Chapter 2 Properties of resistive circuits

2.1-2.2 (optional)

equivalent resistance, resistive ladder, duality, dual entities

2.3 Circuits with controlled sources

controlled sources (VCVS, CCVS, VCCS, CCVS), equivalent resistance

2.4 Linearity and superposition

proportionality principle, superposition theorem

2.5 Thevenin and Norton networks

Thevenin's and Norton's theorems, source conversion

2.1-2.2

Optional

1. **equivalent two-terminal networks: two-terminal networks have the same *i-v* characteristics**
2. series-connected resistors: voltage divider
   series equivalent resistance: $R_{ser} = R_1 + R_2 + ... + R_N$
3. parallel-connected resistors: current divider
   parallel equivalent conductance: $G_{par} = G_1 + G_2 + ... + G_N$
4. resistive ladder: a network consisting entirely of series and parallel resistors
5. **series-parallel reduction method**: to solve branch variables of a resistive ladder
   (1) step 1: find the equivalent resistance of the ladder network
   (2) step 2: calculate the voltage or current at the network terminal
   (3) step 3: use KVL, KCL, Ohm's law to calculate branch variables

6. Ex. 2.4 solve terminal current i, total dissipated power p, branch variables v_x, v_y

(1) step 1

$$R_{eq}=2+10//15=8k\Omega$$

(2) step 2

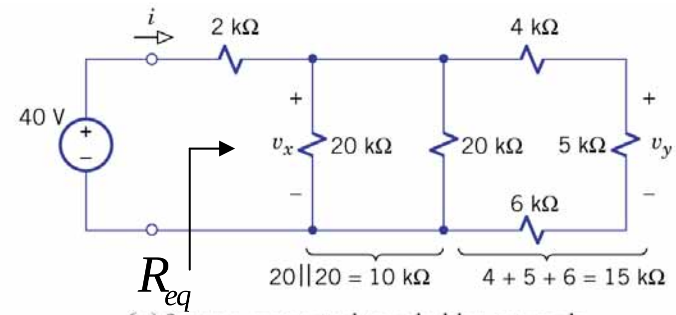
$$i=\frac{40}{8000}=0.005$$

$$p=40\times 0.005=200mW$$

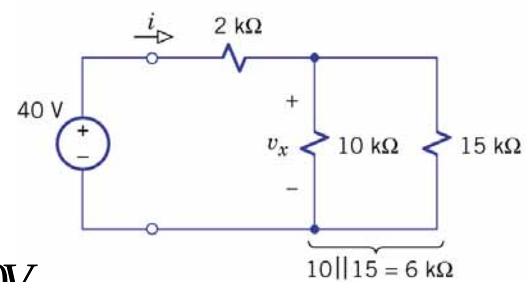
(3) step 3

$$v_x=40-2000\times 0.005=30V$$

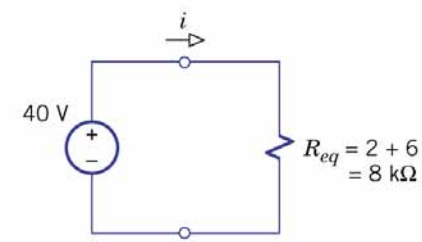
$$v_y=30\times\frac{5}{4+5+6}=10V$$



(a) Source connected to a ladder network



(b) Partitally reduced network



(c) Terminal equivalent diagram

7. dual networks:

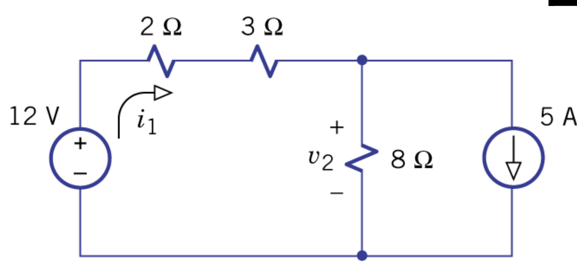
***i-v* equation of network A= *i-v* equation of network B with its *i, v* interchanged**

Series network	Parallel network
KVL loop eq. $v = (R_1 + R_2 + \ldots)\, i$	KCL node eq. $i = (G_1 + G_2 + \ldots) v$
equivalent resistance $R_{ser} = R_1 + R_2 + \ldots$	equivalent conductance $G_{par} = G_1 + G_2 + \ldots$
voltage divider $v_n = \dfrac{R_n}{R_1 + R_2 + \ldots} v$	current divider $i_n = \dfrac{G_n}{G_1 + G_2 + \ldots} i$
open circuit $v_n = v$ when $R_n \to \infty$	short circuit $i_n = i$ when $G_n \to \infty$

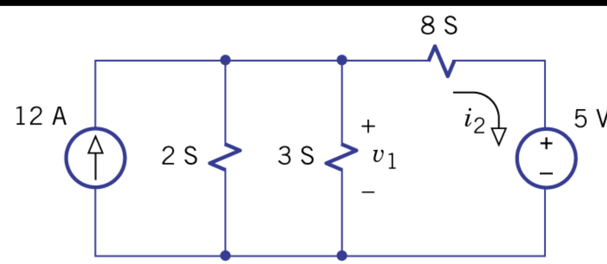
8. Dual pairs

KVL	KCL
voltage	current
resistance	conductance
loop	node
series	parallel
open ckt	short ckt
node voltage	mesh current
capacitance	inductance
impedance	admittance

9. Ex. 2.5 dual networks



(a) Illustrative circuit



(b) Exact dual

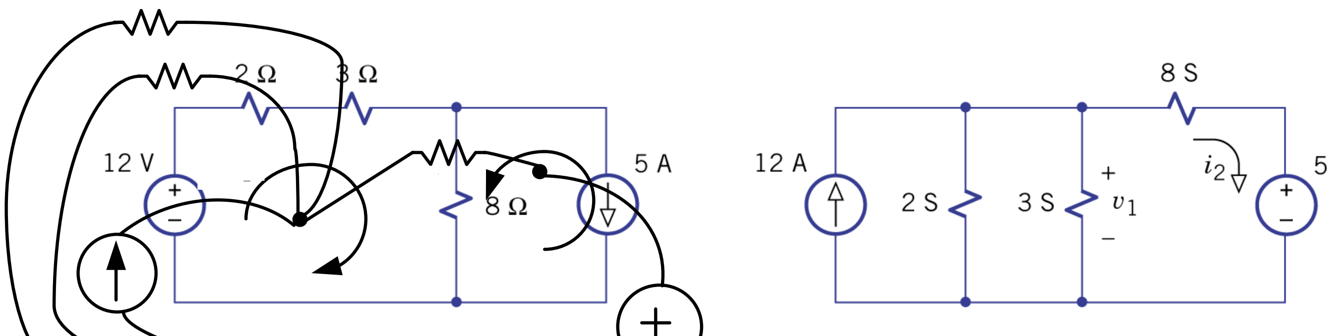
$$i_1 = \frac{v_2}{8} + 5$$

$$v_2 = 12 - (2+3)i_1$$

$$v_1 = \frac{i_2}{8} + 5$$

$$i_2 = 12 - (2+3)v_1$$

(dual circuit conversion)



