Chapter 4 Systematic analysis methods

4.1 **Node analysis**

node voltage, matrix node equation, floating voltage source, supernode

4.2 **Mesh analysis**

mesh current, matrix mesh equation, interior current source, supermesh

4.3 Systematic analysis with controlled sources

mesh analysis, node analysis

4.4 Applications of systematic analysis

equivalent resistance, Thevenin parameters

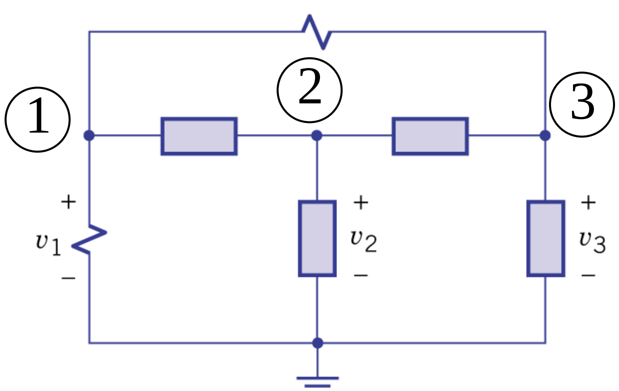
4.5-4.6 (optional)

node analysis with ideal op-amp, Δ-Y transformation

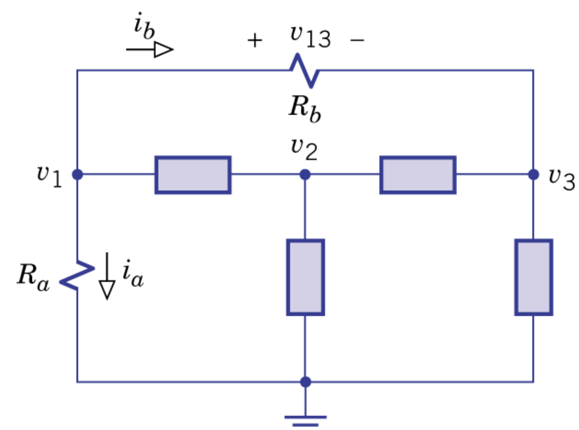
4.1 Node analysis

<u>Basics</u>

1. Node voltage: potential at nonreferenced node to a specified reference node
   - Suppressing sources to identify node numbers, the number of **unknown node voltages** is one less than the node numbers.
   - Select the node with its connection to the maximum voltage sources as the **reference node**.
   - All branch voltages can be determined from the values of node voltages connected.

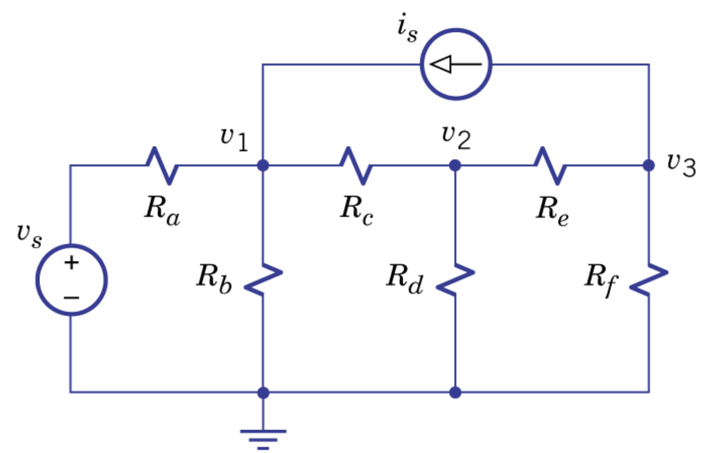


(a) Circuit with four nodes and three node voltages



(b) Relating other branch variables to node voltages

2. An example



$$\text{node analysis} \xrightarrow{KCL} \sum_{\text{node}} \text{flow out } i = 0$$

$$\text{node } 1: G_a(v_1 - v_s) + G_b v_1 + G_c(v_1 - v_2) - i_s = 0$$

$$\text{node } 2: G_c(v_2 - v_1) + G_d v_2 + G_e(v_2 - v_3) = 0$$

$$\text{node } 3: G_e(v_3 - v_2) + G_f v_3 + i_s = 0$$

into node

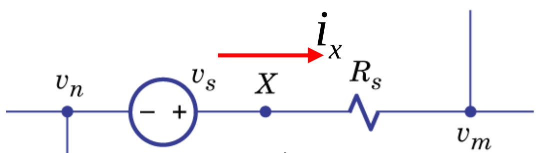
symmetric matrix

$$\begin{bmatrix} G_a + G_b + G_c & -G_c & 0 \\ -G_c & G_c + G_d + G_e & -G_e \\ 0 & -G_e & G_e + G_f \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i_s + G_a v_s \\ 0 \\ -i_s \end{bmatrix}$$

3. Floating voltage source

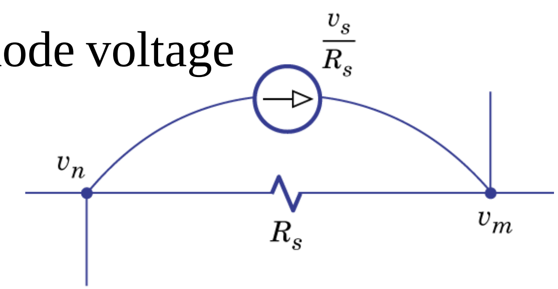
- Floating voltage source with series resistance → convert to current source

$v_x = v_s + v_n \therefore v_x$ is not an independent node voltage

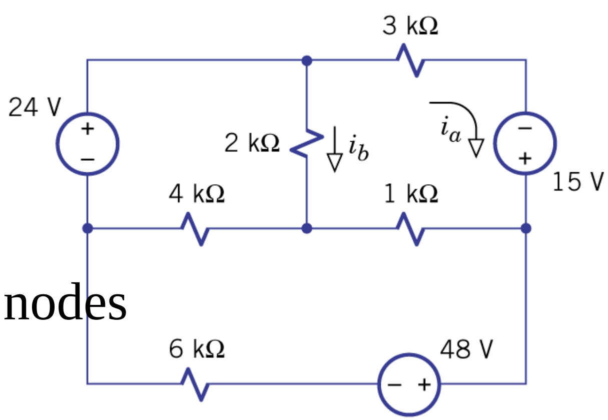


$$i_x = \frac{v_x - v_m}{R_s} = \frac{v_s + v_n - v_m}{R_s} = \frac{v_s}{R_s} + \frac{v_n - v_m}{R_s}$$

(a) Floating voltage source with series resistance

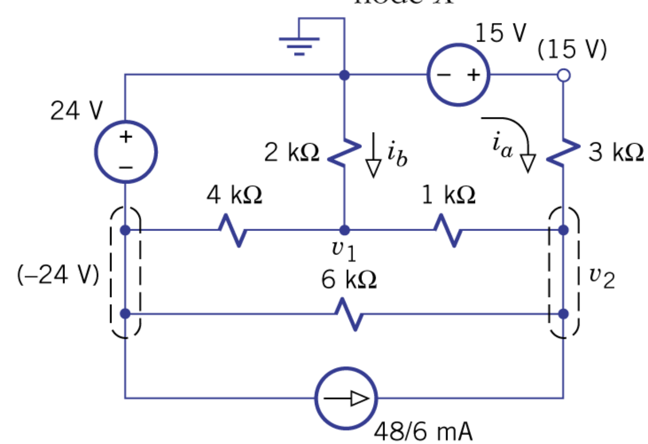


(b) Source conversion eliminating node X



3 nodes

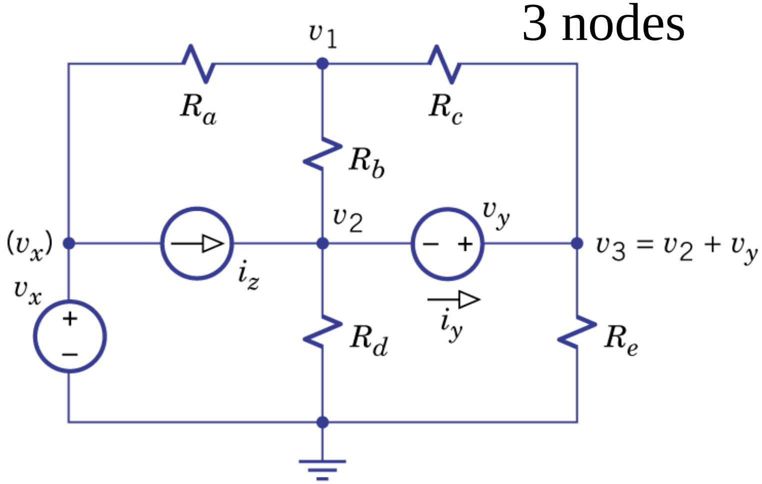
(a) Circuit with four nodes

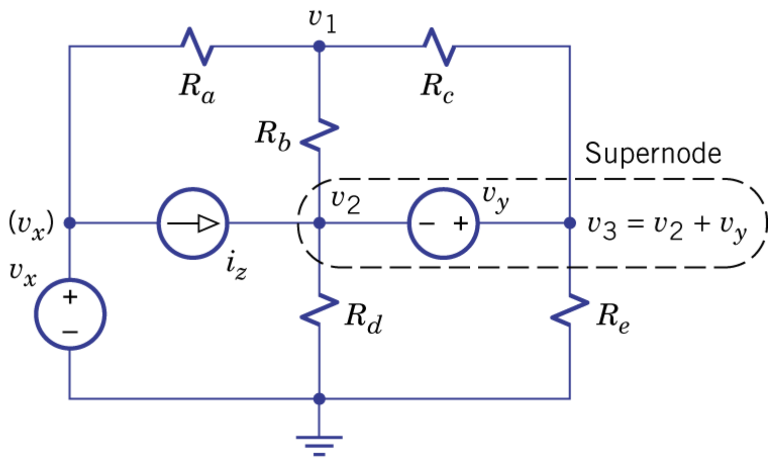


(b) Diagram for node analysis

- Floating ideal voltage source → use supernode

3 nodes





(1) v_2 or v_3 is not an independent unknown, and is difficult to write node equation on i_y

