Chapter 3 Transmission Lines and Waveguides

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3.1 General solutions for TEM, TE and TM waves

- Procedure to analyze a TEM (Ez, Hz=0) line

1) solve $\Phi(x, y)$ from Laplace's equation $\nabla_t^2\Phi(x, y) = 0$

2) apply B.C. $\rightarrow$ $\Phi(x,y)$

3) $\bar{e}(x, y) = -\nabla_t\Phi(x, y) \rightarrow \bar{E}^+(x, y, z) = \bar{e}(x, y)e^{-j\beta z}$

$$\bar{h}(x, y) = \frac{1}{Z_{TEM}}\hat{z}\times\bar{e}(x, y),\ Z_{TEM} = \frac{E_x^+}{H_y^+} = \eta \rightarrow \bar{H}^+(x, y, z) = \bar{h}(x, y)e^{-j\beta z}$$

4) $V^+ = \Phi_1 - \Phi_2 = \int_1^2 \bar{E}^+ \bullet d\bar{l}\,,\quad I^+ = \oint \bar{H}^+ \bullet d\bar{l}$

5) $v_p = \dfrac{c}{\sqrt{\varepsilon_r}},\quad \beta = \dfrac{\omega}{v_p} = \omega\sqrt{\mu\varepsilon}\,,\ Z_o = \dfrac{V^+}{I^+} = \sqrt{\dfrac{L}{C}} = \dfrac{\sqrt{LC}}{C} = \dfrac{1}{v_pC}$

- Procedure to analyze a TE (*Ez*=0) or TM (*Hz*=0) line

1) solve $h_z(x,y)$ from Helmholtz equation $(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + k_C^2)h_z(x,y) = 0$: TE case

solve $e_z(x,y)$ from Helmholtz equation $(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + k_C^2)e_z(x,y) = 0$: TM case

2) apply B.C. to find k_c

3) $H_z^+(x,y,z) = h_z(x,y)e^{-j\beta z}$

$$H_x^+ = \frac{-j\beta}{k_c^2}\frac{\partial H_z^+}{\partial x},\ H_y^+ = \frac{-j\beta}{k_c^2}\frac{\partial H_z^+}{\partial y}, E_x^+ = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z^+}{\partial y},\ H_y^+ = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z^+}{\partial x}: \text{TE case}$$

$E_z^+(x,y,z) = e_z(x,y)e^{-j\beta z}$

$$H_x^+ = \frac{j\omega\varepsilon}{k_c^2}\frac{\partial E_z^+}{\partial y},\ H_y^+ = \frac{-j\omega\varepsilon}{k_c^2}\frac{\partial E_z^+}{\partial x}, E_x^+ = \frac{-j\beta}{k_c^2}\frac{\partial E_z^+}{\partial x},\ H_y^+ = \frac{-j\beta}{k_c^2}\frac{\partial E_z^+}{\partial y}: \text{TM case}$$

4) $\beta = \sqrt{k^2 - k_c^2}, Z_{TM}(= \frac{E_x^+}{H_y^+}) = \frac{\beta}{\omega\varepsilon} = \frac{\beta\eta}{k}, Z_{TE}(= \frac{E_x^+}{H_y^+}) = \frac{\omega\mu}{\beta} = \frac{k\eta}{\beta}$

- Dielectric loss

$$\gamma = \alpha_d + j\beta \rightarrow \alpha_d = \frac{k^2 \tan\delta}{2\beta}(NP/m): \text{TE or TM case}$$

$$\alpha_d = \frac{k \tan\delta}{2}(NP/m): \text{TEM case}$$

(*derivation of the dielectric loss expression*)

$$\textit{wave eq.}\ \nabla^2 \overline{E} + k^2 \overline{E} = 0 \rightarrow (\nabla_{xy}^2 + \nabla_z^2)\overline{E} + k^2 \overline{E} = 0 \rightarrow \nabla_{xy}^2 \overline{E} + (\gamma^2 + k^2)\overline{E} = 0$$

$$\nabla_{xy}^2 \overline{E} + k_c^2 \overline{E} = 0, k_c^2 \equiv \gamma^2 + k^2$$

$$\rightarrow \gamma \equiv \alpha_d + j\beta = \sqrt{k_c^2 - k_d^2}$$

$$\because k_d = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = \omega\sqrt{\mu\varepsilon'(1 - j\tan\delta)} = k\sqrt{1 - j\tan\delta}$$

$$\tan\delta \equiv \frac{\varepsilon''}{\varepsilon'}, k = \omega\sqrt{\mu\varepsilon'}$$