(a) Illustrative circuit (b) Exact dual

Step 1: place a node at the center of each loop and the reference node outside of the given circuit

Step 2: draw a line between the nodes with each line crossing an element, and replace that element by its dual

3S

rule of source: a voltage source that produces a positive (CW) loop current has as its dual a current source whose reference direction is from the ground to nonreference node

2S

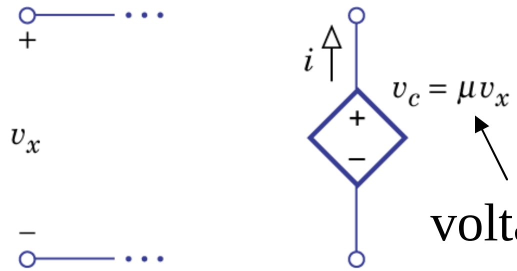
$12V \rightarrow i_1$ *positive or* CW $\rightarrow$ *dual* 12A current source directed from 0 to 1

$5A$ to the ground node $\rightarrow i_2(negative) = -5A$ or CCW $\rightarrow$ *dual* 5V voltage source

2.3 Circuit with controlled sources

<u>Basics</u>

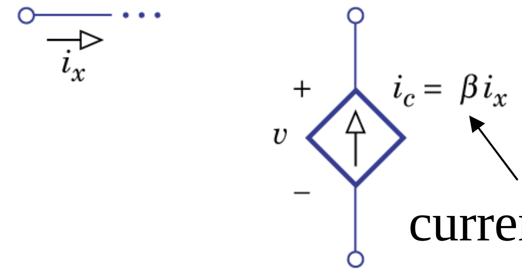
1. **dependent (controlled) sources: linear element**



voltage gain

(a) Voltage-controlled voltage source (VCVS)

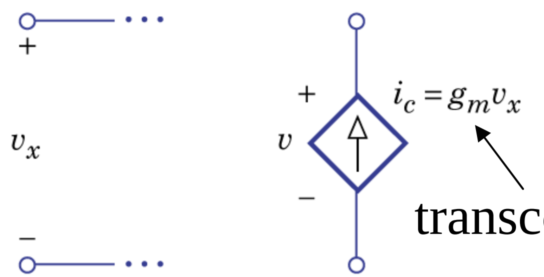
(transformer, voltage amplifier)



current gain

(b) Current-controlled current source (CCCS)

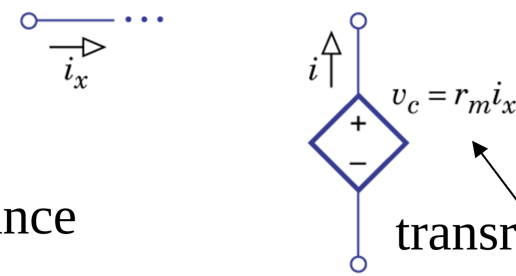
(BJT)



transconductance

(c) Voltage-controlled current source (VCCS)

(FET)

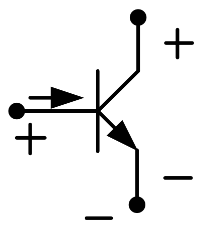


transresistance

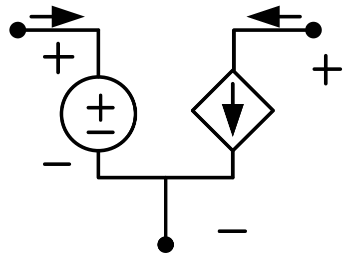
(d) Current-controlled voltage source (CCVS)

(transformer)

(DC transistor circuit)



npn transistor



DC equivalent model

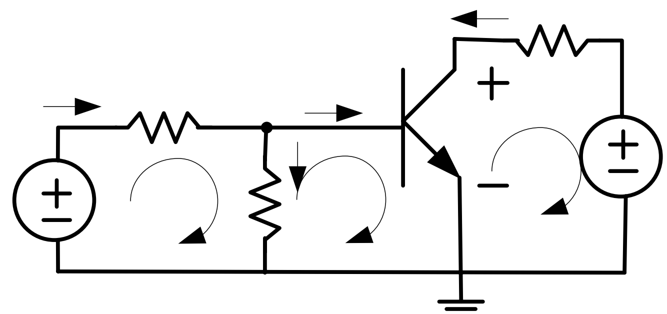
in active mode: $V_{BE} \approx 0.7V$

$I_C = \alpha I_E, \alpha$: CB current gain

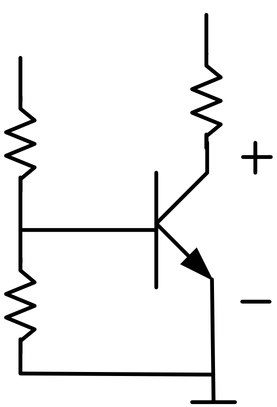
$0.98 < \alpha < 0.999$

$I_C = \beta I_B, \beta$: CE current gain

$50 < \beta < 1000$



method 1



$\beta = 150$

loop 1: $2 = 100\times10^3 i_1 + 200\times10^3 i_2$...(1)

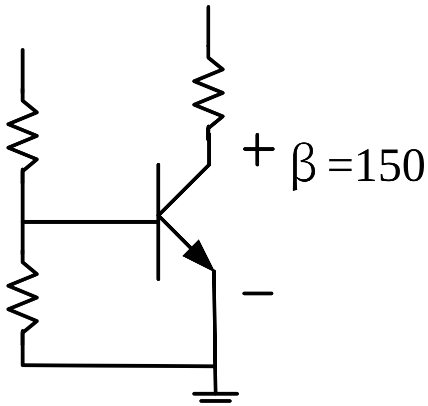
loop 2: $V_{BE} = 0.7 = 200\times10^3 i_2 \rightarrow i_2 = 3.5uA$

