Circuits－#1 Quiz (solution) Oct-20-2006

1. (25%) (a) Find the Thévenin equivalent circuit of the circuit inside the bracket given in Fig.1.
   (5%) (b) Use the result of (a) to solve v and i.

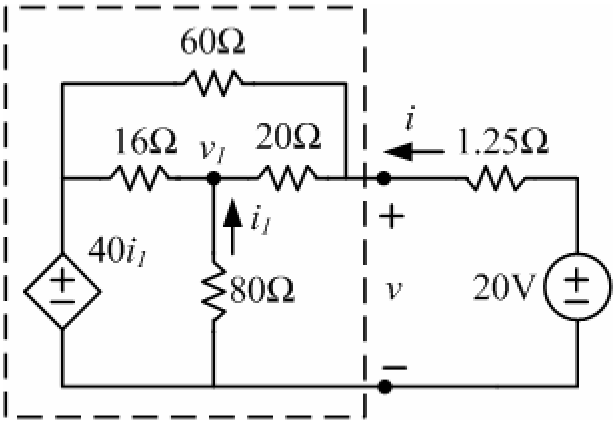
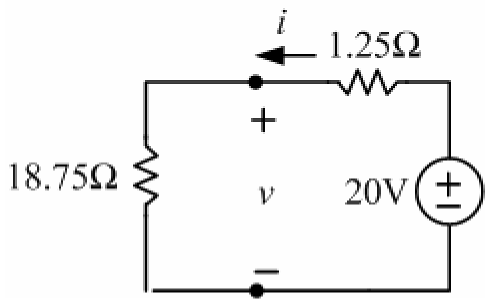


Fig.1



$$i=\frac{v-v_1}{20}+\frac{v-40i_1}{60}\text{....(1)},\ \frac{v_1-40i_1}{16}+\frac{v_1}{80}+\frac{v_1-v}{20}=0\text{....(2)}$$

$$(1)\rightarrow 60i=3v-3v_1+v-40i_1=4v-3v_1-40i_1\text{....(3)}$$

$$(2)\rightarrow 5v_1-200i_1+v_1+4v_1-4v=0\rightarrow 10v_1-200i_1-4v=0\text{....(4)}$$

$$\because i_1=-\frac{v_1}{80},\quad \begin{array}{l}(4)\rightarrow 10v_1-200(-\frac{v_1}{80})=4v\rightarrow 12.5v_1=4v\text{....(5)}\\ (3)\rightarrow 60i=4v-3v_1-40(-\frac{v_1}{80})=4v-2.5v_1\overset{(5)}{=}4v-0.8v=3.2v\end{array}$$

$$R_{th}=\frac{v}{i}=18.75$$

$$i=\frac{20}{18.75+1.25}=1A, v=18.75V$$

2. (30%) For the circuit given in Fig.2, solve v_1, i_1, v_2, and i_2.

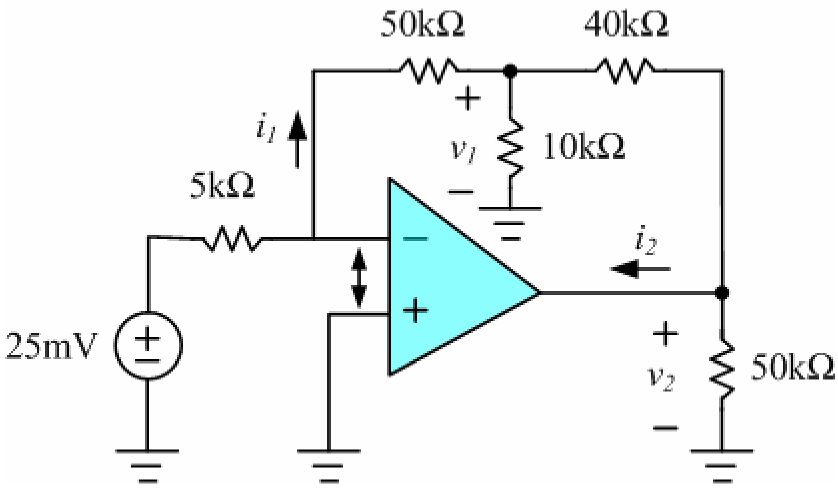


Fig.2

$$i_1 = \frac{25\times10^{-3}}{5000} = 5uA$$

$$v_1 = -50\times10^3 \times i_1 = -0.25V$$

$$-5\times10^{-6} + \frac{v_1}{10\times10^3} + \frac{v_1 - v_2}{40\times10^3} = 0 \rightarrow -0.2 + 4v_1 + v_1 - v_2 = 0 \rightarrow v_2 = 5v_1 - 0.2 = -1.45V$$

$$i_2 = \frac{v_1 - v_2}{40\times10^3} - \frac{v_2}{50\times10^3} = \frac{1.2}{40\times10^3} + \frac{1.45}{50\times10^3} = 59uA$$

3. (40%) Use superposition theorem to solve the voltage v given in Fig.3.

