Chapter 4 Microwave network analysis

4.1 Impedance and equivalent voltages and currents
equivalent transmission line model (β, Zo)
4.2 Impedance and admittance matrices
not applicable in microwave circuits
4.3 The scattering matrix
properties, VNA measurement
4.4 The transmission (ABCD) matrix
cascade network
4.5 Signal flow graph
2-port circuit, TRL calibration
4.6 Discontinuities and modal analysis
microstrip discontinuities and compensation

4.1 Impedance and equivalent voltages and currents

- Equivalent voltages and currents

Microwave circuit approach

Interest: voltage and current at a set of terminals (ports), power flow through a device, and how to find the response of a network

For a certain mode in the line, the line characteristics are represented by it's global quantities Zo, β, l.

Define: equivalent voltage (wave) $\propto$ transverse electric field
equivalent current (wave) $\propto$ transverse magnetic field

$\ni$ voltage (wave)/current (wave) = characteristic impedance or wave impedance of the line

and voltage $\times$ current = power flow of the mode

$\rightarrow$ use transmission line theory to analyze microwave circuit performance at the interested ports

- Impedance

characteristic impedance of the medium $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

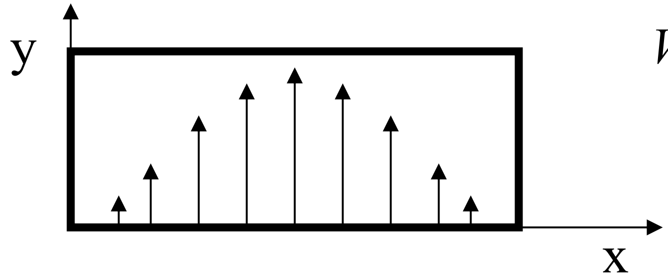
wave impedance of the particular mode of wave $Z_w = \frac{E_t^+}{H_t^+}$

characteristic impedance of the line $Z_o = \frac{V^+}{I^+}$

input impedance at a port of circuit $Z_{in}(z) = \frac{V(z)}{I(z)}$

Discussion

1. Transmission line model for the TE10 mode of a rectangular waveguide



$$V(x,z) \equiv \int \overline{E} \bullet d\bar{l} = \int E_y dy$$

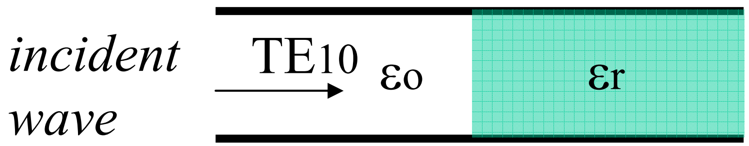
: x – dependent , non - unique value

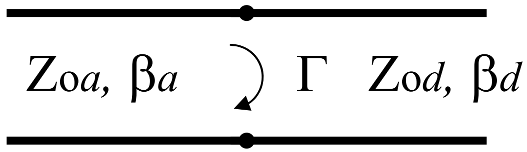
transverse fields	transmission line model
$E_y = (A^+ e^{-j\beta z} + A^- e^{j\beta z}) \sin \frac{\pi x}{a} \equiv C_1 V \sin \frac{\pi x}{a}$	$V = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$
$H_x = -\frac{1}{Z_{TE_{10}}}(A^+ e^{-j\beta z} - A^- e^{j\beta z}) \sin \frac{\pi x}{a} \equiv C_2 I \sin \frac{\pi x}{a}$	$I = I_o^+ e^{-j\beta z} + I_o^- e^{j\beta z} = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{V_o^-}{Z_o} e^{j\beta z}$
$Z_{TE_{10}} = -\frac{E_y^+}{H_x^+} = \frac{k\eta}{\beta} \equiv Z_o$	$Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-}$
$P^+ = -\frac{1}{2}\int E_y^+ H_x^{+*} dxdy$	$P^+ = \frac{1}{2} V_o^+ I_o^{+*}$

$$P^{+}=\frac{ab}{4Z_{TE_{10}}}\left|A^{+}\right|^{2}\equiv\frac{1}{2}V_{o}^{+}I_{o}^{+*}=\frac{1}{2}C_{1}C_{2}^{*}\left|A^{+}\right|^{2}$$

$$Z_{TE_{10}}\equiv Z_{o}=\frac{C_{1}}{C_{2}}\ \Rightarrow\ C_{1}=\sqrt{\frac{ab}{2}},\ C_{2}=\frac{1}{Z_{TE_{10}}}\sqrt{\frac{ab}{2}}$$