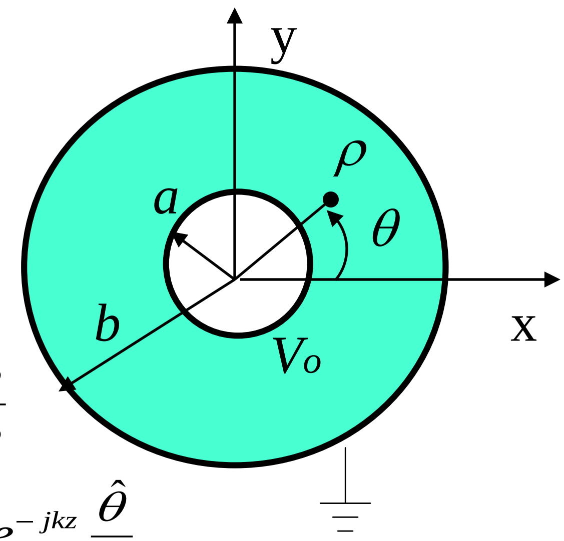
$$\rightarrow \gamma = \sqrt{k_c^2 - k^2(1 - j\tan\delta)} = \sqrt{k_c^2 - k^2 + jk^2\tan\delta}$$

$$\approx \sqrt{k_c^2 - k^2} + \frac{1}{2}\frac{jk^2\tan\delta}{\sqrt{k_c^2 - k^2}} = j\beta + \frac{1}{2}\frac{k^2\tan\delta}{\beta} = j\beta + \alpha_d$$

$$\Rightarrow \alpha_d = \frac{k^2\tan\delta}{2\beta}(NP/m) : \text{TE or TM case}$$

$$\alpha_d = \frac{k\tan\delta}{2}(NP/m) : \text{TEM case} \because k_c = 0$$

3.5 Coaxial line



- TEM mode

$$\Phi(\rho,\theta)=\frac{\ln b/\rho}{\ln b/a}V_o$$

$$\bar{E}^+(\rho,\theta,z)=-\nabla_t\Phi(\rho,\theta)e^{-jkz}=\frac{V_o}{\ln b/a}e^{-jkz}\frac{\hat{\rho}}{\rho}$$

$$\bar{H}^+(\rho,\theta,z)=\frac{1}{Z_{TEM}}\hat{z}\times\bar{E}^+(\rho,\theta,z)=\frac{V_o}{\eta\ln b/a}e^{-jkz}\frac{\hat{\theta}}{\rho}$$

$$V^+(z)=\int_a^b\bar{E}^+(\rho,\theta,z)\bullet d\bar{l}=V_oe^{-jkz}\equiv V_o^+e^{-jkz}$$

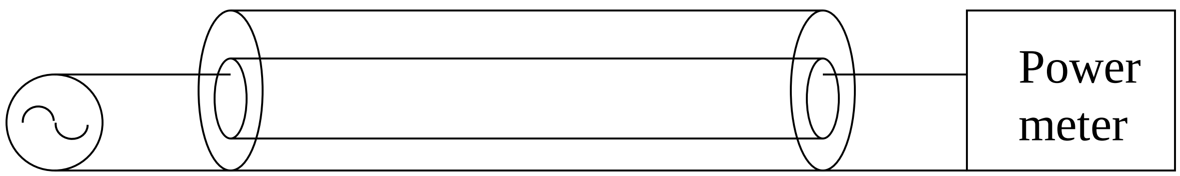
$$I^+(z)=\oint\bar{H}^+(\rho,\theta,z)\bullet d\bar{l}=\frac{2\pi V_o}{\eta\ln b/a}e^{-jkz}\equiv I_o^+e^{-jkz}$$

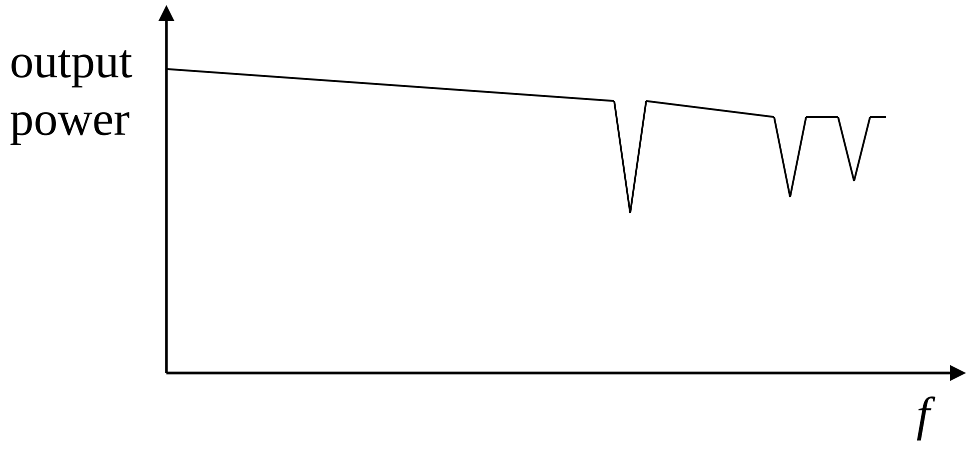
$$Z_o=\frac{V^+(z)}{I^+(z)}=\frac{\eta\ln b/a}{2\pi}=\frac{60}{\sqrt{\varepsilon_r}}\ln\frac{b}{a}=\sqrt{\frac{L}{C}},\beta=k=\omega\sqrt{\mu\varepsilon}$$

$$L=\frac{\mu}{2\pi}\ln\frac{b}{a},C=\frac{2\pi\varepsilon}{\ln b/a}\text{(Table 2.1, p.54)}$$

• Higher order mode effect

TE_{11} mode (p.133 Fig.3.17) $f_{max} < f_c = \dfrac{1}{\pi(a+b)\sqrt{\mu\varepsilon}}$