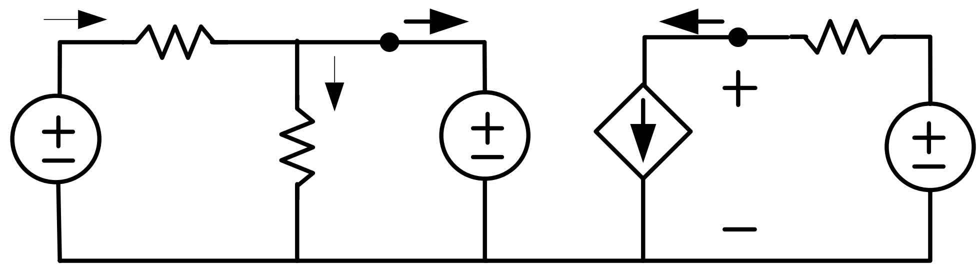
loop 3: $-v_o - 10^3 I_C + 16 = 0$...(2)

(1) $\rightarrow i_1 = 13uA$, node A : $i_1 = i_2 + I_B \rightarrow I_B = 9.5uA$

$I_C = \beta I_B = 1.425mA$, (2) $\rightarrow v_o = 16 - 1.425 = 14.575V$

method 2





$$v_o = 16 - 10^3 \times 150 I_B \ldots (1)$$

$$I_B = i_1 - i_2 = \frac{2-0.7}{100\times 10^3} - \frac{0.7}{200\times 10^3} = 9.5uA$$

$$(1) \rightarrow v_o = 16 - 10^3 \times 150 \times 9.5 \times 10^6 = 14.575V$$

16V

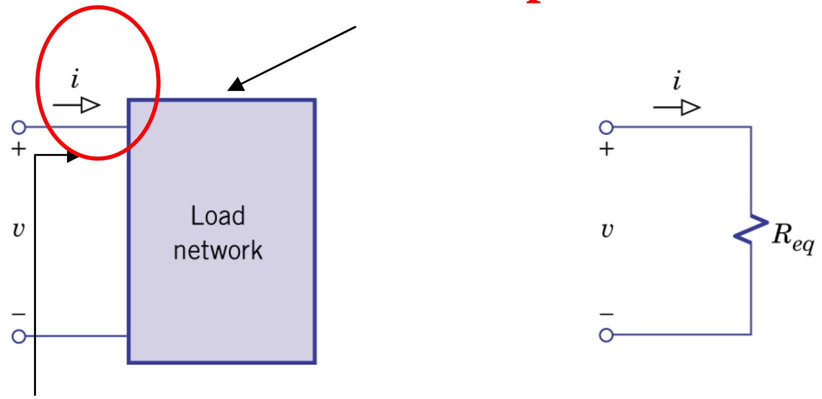
2V

1kΩ

100kΩ

2. equivalent resistance theorem (terminal behavior of a network)
For a two-terminal load network containing resistors and controlled sources, the equivalent resistor is determined by its terminal voltage and current.

no independent source inside



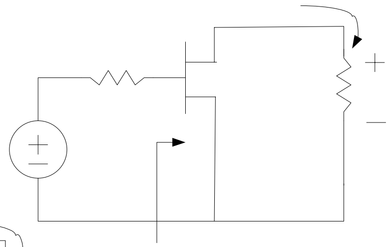
(a) Load network containing resistors and controlled sources

(b) Equivalent resistance $R_{eq} = v/i$

$$R_{eq} = \frac{v}{i}$$

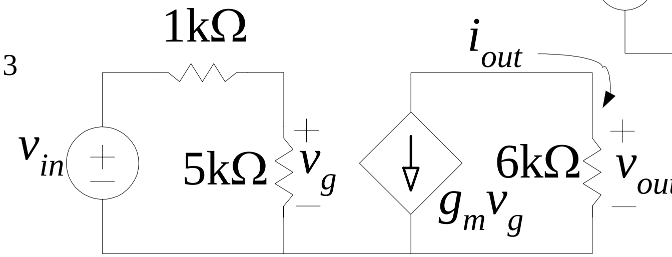
Discussion

1. Ex.2.6 a FET with g_m=5mS, v_{out}=?



$$v_{out} = i_{out}\,6\times10^3 = -g_m v_g\,6\times10^3$$

$$= -g_m \frac{5}{1+5}\,6\times10^3 v_{in} = -25 v_{in}$$



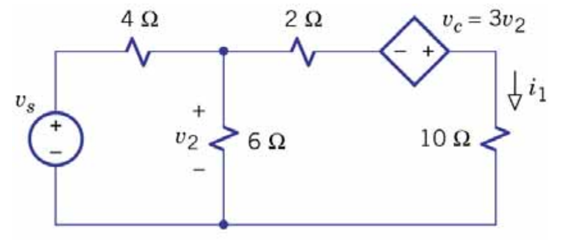
2. Ex.2.7 a VCVS to find $i_1(v_s)$

$$KVL: v_2 + 3v_2 - 12i_1 = 0 \rightarrow v_2 = 3i_1$$

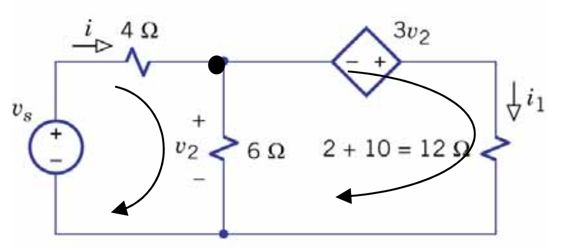
$$KVL: v_s - 4i - v_2 = 0 \rightarrow v_s = 4i + 3i_1$$

$$KCL: i = \frac{v_2}{6} + i_1 = \frac{3}{2} i_1$$

$$\Rightarrow v_s = 6i_1 + 3i_1 = 9i_1 \rightarrow i_1 = \frac{v_s}{9}$$

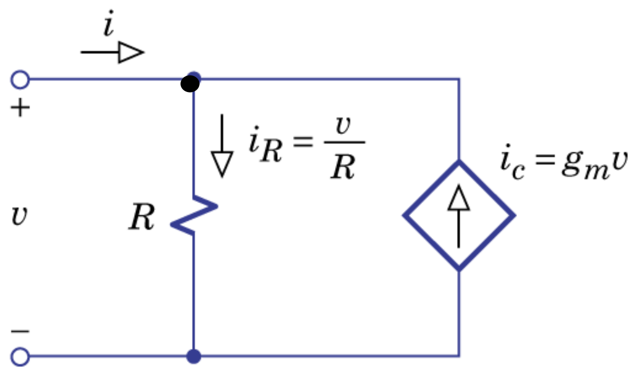


(a) Circuit with a VCVS



(b) Simplified diagram

3. Ex.2.8 find R_{eq}



$$KCL: i + g_m v = \frac{v}{R} \rightarrow i = (-g_m + \frac{1}{R})v = \frac{1 - g_m R}{R} v$$

