Chapter 3 Transmission Lines and Waveguides

3.1 General solutions for TEM, TE and TM waves

procedures, α_d

3.5 Coaxial line (TEM line)

TEM mode, higher order mode effect

3.7 Stripline (TEM line)

conformal mapping solution, electrostatic solution

3.8 Microstrip (quasi-TEM line)

ε_{eff} concept, conformal mapping solution, electrostatic solution

3.11 Summary of transmission lines and waveguides

3.1 General solutions for TEM, TE and TM waves

- Procedure to analyze a TEM (Ez, Hz=0) line

1) solve $\Phi(x, y)$ from Laplace's equation $\nabla_t^2\Phi(x, y) = 0$

2) apply B.C. $\rightarrow$ $\Phi(x,y)$

3) $\bar{e}(x, y) = -\nabla_t\Phi(x, y) \rightarrow \bar{E}(x, y, z) = \bar{e}(x, y)e^{-j\beta z}$

$$\bar{h}(x, y) = \frac{1}{Z_{TEM}}\hat{z}\times\bar{e}(x, y),\ Z_{TEM} = \frac{E_x}{H_y} = \eta \rightarrow \bar{H}(x, y, z) = \bar{h}(x, y)e^{-j\beta z}$$

4) $V^+ = \Phi_1 - \Phi_2 = \int_1^2 \bar{E}\bullet d\bar{l},\quad I^+ = \oint \bar{H}\bullet d\bar{l}$

5) $v_p = \frac{c}{\sqrt{\varepsilon_r}},\ \beta = \frac{\omega}{v_p} = \omega\sqrt{\mu\varepsilon},\ Z_o = \frac{V^+}{I^+} = \sqrt{\frac{L}{C}} = \frac{\sqrt{LC}}{C} = \frac{1}{v_p C}$

- Procedure to analyze a TE (Ez=0) or TM (Hz=0) line

1) solve $h_z(x, y)$ from Helmholtz equation $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_C^2)h_z(x, y) = 0$: TE case

solve $e_z(x, y)$ from Helmholtz equation $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_C^2)e_z(x, y) = 0$: TM case

2) apply B.C. to find k_c

3) $H_z(x,y,z) = h_z(x,y)e^{-j\beta z}$

$$H_x = \frac{-j\beta}{k_c^2}\frac{\partial H_z}{\partial x},\ H_y = \frac{-j\beta}{k_c^2}\frac{\partial H_z}{\partial y}, E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y},\ H_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}: \text{TE case}$$

$E_z(x,y,z) = e_z(x,y)e^{-j\beta z}$

$$H_x = \frac{j\omega\varepsilon}{k_c^2}\frac{\partial E_z}{\partial y},\ H_y = \frac{-j\omega\varepsilon}{k_c^2}\frac{\partial E_z}{\partial x}, E_x = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial x},\ H_y = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial y}: \text{TM case}$$

4) $\beta = \sqrt{k^2 - k_c^2}, Z_{TM}(= \frac{E_x}{H_y}) = \frac{\beta}{\omega\varepsilon} = \frac{\beta\eta}{k}, Z_{TE}(= \frac{E_x}{H_y}) = \frac{\omega\mu}{\beta} = \frac{k\eta}{\beta}$

- Dielectric loss

$$\gamma = \alpha_d + j\beta \rightarrow \alpha_d = \frac{k^2 \tan\delta}{2\beta}(NP/m): \text{TE or TM case}$$

$$\alpha_d = \frac{k \tan\delta}{2}(NP/m): \text{TEM case}$$

(*derivation of the dielectric loss expression*)

$$wave\ eq.\ \nabla^2 \overline{E} + k^2 \overline{E} = 0 \rightarrow (\nabla_{xy}^2 + \nabla_z^2)\overline{E} + k^2 \overline{E} = 0 \rightarrow \nabla_{xy}^2 \overline{E} + (\gamma^2 + k^2)\overline{E} = 0$$

$$\nabla_{xy}^2 \overline{E} + k_c^2 \overline{E} = 0, k_c^2 \equiv \gamma^2 + k^2$$

$$\rightarrow \gamma \equiv \alpha_d + j\beta = \sqrt{k_c^2 - k_d^2}$$

$$\because k_d = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = \omega\sqrt{\mu\varepsilon'(1 - j\tan\delta)} = k\sqrt{1 - j\tan\delta}$$

$$\tan\delta \equiv \frac{\varepsilon''}{\varepsilon'}, k = \omega\sqrt{\mu\varepsilon'}$$