(2) Enclose both terminals of the floating ideal source within a **supernode** → easy to write node equations with 2 unknowns

4. Matrix node equations

Step 1: identify reference node, and identify known node voltages by fixed voltage sources

Step 2: perform floating voltage source conversion

Step 3: apply KCL to write the matrix node equation

$$KCL : [G][v] = [i_s]$$

$$\begin{bmatrix} G_{11} & -G_{12} & \bullet & -G_{1N} \\ -G_{21} & G_{22} & \bullet & -G_{2N} \\ \bullet & \bullet & \bullet & \bullet \\ -G_{N1} & -G_{12} & \bullet & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \bullet \\ v_N \end{bmatrix} = \begin{bmatrix} i_{s1} \\ i_{s2} \\ \bullet \\ i_{sN} \end{bmatrix}$$

$[G]$: conductance matrix, $G_{ij} = G_{ji}$

G_{ii} : sum of conductances connected to node i

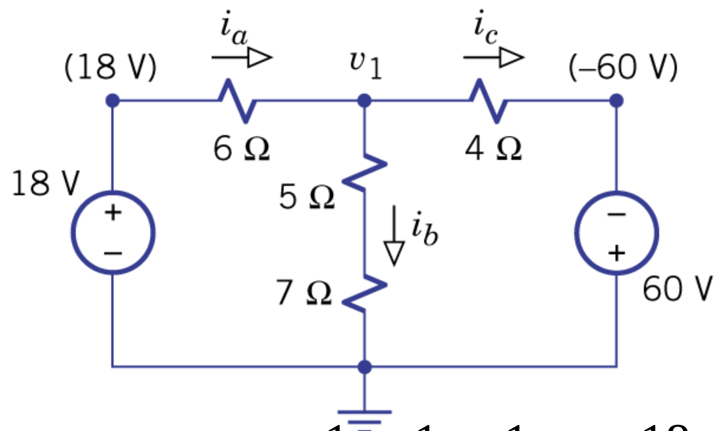
G_{ij} : sum of conductances directly connected to nodes i and j

i_{si} : source-current vector

sum of current sources into node i

Discussion

1. Ex.4.1



one-node equation: $(\frac{1}{6}+\frac{1}{12}+\frac{1}{4})v_1=\frac{18}{6}-\frac{60}{4}$

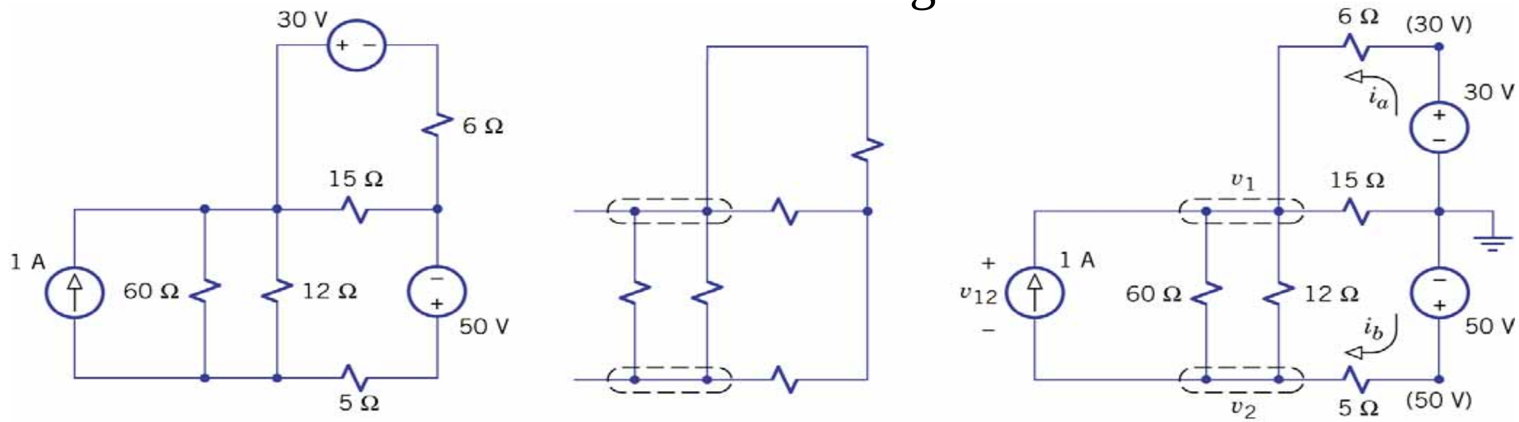
$\rightarrow$ node voltage $v_1=-24$

$$\rightarrow i_a=\frac{18-(-24)}{6}=7$$

$$i_b=\frac{-24}{12}=-2, i_c=\frac{-24-(-60)}{4}=9$$

2. Ex.4.2

3 nodes→2 node voltages



(a) Circuit for node analysis

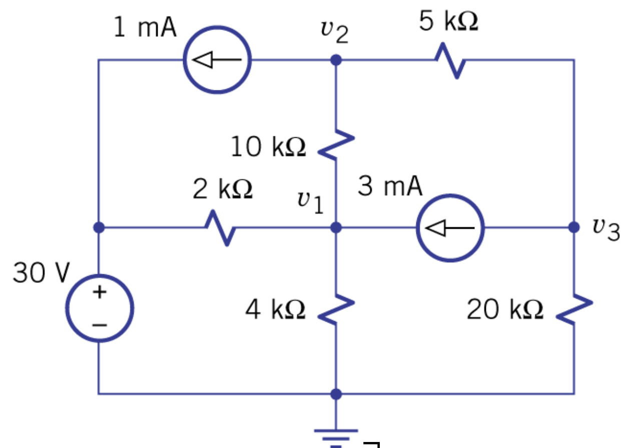
(b) Source suppression leaving three nodes

(c) Diagram with two unknown node voltages

$$\begin{bmatrix} \frac{1}{6}+\frac{1}{15}+\frac{1}{12}+\frac{1}{60} & -(\frac{1}{12}+\frac{1}{60}) \\ -(\frac{1}{12}+\frac{1}{60}) & \frac{1}{5}+\frac{1}{12}+\frac{1}{60} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1+\frac{30}{6} \\ -1+\frac{50}{5} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{3} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{3}{10} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix} \rightarrow \begin{cases} v_1 = 30 \\ v_2 = 40 \end{cases} \rightarrow \begin{cases} i_a = \frac{30-30}{6} = 0 \\ i_b = \frac{50-40}{5} = 2 \end{cases}$$

3. Ex.4.3



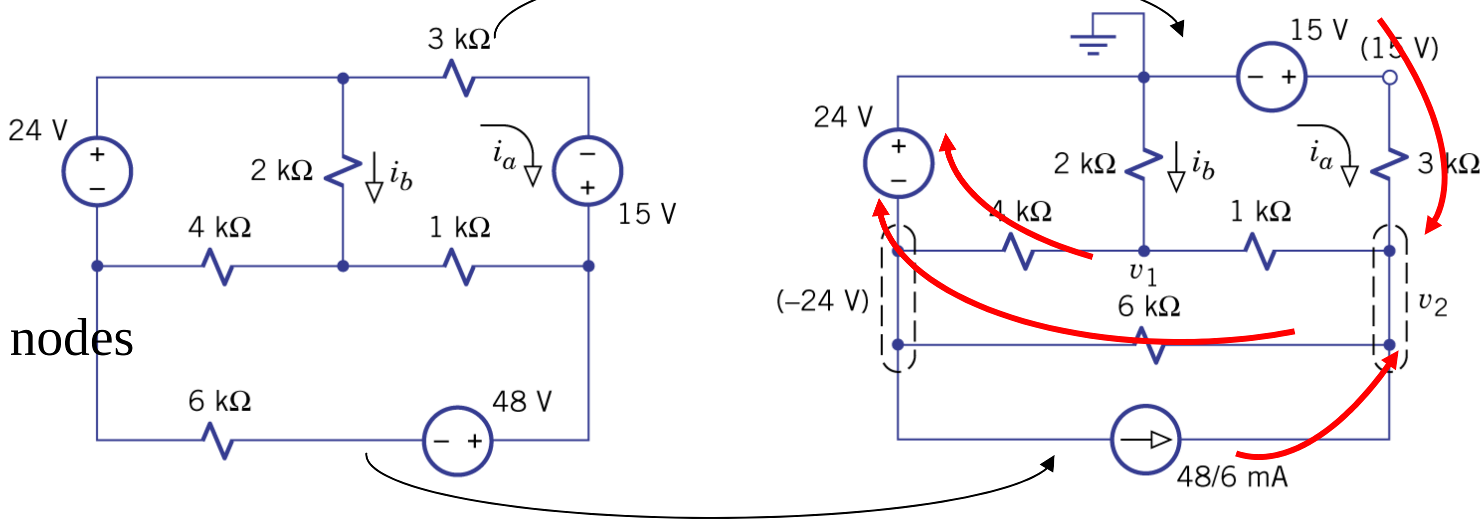
$$\frac{1}{1000}\begin{bmatrix} \frac{1}{2}+\frac{1}{4}+\frac{1}{10} & -\frac{1}{10} & 0 \\ -\frac{1}{10} & \frac{1}{5}+\frac{1}{10} & -\frac{1}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5}+\frac{1}{20} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \frac{1}{1000}\begin{bmatrix} \frac{30}{2}+3 \\ -1 \\ -3 \end{bmatrix} \rightarrow \begin{cases} v_1 = 20 \\ v_2 = -10 \\ v_3 = -20 \end{cases}$$

