微波電路講義

Microwave Circuits Notes

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General Information

1. Class: EE5009(921U0560) Tue. 9:10-12:10, EE-II 144
2. Textbook: *Microwave Engineering*, 4th ed., D.M. Pozar, John Wiley & Sons, 2011
3. Scopes: basic principles and design formula of passive and active microwave linear circuits using transmission line theory in terms of V(z) and I(z) ($V^+(z)$, $V^-(z)$, $I^+(z)$, $I^-(z)$) representation and microwave network analysis in terms of S-parameters
4. Contents:

Ch.2	Transmission line theory, 2.1, 2.3-2.7
Ch.3	Transmission lines and waveguides, 3.1, 3.5, 3.7, 3.8, 3.11
Ch.4	Microwave network analysis, 4.1-4.6
Ch.5	Impedance matching and tuning, 5.1-5.9
Ch.6	Microwave resonators, 6.1, 6.2, 6.5, 6.6
Ch.7	Power dividers and directional couplers, 7.1-7.3, 7.5-7.9

Ch.8 Microwave filters, 8.3-8.8
Ch.9 Theory and design of ferrimagnetic components, 9.6
Ch.10 Noise and nonlinear distortion, 10.1-10.4
Ch.11 Active RF and microwave devices, 11.1-11.4
Ch.12 Microwave amplifier design, 12.1-12.5
Ch.13 Oscillators and mixers, 13.2, 13.5
Ch.14 Introduction to microwave systems, 14.1-14.6

5. Estimated time table: 48hrs

	Date	Notes		Date	Notes
1	9/13	~2-9	10	11/15	university carnival
2	9/20	~2-34	11	11/22	~8-53
3	9/27	~4-11, Quiz#1(ch.2)	12	11/29	~10-16, Quiz#4(ch.7)
4	10/4	~4-27, Quiz#2(ch.3)	13	12/6	APMC
5	10/11	~5-5	14	12/13	~11-11. Quiz#5(ch.8)
6	10/18	~5-36, Quiz#3(ch.4)	15	12/20	~11-38
7	10/25	~6-20	16	12/27	~12-36, Quiz#6(ch.10,11)
8	11/1	~7-25	17	1/3	~14-14
9	11/8	Midterm Exam (~ch.6)	18	1/10	Final Exam

6. Grades: review quiz 15%, midterm exam. (Ch.2-Ch.6) 40%, final exam. (Ch.7-Ch.14) 45%

7. Office hour: Tue. 14:00~16:00 @room 541, or thc@ntu.edu.tw

8. Related readings:

(1) *Foundations of interconnect and microstrip design*, 3rd ed., T.C. Edwards, M.B. Steer, John Wiely & Sons, 2000.

(2) *Microwave engineering using microstrip circuits*, Fooks and Zakarevicius, Prentice Hall, 1990.

(3) *RF/microwave circuit design for wireless applications*, U.L. Rohde and D.P. Newkirk, John Wiley & Sons, 2000.

(4) *Microwave and rf design*, 2nd ed., M. Steer, Scitech, 2013.

(5) *Radio frequency and microwave electronics*, M. M. Radmanesh, Prentice Hall, 2001.

(6) *Microwave and RF engineering*, R. Sorrentino and G. Bianchi, John Wiley & Sons, 2009.

*(7) *Foundations for microwave engineering*, R.E. Collin, McGraw-Hill, 2nd ed., 1992.

9. Notes (including solved problems and ADS examples) are available at http://cc.ee.ntu.edu.tw/~thc/ or comm.ntu.edu.tw/faculty.htm

Chapter 1 Introduction

1. Definition

Microwave: designating or of that part of the electromagnetic spectrum between the far infrared and some lower frequency limit: commonly regarded as extending from 300,000 to 300 megahertz. (Webster's dictionary)

f: 300MHz - 300GHz $\Rightarrow \lambda$: 100cm - 0.1cm

electromagnetic spectrum (p.2, Fig.1.1)

2. Why use microwaves?

(1) Antenna gain is proportional to the electric size of the antenna.

$\Rightarrow$ f ↑, gain ↑(p.665, eq.(14.12))

miniature microwave system possible

$$D_{\max} = \frac{4\pi A}{\lambda^2}$$

(2) f ↑ $\Rightarrow$ available bandwidth ↑(p.671, Sec.14.2)

e.g., TV BW=6MHz

10% BW of VHF @60MHz for 1channel

1% BW of U-band @60GHz for 100 channels

(3) Line of sight propagation and not affected by cloud, fog,…

$\Rightarrow$ frequency reuse in satellite and terrestrial communications (frequency division duplexing, FDD)

(4) Radar cross section (RCS) is proportional to the target electrical size.

$\Rightarrow$ frequency ↑, RCS ↑(p.696, Table 14.3)
radar application, see p.690

$$\sigma = \frac{P_s}{S_t} \quad (14.39)$$

(5) Molecular, atomic and nuclear resonances occur at microwave frequencies (p.703, Fig.14.29)

$\Rightarrow$ astronomy, medical diagnostics and treatment, remote sensing and industrial heating applications

3. Biological effects and safety

non-ionized radiation $\Rightarrow$ thermal effect

IEEE standard C95.1-2005 (p.707, Fig. 14.32)

Excessive radiation may be dangerous to brain, eye, genital, stomach organs,….. $\Rightarrow$ cataract, sterility, cancer,…..

http://www.fda.gov/Radiation-EmittingProducts/RadiationEmittingProductsandProcedures/HomeBusinessandEntertainment/CellPhones/default.htm

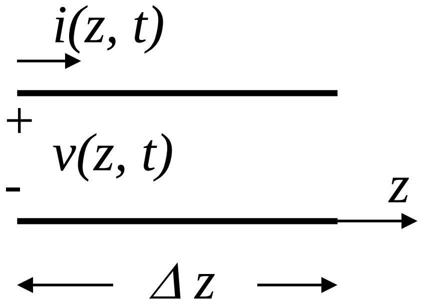
Chapter 2 Transmission Line Theory

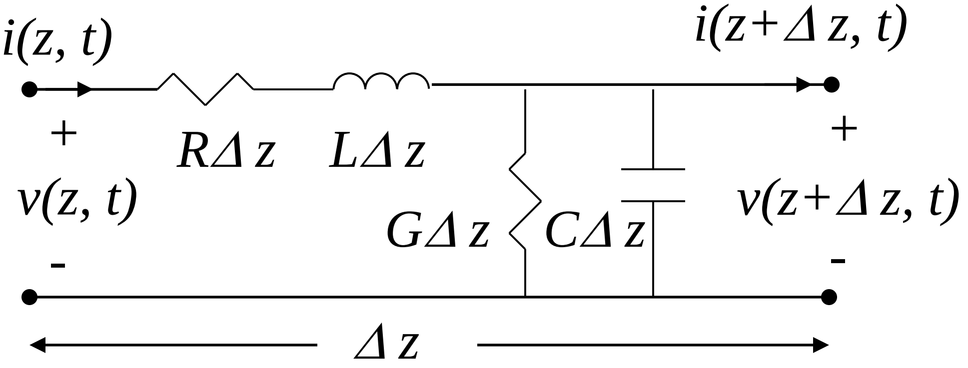
2.1 The lumped-element circuit model for a transmission line
  transmission line or telegrapher equation,
  traveling wave solution
2.3 The terminated lossless transmission line
  Z_{in}, Γ, VSWR
  time-average power flow
2.4 The Smith chart
  Z_{in}-plot conformal mapped on Γ-plot
2.5 The quarter-wave transformer
  frequency response, TDR
2.6 Generator and load mismatches
  impedance match, conjugate match
2.7 Lossy transmission lines
  low loss line, distortionless line
  perturbation method

2.1 The lumped-element circuit model for a transmission line

- Transmission line equation

TEM lines: coaxial line, parallel line and stripline





R, L: conductor resistance, inductance/unit length (series elements)

G, C: dielectric conductance, capacitance/unit length (parallel elements)

KVL, KCL (2.1) $\Rightarrow$

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

time-domain form of transmission line equation, or telegrapher equation

$\Rightarrow$ time-harmonic form e^{jwt}

$$v(z,t)=\mathrm{Re}[V(z)e^{jwt}],\; i(z,t)=\mathrm{Re}[I(z)e^{jwt}]$$

$$\frac{dV(z)}{dz}=-(R+jwL)I(z)$$

$$\frac{dI(z)}{dz}=-(G+jwC)V(z)$$

$\Rightarrow$ wave equation

$$\frac{d^2V(z)}{dz^2}-\gamma^2V(z)=0,\qquad \frac{d^2I(z)}{dz^2}-\gamma^2I(z)=0$$

$$\gamma\equiv\sqrt{(R+jwL)(G+jwC)}\equiv\alpha+j\beta$$

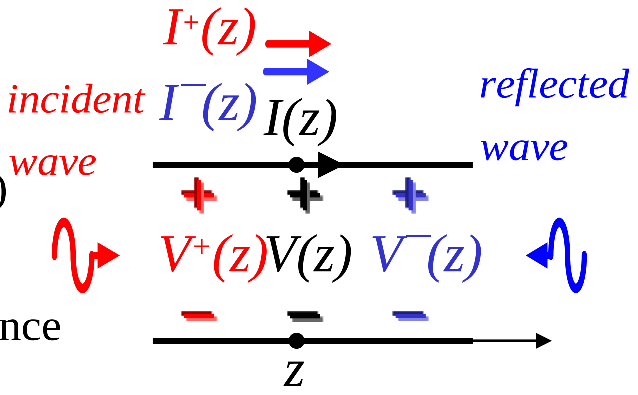
propagation constant

- Traveling wave solution

$$V(z)\equiv V^+(z)+V^-(z)=\;V_o^+e^{-\gamma z}+V_o^-e^{\gamma z}$$

$$I(z)\equiv I^+(z)+I^-(z)=I_o^+e^{-\gamma z}+I_o^-e^{\gamma z}=\frac{1}{Z_o}(V_o^+e^{-\gamma z}-V_o^-e^{\gamma z})$$

$$\Rightarrow\quad Z_o\equiv\sqrt{\frac{R+jwL}{G+jwC}}=\frac{V_o^+}{I_o^+}=-\frac{V_o^-}{I_o^-}$$: characteristic impedance



(*derivation*)

$$\frac{dV(z)}{dz}=-\gamma V_o^+e^{-\gamma z}+\gamma V_o^-e^{\gamma z}=-(R+jwL)I(z)\rightarrow I(z)=\frac{\gamma}{R+jwL}(V_o^+e^{-\gamma z}-V_o^-e^{\gamma z})=\sqrt{\frac{G+jwC}{R+jwL}}(V_o^+e^{-\gamma z}-V_o^-e^{\gamma z})$$

$$\underline{=\frac{1}{Z_o}(V_o^+e^{-\gamma z}-V_o^-e^{\gamma z})=I_o^+e^{-\gamma z}+I_o^-e^{\gamma z}}\equiv I^+(z)+I^-(z)\Rightarrow Z_o\equiv\frac{R+jwL}{\gamma}=\sqrt{\frac{R+jwL}{G+jwC}}$$

time-domain solution

$$v(z,t) = \left|V_o^+\right| e^{-\alpha z}\cos(wt - \beta z + \angle V_o^+) + \left|V_o^-\right| e^{\alpha z}\cos(wt + \beta z + \angle V_o^-)$$