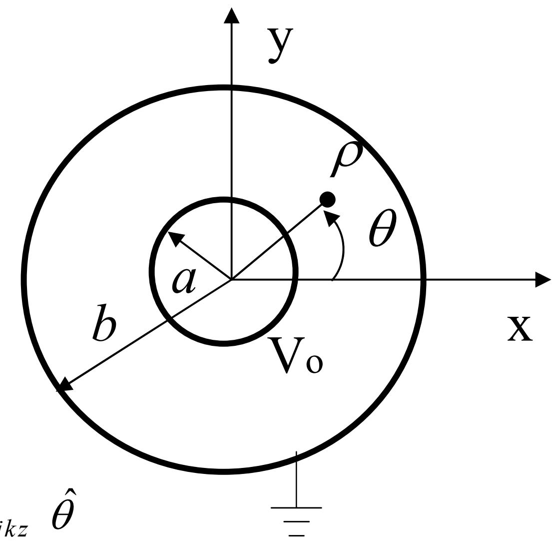
$$\rightarrow \gamma = \sqrt{k_c^2 - k^2(1 - j\tan\delta)} = \sqrt{k_c^2 - k^2 + jk^2\tan\delta}$$

$$\approx \sqrt{k_c^2 - k^2} + \frac{1}{2}\frac{jk^2\tan\delta}{\sqrt{k_c^2 - k^2}} = j\beta + \frac{1}{2}\frac{k^2\tan\delta}{\beta} = j\beta + \alpha_d$$

$$\Rightarrow \alpha_d = \frac{k^2\tan\delta}{2\beta}(NP/m) : \text{TE or TM case}$$

$$\alpha_d = \frac{k\tan\delta}{2}(NP/m) : \text{TEM case} \because k_c = 0$$

3.5 Coaxial line



- TEM mode

$$\Phi(\rho,\theta)=\frac{\ln b/\rho}{\ln b/a}V_o$$

$$\overline{E}(\rho,\theta,z)=-\nabla_t\Phi(\rho,\theta)e^{-jkz}=\frac{V_o}{\ln b/a}e^{-jkz}\frac{\hat{\rho}}{\rho}$$

$$\overline{H}(\rho,\theta,z)=\frac{1}{Z_{TEM}}\hat{z}\times\overline{E}(\rho,\theta,z)=\frac{V_o}{\eta\ln b/a}e^{-jkz}\frac{\hat{\theta}}{\rho}$$

$$V^+(z)=\int_a^b\overline{E}(\rho,\theta,z)\bullet d\overline{l}=V_oe^{-jkz}\equiv V_o^+e^{-jkz}$$

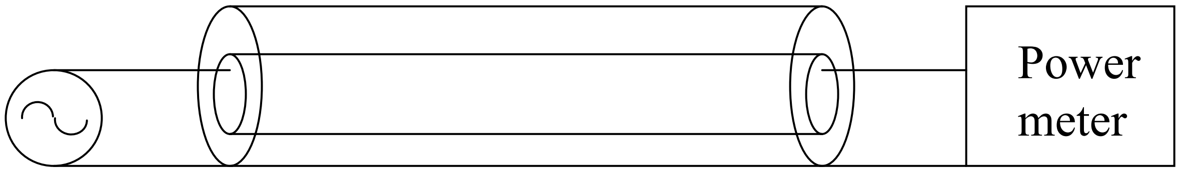
$$I^+(z)=\oint\overline{H}(\rho,\theta,z)\bullet d\overline{l}=\frac{2\pi V_o}{\eta\ln b/a}e^{-jkz}\equiv I_o^+e^{-jkz}$$

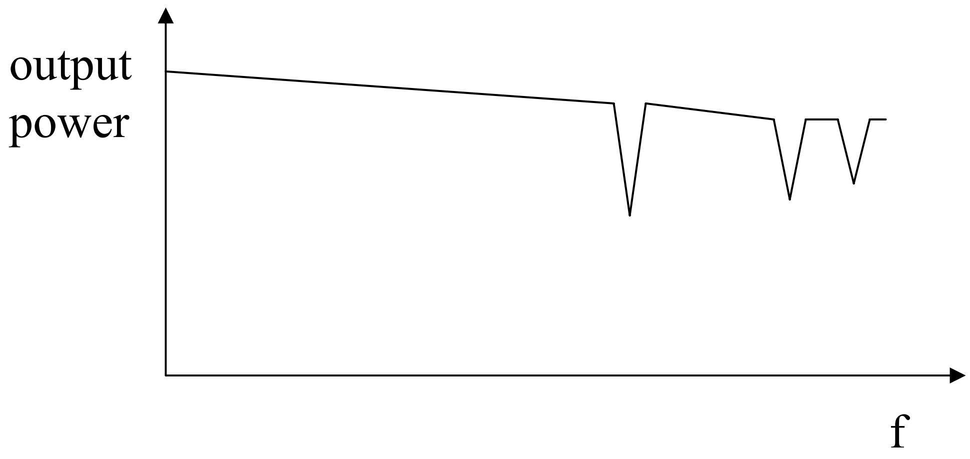
$$Z_o=\frac{V^+(z)}{I^+(z)}=\frac{\eta\ln b/a}{2\pi}=\frac{60}{\sqrt{\varepsilon_r}}\ln\frac{b}{a}=\sqrt{\frac{L}{C}},\beta=k=\omega\sqrt{\mu\varepsilon}$$

$$L=\frac{\mu}{2\pi}\ln\frac{b}{a},C=\frac{2\pi\varepsilon}{\ln b/a}(\text{Table 2.1, p.55})$$