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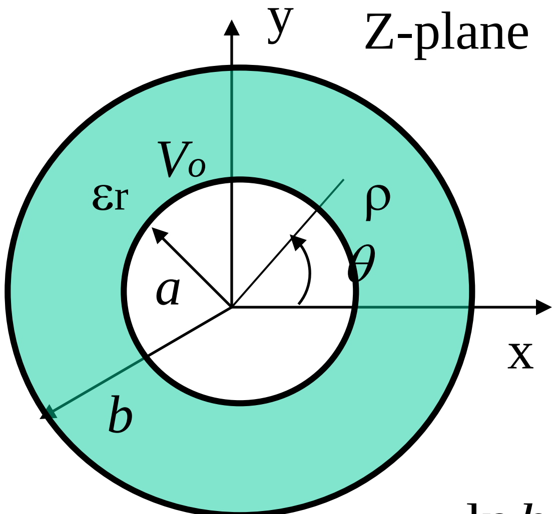
Discussion

1. Conformal mapping solution (Collin, *Field theory of guided waves*, p.262)

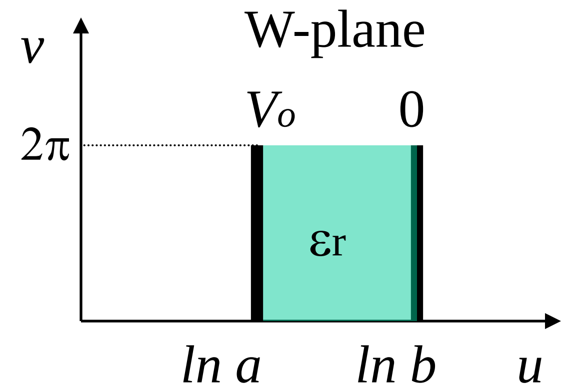


$$Z = x + jy$$

$$W = \ln Z = \ln \rho e^{j\theta}$$

$$= \ln \rho + j\theta$$

$$= u + jv$$



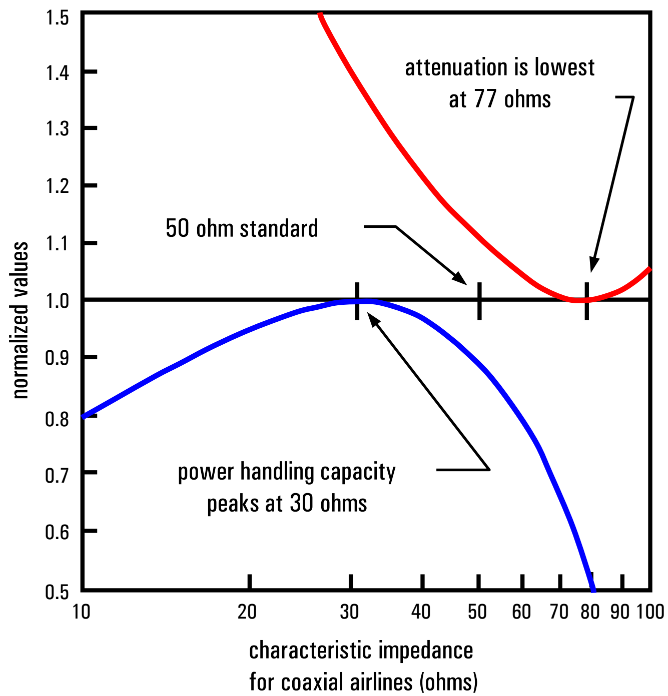
$$\Phi = \frac{\ln b - u}{\ln b - \ln a} V_o$$

$$W_e = \frac{1}{2}\varepsilon \int_0^{2\pi}\int_{\ln a}^{\ln b} \left(\frac{\partial \Phi}{\partial u}\right)^2 + \left(\frac{\partial \Phi}{\partial v}\right)^2 du dv = \frac{\pi \varepsilon V_o^2}{\ln b/a} = \frac{1}{2} C V_o^2$$

$$\Rightarrow C = \frac{2\pi\varepsilon}{\ln b/a}, Z_o = \frac{1}{v_p C} = \frac{\sqrt{\mu\varepsilon}}{C} = \frac{\eta}{2\pi} \ln \frac{b}{a}, \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

2. Reasons for selecting Z_o=50Ω

Coaxial line has minimum attenuation as Z_o=77Ω (Prob. 2-27), and maximum power capacity as Z_o=30Ω (Prob. 3-28).

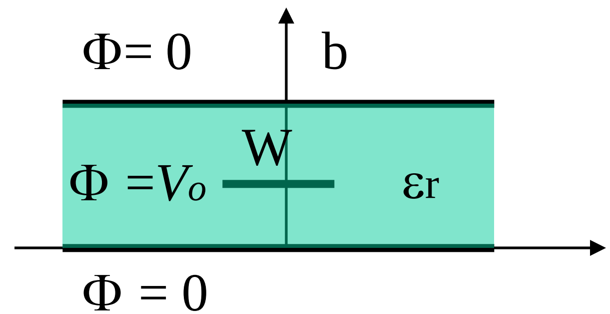


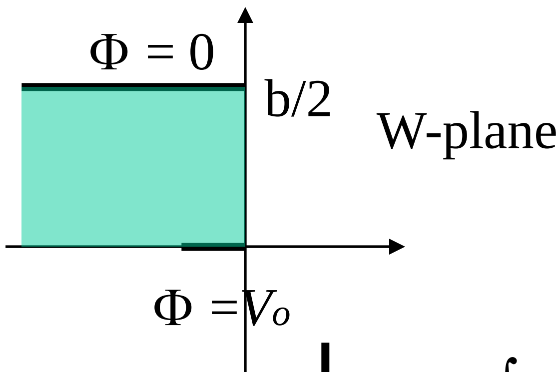
3. Types of coaxial connectors

type-N, SMA, APC3.5, APC2.4,….. (p.134, point of interest)