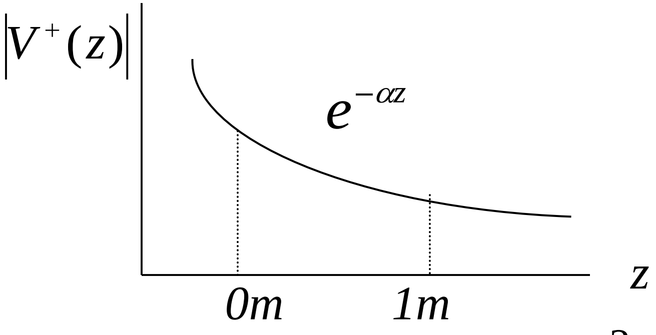
$$i(z,t) = \left|I_o^+\right| e^{-\alpha z}\cos(wt - \beta z + \angle I_o^+) + \left|I_o^-\right| e^{\alpha z}\cos(wt + \beta z + \angle I_o^-)$$

Discussion:

1. attenuation constant

$$\left|V^+(1m)\right| = \left|V^+(0m)\right| e^{-\alpha}$$

$$\alpha = \ln\frac{\left|V^+(0m)\right|}{\left|V^+(1m)\right|}(\text{Neper}) = 20\log\frac{\left|V^+(0m)\right|}{\left|V^+(1m)\right|}(\text{dB}), 1\text{Np} = 20\log e\,\text{dB} = 8.68\text{dB}$$



$$\left|P^+(z)\right| = \left|P^+(0)\right| e^{-2\alpha z}$$

2. phase constant $\beta = \frac{2\pi}{\lambda} = \frac{w}{v_p}$ wavelength $\lambda = \frac{2\pi}{\beta}$

phase velocity $v_p = \frac{w}{\beta}$ group velocity $v_g = (\frac{d\beta}{dw})^{-1}$

3. characteristic impedance (wave impedance)

$$Z_o \equiv \frac{V_o^+}{I_o^+} (= \frac{V^+(z)}{I^+(z)}) = -\frac{V_o^-}{I_o^-} (= -\frac{V^-(z)}{I^-(z)})$$

input impedance

$$Z_{in}(z) \equiv \frac{V(z)}{I(z)} (= \frac{V^+(z)+V^-(z)}{I^+(z)+I^-(z)} = \frac{V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}}{I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z}})$$

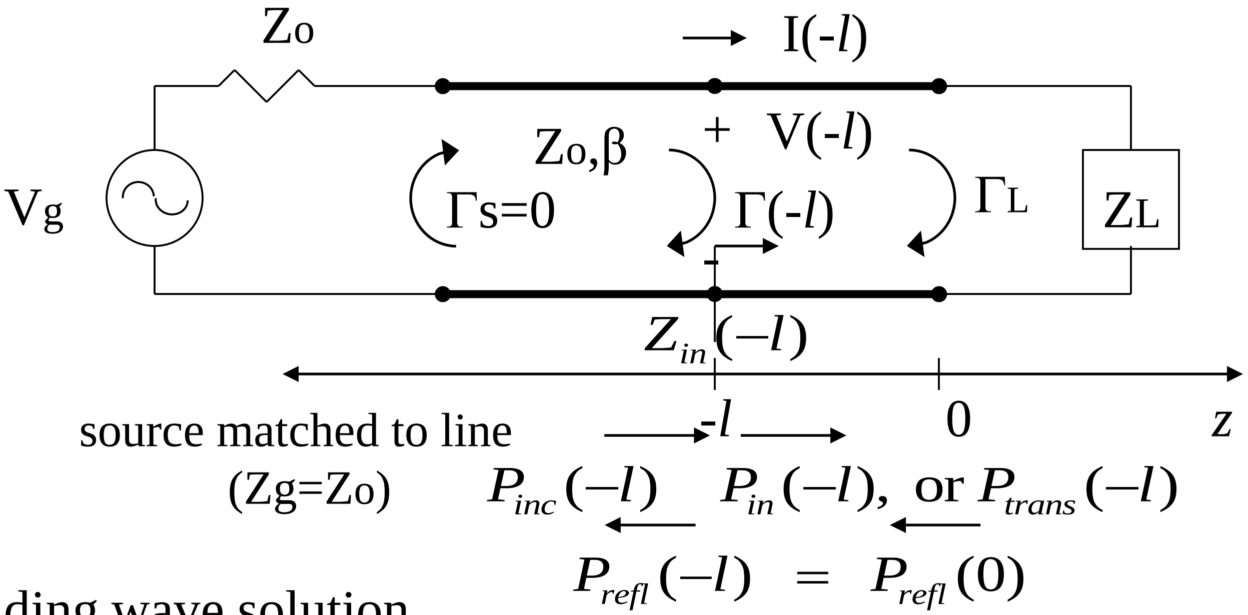
4.

$$P^+(z) \equiv V^+(z) I^{+*}(z) = V_o^+ e^{-\alpha z} I_o^{+*} e^{-\alpha z} = V_o^+ e^{-\alpha z} \frac{V_o^{+*}}{Z_o} e^{-\alpha z} = \frac{\left|V_o^+\right|^2}{Z_o} e^{-2\alpha z}$$

$$P^-(z) \equiv V^-(z) I^{-*}(z) = V_o^- e^{\alpha z} I_o^{-*} e^{\alpha z} = V_o^- e^{\alpha z} (-\frac{V_o^{-*}}{Z_o}) e^{\alpha z} = -\frac{\left|V_o^-\right|^2}{Z_o} e^{2\alpha z}$$

5. Transmission line equation can be derived from the Maxwell's equations (Sec. 2.2 on coaxial line example).

2.3 The terminated lossless transmission line



source matched to line

(Zg=Zo)

$P_{inc}(-l)$ $P_{in}(-l)$, or $P_{trans}(-l)$

$$P_{refl}(-l) = P_{refl}(0)$$

standing wave solution

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

$$I(z) = I_o^+ e^{-j\beta z} + I_o^- e^{j\beta z} = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{V_o^-}{Z_o} e^{j\beta z}$$

- Reflection coefficient (applications in measurement, radar, and remote sensing)

$$\Gamma(-l) \equiv \frac{V^-(-l)}{V^+(-l)} = \frac{V_o^- e^{-j\beta l}}{V_o^+ e^{j\beta l}} = \Gamma_L e^{-j2\beta l} = e^{-j\beta l}\Gamma_L e^{-j\beta l}$$

$$\because \Gamma(0) = \frac{V_o^-}{V_o^+} \equiv \Gamma_L$$

- Input impedance (application in circuit design)

$$Z_{in}(-l) \equiv \frac{V(-l)}{I(-l)} = Z_o \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}$$

$(derivation)$

$$Z_{in}(-l) = \frac{V(-l)}{I(-l)} = Z_o \frac{V_o^+ e^{j\beta l} + V_o^- e^{-j\beta l}}{V_o^+ e^{j\beta l} - V_o^- e^{-j\beta l}} = Z_o \frac{V_o^+ e^{j\beta l}(1+\Gamma_L e^{-2j\beta l})}{V_o^+ e^{j\beta l}(1-\Gamma_L e^{-2j\beta l})} = Z_o \frac{1+\Gamma_L e^{-2j\beta l}}{1-\Gamma_L e^{-2j\beta l}}$$

$$\xrightarrow{l=0} Z_{in}(0) = \frac{V(0)}{I(0)} \equiv Z_L = Z_o \frac{1+\Gamma_L}{1-\Gamma_L} \Rightarrow \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$Z_{in}(-l) = Z_o \frac{e^{j\beta l} + \Gamma_L e^{-j\beta l}}{e^{j\beta l} - \Gamma_L e^{-j\beta l}} = Z_o \frac{(Z_L + Z_o)e^{j\beta l} + (Z_L - Z_o)e^{-j\beta l}}{(Z_L + Z_o)e^{j\beta l} - (Z_L - Z_o)e^{-j\beta l}} = Z_o \frac{Z_L(e^{j\beta l} + e^{-j\beta l}) + Z_o(e^{j\beta l} - e^{-j\beta l})}{Z_L(e^{j\beta l} - e^{-j\beta l}) + Z_o(e^{j\beta l} + e^{-j\beta l})}$$

$$= Z_o \frac{Z_L \cos\beta l + jZ_o \sin\beta l}{jZ_L \sin\beta l + Z_o \cos\beta l} = Z_o \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}$$

- Voltage standing wave ratio

$$\text{VSWR} \equiv \frac{|\text{V}_{max}|}{|\text{V}_{min}|} = \frac{|1+|\Gamma_\text{L}||}{|1-|\Gamma_\text{L}||}$$

$(derivation)$

$$V(-l) = V_o^+ e^{j\beta l} + V_o^- e^{-j\beta l} = V_o^+ e^{j\beta l}(1+\Gamma_L e^{-j2\beta l}) = |V_o^+| e^{j\measuredangle V_o^+} e^{j\beta l}(1+|\Gamma_L| e^{j\measuredangle\Gamma_L} e^{-j2\beta l})$$

$$|V(-l)| = |V_o^+||1+|\Gamma_L| e^{j\measuredangle\Gamma_L} e^{-j2\beta l}|$$

$$\measuredangle\Gamma_L - 2\beta l = \begin{cases} 2n\pi \\ (2n+1)\pi \end{cases} \rightarrow \begin{matrix} |V_{\max}| = |V_o^+||1+|\Gamma_L|| \\ |V_{\min}| = |V_o^+||1-|\Gamma_L|| \end{matrix}, 2\beta\Delta l = \pi \rightarrow \Delta l = \frac{\pi}{2(2\pi/\lambda)} = \frac{\lambda}{2}$$

- Time-average input or transmitted power flow

$$P_{in}(z) \equiv \frac{1}{T}\int_o^T v(z,t)i(z,t)dt = \frac{1}{2}\text{Re}\left[V(z)I^*(z)\right] = \frac{1}{2}\frac{|V_o^+|^2}{Z_o}(1-|\Gamma_L|^2) = P_{inc} - P_{refl} = P_{trans}$$

constant

(lossless line)

time-average incident power

time-average reflected power

(derivation)

$$P_{in}(z) \equiv \frac{1}{T}\int_0^T v(z,t)i(z,t)dt = \frac{1}{T}\int_0^T \mathrm{Re}[V(z)e^{jwt}]\mathrm{Re}[I(z)e^{jwt}]dt$$

$$= \frac{1}{T}\int_0^T \frac{1}{2}[V(z)e^{jwt} + V^*(z)e^{-jwt}]\frac{1}{2}[I(z)e^{jwt} + I^*(z)e^{-jwt}]dt$$

$$= \frac{1}{4T}\int_0^T [V(z)\cancel{I(z)e^{j2wt}} + V^*(z)\cancel{I^*(z)e^{-j2wt}} + V(z)I^*(z) + V^*(z)I(z)]dt$$

$$= \frac{1}{4T}[V(z)I^*(z) + V^*(z)I(z)]\int_0^T dt = \frac{1}{4T}2\,\mathrm{Re}[V(z)I^*(z)]T$$

$$= \frac{1}{2}\mathrm{Re}[V(z)I^*(z)]$$

$$= \frac{1}{2}\mathrm{Re}\{(V^+ + V^-)(I^+ + I^-)^*\} = \frac{1}{2}\mathrm{Re}\{V^+I^{+*} + V^+I^{-*} + V^-I^{+*} + V^-I^{-*}\}$$