2. Ex.4.2





$$\Gamma=\frac{Z_{od}-Z_{oa}}{Z_{od}+Z_{oa}}$$

Q: What if the incident wave is from the other direction? **"N"**

$$Z_{oa}=\frac{k_{o}\eta_{o}}{\beta_{a}},\ Z_{od}=\frac{k\eta}{\beta_{d}},\ k_{o}\eta_{o}=k\eta,\ k=\sqrt{\varepsilon_{r}}k_{o}$$

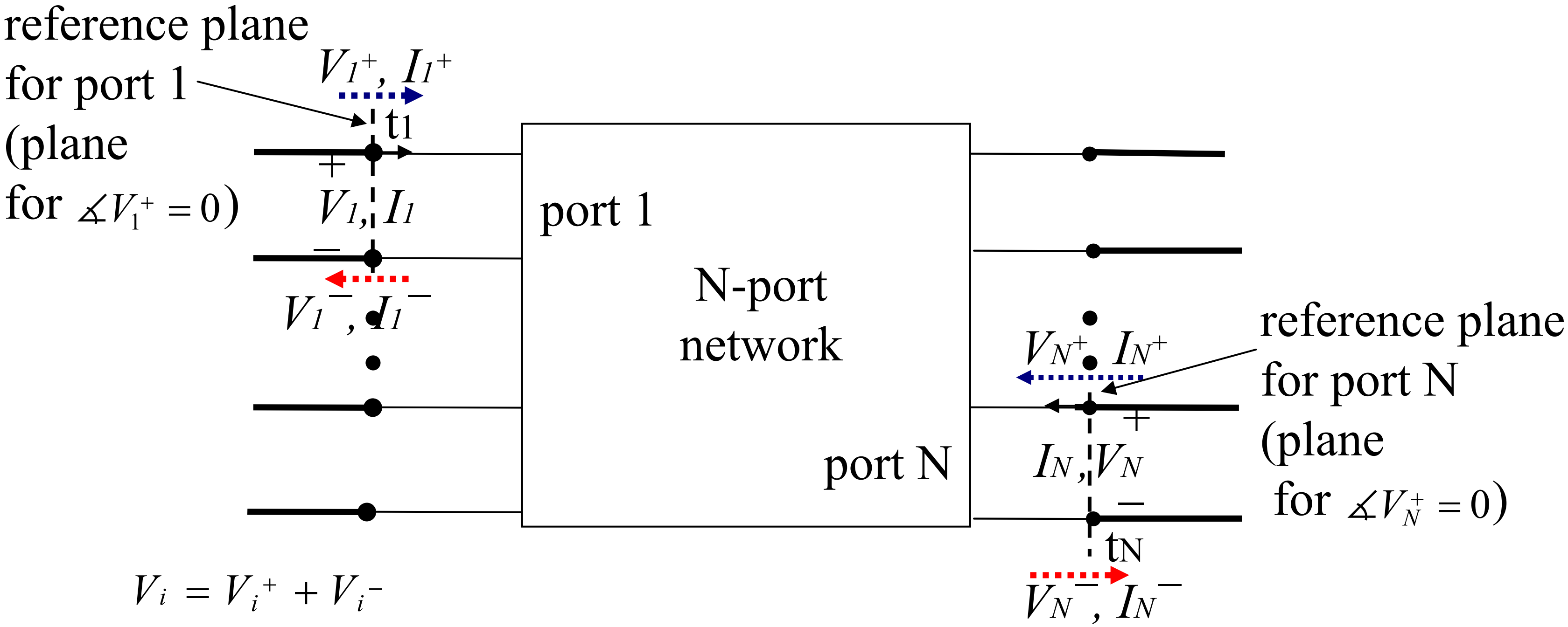
$$\beta_{a}^{2}+k_{c}^{2}=k_{o}^{2},\ \beta_{d}^{2}+k_{c}^{2}=k^{2},\ k_{c}=\frac{\pi}{a}$$