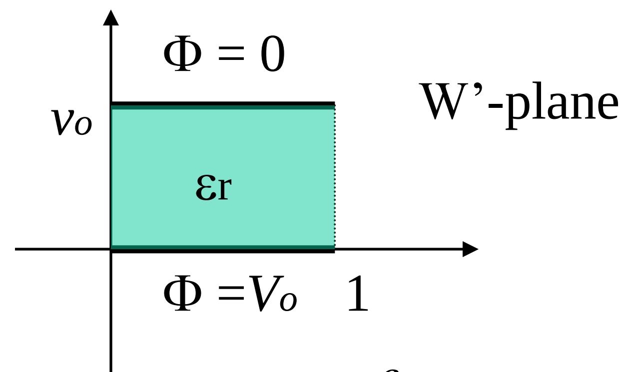
3.7 Stripline

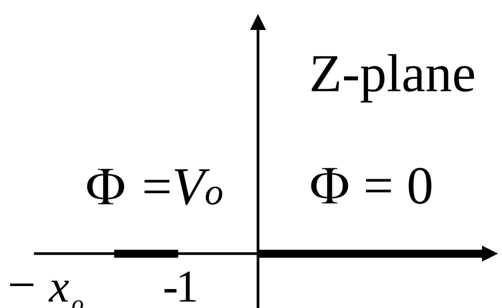
- Conformal mapping solution
  (Collin's book, p.265)





$$W = A\int \frac{dZ}{\sqrt{Z(Z+1)}} + B$$





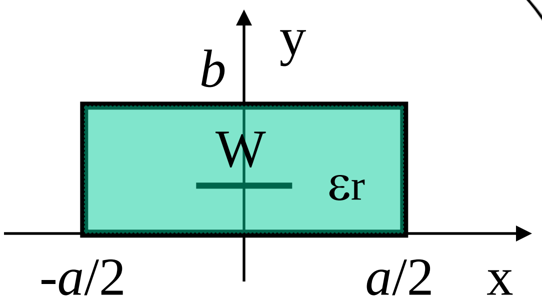
$$-x_o = -\cosh^2 \frac{\pi w}{2b}$$

$$W' = A_1\int \frac{dZ}{\sqrt{Z(Z+x_o)(Z+1)}} + B_o$$

$$C = \frac{\varepsilon}{v_o}$$

$$\rightarrow C = \frac{4\varepsilon}{v_o}, \quad Z_o = \frac{\sqrt{\mu\varepsilon}}{C}$$

- Electrostatic solution



$$\nabla_t^2 \Phi(x,y) = 0 \qquad |x| < a/2, \;\; 0 < y < b$$

$$B.C. \quad \Phi(x,y) = 0 \;\; x = \pm a/2, \;\; y = 0, b$$

$$\Phi(x,y) = \begin{cases} \sum_{odd} A_n \cos\frac{n\pi}{a} x \sinh\frac{n\pi}{a} y & 0 \le y \le b/2 \\ \sum_{odd} A_n \cos\frac{n\pi}{a} x \sinh\frac{n\pi}{a}(b-y) & b/2 \le y \le b \end{cases}$$

$$E_y = -\frac{\partial \Phi}{\partial y} = \begin{cases} -\sum_{odd} A_n (\frac{n\pi}{a}) \cos\frac{n\pi}{a} x \cosh\frac{n\pi}{a} y & 0 \le y \le b/2 \\ \sum_{odd} A_n (\frac{n\pi}{a}) \cos\frac{n\pi}{a} x \cosh\frac{n\pi}{a}(b-y) & b/2 \le y \le b \end{cases}$$

$$\rho_s(x) = D_y(x, y = b^+/2) - D_y(x, y = b^-/2) \cong \begin{cases} 1 & |x| < W/2 \\ 0 & |x| > W/2 \end{cases} \rightarrow A_n \ldots (3.189)$$

$$V = -\int_0^{b/2} E_y(x=0,y)dy = 2\sum_{odd} A_n \sinh\frac{n\pi}{4a} b, \quad Q = \int_{-W/2}^{W/2} \rho_s(x)dx = W$$

$$\Rightarrow \quad C = \frac{Q}{V} \ldots (3.192), \quad Z_o = \sqrt{\frac{L}{C}} = \frac{1}{v_p C} = \frac{\sqrt{\varepsilon_r}}{cC}$$