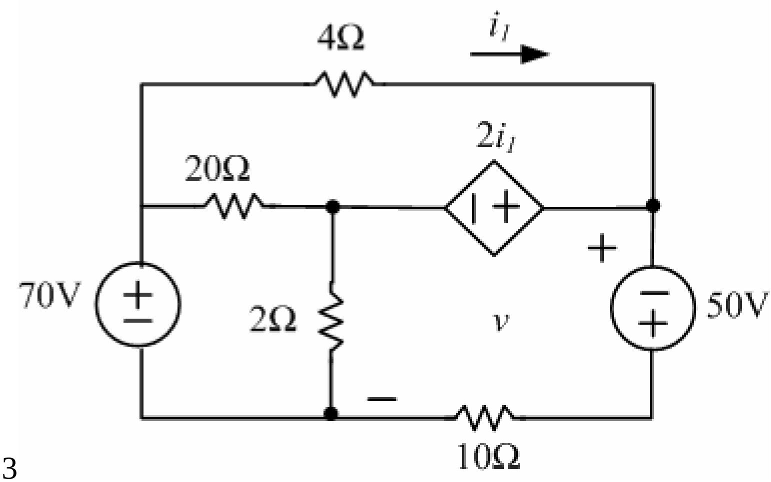
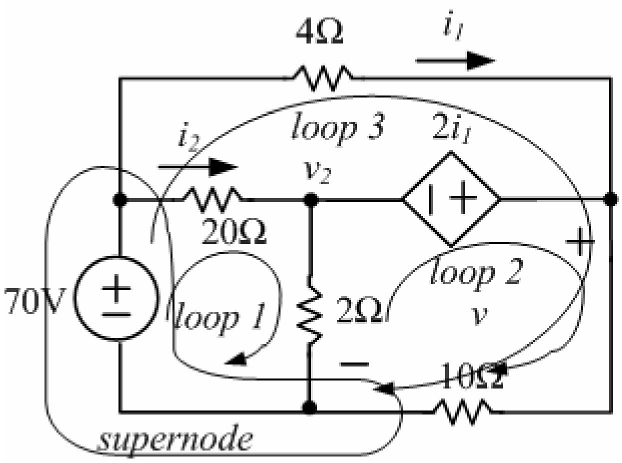


Fig.3



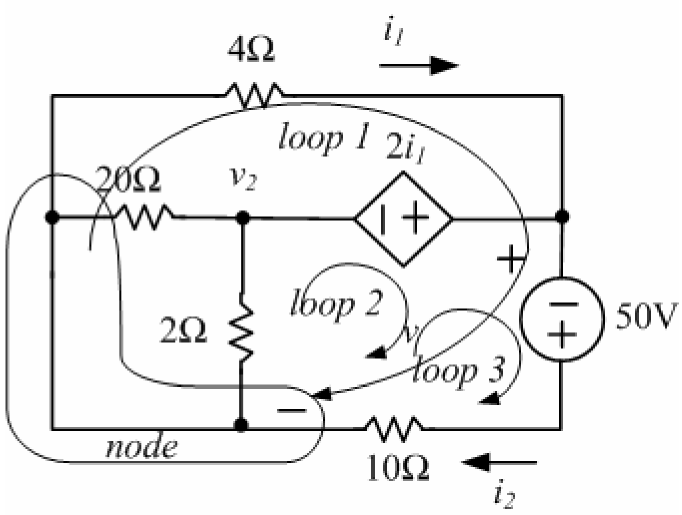
$$loop1: 70 = 20i_2 + v_2 \rightarrow i_2 = \frac{70 - v_2}{20} \overset{(2)}{=} \frac{7}{2} - \frac{1}{20}(\frac{3v}{2} - 35) = -\frac{3v}{40} + \frac{21}{4} ....(3)$$

$$loop2: v_2 + 2i_1 - v = 0 \rightarrow v_2 = v - 2i_1 \overset{(1)}{=} v - 2(\frac{35}{2} - \frac{v}{4}) = \frac{3v}{2} - 35....(2)$$

$$loop3: 70 - 4i_1 - v = 0 \rightarrow i_1 = \frac{35}{2} - \frac{v}{4}....(1)$$

$$\sup ernode: \frac{v_2}{2} + \frac{v}{10} = i_1 + i_2 \overset{(1),(2),(3)}{\rightarrow} \frac{1}{2}(\frac{3v}{2} - 35) + \frac{v}{10} = \frac{35}{2} - \frac{v}{4} - \frac{3v}{40} + \frac{21}{4}$$

$$\rightarrow \frac{47v}{40} = \frac{161}{4} \rightarrow v = \frac{1610}{47}$$



$$loop1: 4i_1 + v = 0 \rightarrow i_1 = -\frac{v}{4} \text{....(1)}$$

$$loop2: v_2 + 2i_1 - v = 0 \rightarrow v_2 = -2i_1 + v = \frac{3v}{2} \text{....(2)}$$

$$loop3: v + 50 - 10i_2 = 0 \rightarrow i_2 = \frac{v}{10} + 5 \text{....(3)}$$

$$node: i_1 = \frac{v_2}{20} + \frac{v_2}{2} + i_2 \overset{(1),(2),(3)}{\rightarrow} -\frac{v}{4} = \frac{11}{20} \times \frac{3v}{2} + \frac{v}{10} + 5 \Rightarrow -\frac{47}{40} v = 5 \rightarrow v = -\frac{200}{47}$$

$$Total\ \ v = \frac{1610}{47} - \frac{200}{47} = \frac{1410}{47} = 30V$$