• Higher order mode effect

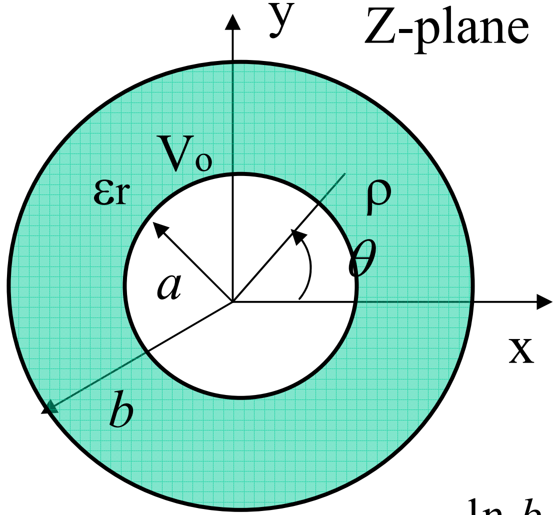
TE_{11} mode (p.129 Fig.3.17) $$f_{max} < f_c = \frac{1}{\pi(a+b)\sqrt{\mu\varepsilon}}$$



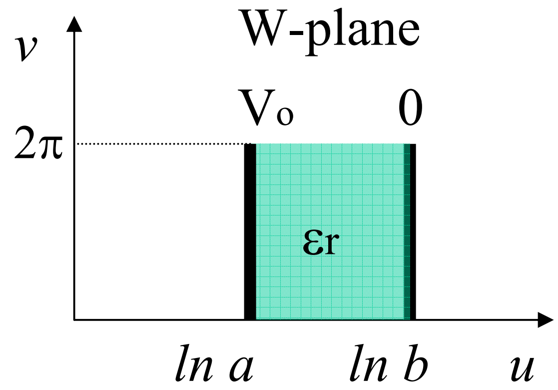


Discussion

1. Conformal mapping solution (Collin, *Field theory of guided waves*, p.262)



$$Z = x + jy$$
$$W = \ln Z = \ln \rho e^{j\theta}$$
$$= \ln \rho + j\theta$$
$$= u + jv$$



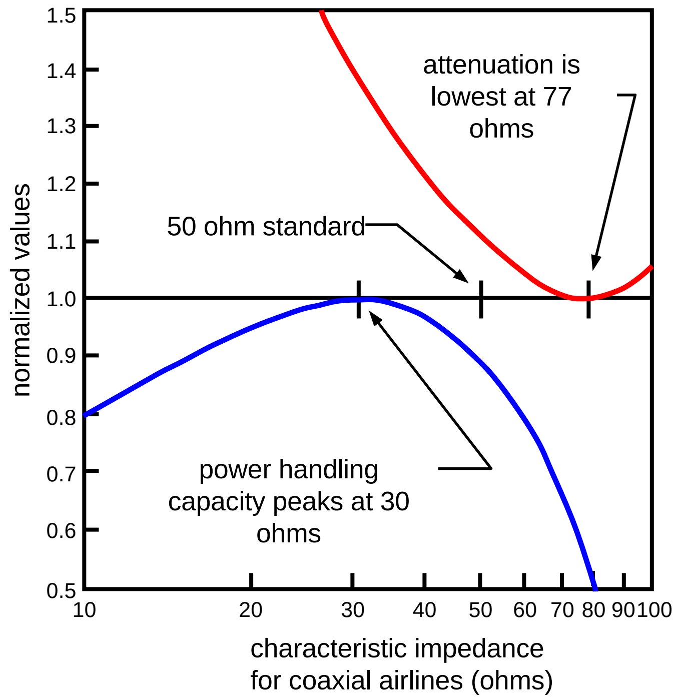
$$\Phi = \frac{\ln b - u}{\ln b - \ln a} V_o$$

$$W_e = \frac{1}{2}\varepsilon \int_0^{2\pi}\int_{\ln a}^{\ln b} \left(\frac{\partial \Phi}{\partial u}\right)^2 + \left(\frac{\partial \Phi}{\partial v}\right)^2 du\, dv = \frac{\pi \varepsilon V_o^2}{\ln b/a} = \frac{1}{2} C V_o^2$$

$$\Rightarrow C = \frac{2\pi\varepsilon}{\ln b/a}, Z_o = \frac{1}{v_p C} = \frac{\sqrt{\mu\varepsilon}}{C} = \frac{\eta}{2\pi}\ln\frac{b}{a}, \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

2. Reasons for selecting Z_o=50Ω

Coaxial line has minimum attenuation as Z_o=77Ω (Prob. 2-28), and maximum power capacity as Z_o=30Ω (Prob. 3-28).

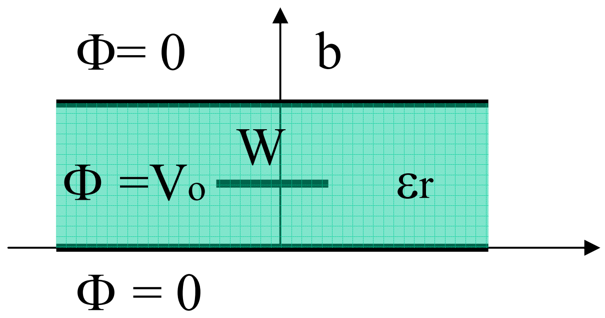


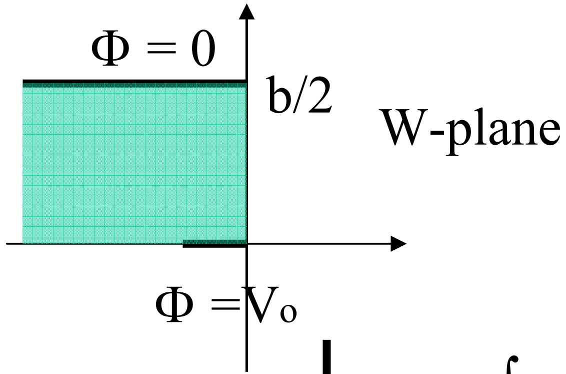
3. Types of coaxial connectors

type-N, SMA, APC3.5, APC2.4,….. (p.130, point of interest)