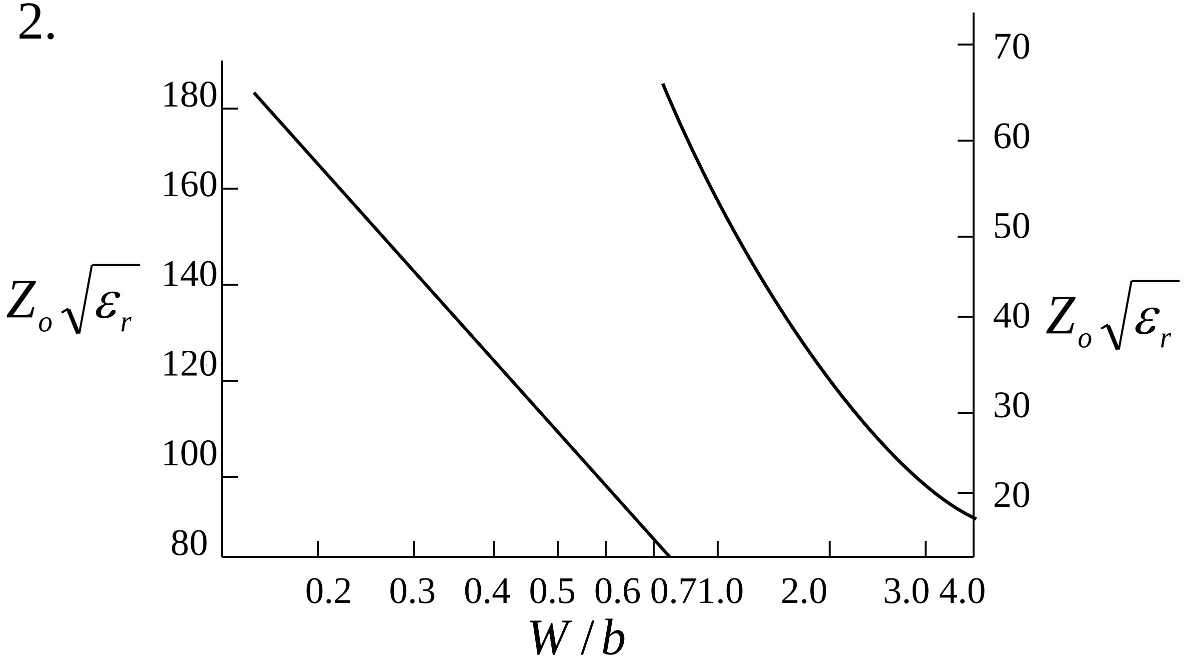
Discussion

1. analysis eq.(3.179) $W, b, \varepsilon \rightarrow Z_o$

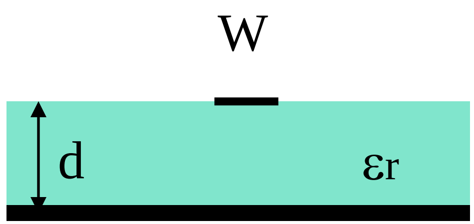
synthesis eq.(3.180) $Z_o, b, \varepsilon \rightarrow W$

$$\alpha_d = \frac{k \tan\delta}{2}, \ \alpha_c \ \text{eq.(3.181)}$$

2.



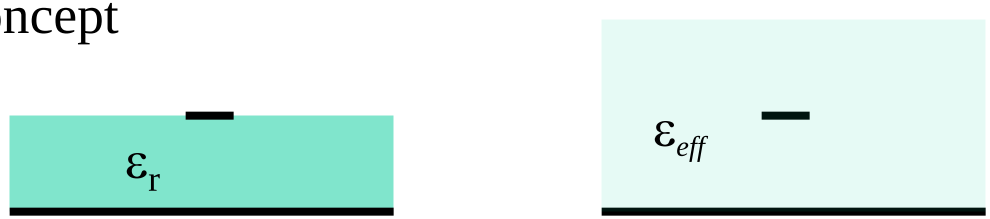
3.8 Microstrip



- Characteristics

  fabrication by printed circuit
  devices can be bonded to strip
  component are accessible
  in-circuit characterization of devices is straightforward to implement
  dc as well as ac signals can be transmitted
  large variation in Z_o
  $\alpha_c > \alpha_d$
  monolithic applications
  structure is rugged and can withstand high voltages and power levels
  power handling is best with BeO substrate
  used up to 300GHz or more
  quasi-TEM mode for $d \ll \lambda$

- ε_{eff} concept



TEM line $Z_o = \sqrt{\dfrac{L}{C}},\ v_p = \dfrac{c}{\sqrt{\varepsilon_r}}, \beta = \omega\sqrt{\mu\varepsilon} = k_o\sqrt{\varepsilon_r}, \lambda = \dfrac{\lambda_o}{\sqrt{\varepsilon_r}}$

air filled microstrip (TEM line) $Z_{oa} = \sqrt{\dfrac{L}{C_a}} = \dfrac{1}{cC_a}$

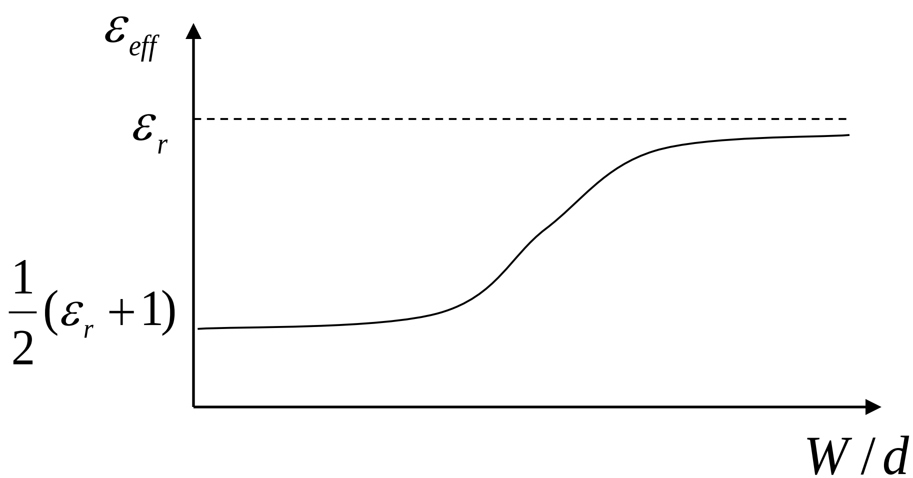
microstrip (quasi-TEM line) $\mathrm{Z_o} = \sqrt{\dfrac{L}{\mathrm{C}}} = \dfrac{1}{v_\mathrm{p}\mathrm{C}},\ v_\mathrm{p} = \dfrac{1}{\sqrt{LC}} = \dfrac{\mathrm{c}}{\sqrt{\varepsilon_{eff}}}, \beta = \mathrm{k_o}\sqrt{\varepsilon_{eff}},$