Note: Inserting a R in series with 3mA current source does not change the results.

$$\text{node 1:}\frac{v_1\text{-}30}{2}+\frac{v_1\text{-}v_2}{10}+\frac{v_1}{4}-3=0$$

$$\text{node 3:}\frac{v_3\text{-}v_2}{5}+\frac{v_3}{20}+3=0$$

4. Ex. 4.4 find i_a, i_b no need for source conversion for a non-floating voltage source



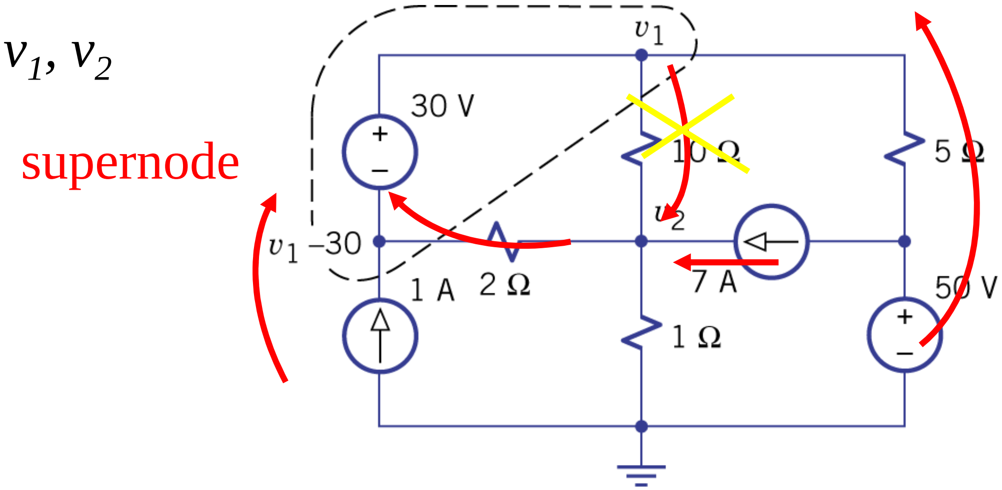
3 nodes

(a) Circuit with four nodes

(b) Diagram for node analysis

$$\frac{1}{1000}\begin{bmatrix} \frac{1}{1}+\frac{1}{2}+\frac{1}{4} & -\frac{1}{1} \\ -\frac{1}{1} & \frac{1}{1}+\frac{1}{3}+\frac{1}{6} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{1000}\begin{bmatrix} -\frac{24}{4} \\ \frac{48}{6}+\frac{15}{3}-\frac{24}{6} \end{bmatrix} \rightarrow \begin{cases} v_1 = 0 \\ v_2 = 6 \end{cases} \rightarrow \begin{cases} i_a = \frac{15-6}{3000} = 3mA \\ i_b = \frac{0}{2000} = 0mA \end{cases}$$

5. Ex. 4.5 find v_1, v_2

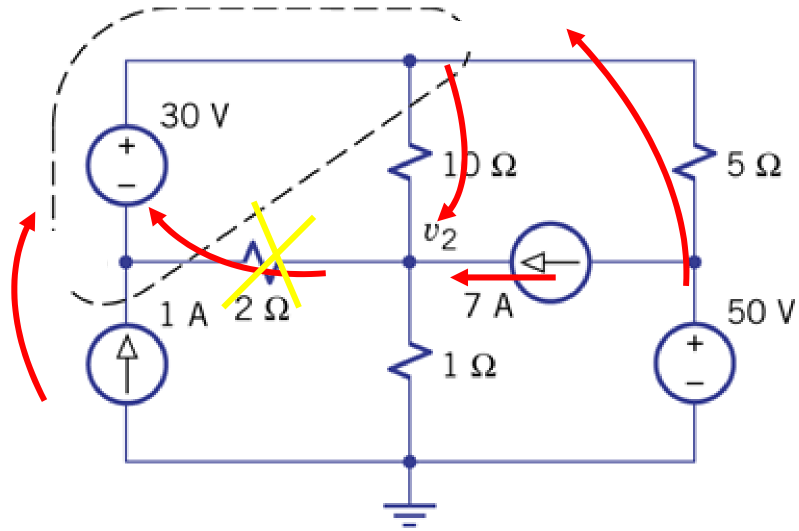


$$\text{supernode 1}: \frac{(v_1-30)-v_2}{2} - 1 + \frac{v_1-v_2}{10} + \frac{v_1-50}{5} = 0 \rightarrow (\frac{1}{10}+\frac{1}{2}+\frac{1}{5})v_1 + (-\frac{1}{10}-\frac{1}{2})v_2 = \frac{30}{2}+1+\frac{50}{5}$$

$$\text{node 2} \rightarrow \frac{v_2-(v_1-30)}{2} + \frac{v_2}{1} - 7 + \frac{v_2-v_1}{10} = 0 \qquad (-\frac{1}{10}-\frac{1}{2})v_1 + (\frac{1}{1}+\frac{1}{10}+\frac{1}{2})v_2 = 7-\frac{30}{2}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{10}+\frac{1}{2}+\frac{1}{5} & -(\frac{1}{10}+\frac{1}{2}) \\ -(\frac{1}{10}+\frac{1}{2}) & \frac{1}{1}+\frac{1}{10}+\frac{1}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{30}{2}+1+\frac{50}{5} \\ 7-\frac{30}{2} \end{bmatrix} \rightarrow \begin{cases} \frac{4}{5}v_1 - \frac{3}{5}v_2 = 26 \\ -\frac{3}{5}v_1 + \frac{8}{5}v_2 = -8 \end{cases} \rightarrow \begin{matrix} v_1 = 40 \\ v_2 = 10 \end{matrix}$$

(alternate)



$$\text{supernode 1}: \frac{v_2 - v_1}{2} - 1 + \frac{v_1 + 30 - v_2}{10} + \frac{v_1 + 30 - 50}{5} = 0$$

$$\text{node 2} \rightarrow \frac{v_2 - v_1}{2} + \frac{v_2}{1} - 7 + \frac{v_2 - (v_1 + 30)}{10} = 0$$

$$\rightarrow \begin{array}{l} (\frac{1}{10} + \frac{1}{2} + \frac{1}{5})v_1 + (-\frac{1}{10} - \frac{1}{2})v_2 = -\frac{30}{10} + 1 + \frac{50-30}{5} \\ (-\frac{1}{10} - \frac{1}{2})v_1 + (\frac{1}{1} + \frac{1}{10} + \frac{1}{2})v_2 = 7 + \frac{30}{10} \end{array}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{10} + \frac{1}{2} + \frac{1}{5} & -(\frac{1}{10} + \frac{1}{2}) \\ -(\frac{1}{10} + \frac{1}{2}) & \frac{1}{1} + \frac{1}{10} + \frac{1}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -\frac{30}{10} + 1 + \frac{50-30}{5} \\ 7 + \frac{30}{10} \end{bmatrix} \rightarrow \begin{cases} \frac{4}{5}v_1 - \frac{3}{5}v_2 = 2 \\ -\frac{3}{5}v_1 + \frac{8}{5}v_2 = 10 \end{cases} \rightarrow \begin{array}{l} v_1 = 10 \\ v_2 = 10 \end{array}$$

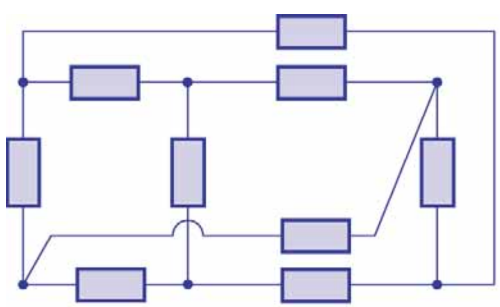
4.2 Mesh analysis

<u>Basics</u>

1. Dual relation

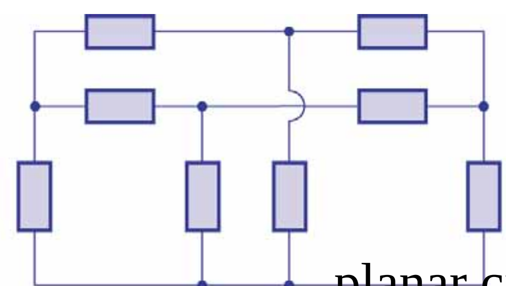
node voltage	mesh current
matrix node equation	matrix mesh equation (for planar circuit only)
KCL	KVL
floating voltage source conversion	interior current source conversion

2. mesh: a closed current path containing no closed paths within it, a special loop