$$a=2.286cm\rightarrow\lambda_{c}=2a=4.472cm\rightarrow k_{c}=137m^{-1},f_{c}=6.56GHz$$

$$\text{if}\ f=10GHz\rightarrow k_{o}=209m^{-1},\beta_{a}=158m^{-1},\varepsilon_{r}=2.54\rightarrow k=333m^{-1},\beta_{d}=304m^{-1}$$

$$\text{if}\ f=6GHz\rightarrow k=201m^{-1}>k_{c},k_{o}=126m^{-1}<k_{c}$$

4.2 Impedance and admittance matrices



$$V_i = V_i^+ + V_i^-$$

$$I_i = I_i^+ + I_i^- = \frac{V_i^+ - V_i^-}{Z_{oi}}$$

$$Z_{oi} = \frac{V_i^+}{I_i^+} = -\frac{V_i^-}{I_i^-}$$

$$P_{inc,i} = \frac{1}{2}\mathrm{Re}\{V_i^+ I_i^+{}^*\},\ P_{in,i} = \frac{1}{2}\mathrm{Re}\{V_i I_i{}^*\}$$

- Impedance matrix

$$[V]=[Z]\ [I]\ ,\ \begin{bmatrix} V_1 \\ V_2 \\ \bullet \\ \bullet \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \bullet & \bullet & Z_{1N} \\ Z_{21} & \bullet & \bullet & \bullet & Z_{2N} \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ Z_{N1} & Z_{N2} & \bullet & \bullet & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \bullet \\ \bullet \\ I_N \end{bmatrix},\ Z_{ij} = \left.\frac{V_i}{I_j}\right|_{I_k=0,k\neq j} = \left.\frac{response_i}{source_j}\right|_{I_k=0,k\neq j}$$

- Admittance matrix

$$[I]=[Y]\ [V],\begin{bmatrix} I_1 \\ I_2 \\ \bullet \\ \bullet \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \bullet & \bullet & Y_{1N} \\ Y_{21} & \bullet & \bullet & \bullet & Y_{2N} \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ Y_{N1} & Y_{N2} & \bullet & \bullet & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \bullet \\ \bullet \\ V_N \end{bmatrix},\ Y_{ij} = \left.\frac{I_i}{V_j}\right|_{V_k=0,k\neq j} = \left.\frac{response_i}{source_j}\right|_{V_k=0,k\neq j}$$

Discussion

1. Reciprocal network

$$\left.\begin{array}{l}[Z]=[Z]^{t},\quad Z_{ij}=Z_{ji}\\ [Y]=[Y]^{t},\quad Y_{ij}=Y_{ji}\end{array}\right., [Z] \text{ and } [\mathrm{Y}] : \text{symmetric matrix}$$

(*derivation*)

source		port 1	port 2
a	$\longrightarrow$	$V1a, I1a$	$V2a, I2a$
b	$\longrightarrow$	$V1b, I1b$	$V2b, I2b$

reciprocity theorem: $V1aI1b + V2aI2b = V1bI1a + V2bI2a$

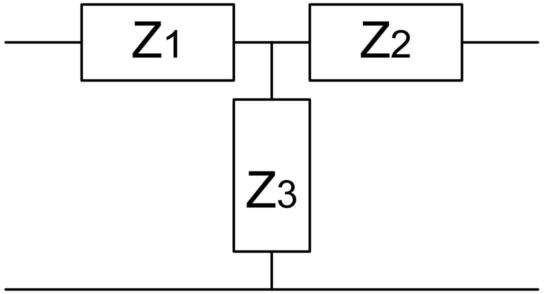
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