Im

$$= \frac{1}{2}\mathrm{Re}\{V^+I^{+*} \cancel{-\frac{V^+V^{-*}}{Z_o} + \frac{V^-V^{+*}}{Z_o}} + V^-I^{-*}\} = \frac{1}{2}\mathrm{Re}\{V^+I^{+*} + V^-I^{-*}\} = \frac{1}{2}\mathrm{Re}\left\{P^+ + P^-\right\}$$

lossless line

$$= \frac{\left|V_o^+\right|^2}{2Z_o} - \frac{\left|V_o^-\right|^2}{2Z_o} = \frac{\left|V_o^+\right|^2}{2Z_o}(1 - \left|\Gamma_L\right|^2) = P_{inc} - P_{refl}, \left|\Gamma_L\right| = \frac{\left|V_o^-\right|}{\left|V_o^+\right|} = \left|\frac{Z_L - Z_o}{Z_L + Z_o}\right|$$

- Discussion

1. Zin ⟷ Γ $\qquad Z_{in}(z)=Z_o\dfrac{1+\Gamma(z)}{1-\Gamma(z)}, \qquad \Gamma(z)=\dfrac{Z_{in}(z)-Z_o}{Z_{in}(z)+Z_o}$

2. Return loss $RL \equiv -20\log\left|\Gamma_L\right|(dB) = -20\log\left|\dfrac{V_o^-}{V_o^+}\right| = -10\log\left|\dfrac{P_o^-}{P_o^+}\right|$

$e.g., \left\|\Gamma_L\right\| =$	1	0.1	0
$RL =$	$0dB$	$20dB$	∞dB
VSWR	∞	1.22	1
	all incident power is reflected "no return loss"		matched load "∞ return loss"

3. $1 \le VSWR \le \infty$

matched load $\left|\Gamma_L\right| = 0 \rightarrow VSWR = 1$

Q:VSWR for a lossy line?

"N"

4. passive load $0 \le \left|\Gamma_L\right| \le 1$, eg. $o.c.\ \Gamma_L = 1$, $s.c.\ \Gamma_L = -1$, matched load $\Gamma_L = 0$

active load $\left|\Gamma_L\right| > 1$

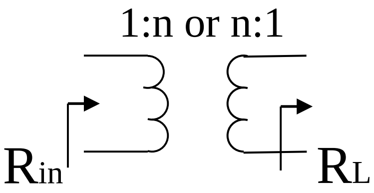
5. Impedance match

$Z_{in}(z)=Z_o \Rightarrow$ no reflected wave $\Gamma(z)=0$, VSWR=1, RL= ∞dB

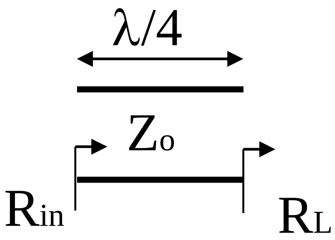
$P_{in}=P_{in,max}$: maximum power delivered to the load

6. $l=\lambda/2$, $Z_{in}(l)=Z_L$,

$l=\lambda/4$, $Z_{in}(l)=Z_o^2/Z_L$ impedance inverter or quarter-wave “transformer”



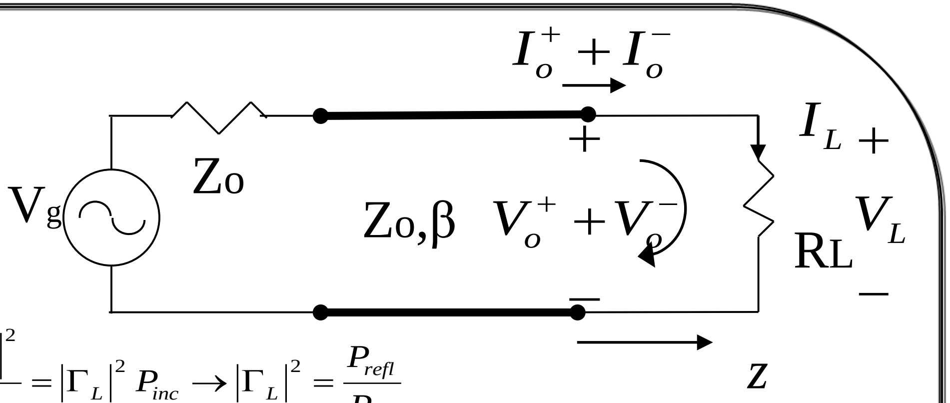
$$n \leftrightarrow VSWR$$



$$1:n \rightarrow R_{in}=\frac{R_L}{n^2}, n:1 \rightarrow R_{in}=n^2R_L$$

$$VSWR=\frac{1+|\Gamma_L|}{1-|\Gamma_L|}=\frac{1+\left|\frac{R_L-Z_o}{R_L+Z_o}\right|}{1-\left|\frac{R_L-Z_o}{R_L+Z_o}\right|}\begin{cases}\overset{R_L>Z_o}{\underset{1:n}{=}}\frac{R_L}{Z_o}, R_{in}=\frac{Z_o^2}{R_L}=\frac{R_L}{R_L^2/Z_o^2}=\frac{R_L}{VSWR^2}\\ \overset{R_L<Z_o}{\underset{n:1}{=}}\frac{Z_o}{R_L}, R_{in}=\frac{Z_o^2}{R_L}=\frac{R_L}{R_L^2/Z_o^2}=VSWR^2R_L\end{cases}$$

7. Mismatch loss (ML)



$$@\, z=0, V_o^+ + V_o^- = V_L$$

$$\Gamma_L \equiv \frac{V_o^-}{V_o^+}, T \equiv \frac{V_L}{V_o^+}$$

$$\rightarrow V_o^+(1+\Gamma_L) = V_o^+ T \rightarrow 1+\Gamma_L = T$$

$$P_{inc} = \frac{1}{2}\frac{\left|V_o^+\right|^2}{Z_o}, P_{refl} = \frac{1}{2}\frac{\left|V_o^-\right|^2}{Z_o} = \frac{\left|V_o^-\right|^2}{\left|V_o^+\right|^2}\frac{1}{2}\frac{\left|V_o^+\right|^2}{Z_o} = \left|\Gamma_L\right|^2 P_{inc} \rightarrow \left|\Gamma_L\right|^2 = \frac{P_{refl}}{P_{inc}}$$

$$P_{trans} = \frac{1}{2}\frac{\left|V_L\right|^2}{R_L} = \frac{Z_o}{R_L}\frac{\left|V_L\right|^2}{\left|V_o^+\right|^2}\frac{1}{2}\frac{\left|V_o^+\right|^2}{Z_o} = \frac{Z_o}{R_L}\left|T\right|^2 P_{inc} \equiv \left|T_P\right|^2 P_{inc} \rightarrow \left|T_P\right|^2 = \frac{P_{trans}}{P_{inc}} = \frac{Z_o}{R_L}\left|T\right|^2$$

$$P_{inc} = P_{refl} + P_{trans} \rightarrow P_{inc} = \left|\Gamma_L\right|^2 P_{inc} + \left|T_P\right|^2 P_{inc} \rightarrow 1 = \left|\Gamma_L\right|^2 + \left|T_P\right|^2 = \left|\Gamma_L\right|^2 + \frac{Z_o}{R_L}\left|T\right|^2$$

$$ML(mismatch\ loss) \equiv -10\log\frac{P_{trans}}{P_{inc}} = -20\log\left|T_P\right|$$

$$RL(return\ loss) \equiv -10\log\frac{P_{refl}}{P_{inc}} = -20\log\left|\Gamma_L\right|$$

$$R_L = \infty, \Gamma_L = 1, RL = 0dB, T = 2, T_P = 0, ML = \infty dB$$

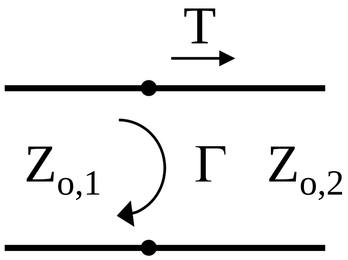
$$R_L = 0, \Gamma_L = -1, RL = 0dB, T = 0, T_P = 0, ML = \infty dB$$

$$R_L = 150\Omega, Z_o = 50\Omega, \Gamma_L = \frac{1}{2}, RL = 6dB, T = \frac{3}{2}, \left|T_P\right|^2 = \frac{50}{150}\frac{9}{4} = \frac{3}{4}, ML = 1.25dB$$

$$R_L = \frac{50}{3}\Omega, Z_o = 50\Omega, \Gamma_L = -\frac{1}{2}, RL = 6dB, T = \frac{1}{2}, \left|T_P\right|^2 = \frac{50}{50/3}\frac{1}{4} = \frac{3}{4}, ML = 1.25dB$$

$$R_L = 50\Omega, Z_o = 50\Omega, \Gamma_L = 0, RL = \infty dB, T = 1, \left|T_P\right|^2 = 1, ML = 0dB$$

e.g. transmission line junction
(p.62, Fig.2.9)



$$@\ junction,\ V_1 = V_2 \rightarrow V_{o,1}^{+} + V_{o,1}^{-} = V_{o,2}^{+}$$

$$\Gamma \equiv \frac{V_{o,1}^{-}}{V_{o,1}^{+}}, T \equiv \frac{V_{o,2}^{+}}{V_{o,1}^{+}} \rightarrow V_{o,1}^{+}(1+\Gamma) = V_{o,2}^{+} = V_{o,1}^{+}T \rightarrow 1+\Gamma = T$$

$$P_{inc} = \frac{1}{2}\frac{\left|V_{o,1}^{+}\right|^2}{Z_{o,1}}, P_{refl} = \frac{1}{2}\frac{\left|V_{o,1}^{-}\right|^2}{Z_{o,1}} = \frac{\left|V_{o,1}^{-}\right|^2}{\left|V_{o,1}^{+}\right|^2}\frac{1}{2}\frac{\left|V_{o,1}^{+}\right|^2}{Z_{o,1}} = \left|\Gamma\right|^2 P_{inc}, \left|\Gamma\right|^2 = \frac{P_{refl}}{P_{inc}}$$