$$\lambda_\mathrm{g} = \frac{\lambda_o}{\sqrt{\varepsilon_{eff}}} \quad \Rightarrow \quad \frac{\mathrm{Z_o}}{Z_{oa}} = \sqrt{\frac{C_a}{\mathrm{C}}} = \frac{1}{\sqrt{\varepsilon_{eff}}}, \qquad \frac{\mathrm{C}}{C_a} = \left(\frac{c}{v_\mathrm{p}}\right)^2 \equiv \varepsilon_{eff}$$

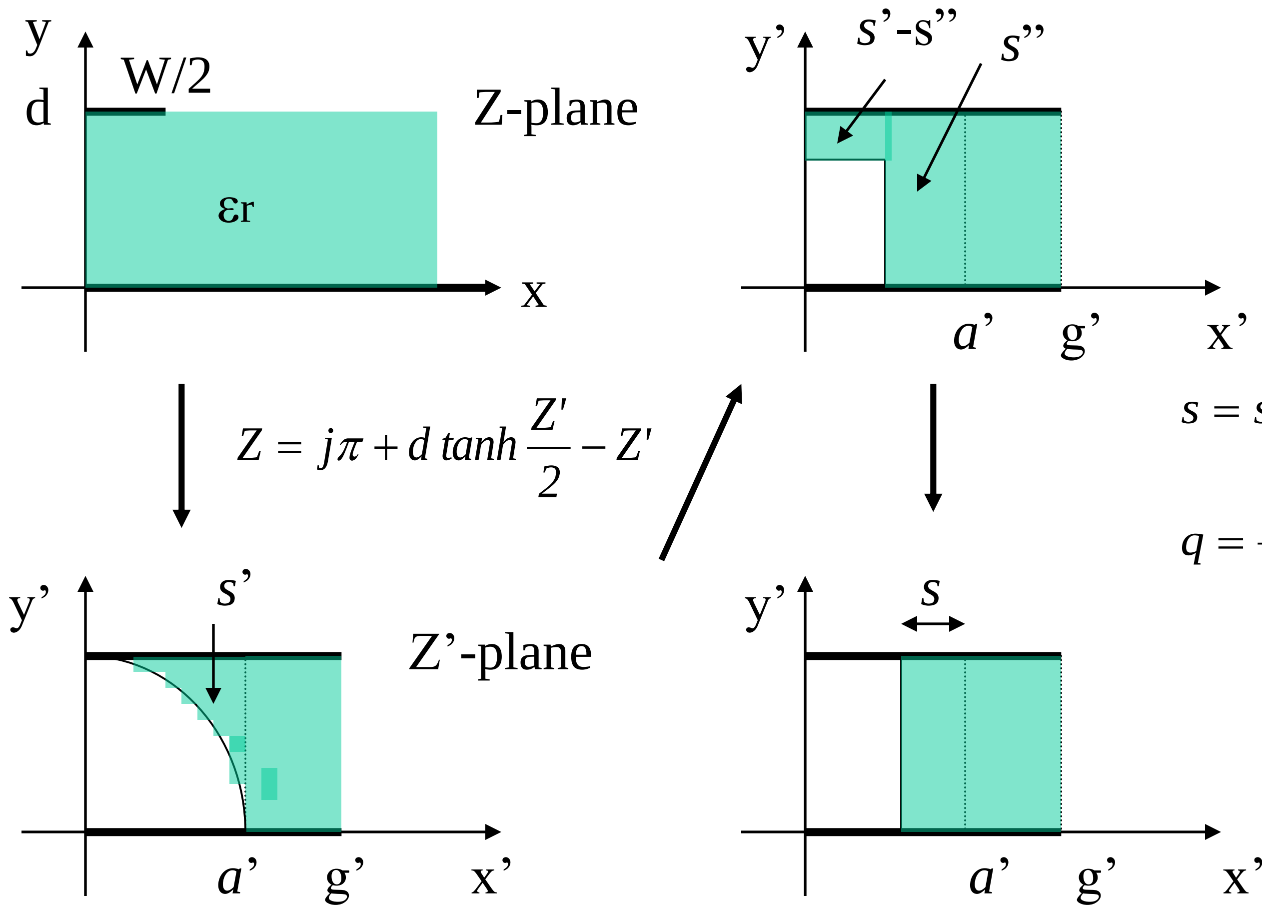
$$\varepsilon_{eff} \to \frac{1}{2}(\varepsilon_r + 1) \qquad\qquad \varepsilon_{eff} \to \varepsilon_r$$

$$\frac{1}{2}(\varepsilon_r + 1) \le \varepsilon_{eff} \le \varepsilon_r$$

filling factor q $\qquad \varepsilon_{eff} = q\varepsilon_r + (1-q)1 = 1 + q(\varepsilon_r - 1)\,, \quad \frac{1}{2} \le q \le 1$