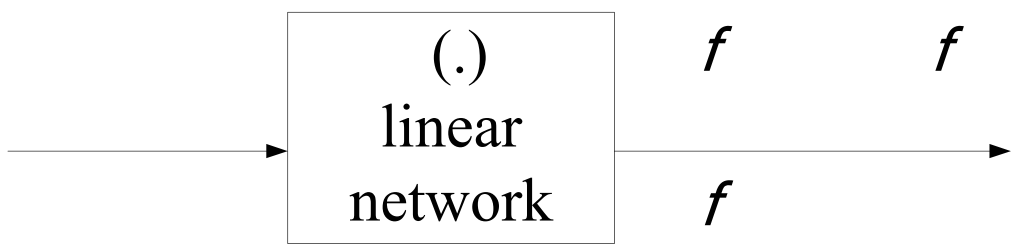
$$R_{eq} = \frac{v}{i} = \frac{R}{1 - g_m R}$$

$$if \;\; R = \infty, \;\; i = -i_c = -g_m v \rightarrow R_{eq} = \frac{v}{i} = -\frac{1}{g_m} (active)$$

2.4 Linearity and superposition

<u>Basics</u>

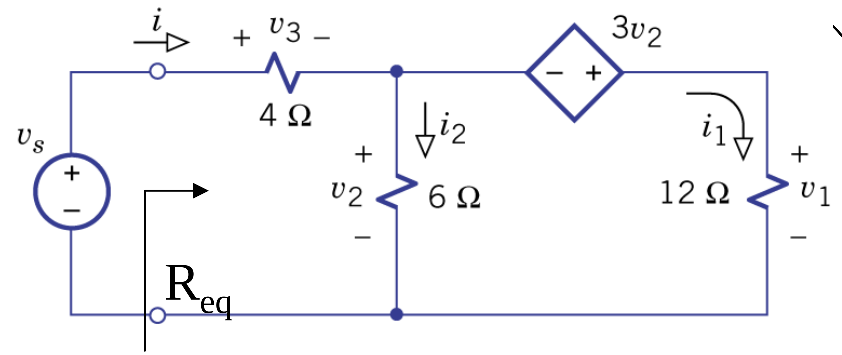
1. Linear resistive circuit: circuit contains linear resistors and independent sources.
2. Proportionality principle **(single independent** source)
   if $y=f(x)$, then $f(Kx)=Kf(x)$
3. Superposition theorem **(multiple independent** sources)
   For a linear network, response of a sum of signals = sum of signal responses



4. **Controlled sources** are **not** applied for the superposition theorem.
5. suppressed independent **voltage source $v_s=0$ → short circuit**
   suppressed independent **current source $i_s=0$ → open circuit**

Discussion

1. Ex.2.9 (ladder network) v_s =72 use proportionality principle to find i, v_2, i_1



step 1: assume a convenient value for a branch variable farthest from the source

$$\text{let } i_1 = 1 \rightarrow v_1 = 12$$

step 2: work toward source to obtain the source value x

$$v_2 = -3v_2 + v_1 \rightarrow v_2 = \frac{v_1}{4} = 3 \rightarrow i_2 = \frac{3}{6} \rightarrow i = i_2 + i_1 = \frac{3}{2}$$

