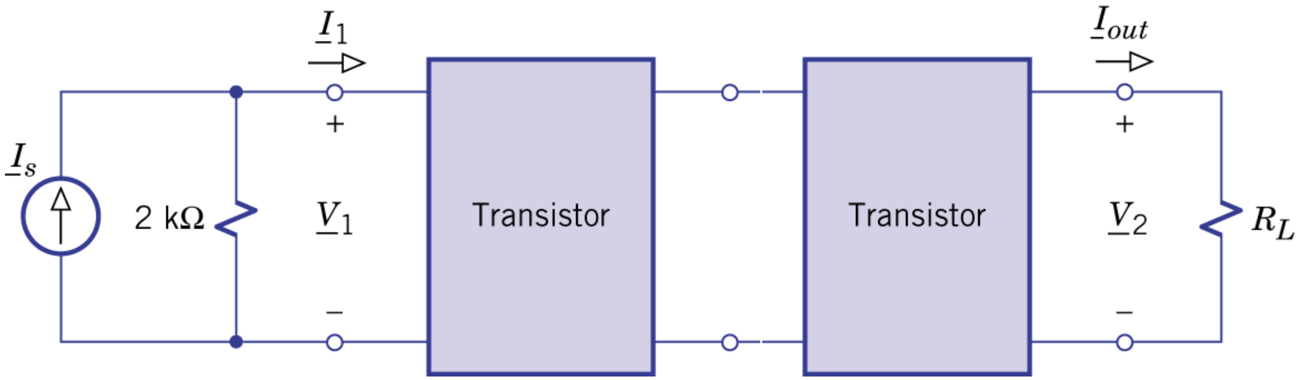
$$H_i(s)=\frac{I_2}{I_1}=\frac{h_{21}Y_L}{h_{22}+Y_L}$$

$$A_i=\frac{I_{out}}{I_s}=\frac{-I_2}{I_1}\frac{I_1}{I_s}=-H_i\frac{R_s}{R_s+Z_i}$$

$$Z_i(s)=\frac{0.05R_L+1}{0.1R_L+1},\ H_i(s)=\frac{50}{0.1R_L+1}$$

$$A_i=\frac{-100}{3+0.25R_L}=-25\rightarrow R_L=4k\Omega$$

4. Ex. 14.11 find A_i of two amplifiers of ex.14.10 in cascade



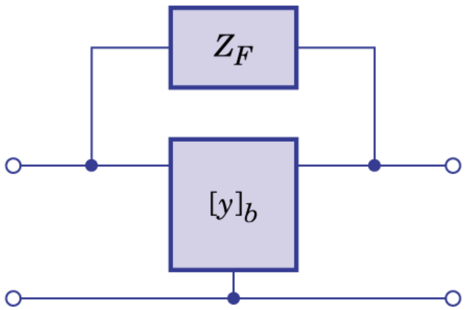
$$[h]=\begin{bmatrix}1000 & 10^{-3}\\ 50 & 0.1\times10^{-3}\end{bmatrix},[T]_a=[T]_b=\begin{bmatrix}\frac{-\Delta_h}{h_{21}} & \frac{-h_{11}}{h_{21}}\\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}}\end{bmatrix}=\begin{bmatrix}-10^{-3} & -20\\ -2\times10^{-6} & -0.02\end{bmatrix}$$

$$\rightarrow [T]_{cas}=[T]_a[T]_b=\begin{bmatrix}41\times10^{-6} & 0.42\\ 0.042\times10^{-6} & 440\times10^{-6}\end{bmatrix}$$

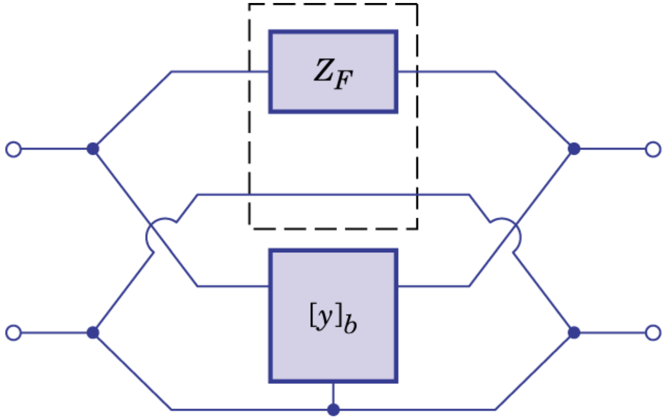
$$Z_i(s)=\frac{AR_L+B}{CR_L+D}=961, H_i(s)=\frac{-1}{CR_L+D}=-1645$$

$$A_i=\frac{I_{out}}{I_s}=\frac{-I_2}{I_1}\frac{I_1}{I_s}=-H_i\frac{R_s}{R_s+Z_i}=1100$$

5. Bridged-T connection



(a) Network with bridging impedance



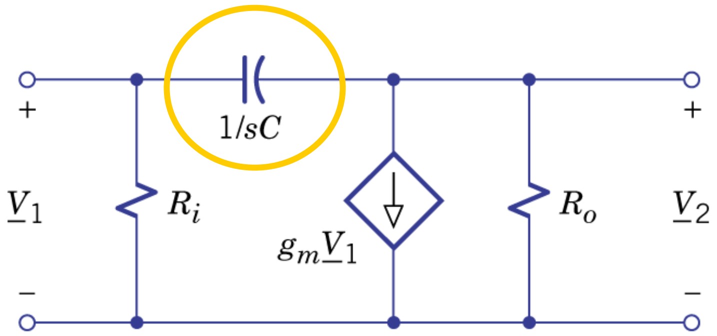
(b) Model as parallel two-ports

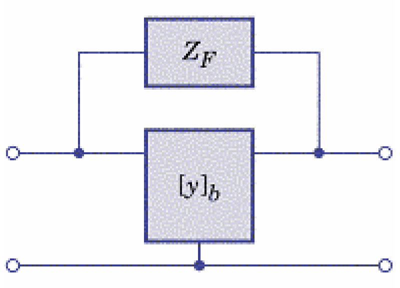
$$[y]=\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}, y_{11}=\left.\frac{I_1}{V_1}\right|_{V_2=0}, y_{22}=\left.\frac{I_2}{V_2}\right|_{V_1=0}, y_{12}=\left.\frac{I_1}{V_2}\right|_{V_1=0}, y_{21}=\left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{11}=y_{22}=Y_F, y_{12}=y_{21}=-Y_F$$

$$\Rightarrow [y]=\begin{bmatrix} Y_F+y_{11b} & -Y_F+y_{12b} \\ -Y_F+y_{21b} & Y_F+y_{22b} \end{bmatrix}$$

6. Ex.14.12 find $H_v = \underline{V}_2/\underline{V}_1$ of a high frequency transistor





$$Z_F = \frac{1}{sC}, [y]_a = \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix}, [y]_b = \begin{bmatrix} \frac{1}{R_i} & 0 \\ g_m & \frac{1}{R_o} \end{bmatrix} \Rightarrow [y] = \begin{bmatrix} sC + \frac{1}{R_i} & -sC \\ -sCg_m & sC + \frac{1}{R_o} \end{bmatrix}$$

$$H_v = \frac{-y_{21}}{y_{22} + Y_L} = \frac{sC - g_m}{sC + 1/R_o + 1/R_L} = \frac{s - g_m/C}{s + G/C}$$

$$G = 1/R_o + 1/R_L = 1/R_o // R_L$$