(a) Diagram of a nonplanar circuit

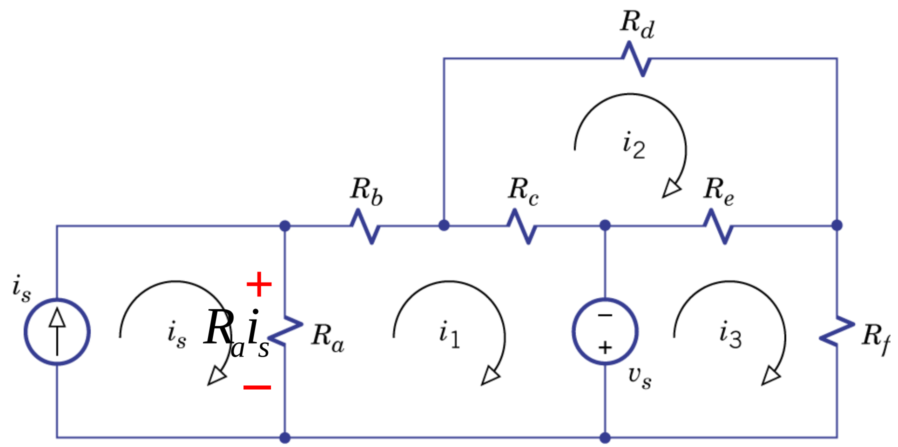
non-planar circuit



planar circuit

(b) Diagram of a planar circuit that could be redrawn without a hop-over

3. The minimum number of meshes is determined by suppressing all sources.
4. An example



$$\text{mesh analysis} \xrightarrow{KVL} \sum_{\text{mesh}} \text{voltage drop in mesh } i = 0$$

$$\text{mesh 1}: R_a(i_1 - i_s) + R_b i_1 + R_c(i_1 - i_2) - v_s = 0$$

$$\text{mesh 2}: R_c(i_2 - i_1) + R_d i_2 + R_e(i_2 - i_3) = 0$$

$$\text{node 3}: R_e(i_3 - i_2) + R_f i_3 + v_s = 0$$

same direction of mesh current

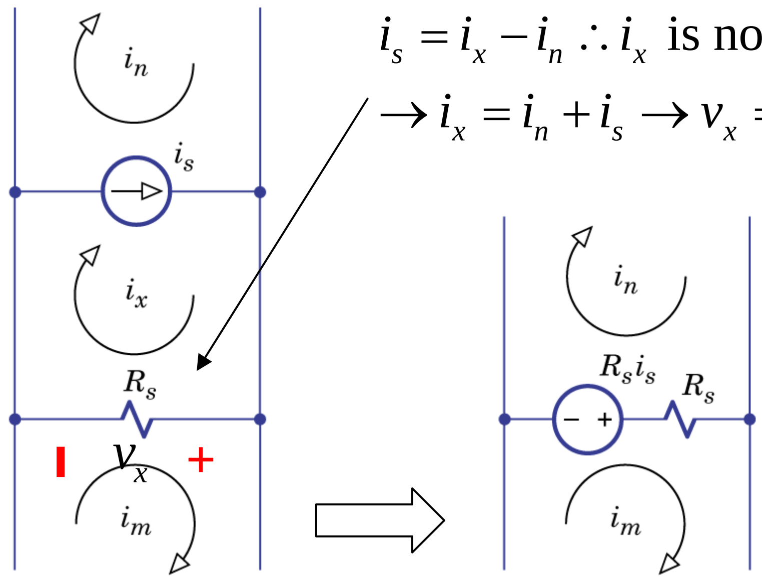
$$\begin{bmatrix} R_a + R_b + R_c & -R_c & 0 \\ -R_c & R_c + R_d + R_e & -R_e \\ 0 & -R_e & R_e + R_f \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_s + R_a i_s \\ 0 \\ -v_s \end{bmatrix}$$

5. Interior current source

- Interior current sources with parallel resistance → convert to voltage source

$$i_s = i_x - i_n \therefore i_x$$ is not an independent mesh current

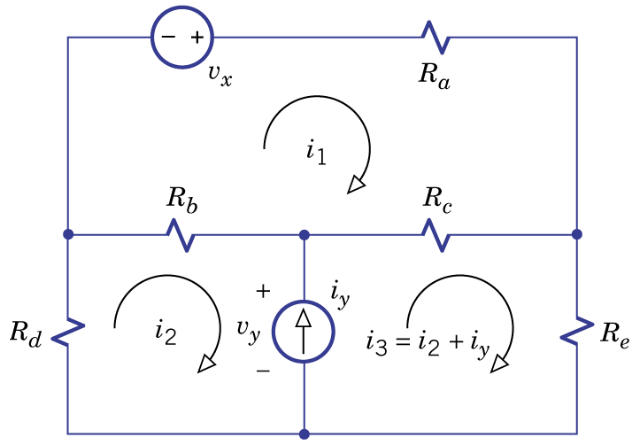
$$\rightarrow i_x = i_n + i_s \rightarrow v_x = i_x R_s = i_n R_s + i_s R_s$$

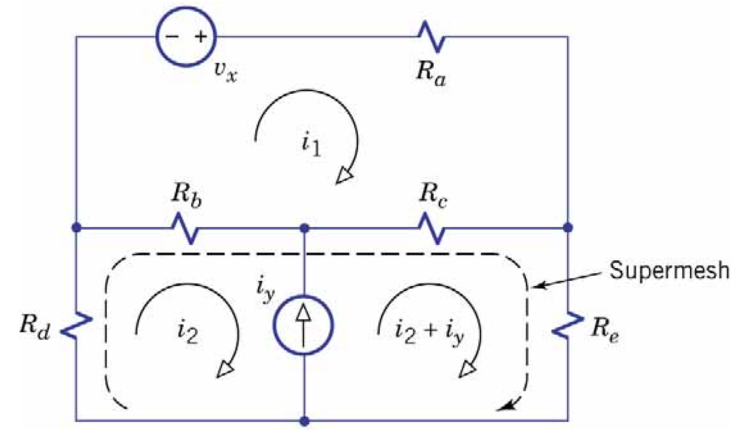


(a) Interior current source with parallel resistance

(b) Source conversion eliminating i_x

- Interior ideal current source $\rightarrow$ use supermesh





$$\begin{cases} R_d i_2 + R_b(i_2 - i_1) - v_y = 0 \\ v_y + R_c(i_2 + i_y - i_1) + R_e(i_2 + i_y) = 0 \end{cases}$$

$$R_d i_2 + R_b(i_2 - i_1) + R_c(i_2 + i_y - i_1) + R_e(i_2 + i_y) = 0$$

$$\rightarrow R_d i_2 + R_b(i_2 - i_1) + R_c(i_2 + i_y - i_1) + R_e(i_2 + i_y) = 0$$

(1) Two meshes exist with source compressed.

(2) i_2 or i_3 is not an independent unknown.

(3) Use **supermesh** to enclose both meshes of the interior ideal source $\rightarrow$ write mesh equation with unknown mesh currents

6. Matrix mesh equations

Step 1: identify known mesh currents fixed by current source

Step 2: label the remaining unknown mesh currents

Step 3: apply KVL to write the matrix mesh equation

$KVL: [R][i] = [v_s]$

$$\begin{bmatrix} R_{11} & -R_{12} & \bullet & -R_{1N} \\ -R_{21} & R_{22} & \bullet & -R_{2N} \\ \bullet & \bullet & \bullet & \bullet \\ -R_{N1} & -R_{12} & \bullet & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \bullet \\ i_N \end{bmatrix} = \begin{bmatrix} v_{s1} \\ v_{s2} \\ \bullet \\ v_{sN} \end{bmatrix}$$

$[R]$: resistance matrix, $R_{ij} = R_{ji}$

R_{ii} : sum of resistances connected to mesh i

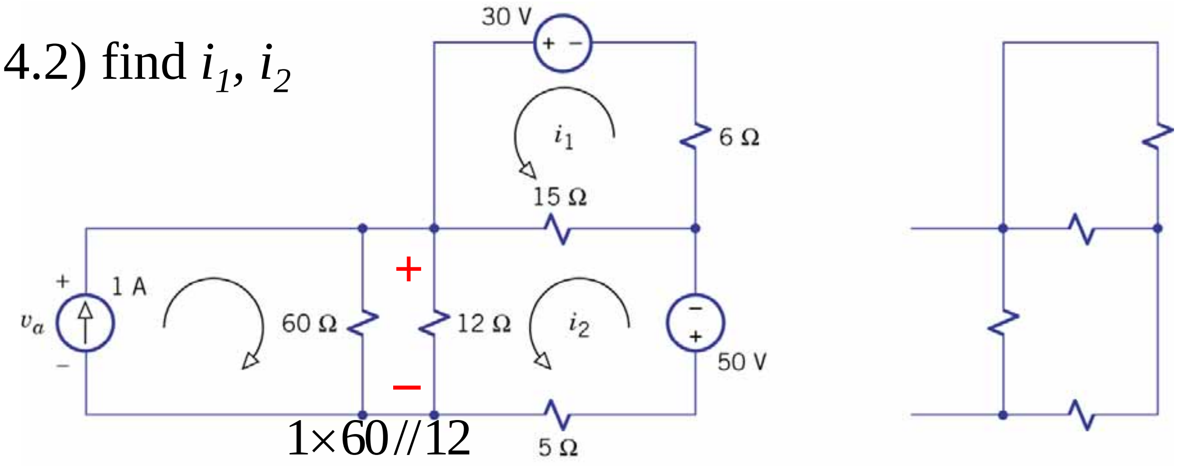
R_{ij} : sum of resistances shared by meshes i and j

v_{si} : source-voltage vector

sum of voltage sources in mesh i to give current in the same direction of mesh current