$$v_3 = 4i = 6 \rightarrow v_s = v_3 + v_2 = 9$$

step 3: calculate the scaling factor K $\quad K \times 9 = 72 \rightarrow K = 8$

step 4: calculate the actual values by the scaling factor K

$$i = K \times \frac{3}{2} = 12, v_2 = K \times 3 = 24, i_1 = K \times 1 = 8$$

$$R_{eq} = \frac{v_s}{i} = 6$$

Ex.2.10 use superposition to find i_1

from source 1 of 30V

$$i_{1-1} = \frac{30}{6+4+2} = \frac{5}{2}$$

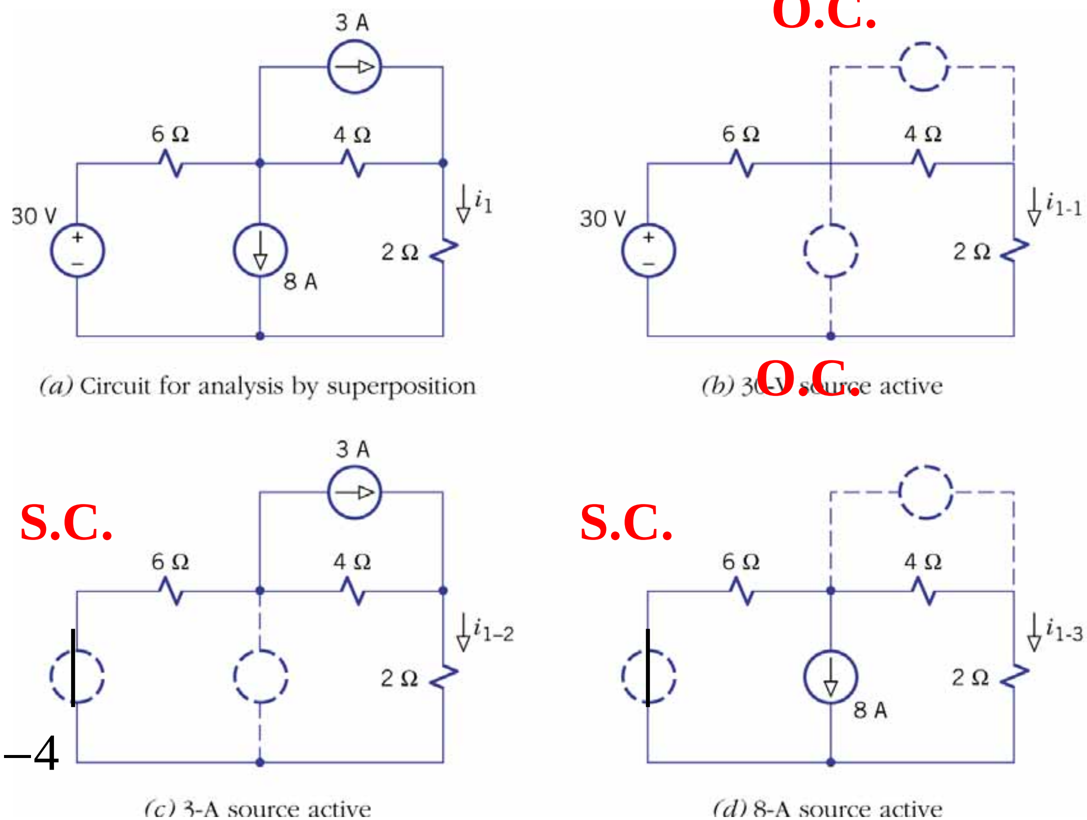
from source 2 of 3A

$$i_{1-2} = \frac{4}{(6+2)+4} \times 3 = 1$$

from source 3 of 8A

$$i_{1-3} = \frac{6}{6+(2+4)} \times (-8) = -4$$

$$i_1 = 2.5 + 1 - 4 = -0.5$$



(a) Circuit for analysis by superposition

(b) 30-V source active

(c) 3-A source active

(d) 8-A source active

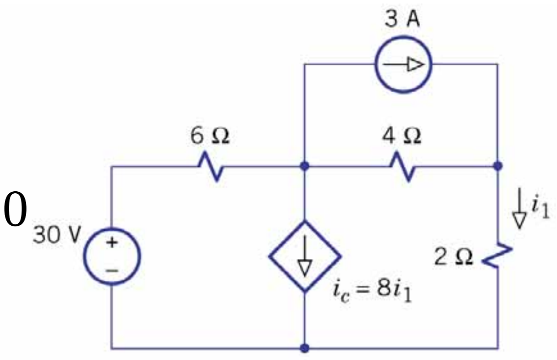
Ex.2.11 use superposition to find i_1

from source 1 of 30V

$KCL \rightarrow 9i_{1-1}$

$KVL \rightarrow 30-6(9i_{1-1})-(4+2)i_{1-1}=0$

$i_{1-1}=0.5$



(a) Circuit with a CCCS

from source 2 of 3A

$KCL \rightarrow 9i_{1-2}$

$KCL \rightarrow$

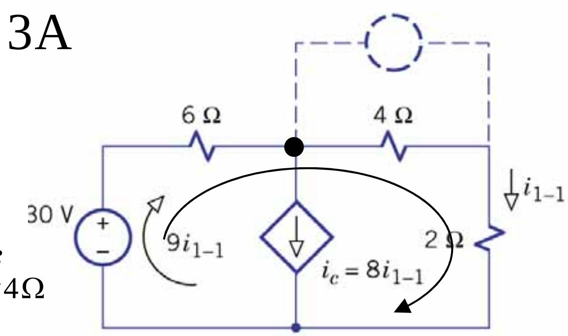
$9i_{1-2}=8i_{1-2}+3+i_{4\Omega}$

$\rightarrow i_{4\Omega}=i_{1-2}-3$

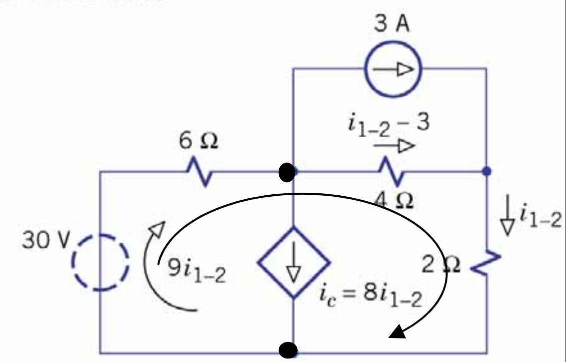
$KVL \rightarrow 6(9i_{1-2})+4(i_{1-2}-3)+2i_{1-2}=0$

$\rightarrow i_{1-2}=0.2$

$i_1=i_{1-1}+i_{1-2}=0.7A$



(b) 30-V source active

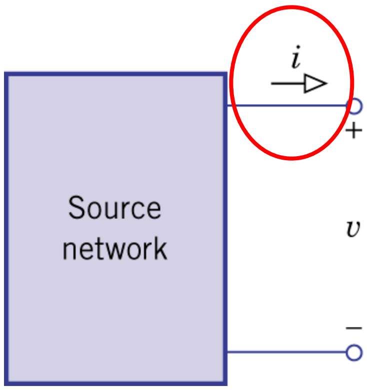


(c) 3-A source active

2.5 Thevenin and Norton networks

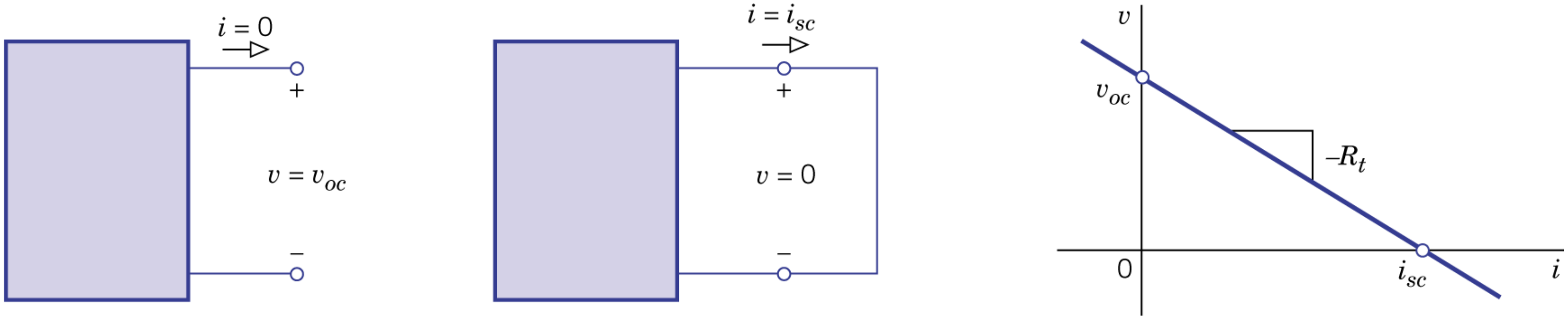
Basics

1. source network: a two-terminal network containing linear resistors (controlled sources) and at least one independent source.



2. Thevenin parameters (terminal behavior of a network): open-circuit voltage v_{oc}, short-circuit current i_{sc}, and Thevenin resistance R_t

$$R_t \equiv \frac{v_{oc}}{i_{sc}}$$

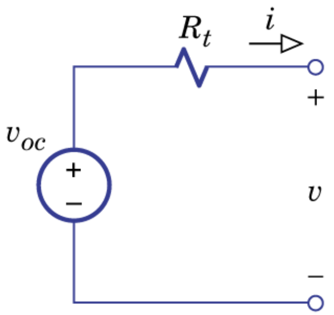


(a) Open-circuit voltage (b) Short-circuit current (c) v–i curve

Discussion

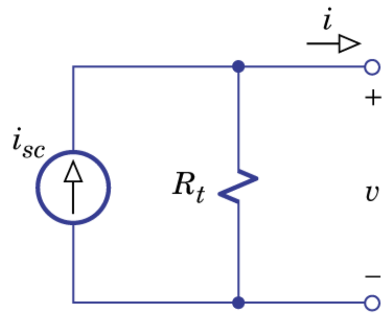
1. **Thevenin's theorem**: the equivalent circuit of a linear resistive source network is a voltage source v_{oc} in series with a resistor R_t.
2. **Norton's theorem**: the equivalent circuit of a linear resistive source network is a current source i_{sc} in parallel with a resistor R_t.

$$\boxed{v_{oc} \equiv v\big|_{i=0},\ i_{sc} \equiv i\big|_{v=0},\ R_t \equiv \frac{v_{oc}}{i_{sc}}}$$



(a) Thévenin equivalent network

$$\boxed{v = v_{oc} - R_t i}$$



(b) Norton equivalent network

$$\boxed{i = i_{sc} - \frac{v}{R_t}}$$

if load SC, $v = 0 = v_{oc} - R_t i_{sc}$

if load OC, $i = 0 = i_{sc} - \dfrac{v_{oc}}{R_t}$

$$\rightarrow R_t = \frac{v_{oc}}{i_{sc}}$$

3. Thevenin and Norton equivalent circuits are useful to solve the **terminal** behavior of a source network, but **not internal branches**.