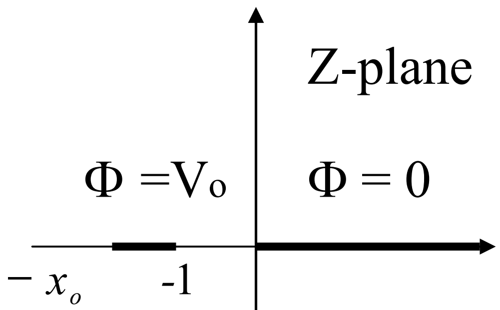
3.7 Stripline

- Conformal mapping solution
  (Collin's book, p.265)



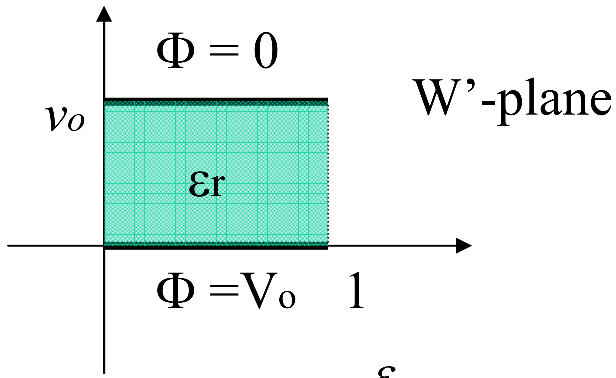


$$W = A\int \frac{dZ}{\sqrt{Z(Z+1)}} + B$$



$$-x_o = -\cosh^2\frac{\pi w}{2b}$$

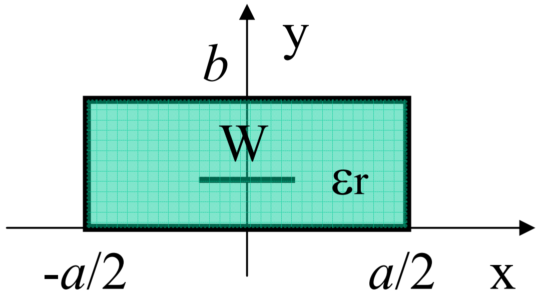
$$W' = A_1\int \frac{dZ}{\sqrt{Z(Z+x_o)(Z+1)}} + B_o$$



$$C = \frac{\varepsilon}{v_o}$$

$$\rightarrow C = \frac{4\varepsilon}{v_o},\quad Z_o = \frac{\sqrt{\mu\varepsilon}}{C}$$

- Electrostatic solution



$$\nabla_t^2 \Phi(x,y) = 0 \qquad |x| < a/2, \;\; 0 < y < b$$

$$B.C. \quad \Phi(x,y) = 0 \;\; x = \pm a/2, \;\; y = 0, b$$

$$\Phi(x,y) = \begin{cases} \sum_{odd} A_n \cos\frac{n\pi}{a}x \sinh\frac{n\pi}{a}y & 0 \le y \le b/2 \\ \sum_{odd} A_n \cos\frac{n\pi}{a}x \sinh\frac{n\pi}{a}(b-y) & b/2 \le y \le b \end{cases}$$

$$E_y = -\frac{\partial \Phi}{\partial y} = \begin{cases} -\sum_{odd} A_n (\frac{n\pi}{a}) \cos\frac{n\pi}{a}x \cosh\frac{n\pi}{a}y & 0 \le y \le b/2 \\ \sum_{odd} A_n (\frac{n\pi}{a}) \cos\frac{n\pi}{a}x \cosh\frac{n\pi}{a}(b-y) & b/2 \le y \le b \end{cases}$$

$$\rho_s(x) = D_y(x, y = b^+/2) - D_y(x, y = b^-/2) \cong \begin{cases} 1 & |x| < W/2 \\ 0 & |x| > W/2 \end{cases} \rightarrow A_n \ldots(3.189)$$

$$V = -\int_0^{b/2} E_y(x=0,y)dy = 2\sum_{odd} A_n \sinh\frac{n\pi}{4a}b, \quad Q = \int_{-W/2}^{W/2} \rho_s(x)dx = W$$

$$\Rightarrow \quad C = \frac{Q}{V} \ldots(3.192), \quad Z_o = \sqrt{\frac{L}{C}} = \frac{1}{v_p C} = \frac{\sqrt{\varepsilon_r}}{cC}$$