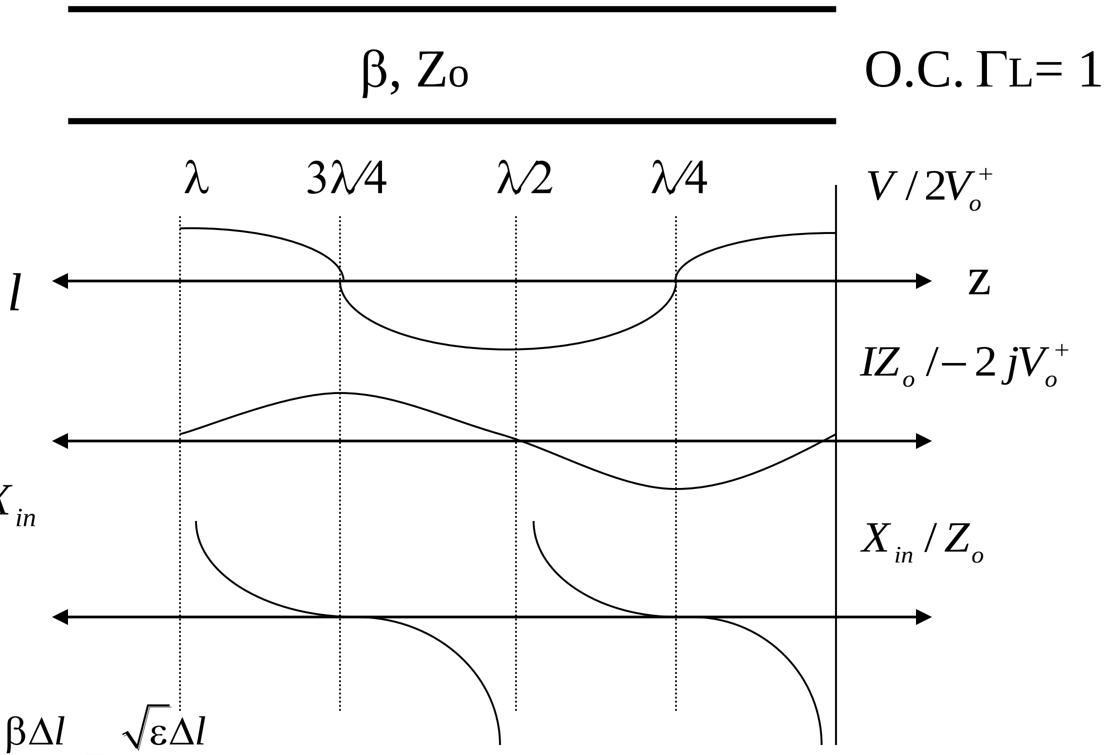
$$P_{trans} = \frac{1}{2}\frac{\left|V_{o,2}^{+}\right|^2}{Z_{o,2}} = \frac{Z_{o,1}}{Z_{o,2}}\frac{\left|V_{o,2}^{+}\right|^2}{\left|V_{o,1}^{+}\right|^2}\frac{1}{2}\frac{\left|V_{o,1}^{+}\right|^2}{Z_{o,1}} = \frac{Z_{o,1}}{Z_{o,2}}\left|T\right|^2 P_{inc} = \left|T_P\right|^2 P_{inc}, \left|T_P\right|^2 \equiv \frac{P_{trans}}{P_{inc}} = \frac{Z_{o,1}}{Z_{o,2}}\left|T\right|^2$$

$$P_{inc} = P_{refl} + P_{trans} \rightarrow P_{inc} = \left|\Gamma\right|^2 P_{inc} + \left|T_P\right|^2 P_{inc} \rightarrow 1 = \left|\Gamma\right|^2 + \left|T_P\right|^2 = \left|\Gamma\right|^2 + \frac{Z_{o,1}}{Z_{o,2}}\left|T\right|^2$$

$$Z_{o,1} = 50\Omega, Z_{o,2} = 50\Omega, \Gamma = 0, RL = \infty dB, T = 1, \left|T_P\right|^2 = 1, ML = 0dB$$

$$Z_{o,1} = 50\Omega, Z_{o,2} = 150\Omega, \Gamma = \frac{1}{2}, RL = 6dB, T = \frac{3}{2}, \left|T_P\right|^2 = \frac{3}{4}, ML = 1.25dB$$

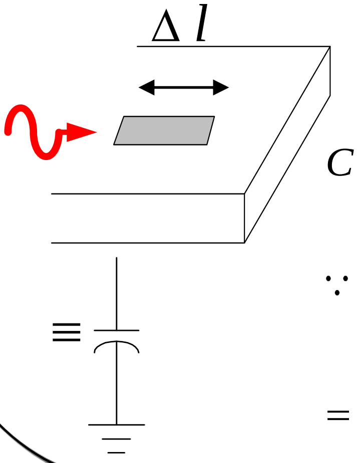
8. Open-circuited line



$$V = 2V_o^+ \cos\beta l$$

$$I = \frac{j2V_o^+}{Z_o}\sin\beta l$$

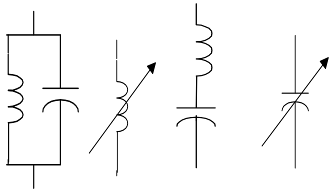
$$Z_{in} = \frac{Z_0}{j\tan\beta l} = -jX_{in}$$



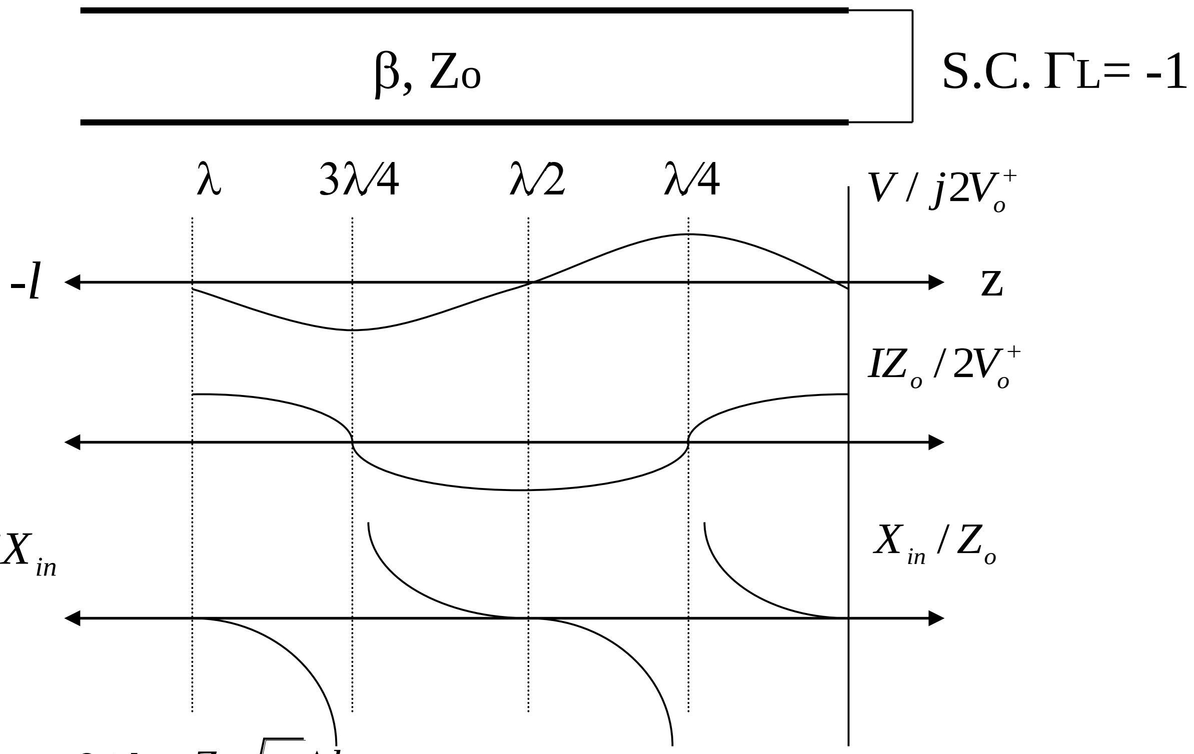
$$C = \frac{\beta\Delta l}{wZ_o} = \frac{\sqrt{\varepsilon}\Delta l}{Z_o c}$$

$$\because Z_{in} = \frac{Z_o}{j\tan\beta\Delta l} \approx -j\frac{Z_o}{\beta\Delta l}$$

$$= -j\frac{1}{wC},\ \beta = \frac{w}{v_p} = \frac{w\sqrt{\varepsilon_r}}{c}$$



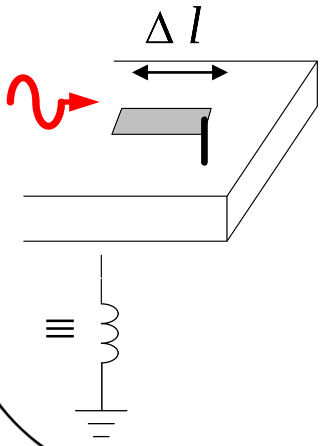
9. Short-circuited line



$$V = j2V_o^+ \sin \beta l$$

$$I = \frac{2V_o^+}{Z_o} \cos \beta l$$

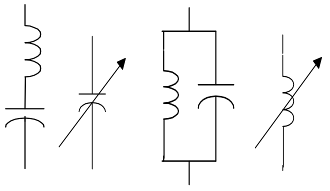
$$Z_{in} = jZ_o \tan \beta l = jX_{in}$$



$$L \approx \frac{Z_o \beta \Delta l}{w} = \frac{Z_o \sqrt{\varepsilon_r} \Delta l}{c}$$

$$\because Z_{in} = jwL = jZ_o \tan \beta \Delta l$$

$$\approx jZ_o \beta \Delta l, \ \beta = \frac{w}{v_p} = \frac{w\sqrt{\varepsilon_r}}{c}$$



(*derivation of the y-axis scales in Figs.2-6 and 2-8*)

open – circuited transmission line

$$V = V_o^+ (e^{-j\beta z} + e^{j\beta z}) = 2V_o^+ \cos\beta z \quad z < 0$$

$$\because z = -l,\ V = 2V_o^+ \cos\beta l, \ \text{ or } \frac{V}{2V_o^+} = \cos\beta l$$

$$I = \frac{V_o^+}{Z_o}(e^{-j\beta z} - e^{j\beta z}) = \frac{-j2V_o^+}{Z_o}\sin\beta z = \frac{j2V_o^+}{Z_o}\sin\beta l, \ \text{ or } \frac{IZ_o}{\text{-}j2V_o^+} = -\sin\beta l$$

$$Z_{in} = \frac{Z_0}{j\tan\beta l} = -jX_{in}, \text{ or } \frac{X_{in}}{Z_0} = \frac{1}{\tan\beta l}$$

similarly, for short – circuited transmission line

$$V = V_o^+ (e^{-j\beta z} - e^{j\beta z}) = -j2V_o^+ \sin\beta z = j2V_o^+ \sin\beta l, \ \text{ or } \frac{V}{j2V_o^+} = \sin\beta l$$

$$I = \frac{V_o^+}{Z_o}(e^{-j\beta z} + e^{j\beta z}) = \frac{2V_o^+}{Z_o}\cos\beta z = \frac{2V_o^+}{Z_o}\cos\beta l, \ \text{ or } \frac{IZ_o}{2V_o^+} = \cos\beta l$$

$$Z_{in} = jZ_o \ \tan\beta l = jX_{in}, \text{ or } \frac{X_{in}}{Z_0} = \tan\beta l$$

10. Standing wave expression

$$V(z)=V_o^+e^{-j\beta z}+V_o^-e^{j\beta z}=V_o^+e^{-j\beta z}(1+\Gamma_Le^{j2\beta z})=\left|V_o^+\right|e^{j\angle V_o^+}e^{-j\beta z}(1+\left|\Gamma_L\right|e^{j\angle\Gamma_L+j2\beta z})$$

$$\left|V(z)\right|=\left|V_o^+\right|\left|1+\left|\Gamma_L\right|e^{j\angle\Gamma_L+j2\beta z}\right|$$

$$=\left|V_o^+\right|\sqrt{[1+\left|\Gamma_L\right|\cos(\angle\Gamma_L+2\beta z)]^2+\left|\Gamma_L\right|^2\sin^2(\angle\Gamma_L+2\beta z)}$$

$$=\left|V_o^+\right|\sqrt{(1+\left|\Gamma_L\right|)^2-2\left|\Gamma_L\right|[1-\cos(\angle\Gamma_L+2\beta z)]}$$

$$=\left|V_o^+\right|\sqrt{(1+\left|\Gamma_L\right|)^2-4\left|\Gamma_L\right|\sin^2(\beta z+\frac{\angle\Gamma_L}{2})}$$

$$\left|V(z)\right|=\begin{matrix}\left|V_{\max}\right|=\left|V_o^+\right|(1+\left|\Gamma_L\right|)\\ \left|V_{\min}\right|=\left|V_o^+\right|(1-\left|\Gamma_L\right|)\end{matrix},\beta z+\frac{\angle\Gamma_L}{2}=\begin{cases}n\pi\\ (n+\frac{1}{2})\pi\end{cases}$$

$$\rightarrow\angle\Gamma_L+2\beta z=\angle\Gamma_L-2\beta l=\begin{cases}2n\pi\\ (2n+1)\pi\end{cases},\Delta l=\frac{\pi}{2\beta}=\frac{\pi}{2(2\pi/\lambda)}=\frac{\lambda}{2}$$

An ADS example of a terminated transmission line is given in Ex1 in Ch2_prj.