Discussion

1. Ex.4.6 (Ex.4.2) find i_1, i_2



(a) Circuit for mesh analysis

(b) Source suppression leaving two meshes

$$\text{mesh analysis} \xrightarrow{KVL} \sum_{mesh} v = 0$$

$$\text{mesh 1}: 15(i_1 - i_2) + 6i_1 - 30 = 0$$

same direction as mesh

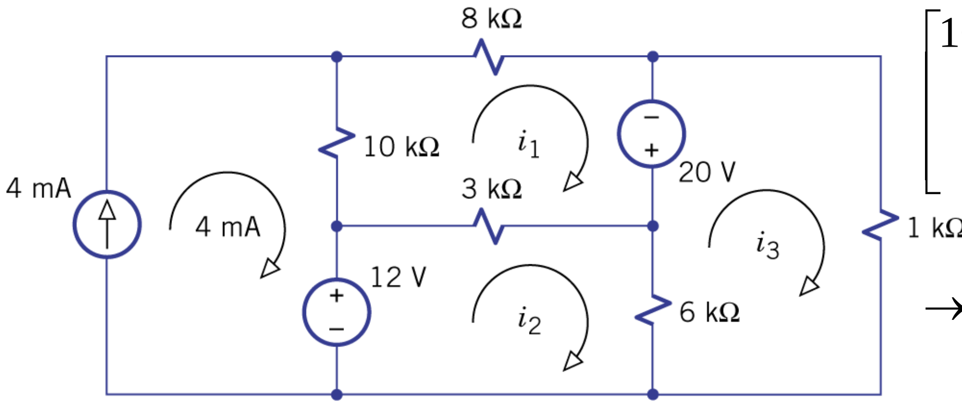
$$\text{mesh 2}: 15(i_2 - i_1) + (60//12)(i_2 + 1) + 5i_2 + 50 = 0$$

$$\begin{bmatrix} 15+6 & -15 \\ -15 & 15+60//12+5 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 30 \\ -50-1\times 60//12 \end{bmatrix} \rightarrow \begin{cases} i_1 = 0 \\ i_2 = -2 \end{cases}$$

$$\rightarrow \text{source 1}: 30\times i_1 = 0, \text{source 2}: 50\times(-i_2) = 100\text{W}$$

$$\text{source 3}: [(60//12)\times(1-2)]\times 1 = \text{-}10\text{W}$$

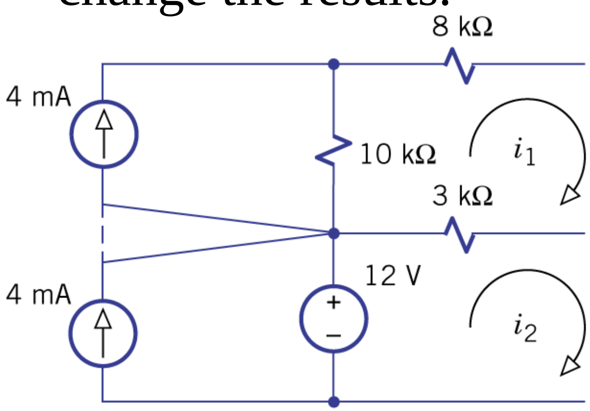
2. Ex.4.7 find i_1, i_2, i_3



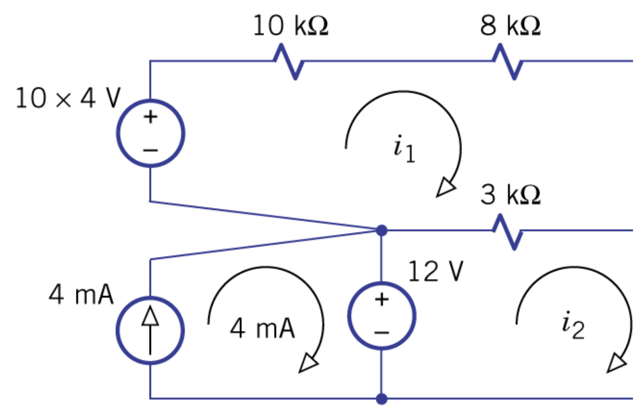
$$\begin{bmatrix} 10+8+3 & -3 & 0 \\ -3 & 3+6 & -6 \\ 0 & -6 & 6+1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 20+10\times 4 \\ 12 \\ -20 \end{bmatrix}$$

$$\rightarrow \begin{cases} i_1 = 3mA \\ i_2 = 1mA \\ i_3 = -2mA \end{cases}$$

Note: Inserting a R in parallel with 20V voltage source does not change the results.

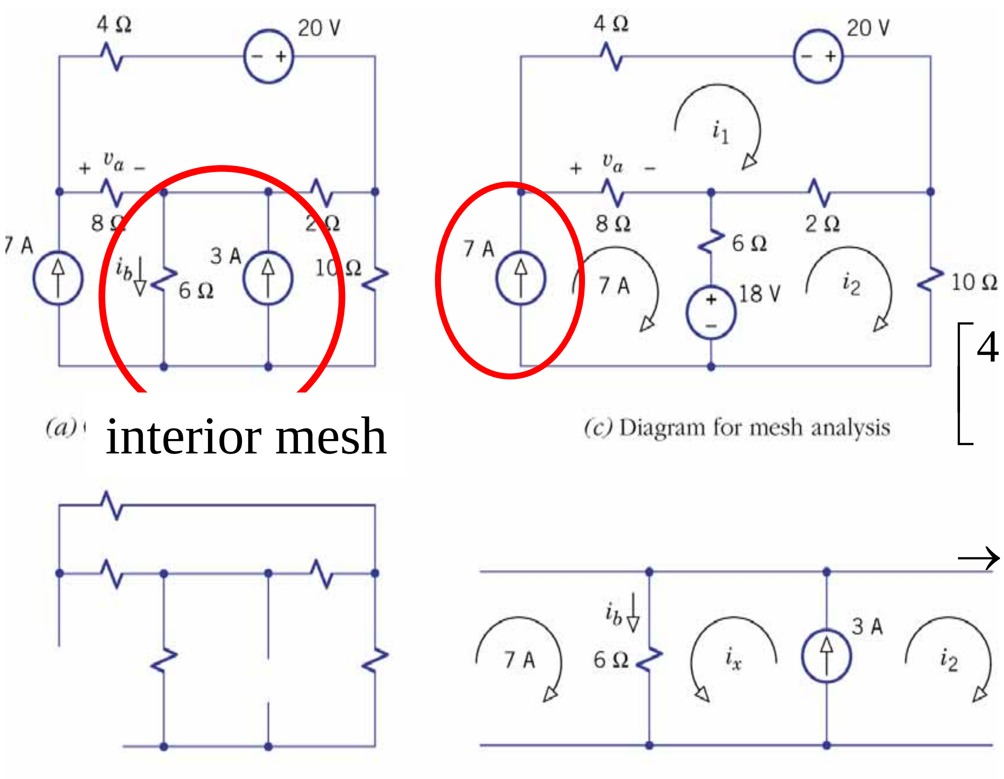


(a) Adding series current source



(b) After source conversion

3. Ex.4.8 (interior current source) to find v_a, i_b



(a)

(c) Diagram for mesh analysis

(b) Source suppression leaving two meshes

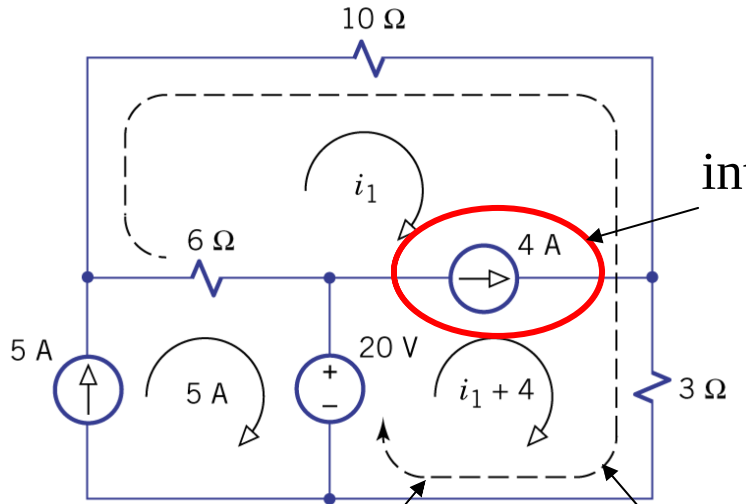
(d) Partial diagram for i_x

$$\begin{bmatrix} 4+2+8 & -2 \\ -2 & 6+2+10 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 20+8\times 7 \\ 18+6\times 7 \end{bmatrix}$$

$$\rightarrow \begin{cases} i_1 = 6 \\ i_2 = 4 \end{cases} \rightarrow \begin{cases} v_a = 8(7-i_1) = 8 \\ i_b = 7 + i_x, 3 = i_x + i_2 \\ \therefore i_b = 6 \end{cases}$$

3 node eqs, 2 mesh eqs→mesh analysis easier

4. Ex4.9 (interior ideal current source) to find i_1



interior current source

2 nodes, 1 mesh

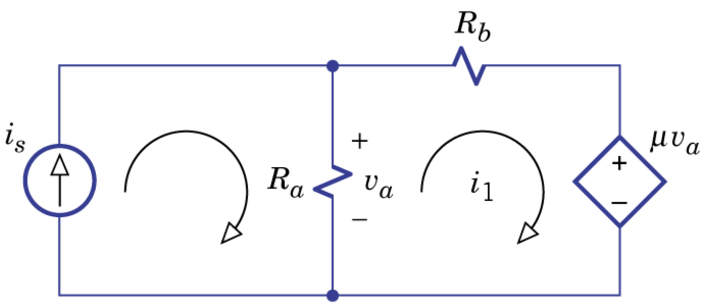
supermesh

$$6(i_1 - 5) + 10i_1 + 3(i_1 + 4) = 20 \rightarrow i_1 = 2$$

4.3 Systematic analysis with controlled sources

Basics

1. An example on mesh analysis



$KVL \rightarrow R_a(i_1 - i_s) + R_b i_1 + \mu v_a = 0$

mesh equation $\rightarrow (R_a + R_b)i_1 = i_s R_a - \mu v_a$

$v_a = R_a(i_s - i_1)$: constraint equation

$\Rightarrow (R_a + R_b)i_1 = i_s R_a - \mu v_a = i_s R_a - \mu R_a(i_s - i_1)$

$\rightarrow (R_a + R_b)i_1 - \mu R_a i_1 = i_s R_a - \mu R_a i_s$

$\rightarrow (R_a + R_b - \mu R_a)i_1 = (1-\mu)R_a i_s$

$\rightarrow (R_{11} - \tilde{R}_{11})i_1 = \tilde{v}_s, R_{11} = R_a + R_b, \tilde{R}_{11} = \mu R_a, \tilde{v}_s = (1-\mu)R_a i_s$

reformulate as the usual mesh eq. $R_{11}i_1 = v_s$

$v_s = i_s R_a - \mu v_a \overset{v_a = R_a(i_s - i_1)}{=} i_s R_a - \mu R_a(i_s - i_1) = (1-\mu)R_a i_s + \mu R_a i_1$