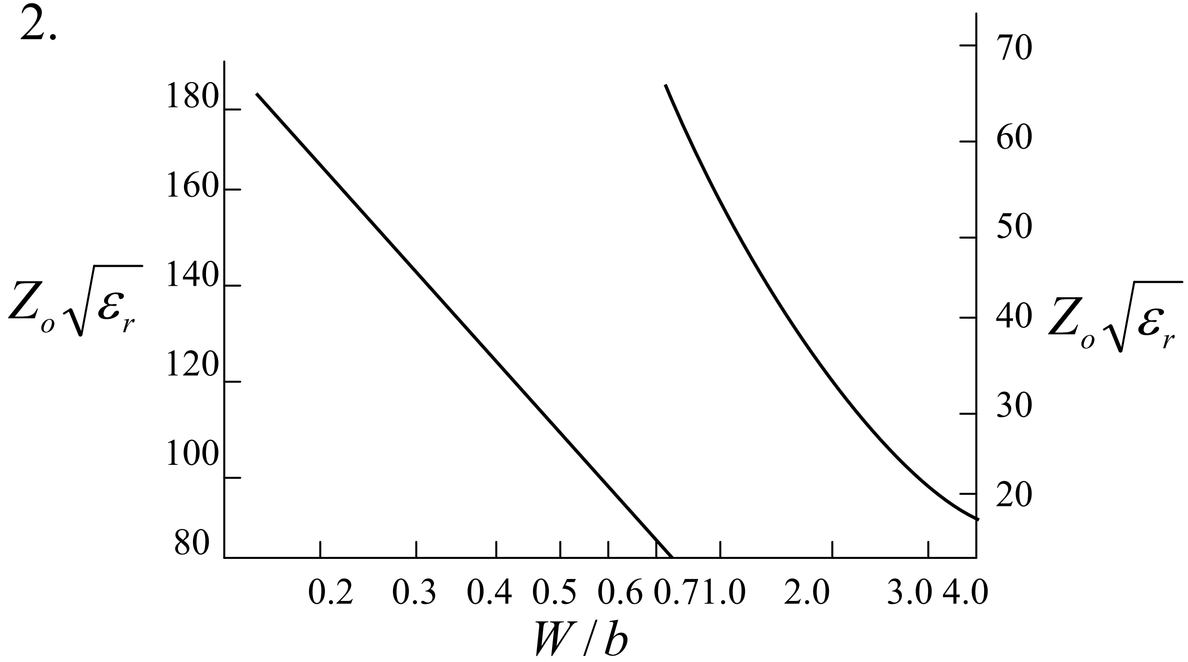
Discussion

1. analysis eq.(3.179) $W, b, \varepsilon \rightarrow Z_o$

   synthesis eq.(3.180) $Z_o, b, \varepsilon \rightarrow W$

$$\alpha_d = \frac{k \tan\delta}{2}, \quad \alpha_c \text{ eq.(3.181)}$$

2.



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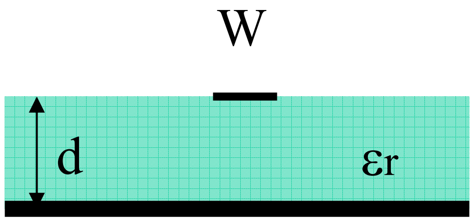
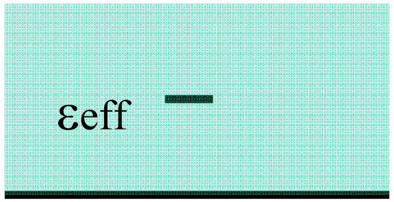
3.8 Microstrip



- Characteristics

  fabrication by printed circuit
  devices can be bonded to strip
  component are accessible
  in-circuit characterization of devices is straightforward to implement
  dc as well as ac signals can be transmitted
  large variation in Z_o
  $\alpha_c > \alpha_d$
  monolithic applications
  structure is rugged and can withstand high voltages and power levels
  power handling is best with BeO substrate
  used up to 300GHz or more
  quasi-TEM mode for $d \ll \lambda$

• ε_{eff} concept

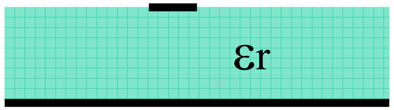


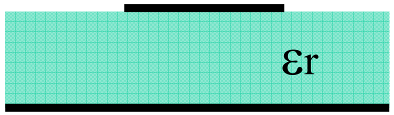
$$\text{TEM line } Z_o = \sqrt{\frac{L}{C}},\ v_p = \frac{c}{\sqrt{\varepsilon_r}}, \beta = \omega\sqrt{\mu\varepsilon} = k_o\sqrt{\varepsilon_r}, \lambda = \frac{\lambda_o}{\sqrt{\varepsilon_r}}$$

$$\text{air filled microstrip (TEM line) } Z_{oa} = \sqrt{\frac{L}{C_a}} = \frac{1}{cC_a}$$

$$\text{microstrip (quasi-TEM line) } Z_o = \sqrt{\frac{L}{C}} = \frac{1}{v_p C},\ v_p = \frac{1}{\sqrt{LC}} = \frac{c}{\sqrt{\varepsilon_{eff}}}, \beta = k_o\sqrt{\varepsilon_{eff}},$$

$$\lambda_g = \frac{\lambda_o}{\sqrt{\varepsilon_{eff}}} \quad \Rightarrow \quad \frac{Z_o}{Z_{oa}} = \sqrt{\frac{C_a}{C}} = \frac{1}{\sqrt{\varepsilon_{eff}}}, \quad \frac{C}{C_a} = \left(\frac{c}{v_p}\right)^2 \equiv \varepsilon_{eff}$$