11. Measurable quantities

eg. an open – circuited transmission line

$$V(-l) = V_o^+ e^{j\beta l} + V_o^+ e^{-j\beta l} = 2V_o^+ \cos\beta l \rightarrow v(-l,t) = \mathrm{Re}[Ve^{jwt}] = 2\left|V_o^+\right| \cos\beta l \cos(wt + \angle V_o^+)$$

$$I(-l) = \frac{j2V_o^+}{Z_o}\sin\beta l \rightarrow i(-l,t) = \mathrm{Re}[Ie^{jwt}] = -\frac{2\left|V_o^+\right|}{Z_o}\sin\beta l \sin(wt + \angle V_o^+) = \frac{2\left|V_o^+\right|}{Z_o}\sin\beta l \cos(wt + \angle V_o^+ + \frac{\pi}{2})$$

(1) $Z_{in}(-l)$: most difficult

$$measure\ v(-l,t) \text{ and } i(-l,t) \rightarrow V(-l) \text{ and } I(-l) \rightarrow Z_{in}(-l) = \frac{V(-l)}{I(-l)}$$

(2) $\Gamma_{in}(-l)$: less difficult

$$separate\ v(-l,t) \text{ into } v^+(-l,t) \text{ and } v^-(-l,t) \rightarrow measure\ V^+(-l) \text{ and } V^-(-l) \rightarrow \Gamma_{in}(-l) = \frac{V^-(-l)}{V^+(-l)} = 1e^{-j2\beta l}$$

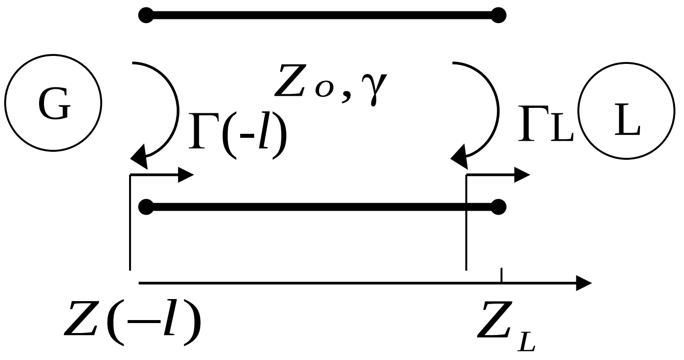
(3) VSWR: least difficult

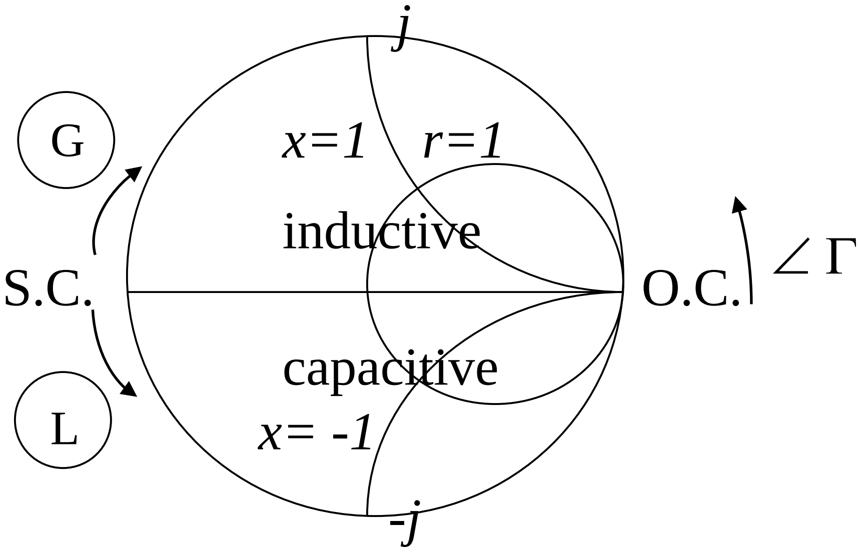
$$measure\ \ \left|v_{amp,\max}\right|, \left|v_{amp,\min}\right|, \left|v_{amp}(-l)\right| = 2\left|V_o^+\right|\left|\cos\beta l\right| \rightarrow VSWR = \frac{\left|V_{\max}\right|}{\left|V_{\min}\right|} = \frac{\left|v_{amp,\max}\right|}{\left|v_{amp,\min}\right|}$$

2.4 The Smith chart

Map rectangular plot of $z = Z/Z_o = r + jx$ onto the polar plot of

$$\Gamma = |\Gamma| e^{j\angle\Gamma} (= \Gamma_r + j\Gamma_i), |\Gamma| \le 1, -180^o \le \angle\Gamma \le 180^o$$

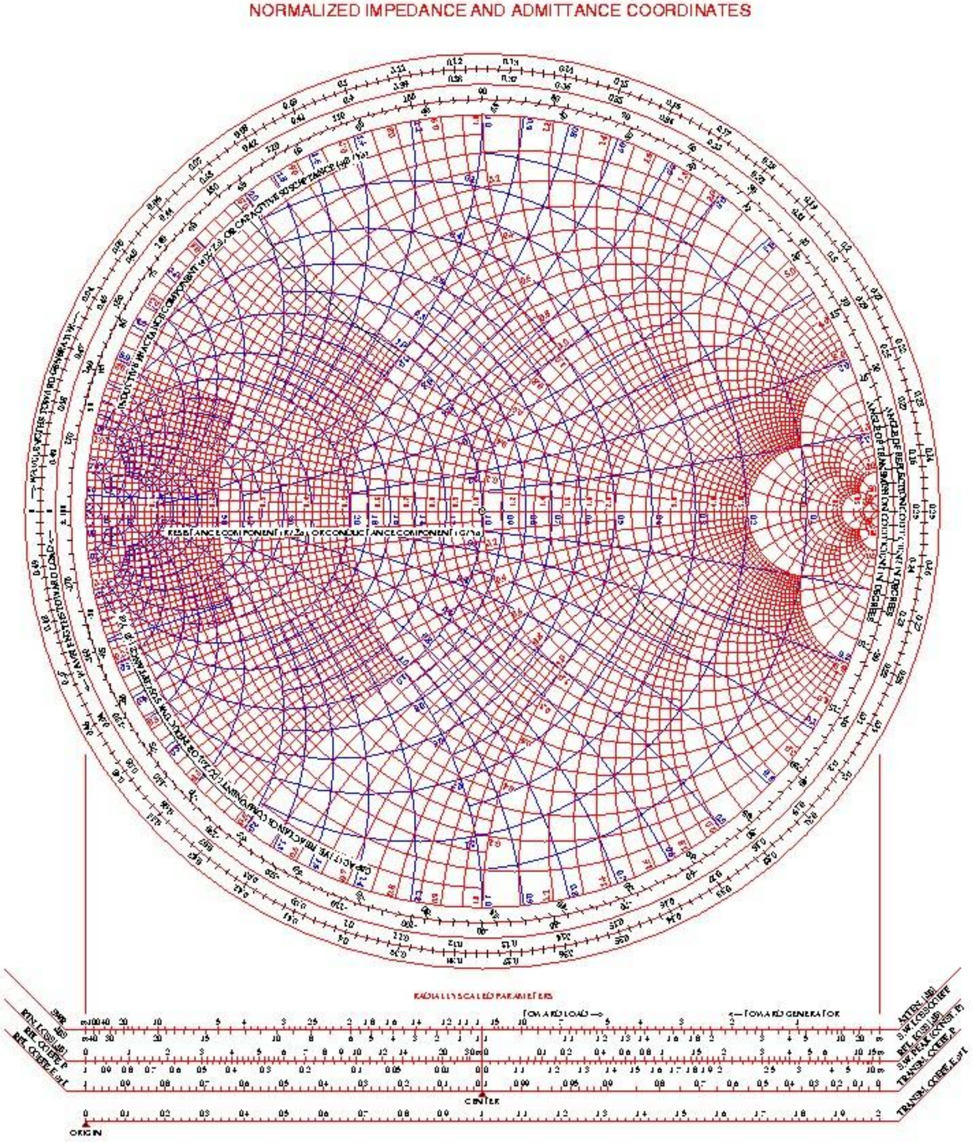




(*derivation*)

$$\Gamma = \frac{Z - Z_o}{Z + Z_o} = \frac{z-1}{z+1} \rightarrow z = r + jx = \frac{1+\Gamma}{1-\Gamma} = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i} = \frac{1-\Gamma_r^2 - \Gamma_i^2 + j2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$\Rightarrow (\Gamma_r - \frac{r}{1+r})^2 + \Gamma_i^2 = (\frac{1}{1+r})^2, (\Gamma_r - 1)^2 + (\Gamma_i - \frac{1}{x})^2 = (\frac{1}{x})^2$$


NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES
RESISTANCE COMPONENT (R/Z₀), OR CONDUCTANCE COMPONENT (G/Y₀)
INDUCTIVE REACTANCE COMPONENT (+jX/Z₀), OR CAPACITIVE SUSCEPTANCE (+jB/Y₀)
CAPACITIVE REACTANCE COMPONENT (-jX/Z₀), OR INDUCTIVE SUSCEPTANCE (-jB/Y₀)
WAVELENGTHS TOWARD GENERATOR →
← WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD →
← TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF. P
RFL. COEFF. E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF. P
TRANSM. COEFF. E or I
CENTER
ORIGIN

Discussion

1. $Z_L \rightarrow z_L \rightarrow \Gamma_L \xrightarrow{\Gamma(-l)=\Gamma_L e^{-j2\beta l}} \Gamma(-l) \rightarrow z(-l) \rightarrow Z(-l)$