4. Ex.2.12 find the Thevenin parameters

$O.C.$ load

$$KCL \rightarrow \frac{1-v_x}{250} = \frac{v_x}{4000} + \frac{v_x-(-100v_x)}{2080}$$

$$\rightarrow v_x = 0.0757$$

$$v_{oc} = \frac{v_x+100v_x}{2080} \times 80 + (-100v_x) = -7.28$$

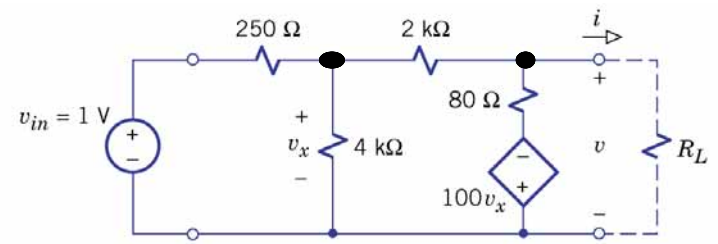
$S.C.$ load

$$KCL \rightarrow \frac{1-v_x}{250} = \frac{v_x}{4000} + \frac{v_x}{2000}$$

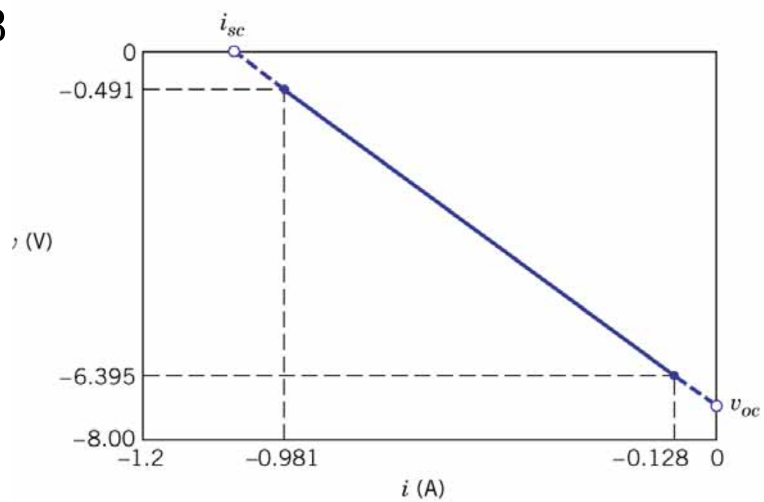
$$\rightarrow v_x = 0.842$$

$$i_{sc} = \frac{v_x}{2000} - \frac{100v_x}{80} = -1.052$$

$$R_t = \frac{v_{oc}}{i_{sc}} = 6.92\Omega$$



(a) Circuit diagram of an inverting voltage amplifier



(b) v-i curve obtained by PSpice simulation

5. Ex.2.13 use equivalent source circuits to design R_L in two cases to give v=24, and i=8

Thevinin and Norton equivalent circuits

$$v_{oc} = 50 \times \frac{20}{5+20} = 40$$

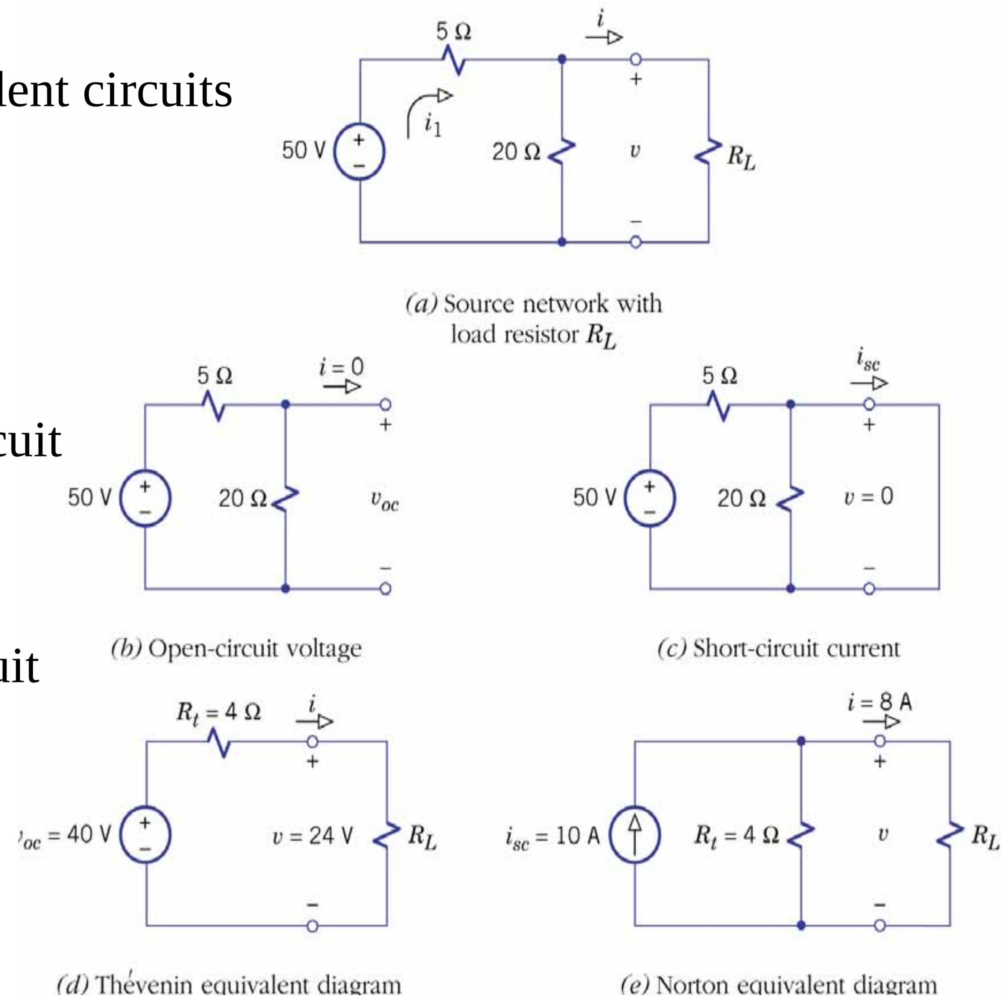
$$i_{sc} = \frac{50}{5} = 10, R_t = \frac{40}{10} = 4$$