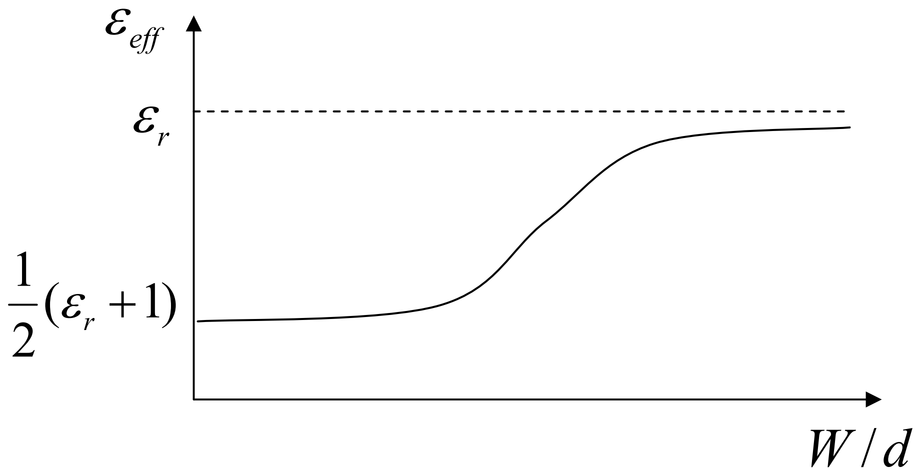




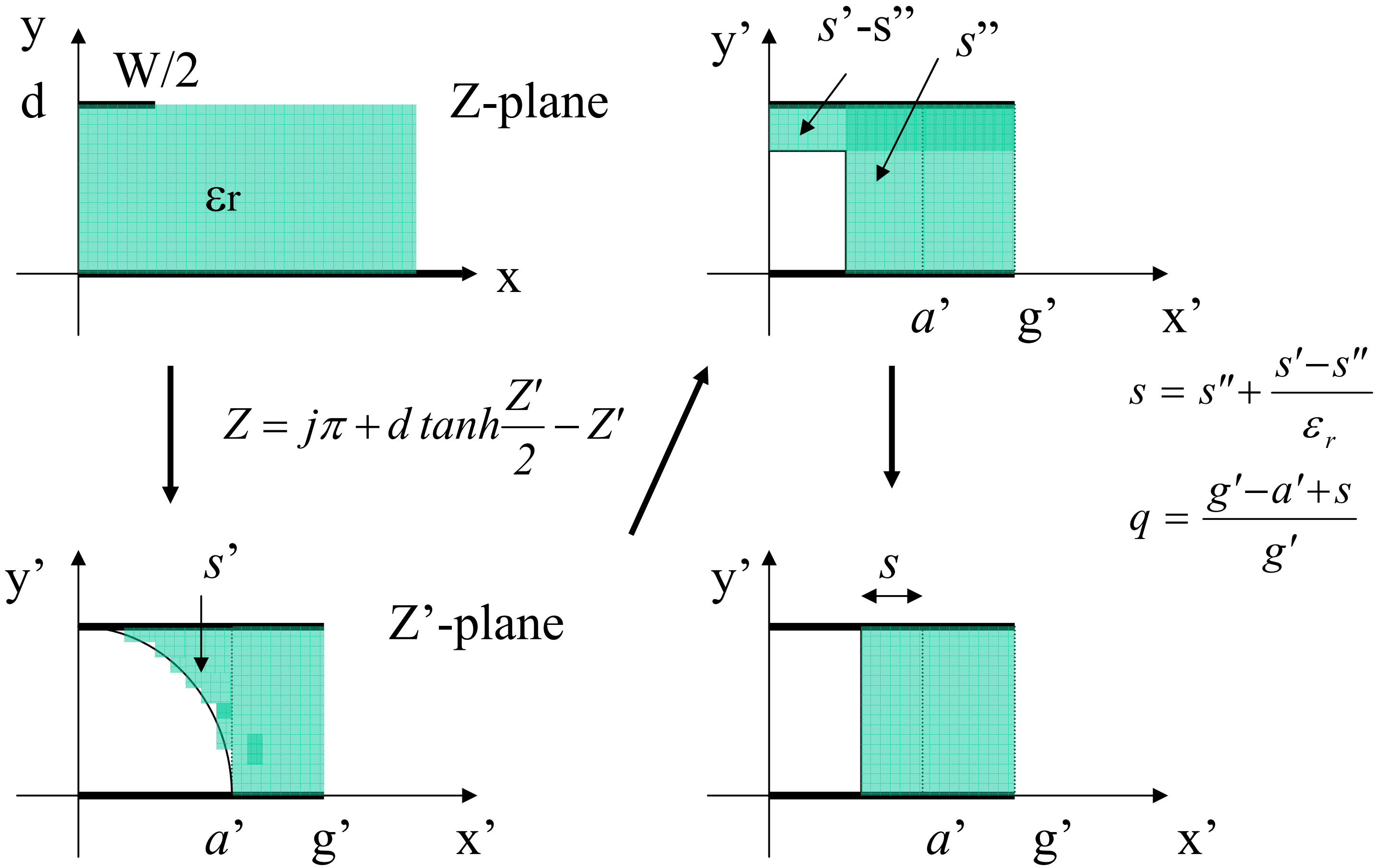
$$\varepsilon_{eff} \to \frac{1}{2}(\varepsilon_r + 1) \qquad\qquad \varepsilon_{eff} \to \varepsilon_r$$