2. r-circle $\perp$ x-circle

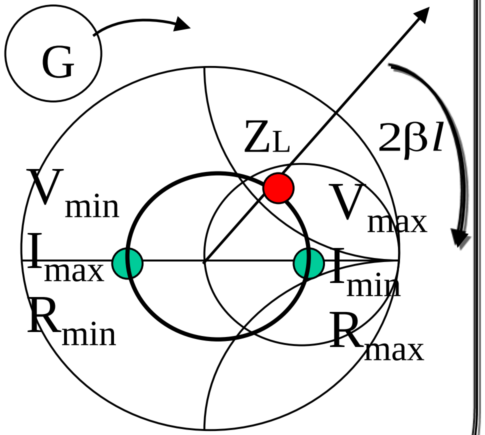
3. $VSWR = r_{\max}$

$(derivation)$

$$VSWR = \frac{|V_{\max}|}{|V_{\min}|} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}, r_{\max} = \frac{R_{\max}}{Z_o} = \frac{|V_{\max}|}{|I_{\min}|Z_o} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$\because V(-l) = V_o^+ e^{j\beta l} + V_o^- e^{-j\beta l} = V_o^+ e^{j\beta l}(1+\Gamma_L e^{-j2\beta l}) = |V_o^+| e^{j\measuredangle V_o^+} e^{j\beta l}(1+|\Gamma_L| e^{j\measuredangle\Gamma_L} e^{-j2\beta l})$$

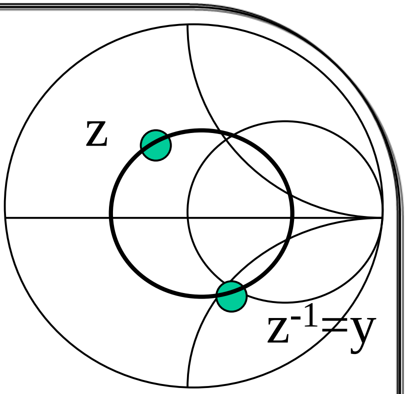
$$|V(-l)| = |V_o^+| \left|1+|\Gamma_L| e^{j\measuredangle\Gamma_L - j2\beta l}\right|, |I(-l)| = \frac{|V_o^+|}{Z_o}\left|1-|\Gamma_L| e^{j\measuredangle\Gamma_L - j2\beta l}\right|$$



$$\angle\Gamma_L - 2\beta l = \begin{cases} 0 \rightarrow |V_{\max}| = |V_o^+|(1+|\Gamma_L|), |I_{\min}| = \frac{|V_o^+|}{Z_o}(1-|\Gamma_L|) \\ \pi \rightarrow |V_{\min}| = |V_o^+|(1-|\Gamma_L|), |I_{\max}| = \frac{|V_o^+|}{Z_o}(1+|\Gamma_L|) \end{cases}$$

4. $\Delta l = \lambda/2 \rightarrow \Delta\angle\Gamma = 360^o$

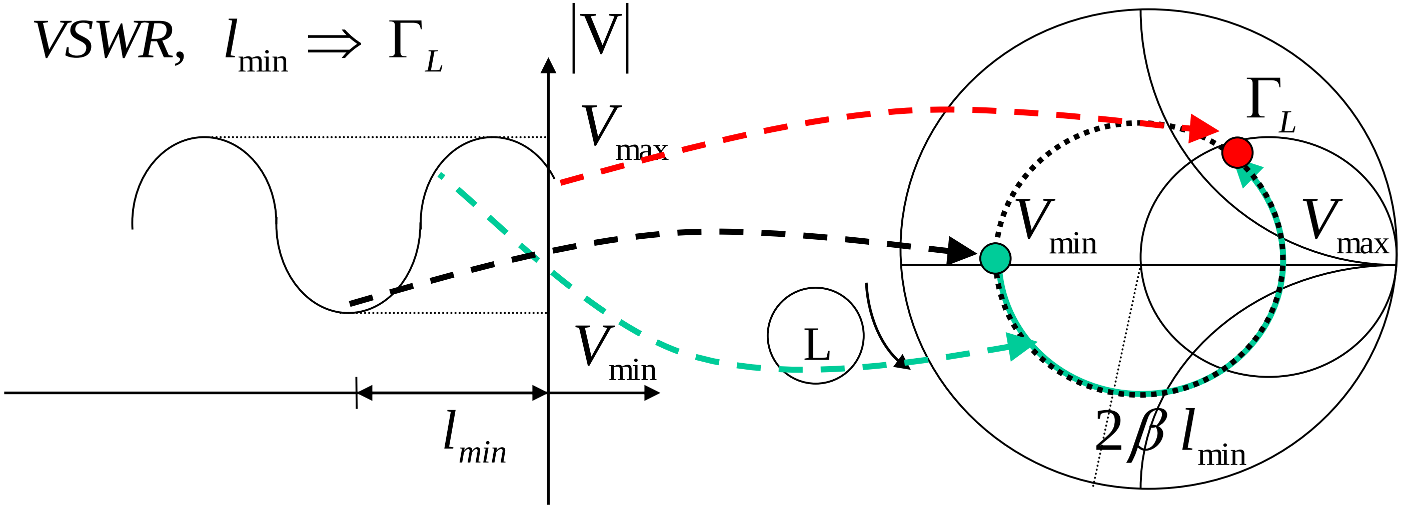
5. $z^{-1}(=y=\frac{Y}{Y_o}=YZ_o)\rightarrow\angle\Gamma+180^o$



(*derivation*)

$$z=\frac{1+\Gamma}{1-\Gamma},z^{-1}=\frac{1}{z}=\frac{1-\Gamma}{1+\Gamma}=\frac{1+\Gamma e^{j\measuredangle180^o}}{1-\Gamma e^{j\measuredangle180^o}}=\frac{1+\Gamma'}{1-\Gamma'},\Gamma'=\Gamma e^{j\measuredangle180^o}$$

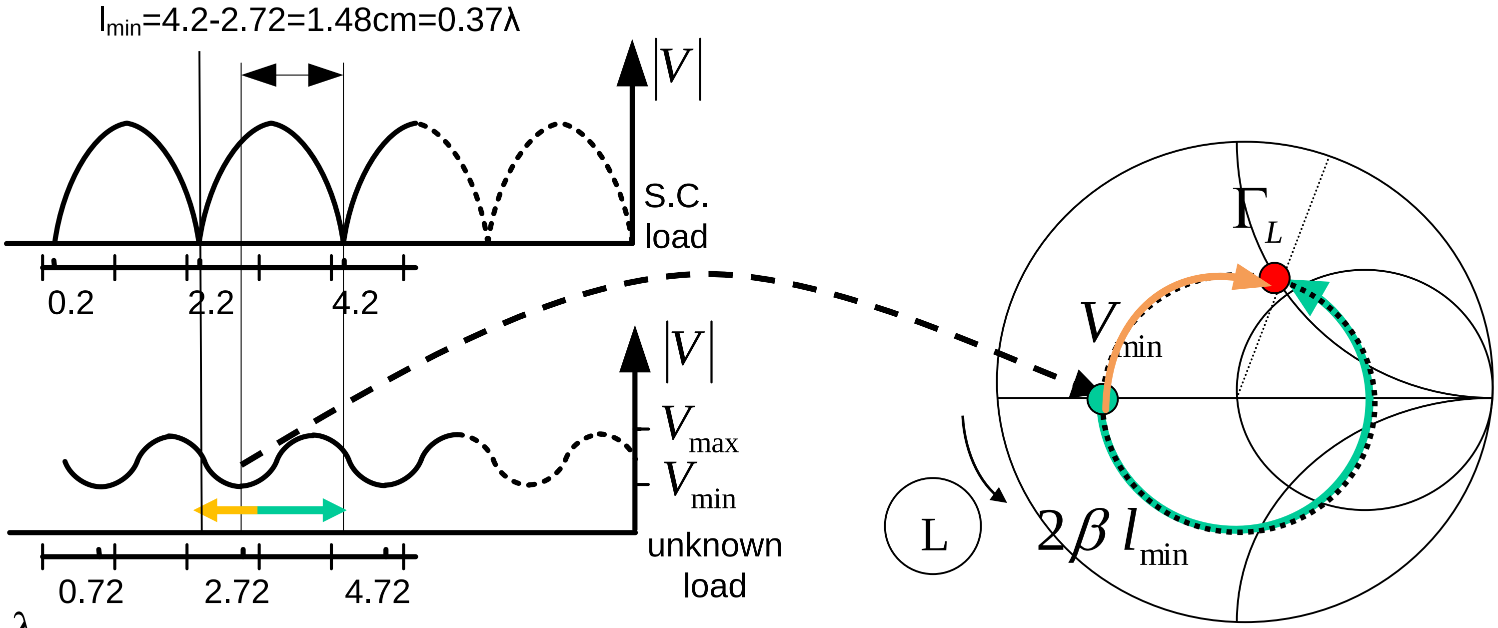
6. $VSWR,\ l_{\text{min}}\Rightarrow\Gamma_L$



$$VSWR=\frac{1+|\Gamma_L|}{1-|\Gamma_L|}\rightarrow|\Gamma_L|=\frac{VSWR-1}{VWSR+1}$$

$$180^o+2\beta l_{\text{min}}=\angle\Gamma_L$$

7. Ex. 2.4 VSWR=1.5, find Γ_L



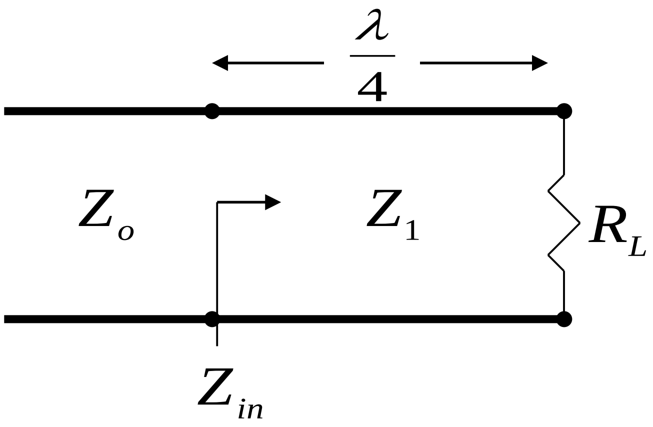
$$\frac{\lambda}{2} = 2cm \rightarrow \lambda = 4cm$$