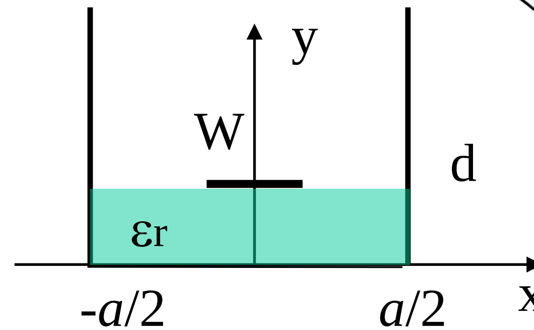
• Conformal mapping solution (Gupta, Garg and Bahl, *Microstrip lines and slotlines*, p.9)



$$s = s'' + \frac{s' - s''}{\varepsilon_r}$$

$$q = \frac{g' - a' + s}{g'}$$

- Electrostatic solution



$$\nabla_t^2 \Phi(x,y) = 0 \qquad |x| \le a/2,\ \ 0 \le y < \infty$$

$$B.C. \quad \Phi(x,y) = 0 \ \ x = \pm a/2, \ \ y = 0, \infty$$

$$\Phi(x,y) = \begin{cases} \sum_{odd} A_n \cos\frac{n\pi}{a} x \sinh\frac{n\pi}{a} y & 0 \le y < d \\ \sum_{odd} A_n \cos\frac{n\pi}{a} x \sinh\frac{n\pi}{a} d e^{-n\pi(y-d)/a} & d \le y < \infty \end{cases}$$

$$E_y = -\frac{\partial \Phi}{\partial y} = \begin{cases} -\sum_{odd} A_n (\frac{n\pi}{a}) \cos\frac{n\pi}{a} x \cosh\frac{n\pi}{a} y & 0 \le y < d \\ \sum_{odd} A_n (\frac{n\pi}{a}) \cos\frac{n\pi}{a} x \sinh\frac{n\pi}{a} d\ e^{-n\pi(y-d)/a} & d \le y < \infty \end{cases}$$

$$\rho_s(x) = D_y(x, y = d^+) - D_y(x, y = d^-) = \varepsilon_o E_y(x, y = d^+) - \varepsilon_o \varepsilon_r E_y(x, y = d^-) \cong \begin{cases} 1 & |x| < W/2 \\ 0 & |x| > W/2 \end{cases} \to A_n$$

$$V = -\int_0^d E_y(x=0, y) dy = \sum_{odd} A_n \sinh\frac{n\pi}{a} d, \ \ Q = \int_{-W/2}^{W/2} \rho_s(x) dx = W$$

$$\Rightarrow \ C = \frac{Q}{V}, \ \ \varepsilon_{eff} = \frac{C}{C_a}, \qquad Z_o = \sqrt{\frac{L}{C}} = \frac{1}{v_p C} = \frac{\sqrt{\varepsilon_{eff}}}{cC}$$

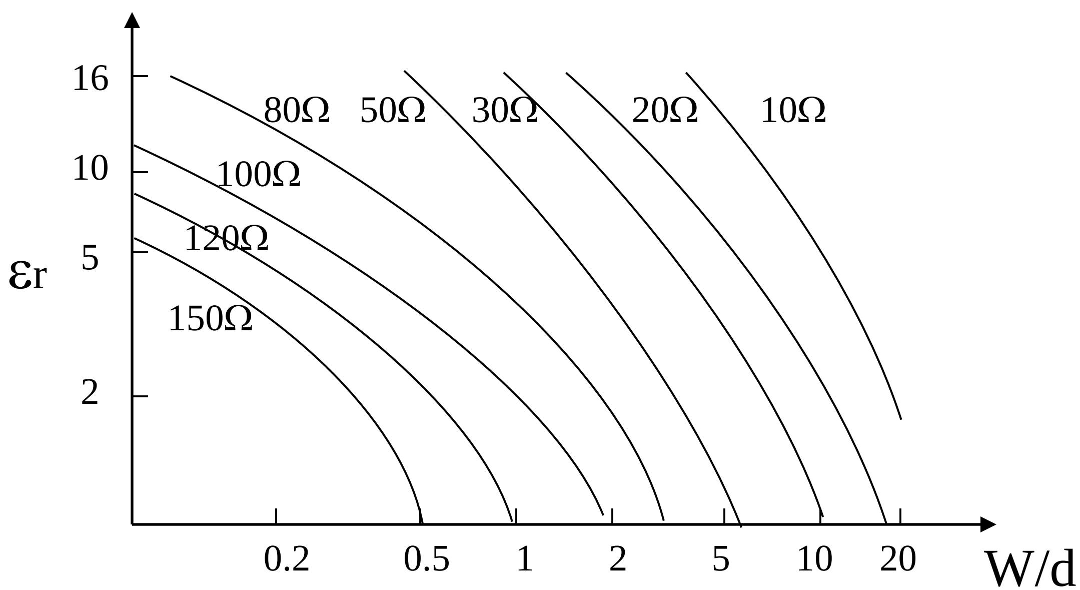
<u>Discussion</u>

1. analysis eq.(3.196, 195) $W, d, \varepsilon \rightarrow Z_o, \varepsilon_{eff}$

 synthesis eq.(3.197, 195) $Z_o, d, \varepsilon \rightarrow W, \varepsilon_{eff}$

$$\alpha_d = \frac{k_o \varepsilon_r (\varepsilon_{eff} - 1) \tan\delta}{2\sqrt{\varepsilon_{eff}}(\varepsilon_r - 1)} (3.198), \quad \alpha_c = \frac{R_s}{Z_o W} (3.199)$$

2.