$\rightarrow R_{11}i_1 = \tilde{v}_s + \tilde{R}_{11}i_1, \tilde{v}_s = (1-\mu)R_a i_s, \tilde{R}_{11} = \mu R_a$

$\rightarrow (R_{11} - \tilde{R}_{11})i_1 = \tilde{v}_s$

2. **Mesh analysis**

- step 1 : identify unknown mesh currents $\rightarrow$ resistance matrix $[R]$
- step 2 : for each controlled source $\rightarrow$ constraint equation
- step 3 : write source - voltage vector with constraint equations

  $[v_s] = [\tilde{v}_s] + [\tilde{R}][i]$, $[\tilde{v}_s]$: independent source terms
- step 4 : $[\mathrm{R}][i] = [v_s] \rightarrow [R - \tilde{R}][i] = [\tilde{v}_s]$

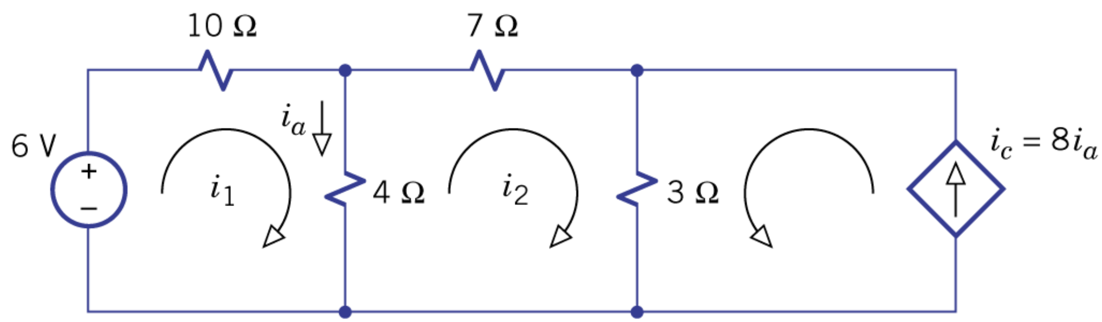
3. **Node analysis**

- step 1 : identify unknown node voltages $\rightarrow$ conductance matrix $[G]$
- step 2 : for each controlled source $\rightarrow$ constraint equation
- step 3 : write source - current vector with constraint equations

  $[i_s] = [\tilde{i}_s] + [\tilde{G}][v]$, $[\tilde{i}_s]$: independent source terms
- step 4 : $[\mathrm{G}][v] = [i_s] \rightarrow [G - \tilde{G}][v] = [\tilde{i}_s]$

Discussion

1. Ex.4.10 find i_1, i_2



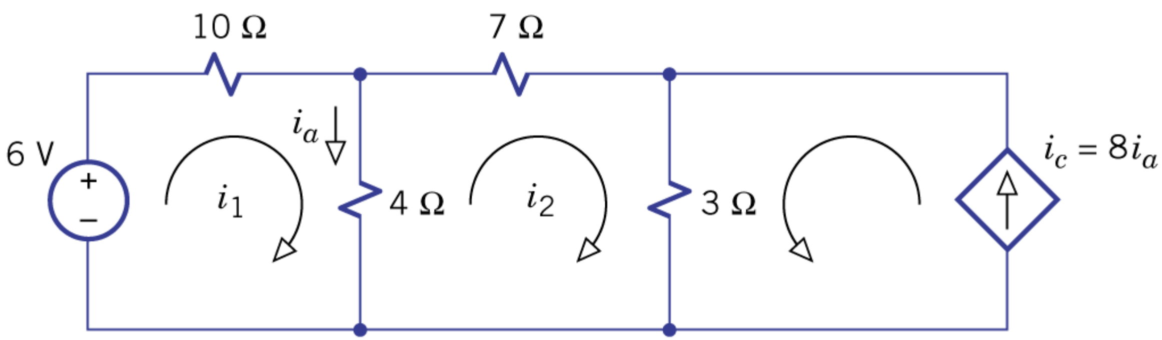
$$\text{mesh 1} \rightarrow \; 6 = 10i_1 + 4(i_1 - i_2) \rightarrow 14i_1 - 4i_2 = 6$$

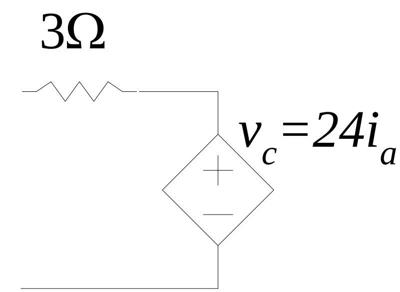
$$\text{mesh 2} \rightarrow \; 4(i_2 - i_1) + 7i_2 + 3(i_2 + 8i_a) = 0, i_a = i_1 - i_2$$

$$\rightarrow 4i_2 - 4i_1 + 7i_2 + 3i_2 + 24i_1 - 24i_2 = 0 \rightarrow 20i_1 - 10i_2 = 0$$

$$\begin{bmatrix} 14 & -4 \\ 20 & -10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix} \rightarrow i_1 = 1, i_2 = 2$$

2. Ex.4.10 find i_1, i_2



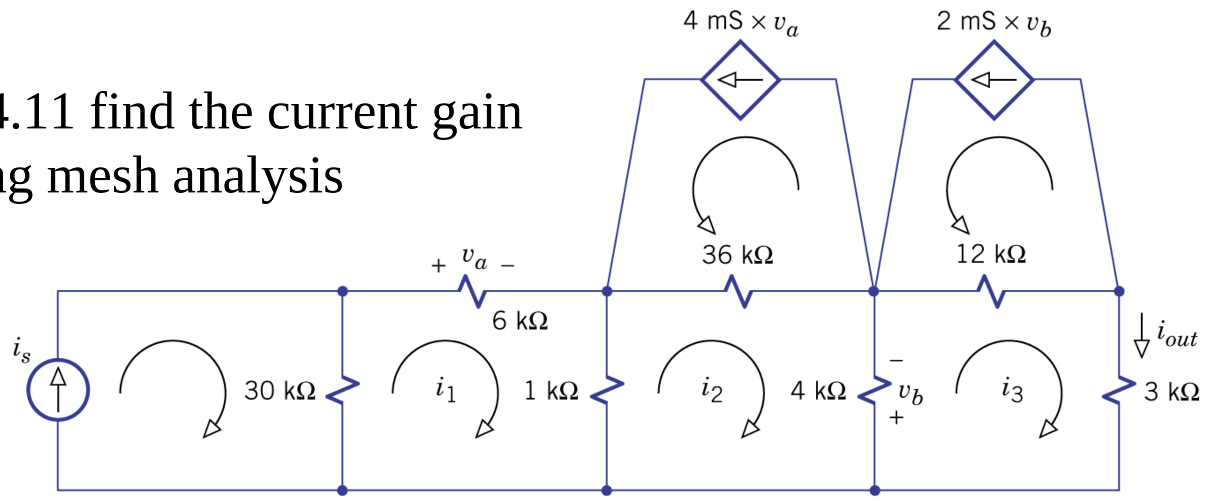


$$[R]=\begin{bmatrix}10+4 & -4\\ -4 & 4+7+3\end{bmatrix}=\begin{bmatrix}14 & -4\\ -4 & 14\end{bmatrix}, \text{constraint equation } i_a = i_1 - i_2$$

$$\text{source - voltage vector } [v_s]=\begin{bmatrix}6\\ -3\times 8i_a\end{bmatrix}=\begin{bmatrix}6\\ -24i_1+24i_2\end{bmatrix}=\begin{bmatrix}6\\ 0\end{bmatrix}+\begin{bmatrix}0 & 0\\ -24 & 24\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix}=[\tilde{v}_s]+[\tilde{R}][i]$$

$$[R-\tilde{R}]=\begin{bmatrix}14 & -4\\ 20 & -10\end{bmatrix}\Rightarrow [R-\tilde{R}][i]=[\tilde{v}_s], \begin{bmatrix}14 & -4\\ 20 & -10\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix}=\begin{bmatrix}6\\ 0\end{bmatrix}\rightarrow i_1=1, i_2=2$$