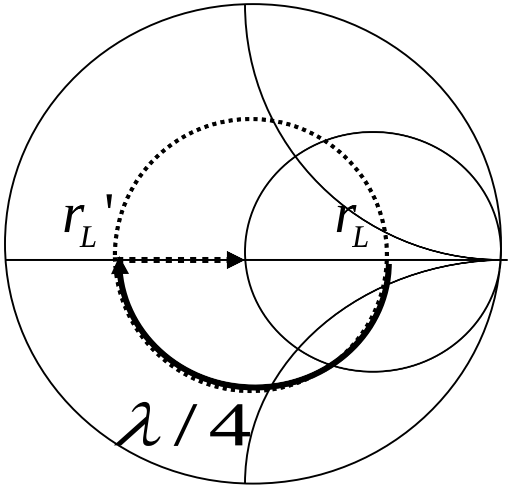
$$VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \rightarrow |\Gamma_L| = \frac{VSWR-1}{VWSR+1} = \frac{1.5-1}{1.5+1} = 0.2$$

$$\angle\Gamma_L = 180^o + 2\beta l_{min} = 180^o + 2\frac{2\times 180^o}{4}\times 1.48 - 360^o$$

$$= 180^o + 266.4^o - 360^o = 86.4^o$$

2.5 The quarter-wave transformer





matched condition: $Z_{in}(f_o) = \frac{Z_1^2}{R_L} = Z_o$

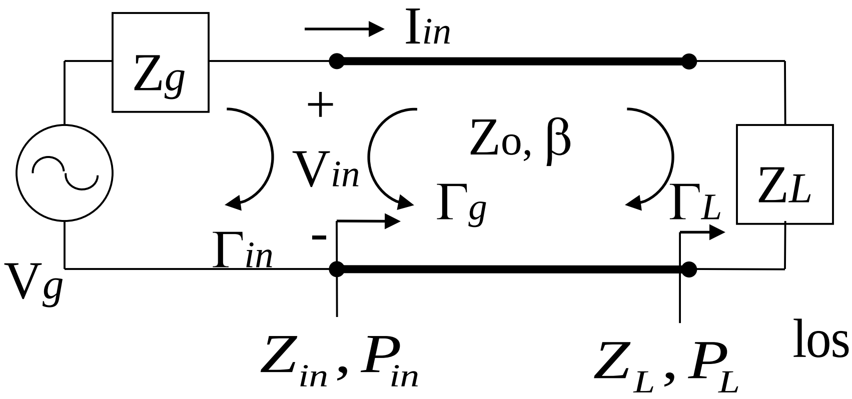
$$r_L = \frac{R_L}{Z_1} \rightarrow r_L' = r_L^{-1} \rightarrow R_L' = r_L' Z_1$$

$$\rightarrow \frac{R_L'}{Z_o} = \frac{r_L' Z_1}{Z_o} = \frac{Z_1}{Z_o r_L} = \frac{Z_1^2}{Z_o R_L} = 1$$

Discussion

1. Ex.2.5, frequency response (p.73, Fig.2.17)
2. Analysis from multiple-reflection viewpoint (slide 2-28)

2.6 Generator and load mismatches



$$P_{in} = \frac{1}{2}\mathrm{Re}(V_{in}I_{in}^*)$$

$$= \frac{1}{2}|V_g|^2 \frac{R_{in}}{(R_{in}+R_g)^2 + (X_{in}+X_g)^2}$$

lossless line → power delivered to the load

$$P_L = P_{in}$$

Discussion

1. Impedance match

load matched to the line $Z_L = Z_o \; (\Gamma_L = 0) \Rightarrow Z_{in} = Z_o$

$$P_{in} = \frac{1}{2}|V_g|^2 \frac{Z_o}{(Z_o+R_g)^2 + X_g{}^2}$$

source matched to the loaded line $Zg = Zin \; (\Gamma in = 0)$

$$P_{in} = \frac{1}{2}|V_g|^2 \frac{R_g}{4(R_g^2 + X_g^2)}$$

$(derivation)$

$time - average\ input\ power$

$$P_{in} = \frac{1}{2}\mathrm{Re}(V_{in}I_{in}^{*}) = \frac{1}{2}\mathrm{Re}(V_{in}\frac{V_{in}^{*}}{Z_{in}^{*}}) = \frac{1}{2}\left|V_{in}\right|^{2}\mathrm{Re}(\frac{1}{Z_{in}^{*}})$$

$$= \frac{1}{2}\left|V_{g}\right|^{2}\frac{\left|Z_{in}\right|^{2}}{\left|Z_{in}+Z_{g}\right|^{2}}\mathrm{Re}(\frac{Z_{in}}{\left|Z_{in}\right|^{2}}) = \frac{1}{2}\left|V_{g}\right|^{2}\frac{R_{in}}{(R_{in}+R_{g})^{2}+(X_{in}+X_{g})^{2}}$$

$load\ matched\ to\ the\ line\ (Z_{L} = Z_{o})$

$$Z_{in} = Z_{o}, P_{in} = \frac{1}{2}\left|V_{g}\right|^{2}\frac{R_{in}}{(Z_{o}+R_{g})^{2}+X_{g}{}^{2}}$$

$source\ matched\ to\ the\ loaded\ line\ (Z_{g} = Z_{in})$

$$P_{in} = \frac{1}{2}\left|V_{g}\right|^{2}\frac{R_{g}}{(R_{g}+R_{g})^{2}+(X_{g}+X_{g})^{2}} = \frac{1}{2}\left|V_{g}\right|^{2}\frac{R_{g}}{4(R_{g}{}^{2}+X_{g}{}^{2})}$$

2. Conjugate match

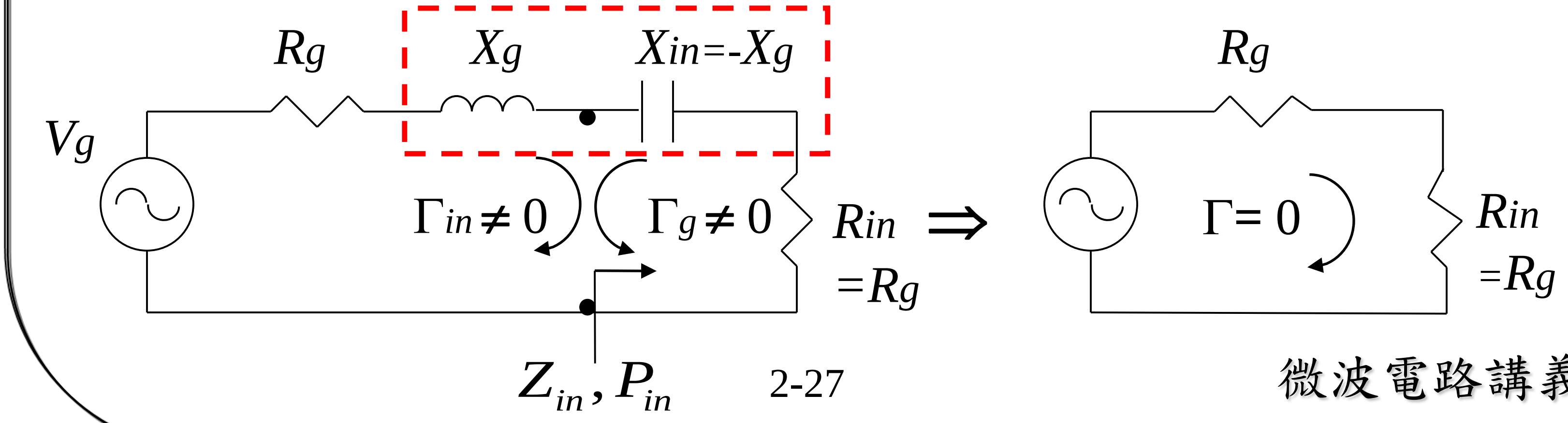
(given a fixed Z_g)

$Z_{in} = Z_g^*(\Gamma_g \neq 0,\ \Gamma_{in} \neq 0)$ $\Rightarrow$ maximum power transfer from source

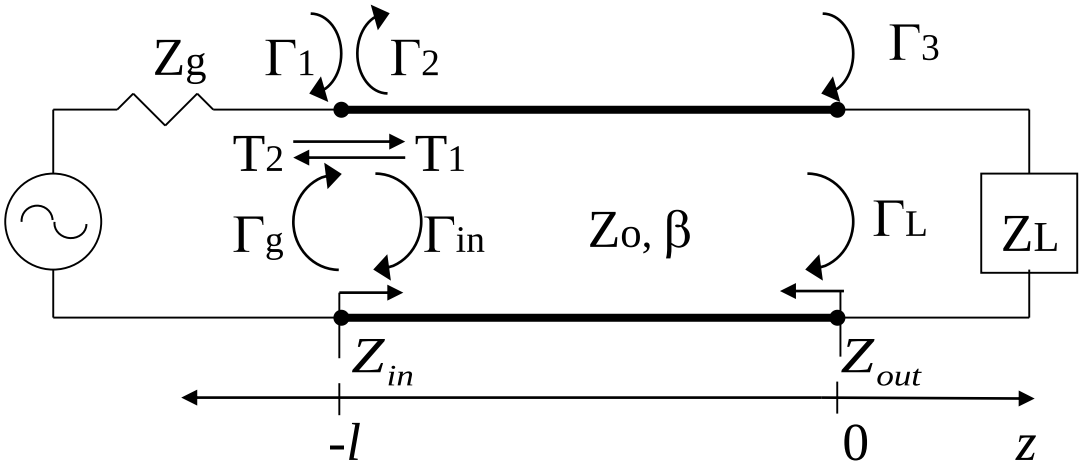
$$\left(\frac{\partial P_{in}}{\partial R_{in}} = 0, \quad \frac{\partial P_{in}}{\partial X_{in}} = 0\right)$$

$$P_{in,\max} = \frac{1}{2}\left|V_g\right|^2 \frac{1}{4R_g} \tag{12.9}$$

$\equiv P_{avs}(P_{inc}) = P_{in}\big|_{Z_{in}=Z_g^*}$: maximum available power from source



3. Reflection coefficient (from multiple-reflection viewpoint)



$$\Gamma_1 = \frac{Z_o - Z_g}{Z_0 + Z_g}$$

$$T_1 = 1 + \Gamma_1$$

$$\Gamma_2 = \frac{Z_g - Z_0}{Z_g + Z_0} = -\Gamma_1$$

$$T_2 = 1 + \Gamma_2$$

$$\Gamma_3 = \frac{Z_L - Z_0}{Z_L + Z_0} \neq \Gamma_L$$

$$\Gamma_{\text{in}} = \Gamma_1 + \underline{T_1 e^{-j2\beta l}\Gamma_3 T_2} + \underline{T_1 e^{-j2\beta l}}\Gamma_3\Gamma_2 e^{-j2\beta l}\Gamma_3 \underline{T_2} + \underline{T_1 e^{-j2\beta l}}\Gamma_3\Gamma_2 e^{-j2\beta l}\Gamma_3\Gamma_2 e^{-j2\beta l}\Gamma_3 \underline{T_2} + \ldots$$

$$= \Gamma_1 + T_1 e^{-j2\beta l}\Gamma_3 T_2 \sum (e^{-j2\beta l}\Gamma_2\Gamma_3)^n = \Gamma_1 + \frac{T_1\Gamma_3 T_2 e^{-j2\beta l}}{1 - \Gamma_2\Gamma_3 e^{-j2\beta l}}$$

$$= \Gamma_1 + \frac{(1+\Gamma_1)\Gamma_3(1-\Gamma_1)e^{-j2\beta l}}{1+\Gamma_1\Gamma_3 e^{-j2\beta l}} = \frac{\Gamma_1 + \Gamma_3 e^{-j2\beta l}}{1+\Gamma_1\Gamma_3 e^{-j2\beta l}}$$

$$\underline{\Gamma_{\mathrm{in}}}=\frac{\Gamma_1+\Gamma_3e^{-j2\beta l}}{1+\Gamma_1\Gamma_3e^{-j2\beta l}}=\frac{\dfrac{Z_o-Z_g}{Z_o+Z_g}+\dfrac{Z_L-Z_o}{Z_L+Z_o}e^{-j2\beta l}}{1+\dfrac{Z_o-Z_g}{Z_o+Z_g}\dfrac{Z_L-Z_o}{Z_L+Z_o}e^{-j2\beta l}}$$