$(derivation\ of\ eq.(3.198))$

TEM line $\alpha_d = \dfrac{k \tan\delta}{2} = \dfrac{k_o\sqrt{\varepsilon_r}\tan\delta}{2}$

microstrip line:

(1)$\varepsilon_{eff} = q\varepsilon_r + (1-q) = q(\varepsilon_r - 1) + 1 \rightarrow q = \dfrac{\varepsilon_{eff} - 1}{\varepsilon_r - 1}$

(2) $\sqrt{\varepsilon_r} \rightarrow \sqrt{\varepsilon_{eff}}$

(3)$\tan\delta \rightarrow \tan\delta_{eff} = \dfrac{\varepsilon''_{eff}}{\varepsilon_{eff}}, \varepsilon''_{eff} = q\varepsilon'' + 0(1-q) = q\varepsilon''$

$$\alpha_d = \frac{k_o\sqrt{\varepsilon_{eff}}\tan\delta_{eff}}{2} = \frac{k_o\sqrt{\varepsilon_{eff}}}{2}\frac{q\varepsilon''}{\varepsilon_{eff}} = \frac{k_o q\varepsilon''}{2\sqrt{\varepsilon_{eff}}} = \frac{k_o q}{2\sqrt{\varepsilon_{eff}}}\frac{\varepsilon''}{\varepsilon_r}\varepsilon_r$$

$$= \frac{k_o}{2\sqrt{\varepsilon_{eff}}}\frac{\varepsilon_{eff}-1}{\varepsilon_r - 1}\tan\delta\varepsilon_r = \frac{k_o\varepsilon_r\tan\delta}{2\sqrt{\varepsilon_{eff}}}\frac{\varepsilon_{eff}-1}{\varepsilon_r-1}$$

4. substrate material

	ε_r	tanδ@10GHz
GaAs	12.9	0.002
Al_2O_3(99.5%)	10.1±0.25	0.0002
quartz	3.8	0.0001
RT/duroid 5880	2.2 ±0.02	0.0009
RT/duroid 6006	6.15±0.15	0.0027
BeO	6.8	0.0003
FR4 (G-10)	4-4.6	0.018-0.025
Si	11.7	0.005
* SiO_2	4	
* Si_3N_4	7.6 (vapor phase) 6.5 (sputtering)	

*: MIM capacitor

conductor material

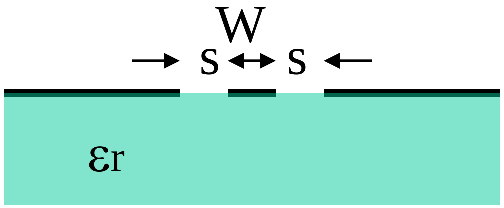
	R_s (Ω/sq $\times 10^{-7}\sqrt{f}$)	δ_s(um)@2GHz
Ag	2.5	1.4
Cu	2.6	1.5
Au	3.0	1.7
Al	3.3	1.9
Cr	4.7	2.7
Ta	7.2	4.0

1 oz Cu $\rightarrow$ 1.4 mil (35 um) thickness

skin depth $\delta_s = \sqrt{\dfrac{2}{\omega\mu\sigma}}$

3.11 Summary of transmission lines and waveguides

- comparison of coaxial line, stripline and microstrip, (p.158, Table 3.6)
- CPW (coplanar waveguide)



  fabrication easy
  quasi-TEM operation
  radiation problem when gap width approaches $\lambda/2$
  monolithic applications
  less radiation than microstrip if well-balanced
  higher order modes (coupled slot mode)
- www.appwave.com for computer assisted transmission line analysis

Solved Problems

Prob. 3.28 Find Z_o of a coaxial line to have maximum power capacity

breakdown field strength of air $E_d = 3\times 10^6 V/m = \dfrac{V_o}{\rho \ln \frac{b}{a}} \to V_{max} = E_d a \ln \dfrac{b}{a}$

maximum power capacity $P_{max} = \dfrac{1}{2}\dfrac{V_{max}^2}{Z_o}, Z_o = \dfrac{\eta_o}{2\pi}\ln\dfrac{b}{a} \to P_{max} = \dfrac{\pi a^2 E_d^2}{\eta_o}\ln\dfrac{b}{a}$

$$\frac{dP_{max}}{da} = 0 \to \ln\frac{b}{a} = \frac{1}{2} \Rightarrow Z_o = \frac{\eta_o}{2\pi}\ln\frac{b}{a} = \frac{377}{2\pi}\frac{1}{2} \approx 30\Omega$$

ADS examples: Ch3_prj