$$\frac{1}{2}(\varepsilon_r + 1) \le \varepsilon_{eff} \le \varepsilon_r$$

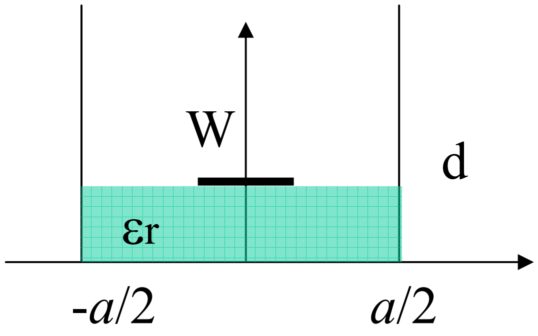
filling factor q $\quad \varepsilon_{eff} = q\varepsilon_r + (1-q)1 = 1 + q(\varepsilon_r - 1)\,,\quad \frac{1}{2} \le q \le 1$



- Conformal mapping solution (see Gupta, Garg and Bahl, *Microstrip lines and slotlines*, p.9)



• Electrostatic solution



$$\nabla_t^2\Phi(x,y)=0 \qquad |x|\le a/2,\quad 0\le y<\infty$$

$$B.C.\quad \Phi(x,y)=0\ \ x=\pm a/2,\quad y=0,\infty$$

$$\Phi(x,y)=\begin{cases}\sum_{odd} A_n\cos\frac{n\pi}{a}x\sinh\frac{n\pi}{a}y & 0\le y<d\\ \sum_{odd} A_n\cos\frac{n\pi}{a}x\sinh\frac{n\pi}{a}d e^{-n\pi(y-d)/a} & d\le y<\infty\end{cases}$$

$$E_y=-\frac{\partial\Phi}{\partial y}=\begin{cases}-\sum_{odd} A_n(\frac{n\pi}{a})\cos\frac{n\pi}{a}x\cosh\frac{n\pi}{a}y & 0\le y<d\\ \sum_{odd} A_n(\frac{n\pi}{a})\cos\frac{n\pi}{a}x\sinh\frac{n\pi}{a}d\, e^{-n\pi(y-d)/a} & d\le y<\infty\end{cases}$$

$$\rho_s(x)=D_y(x,y=d^+)-D_y(x,y=d^-)=\varepsilon_o E_y(x,y=d^+)-\varepsilon_o\varepsilon_r E_y(x,y=d^-)\cong\begin{cases}1 & |x|<W/2\\ 0 & |x|>W/2\end{cases}\rightarrow A_n$$

$$V=-\int_0^d E_y(x=0,y)dy=\sum_{odd} A_n\sinh\frac{n\pi}{a}d,\quad Q=\int_{-W/2}^{W/2}\rho_s(x)dx=W$$

$$\Rightarrow\quad C=\frac{Q}{V},\quad \varepsilon_{eff}=\frac{C}{C_a},\qquad Z_o=\sqrt{\frac{L}{C}}=\frac{1}{v_pC}=\frac{\sqrt{\varepsilon_{eff}}}{cC}$$

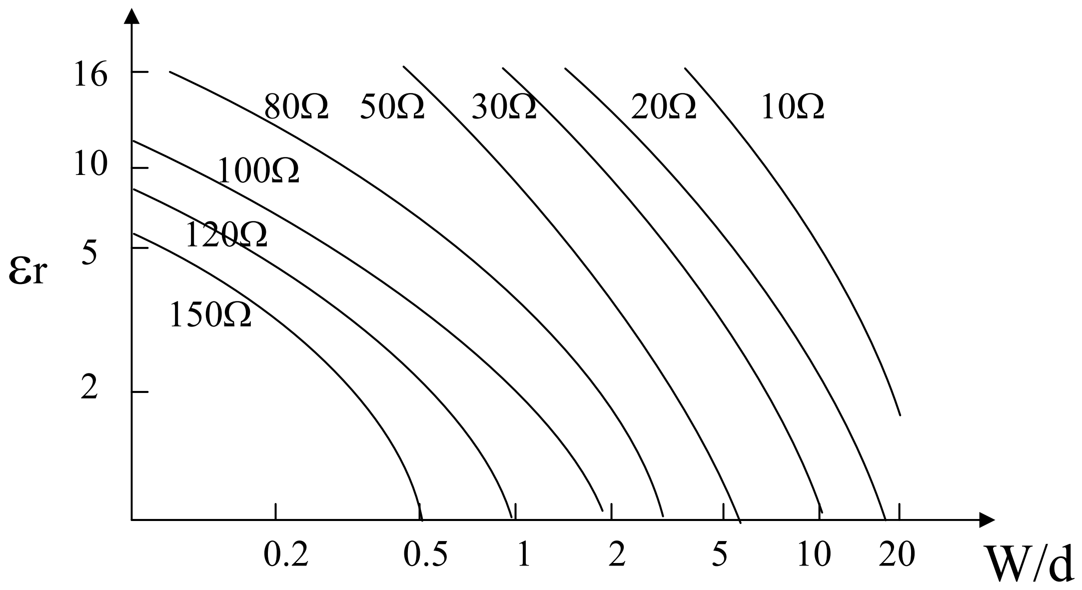
Discussion

1. analysis eq.(3.196, 195) $W, d, \varepsilon \rightarrow Z_o, \varepsilon_{eff}$