3. Ex.4.11 find the current gain using mesh analysis

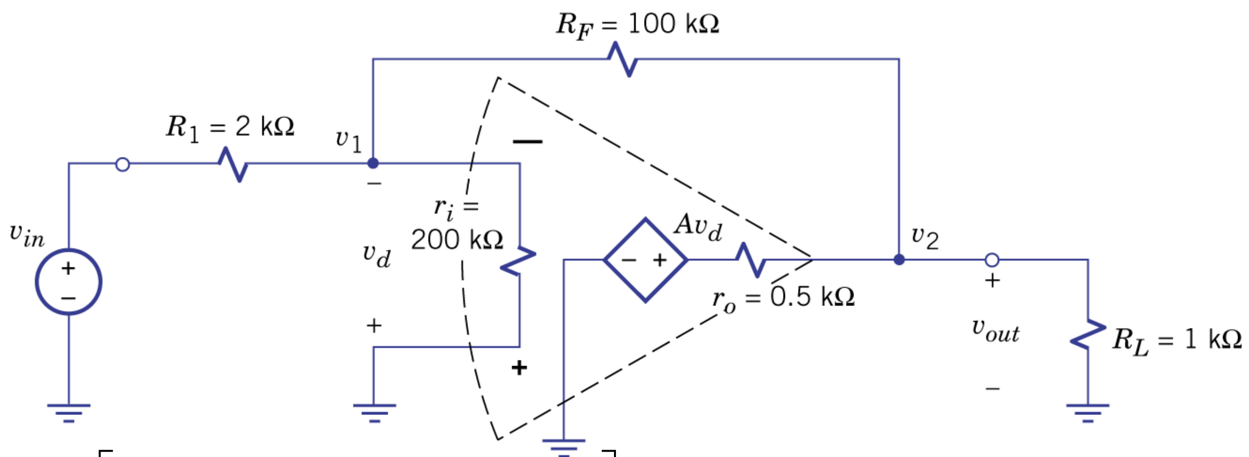


$$[R]=\begin{bmatrix}37 & -1 & 0\\ -1 & 41 & -4\\ 0 & -4 & 19\end{bmatrix}, \text{constraint equation} \begin{cases} v_a = 6i_1 \\ v_b = 4(i_3 - i_2)\end{cases}$$

$$[v_s]=\begin{bmatrix}30i_s\\ -36\times 4v_a\\ -12\times 2v_b\end{bmatrix}=\begin{bmatrix}30i_s\\ -36\times 4\times 6i_1\\ -12\times 2\times 4(i_3-i_2)\end{bmatrix}=\begin{bmatrix}30i_s\\ 0\\ 0\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\ -864 & 0 & 0\\ 0 & 96 & -96\end{bmatrix}\begin{bmatrix}i_1\\ i_2\\ i_3\end{bmatrix}=[\tilde{v}_s]+[\tilde{R}][i]$$

$$[R-\tilde{R}]=\begin{bmatrix}37 & -1 & 0\\ 863 & 41 & -4\\ 0 & -100 & 105\end{bmatrix}\Rightarrow [R-\tilde{R}][i]=[\tilde{v}_s]\xrightarrow[\text{if } i_s=1mA]{}\begin{cases} i_1 = 0.5mA \\ i_2 = -11.5mA \rightarrow A_i = \dfrac{i_{out}}{i_s} = \dfrac{-10}{1} = -10 \\ i_3 = -10mA\end{cases}$$

4. Ex.4.12 find the voltage gain using node analysis



$$[G]=\begin{bmatrix}\frac{1}{2}+\frac{1}{100}+\frac{1}{200} & -\frac{1}{100}\\ -\frac{1}{100} & \frac{1}{100}+\frac{1}{0.5}+1\end{bmatrix}=\begin{bmatrix}0.515 & -0.01\\ -0.01 & 3.01\end{bmatrix}, \text{constraint equation } v_d=-v_1$$

$$\text{source-current vector } [i_s]=\begin{bmatrix}\frac{v_{in}}{2}\\ \frac{Av_d}{0.5}\end{bmatrix}=\begin{bmatrix}\frac{v_{in}}{2}\\ -\frac{Av_1}{0.5}\end{bmatrix}=\begin{bmatrix}\frac{v_{in}}{2}\\ 0\end{bmatrix}+\begin{bmatrix}0 & 0\\ -2A & 0\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=[\tilde{i}_s]+[\tilde{G}][v]$$

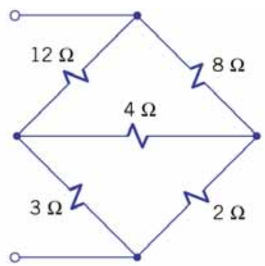
$$[G-\tilde{G}]=\begin{bmatrix}0.515 & -0.01\\ 2A-0.01 & 3.01\end{bmatrix}\Rightarrow[G-\tilde{G}][v]=[\tilde{i}_s], \begin{bmatrix}0.515 & -0.01\\ 2A-0.01 & 3.01\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}0.5v_{in}\\ 0\end{bmatrix}$$

$$\rightarrow A_v=\frac{v_2}{v_{in}}=\frac{0.05-A}{1.55+0.02A}$$

4.4 Applications of systematic analysis

<u>Discussion</u>

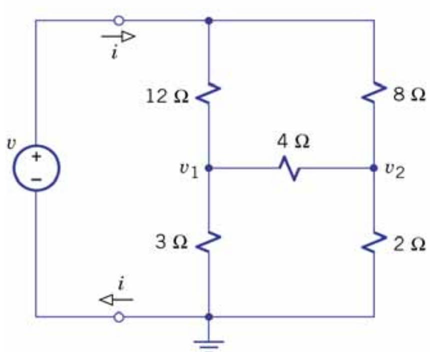
1. Examples of determining equivalent resistance or Thevenin parameters are given to use systematic analysis methods.
2. Ex. 4.13 find the equivalent resistance of a resistive bridge



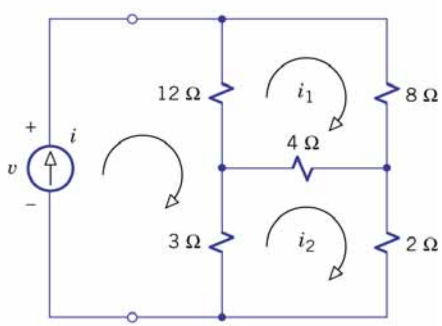
(a) Resistive bridge network

$$i_{4\Omega} = 0$$

$$\rightarrow R_{eq} = 10 // 15 = 6\Omega$$



(b) Test voltage source for node analysis



(c) Test current source for mesh analysis

node analysis

$$\begin{bmatrix} \frac{1}{12}+\frac{1}{4}+\frac{1}{3} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{8}+\frac{1}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{v}{12} \\ \frac{v}{8} \end{bmatrix}$$

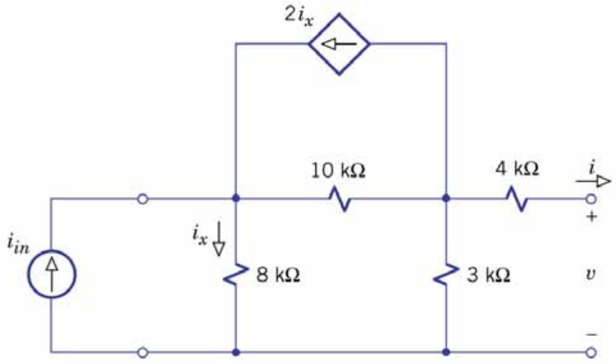
$$\rightarrow v_1 = v_2 = \frac{v}{5} \rightarrow i = \frac{v_1}{3} + \frac{v_2}{2} = \frac{v}{6} \rightarrow R_{eq} = 6$$

mesh analysis

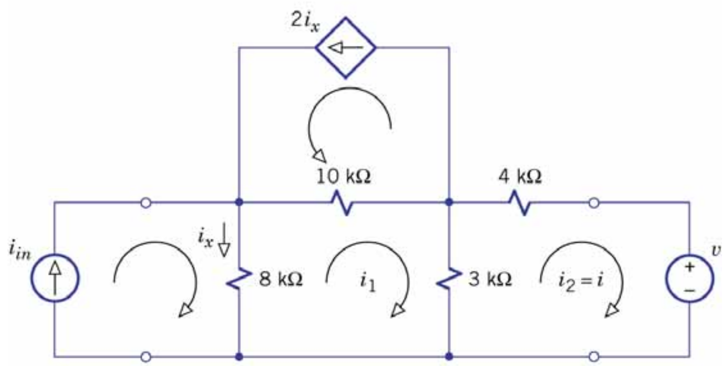
$$\begin{bmatrix} 12+8+4 & -4 \\ -4 & 3+4+2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 12i \\ 3i \end{bmatrix}$$

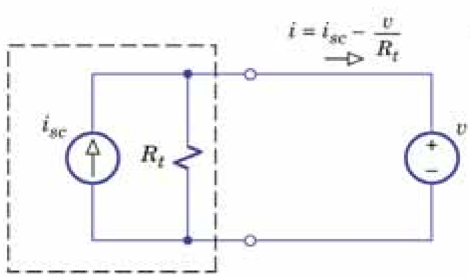
$$\rightarrow i_1 = i_2 = \frac{5i}{3} \rightarrow v = 8i_1 + 2i_2 = 6i \rightarrow R_{eq} = 6$$

3. Ex. 4.14 find the Thevenin parameters of a current amplifier



(a) Circuit model of a current amplifier





node analysis with current source: 3 nodes

mesh analysis: 2 meshes

$$[R]=\begin{bmatrix}21 & -3\\ -3 & 7\end{bmatrix}, \text{constraint equation } i_x = i_{in} - i_1$$

$$[v_s]=\begin{bmatrix}8i_{in}-10\times 2i_x\\ -v\end{bmatrix}=\begin{bmatrix}8i_{in}-20i_{in}+20i_1\\ -v\end{bmatrix}$$

$$=\begin{bmatrix}-12i_{in}\\ -v\end{bmatrix}+\begin{bmatrix}20 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}i_1\\ i\end{bmatrix}=[\tilde{v}_s]+[\tilde{R}][i]$$

$$[R-\tilde{R}]=\begin{bmatrix}1 & -3\\ -3 & 7\end{bmatrix},$$

$$\begin{bmatrix}1 & -3\\ -3 & 7\end{bmatrix}\begin{bmatrix}i_1\\ i\end{bmatrix}=\begin{bmatrix}-12i_{in}\\ -v\end{bmatrix}\rightarrow\ i=\frac{-v-36i_{in}}{-2}=18i_{in}+\frac{v}{2}$$