from Tevenin equivalent circuit

$$24 = v_{oc} \frac{R_L}{R_t + R_L} \rightarrow R_L = 6\Omega$$

from Norton equivalent circuit

$$8 = i_{sc} \frac{R_t}{R_t + R_L} \rightarrow R_L = 1\Omega$$



(a) Source network with load resistor R_L

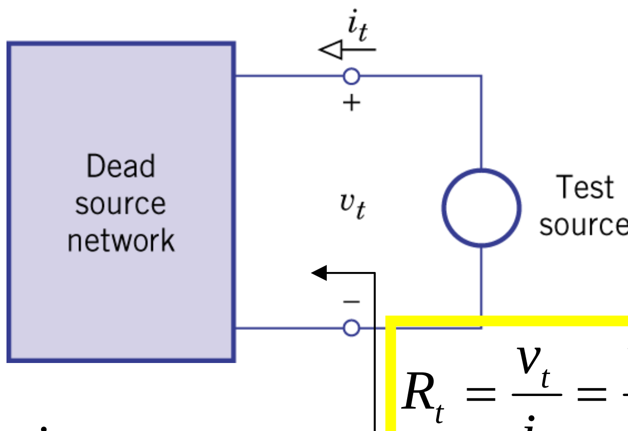
(b) Open-circuit voltage

(c) Short-circuit current

(d) Thévenin equivalent diagram

(e) Norton equivalent diagram

6. Thevenin resistance = the equivalent resistance of the source network with all independent sources suppressed. (another way to find R_t)



(independent voltage source or current source)

(contains only resistors and dependent sources)

$$\boxed{R_t = \frac{v_t}{i_t} = \frac{v_{oc}}{i_{sc}}}$$

$$\because \begin{cases} v_t = v_{oc} + R_t i_t, \text{if } v_{oc} = 0 \\ -i_t = i_{sc} - \dfrac{v_t}{R_t}, \text{if } i_{sc} = 0 \end{cases} \quad \text{then } R_t = \frac{v_t}{i_t}$$

Equivalent resistance theorem is the special case of Thevenin or Norton theorem.

7. A source network can be described by any two of the Thevenin parameters: v_{oc}, i_{sc}, R_t

8. Ex. 2.14 find Thevenin parameters

step 1: 40kΩ→easier to find i_{sc}

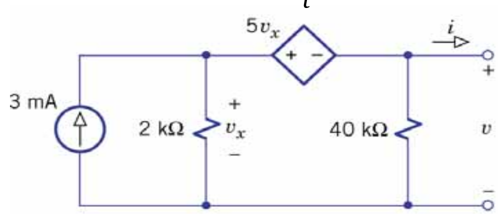
$$KVL \rightarrow v_x - 5v_x = 0 \rightarrow v_x = 0 \rightarrow i_{sc} = 0.003$$

step 2: find R_t

$$KVL \rightarrow v_x - 5v_x - v_t = 0 \rightarrow v_x = -0.25v_t$$

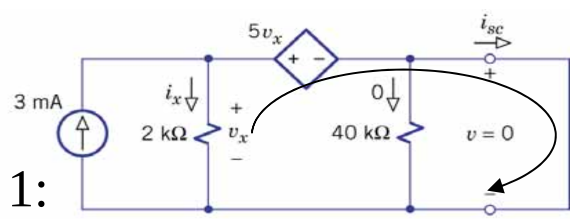
$$KCL \rightarrow i_t = \frac{v_t}{40000} + \frac{v_x}{2000} = -\frac{v_t}{10000}$$

$$R_t = \frac{v_t}{i_t} = -10000 \; (\because \text{controlled source}) \rightarrow v_{oc} = R_t i_{sc} = -30$$



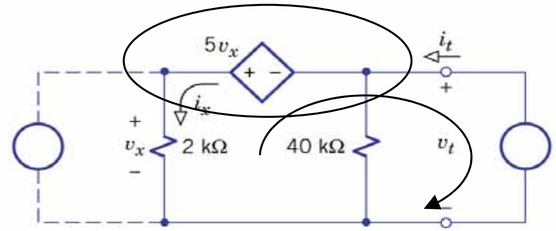
(a) Source network with a VCVS

step 1:



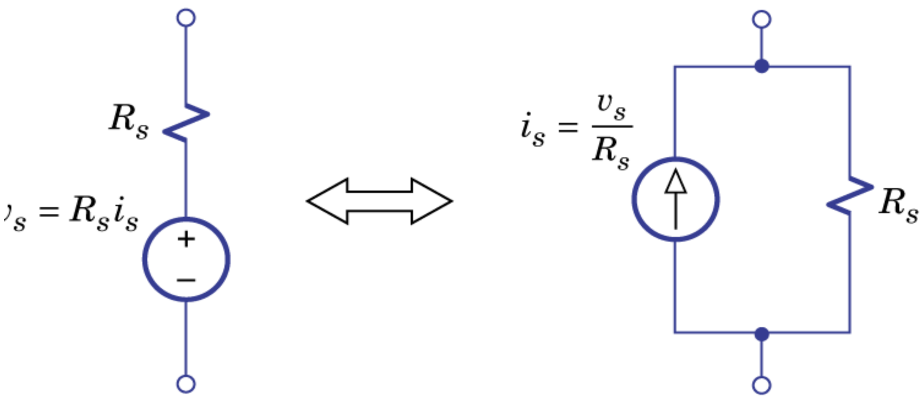
(b) Short circuit at the output

step 2:



(c) Dead network with test source

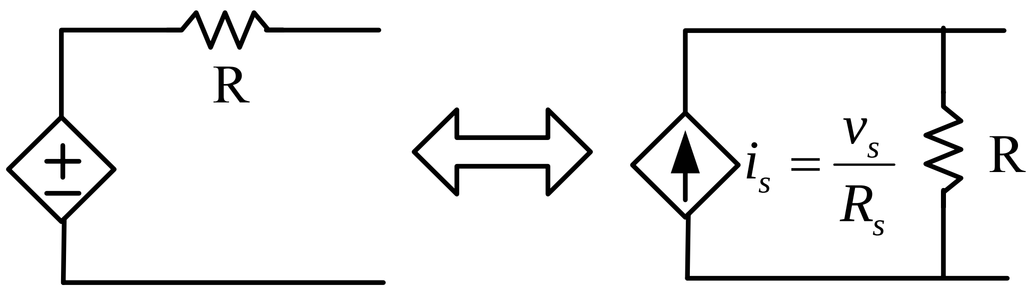
9. equivalent source conversion (**including dependent sources**)