   synthesis eq.(3.197, 195) $Z_o, d, \varepsilon \rightarrow W, \varepsilon_{eff}$

$$\alpha_d = \frac{k_o \varepsilon_r (\varepsilon_{eff} - 1) \tan \delta}{2\sqrt{\varepsilon_{eff}} (\varepsilon_r - 1)} \text{(3.198)}, \quad \alpha_c = \frac{R_s}{Z_o W} \text{(3.199)}$$

2.



(*derivation of eq*.(3.198))

$$\text{TEM line } \alpha_d = \frac{k \tan\delta}{2} = \frac{k_o\sqrt{\varepsilon_r}\tan\delta}{2}$$

microstrip line:

$$(1)\varepsilon_{eff} = q\varepsilon_r + (1-q) = q(\varepsilon_r - 1) + 1 \rightarrow q = \frac{\varepsilon_{eff}-1}{\varepsilon_r - 1}$$

$$(2)\ \sqrt{\varepsilon_r} \rightarrow \sqrt{\varepsilon_{eff}}$$

$$(3)\tan\delta = \frac{\varepsilon''}{\varepsilon_r} \rightarrow q\tan\delta' = q\frac{\varepsilon''}{\varepsilon_{eff}} = q\frac{\varepsilon''}{\varepsilon_r}\frac{\varepsilon_r}{\varepsilon_{eff}} = q\tan\delta\frac{\varepsilon_r}{\varepsilon_{eff}}$$

$$\alpha_d = \frac{k_o\sqrt{\varepsilon_{eff}}\,q\tan\delta}{2}\frac{\varepsilon_r}{\varepsilon_{eff}} = \frac{k_o\varepsilon_r\tan\delta}{2\sqrt{\varepsilon_{eff}}}\frac{\varepsilon_{eff}-1}{\varepsilon_r - 1}$$

4. substrate material

	ε_r	$\tan\delta$ @10GHz
GaAs	12.9	0.002
Al_2O_3(99.5%)	10.1 ± 0.25	0.0002
quartz	3.8	0.0001
RT/duriod 5880	2.2 ± 0.02	0.0009
RT/duroid 6006	6.15 ± 0.15	0.0027
BeO	6.8	0.0003
FR4 (G-10)	4-4.6	0.018-0.025
Si	11.7	0.005
*SiO_2	4	
*Si_3N_4	7.6 (vapor phase) 6.5 (sputtering)	

* : MIM capacitor

conductor material

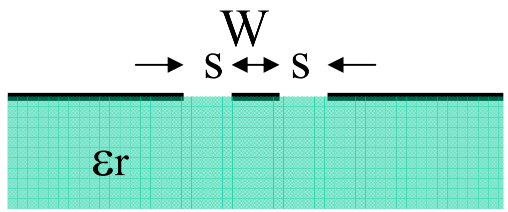
	R_s (Ω/sq $\times 10^{-7}\sqrt{f}$)	δ_s (um)@2GHz
Ag	2.5	1.4
Cu	2.6	1.5
Au	3.0	1.7
Al	3.3	1.9
Cr	4.7	2.7
Ta	7.2	4.0

1 oz Cu $\rightarrow$ 1.4 mil (35 um) thickness

skin depth $\delta_s = \sqrt{\dfrac{2}{\omega\mu\sigma}}$

3.11 Summary of transmission lines and waveguides

- comparison of coaxial line, stripline and microstrip, (p.154, Table 3.6)
- CPW (coplanar waveguide)



  fabrication easy
  quasi-TEM operation
  radiation problem when gap width approaches $\lambda/2$
  monolithic applications
  less radiation than microstrip if well-balanced
  higher order modes (coupled slot mode)
- www.appwave.com for computer assisted transmission line analysis

Solved Problems

Prob. 3.28 Find Z_o of a coaxial line to have maximum power capacity

breakdown field strength of air $E_d = 3\times10^6 V/m = \dfrac{V_o}{\rho\, ln\frac{b}{a}} \rightarrow V_{max} = E_d a\, ln\dfrac{b}{a}$

maximum power capacity $P_{max} = \dfrac{1}{2}\dfrac{V_{max}}{Z_o^2}, Z_o = \dfrac{\eta_o}{2\pi} ln\dfrac{b}{a} \rightarrow P_{max} = \dfrac{\pi a^2 E_d^2}{\eta_o} ln\dfrac{b}{a}$

$$\frac{dP_{max}}{da} = 0 \rightarrow ln\frac{b}{a} = \frac{1}{2} \Rightarrow Z_o = \frac{\eta_o}{2\pi} ln\frac{b}{a} = \frac{377}{2\pi}\frac{1}{2} \approx 30\Omega$$

Suggested homework (due 2 weeks): 5, 22

ADS examples: Ch3_prj