$$=\frac{(Z_o-Z_g)(Z_L+Z_o)e^{j\beta l}+(Z_o+Z_g)(Z_L-Z_o)e^{-j\beta l}}{(Z_o+Z_g)(Z_L+Z_o)e^{j\beta l}+(Z_o-Z_g)(Z_L-Z_o)e^{j\beta l}}$$

$$=\frac{Z_oZ_L\cos\beta l+Z_o^2j\sin\beta l-Z_gZ_Lj\sin\beta l-Z_gZ_o\cos\beta l}{Z_oZ_L\cos\beta l+Z_o^2j\sin\beta l+Z_gZ_Lj\sin\beta l+Z_gZ_o\cos\beta l}\times\frac{1/\cos\beta l}{1/\cos\beta l}$$

$$=\frac{Z_o(Z_L+jZ_o\tan\beta l)-Z_g(Z_o+jZ_L\tan\beta l)}{Z_o(Z_L+jZ_o\tan\beta l)+Z_g(Z_o+jZ_L\tan\beta l)}$$

$$=\frac{Z_o\dfrac{Z_L+jZ_o\tan\beta l}{Z_o+jZ_L\tan\beta l}-Z_g}{Z_o\dfrac{Z_L+jZ_o\tan\beta l}{Z_o+jZ_L\tan\beta l}+Z_g}=\underline{\frac{Z_{in}-Z_g}{Z_{in}+Z_g}},Z_{in}=Z_o\frac{Z_L+jZ_o\tan\beta l}{Z_o+jZ_L\tan\beta l}$$

$$\underline{\Gamma_{\mathrm{L}}=\frac{Z_L-Z_{out}}{Z_L+Z_{out}}},Z_{out}=Z_o\frac{Z_g+jZ_o\tan\beta l}{Z_o+jZ_g\tan\beta l}$$

4. Impedance match (source is matched to the loaded line)
   $Z_{\mathrm{in}}(z)=Z_{\mathrm{g}}(z)$: no reflected wave, $\Gamma_{\mathrm{in}}(z)=0$, VSWR=1, RL= ∞dB

2.7 Lossy transmission lines

- low-loss line, R<<wL, G<<wC

$$\alpha \approx \frac{1}{2}(\frac{R}{Z_o}+GZ_o), \beta \approx w\sqrt{LC}, Z_o \approx \sqrt{\frac{L}{C}}$$

- distortionless line RC=LG

$$\alpha = R\sqrt{\frac{C}{L}} : \text{constant}, \beta = w\sqrt{LC} \rightarrow v_p = \frac{1}{\sqrt{LC}} : \text{constant}, \Delta t = \frac{\Delta l}{v_p} : \text{constant}$$

$$Z_o = \sqrt{\frac{L}{C}}$$

- perturbation method

  low-loss line (assume $\Gamma(z)=0$)

$$P(z) = P_o e^{-2\alpha z} \rightarrow \text{power loss per unit length: } P_l \equiv -\frac{\partial P}{\partial z} = 2\alpha P(z)$$

$$\Rightarrow \alpha = \frac{P_l(z)}{2P(z)} = \frac{P_l(z=0)}{2P_o}$$

Discussion

1. $\alpha = \alpha_c + \alpha_d$ (Ex. 2.6, 2.7 for a coaxial line, p.721 Appendix J)

a coaxial line

$$\alpha = \frac{1}{2}(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}) = \frac{R_s}{2\eta \ln\frac{b}{a}}(\frac{1}{a} + \frac{1}{b}) + \frac{w\varepsilon''\eta}{2} = \alpha_c + \alpha_d$$

2. Wheeler incremental inductance rule

$\alpha_c \rightarrow$ conductor loss $\rightarrow$ current flow inside conductor

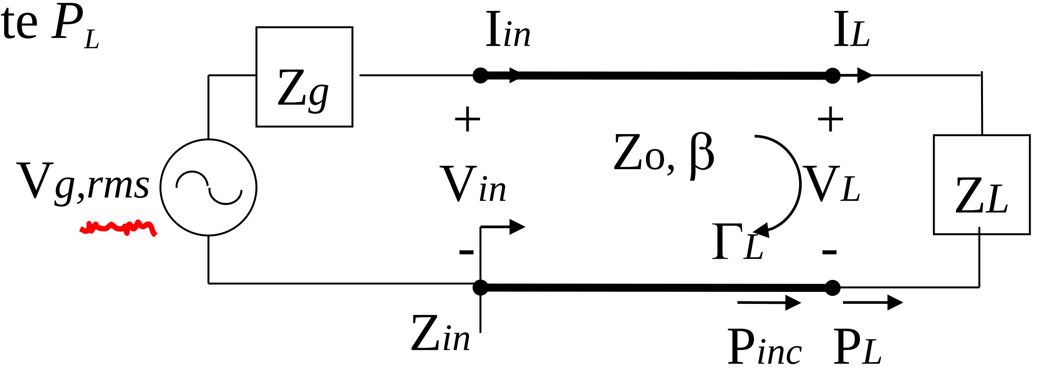
$\rightarrow$ ΔH in the conductor $\rightarrow$ $\Delta L \rightarrow \Delta Z_o$

$$\alpha_c = \frac{\omega \Delta L}{2Z_o} = \frac{R_s}{2Z_o\eta}\frac{dZ_o}{dl}$$

Solved Problems

Prob. 2.16 $V_{g,rms} = 15V, Z_g = 75\Omega, Z_o = 75\Omega, Z_L = 60 - j40\Omega, l = 0.7\lambda$

calculate P_L



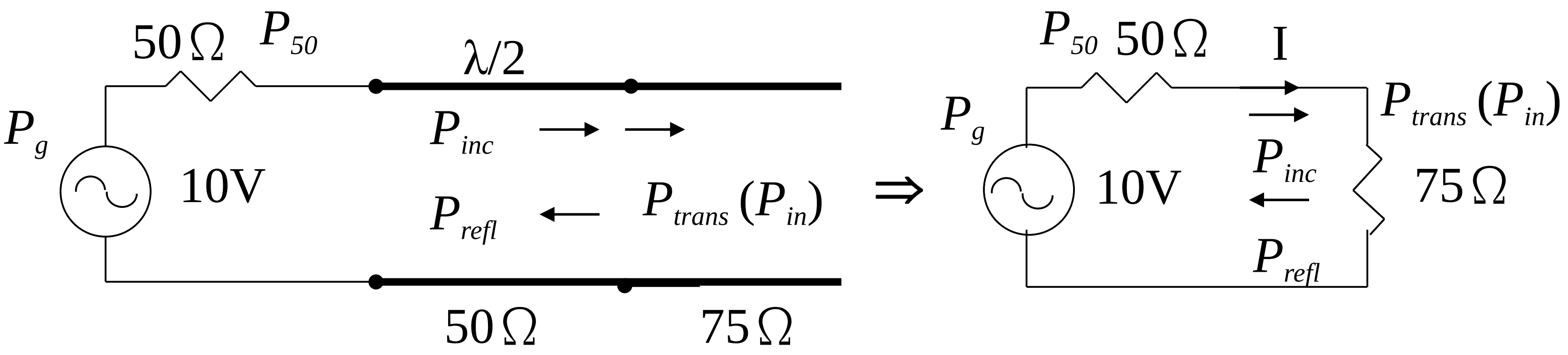
$$(1) P_L = P_{inc}(1 - |\Gamma_L|^2), P_{inc} = \frac{(V_g/2)^2}{Z_o}, \Gamma_L = \frac{Z_L - Z_o}{Z_L - Z_o} (\because Z_g = Z_o)$$

$$(2) P_L = |I_{in}|^2 R_{in} = \left| \frac{V_g}{Z_g + Z_{in}} \right|^2 \mathrm{Re}(Z_{in}), Z_{in} = Z_o \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}$$

$$(3) P_L = |I_L|^2 R_L = \left| \frac{V_L}{Z_L} \right|^2 \mathrm{Re}(Z_L), V_L = V_0^+(1 + \Gamma_L), V_0^+ = \frac{V_g}{2} (\because Z_g = Z_o)$$

for a lossy line: $V_{in} = V_0^+(e^{\gamma l} + \Gamma_L e^{-\gamma l}) = V_g \frac{Z_{in}}{Z_{in} + Z_g}, Z_{in} = Z_o \frac{Z_L + Z_o \tanh\gamma l}{Z_o + Z_L \tanh\gamma l} \rightarrow V_0^+ \xrightarrow{(3)} V_L \rightarrow P_L$

Prob. 2.18 Calculate P_{inc}, P_{refl}, P_{trans}



$$P_g = \frac{1}{2}\frac{10}{50+75}10 = 0.4W, P_{inc} = \frac{1}{2}(\frac{10}{50+50})^2 50 = 0.25W$$

$$P_{50\Omega} = \frac{1}{2}(\frac{10}{50+75})^2 50 = 0.16W, P_{trans} = P_{in} = \frac{1}{2}(\frac{10}{50+75})^2 75 = 0.24W$$

$$P_{refl} = P_{inc}\left|\Gamma\right|^2 = 0.25\left|\frac{75-50}{75+50}\right|^2 = 0.01W$$

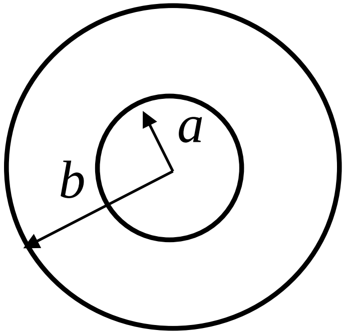
$$P_{inc} = P_{refl} + P_{trans}, P_g = P_{50\Omega} + P_{trans}(P_{in}) = P_{50\Omega} + P_{inc} - P_{refl} \Rightarrow \underline{P_g > P_{inc} \ge P_{in}}$$

For conjugate match: $Z_{in} = Z_g{}^*$, ($R_{in} = R_g$, impedance match)

$$P_{avs} = P_{in}\Big|_{Z_{in}=Z_g{}^*} = 0.25W(>0.24W) \to P_{50\Omega} = 0.25W, P_g = 0.5W, P_{refl} = 0W$$

Prob. 2.27 Find b/a and Zo for a coaxial line to have minimal α_c

$$\alpha_c = \frac{R_s}{2\eta \ln b/a}(\frac{1}{a}+\frac{1}{b})$$



$$\frac{\partial\alpha_c}{\partial a} = \frac{R_s}{2\eta}[\frac{1}{a}(\frac{1}{\ln b/a})^2(\frac{1}{a}+\frac{1}{b}) + \frac{1}{\ln b/a}(-\frac{1}{a^2})] = 0$$

$$a(\frac{1}{a}+\frac{1}{b}) = \ln\frac{b}{a}, 1+\frac{b}{a} = \frac{b}{a}\ln\frac{b}{a} \rightarrow \frac{b}{a} = 3.59$$

$$Z_o = \frac{\eta}{2\pi}\ln\frac{b}{a} \approx 77\Omega,\ \text{for } \varepsilon_r = 1$$

<u>ADS examples: Ch2_prj</u>