$\because$ (1) sources are off $\rightarrow$ same R_s

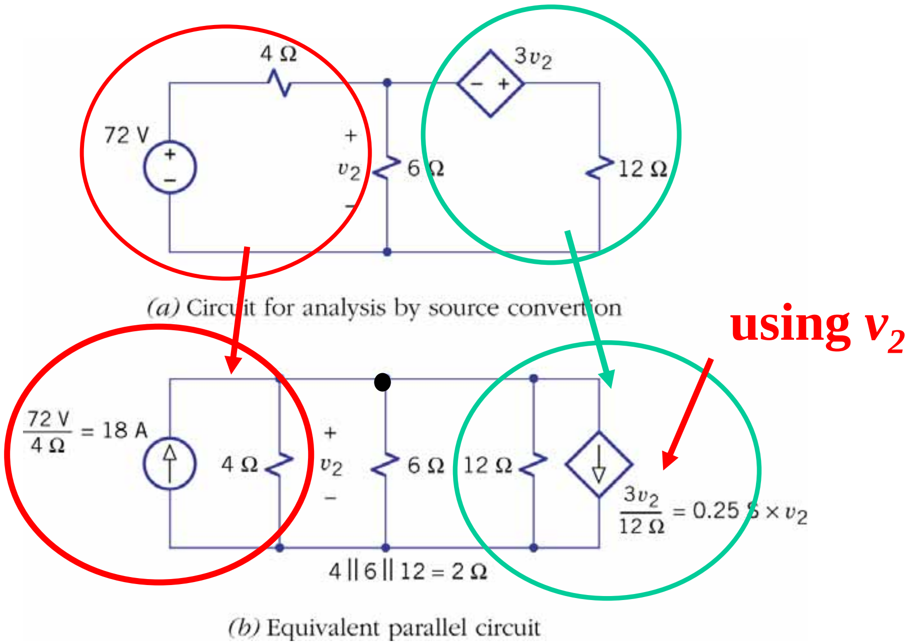
(2) $S.C. \rightarrow$ same i_s

(3) $O.C. \rightarrow$ same v_s



10. Source conversion can ease circuit analysis.

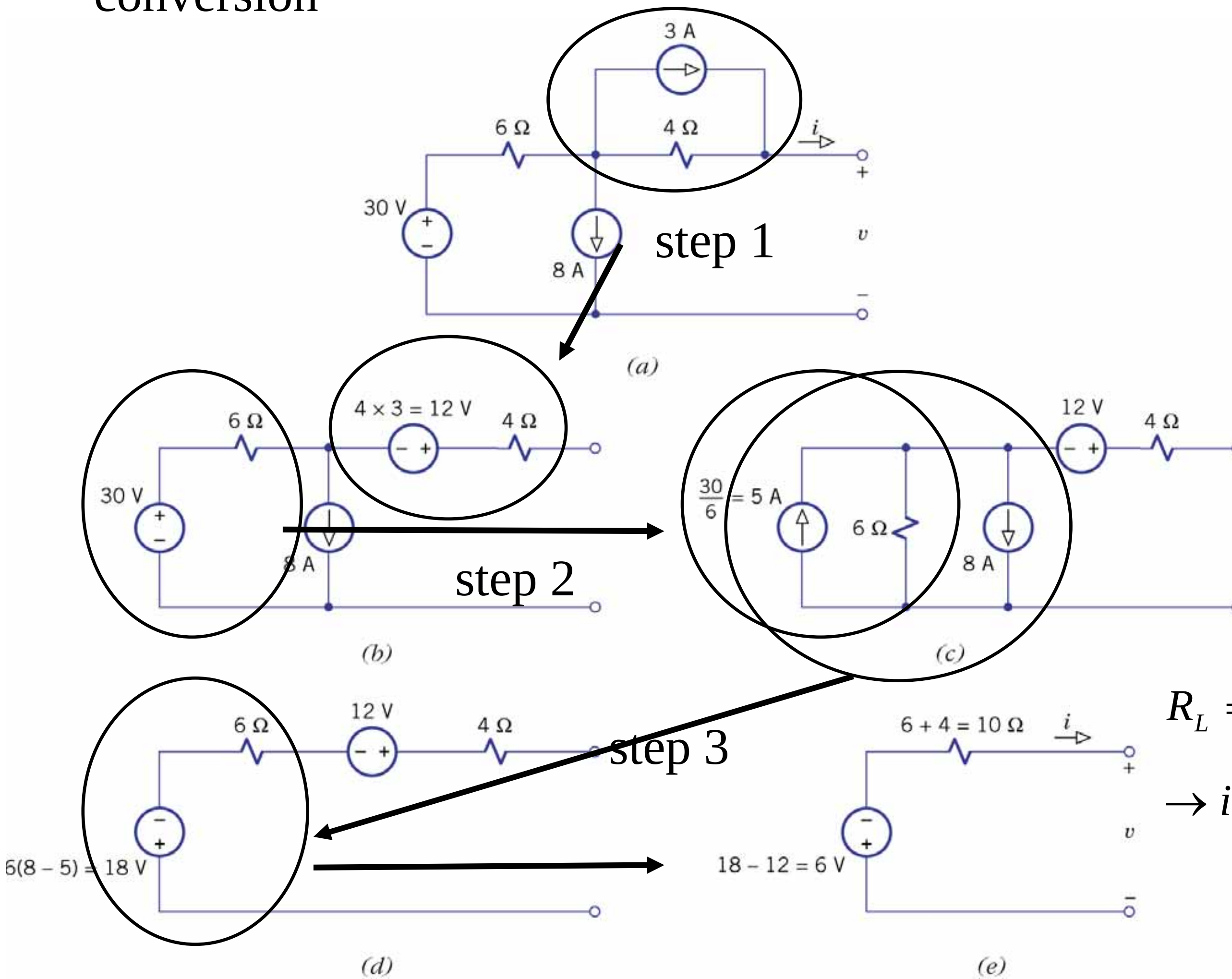
11. Ex. 2.15(ex. 2.9) solve v_2 using equivalent source conversion



$$KCL \rightarrow 18 = \frac{v_2}{4//6//12} + 0.25v_2 \rightarrow v_2 = 24V$$

When controlled sources are present, source conversion must **not obliterate** the identity of any controlled variables.

12. Ex. 2.16 (ex.2.10) find Thevenin equivalent circuit using source conversion



$R_L = 2$

$\rightarrow i = \dfrac{-6}{10+2} = -0.5A$