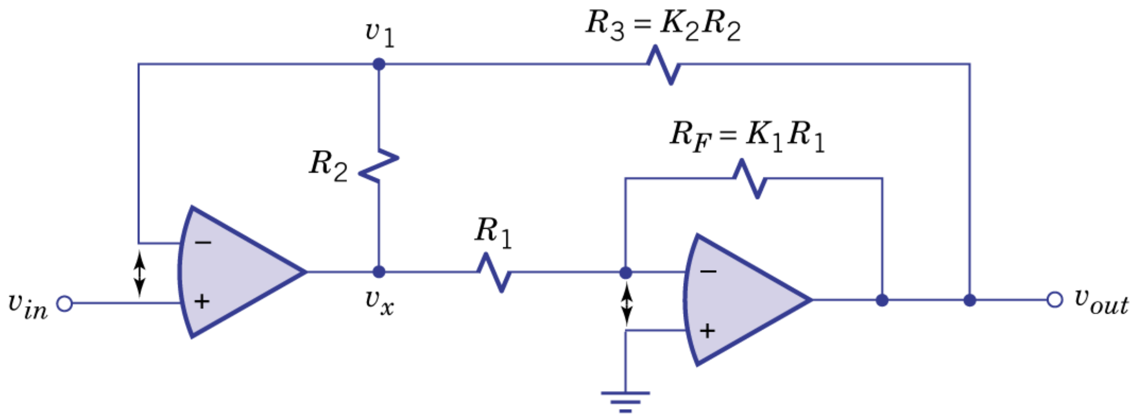
Norton model $i = i_{sc} - \dfrac{v}{R_t}$

$$\rightarrow i_{sc}=18i_{in},\ R_t=-2k, v_{oc}=i_{sc}R_t=-36k\times i_{in}$$

4.5-4.6

Optional

1. Ex. 4.15 find A_v of an op-amp circuit



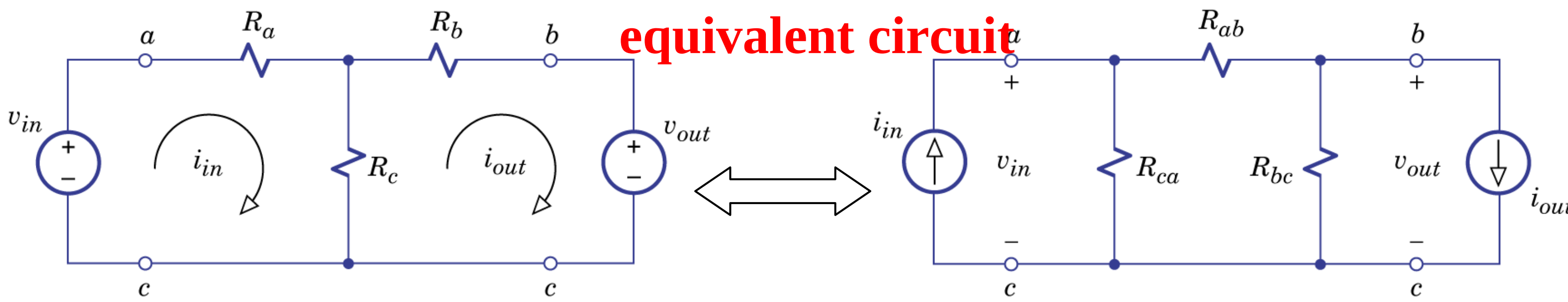
node analysis: $\frac{1}{R_2}(v_1 - v_x) + \frac{1}{K_2R_2}(v_1 - v_{out}) = 0 \ldots (1)$

virtual short: $v_{in} = v_1, \frac{v_x}{R_1} + \frac{v_{out}}{K_1R_1} = 0 \rightarrow v_x = -\frac{v_{out}}{K_1}$

$(1) \rightarrow \frac{1}{R_2}(v_{in} + \frac{1}{K_1}v_{out}) + \frac{1}{K_2R_2}(v_{in} - v_{out}) = 0 \rightarrow (1 + \frac{1}{K_2})v_{in} + (\frac{1}{K_1} - \frac{1}{K_2})v_{out} = 0 \rightarrow \frac{v_{out}}{v_{in}} = \frac{K_1(K_2+1)}{K_1 - K_2}$

2. Y-Δ (T-Π) transformation

equivalent circuit



(a) Wye network connected as a tee (T) network *(b)* Delta network connected as a pi (Π) network

mesh analysis: $\begin{bmatrix} R_a+R_c & -R_c \\ -R_c & R_b+R_c \end{bmatrix}\begin{bmatrix} i_{in} \\ i_{out} \end{bmatrix}=\begin{bmatrix} v_{in} \\ -v_{out} \end{bmatrix}\rightarrow$

$$i_{in}=\frac{(R_b+R_c)v_{in}-R_c v_{out}}{R_aR_b+R_bR_c+R_cR_a}$$

$$i_{out}=\frac{R_c v_{in}-(R_a+R_c)v_{out}}{R_aR_b+R_bR_c+R_cR_a}$$

node analysis: $\begin{bmatrix} \frac{1}{R_{ca}}+\frac{1}{R_{ab}} & -\frac{1}{R_{ab}} \\ -\frac{1}{R_{ab}} & \frac{1}{R_{ca}}+\frac{1}{R_{ab}} \end{bmatrix}\begin{bmatrix} v_{in} \\ v_{out} \end{bmatrix}=\begin{bmatrix} i_{in} \\ -i_{out} \end{bmatrix}\rightarrow$

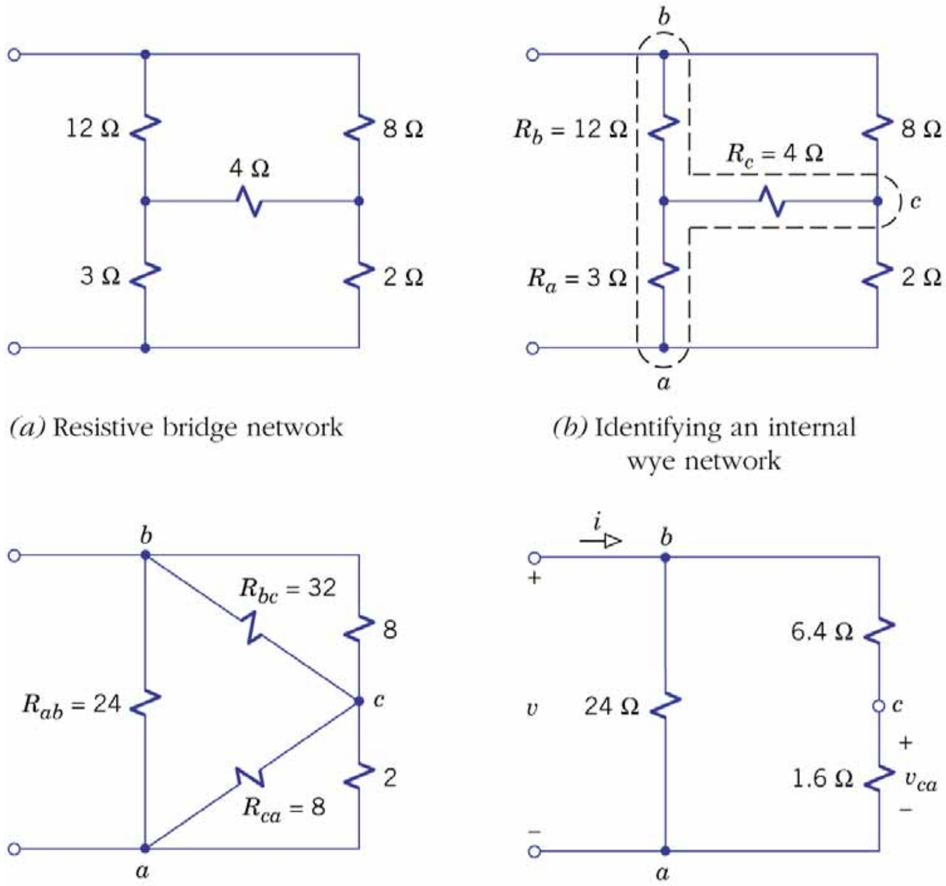
$$v_{in}=\frac{(R_{ab}R_{ca}+R_{ca}R_{bc})i_{in}-R_{ca}R_{bc}i_{out}}{R_{ab}+R_{bc}+R_{ca}}$$

$$v_{out}=\frac{R_{ca}R_{bc}i_{in}-(R_{bc}R_{ab}+R_{ca}R_{bc})i_{out}}{R_{ab}+R_{bc}+R_{ca}}$$

$$R_a=\frac{R_{ab}R_{ca}}{\sum R},R_b=\frac{R_{bc}R_{ab}}{\sum R},R_c=\frac{R_{ca}R_{bc}}{\sum R}\Leftrightarrow R_{ab}=\frac{R^2}{R_c},R_{bc}=\frac{R^2}{R_a},R_{ca}=\frac{R^2}{R_b}$$

$$\sum R\equiv R_{ab}+R_{bc}+R_{ca} \qquad R^2\equiv R_aR_b+R_bR_c+R_cR_a$$

3. Ex.4.16 (Ex.4.13)



(a) Resistive bridge network

(b) Identifying an internal wye network

(c) After wye-to-delta transformation

(d) Reduced equivalent network

$$R^2 \equiv R_a R_b + R_b R_c + R_c R_a = 96$$

$$R_{ab} = \frac{R^2}{R_c} = 24, R_{bc} = \frac{R^2}{R_a} = 32, R_{ca} = \frac{R^2}{R_b} = 8$$

$$R_{eq} = 24 // (6.4 + 1.6) = 6$$