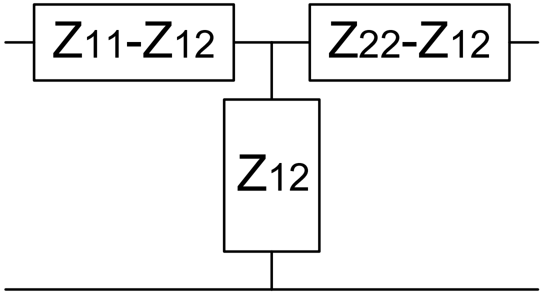
$$(\cancel{Z_{11}I_{1a}} + Z_{12}I_{2a})I_{1b} + (Z_{21}I_{1a} + \cancel{Z_{22}I_{2a}})I_{2b} = (\cancel{Z_{11}I_{1b}} + Z_{12}I_{2b})I_{1a} + (Z_{21}I_{1b} + \cancel{Z_{22}I_{2b}})I_{2a}$$

$$Z_{12}I_{2a}I_{1b} + Z_{21}I_{1a}I_{2b} = Z_{12}I_{2b}I_{1a} + Z_{21}I_{1b}I_{2a}$$

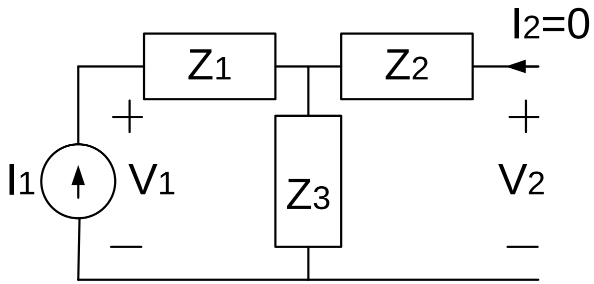
$$(Z_{12} - Z_{21})(I_{2a}I_{1b} - I_{2b}I_{1a}) = 0 \Rightarrow Z_{12} = Z_{21}$$

2. T and Π networks



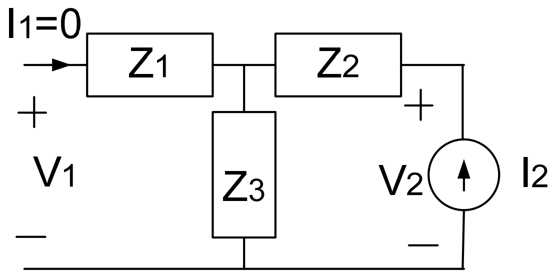


(*derivation*)



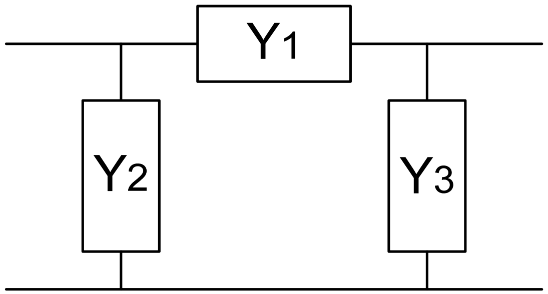
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

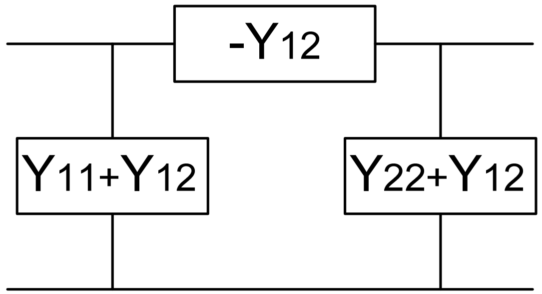
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_1 + Z_3, Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_3 = Z_{12}$$



$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_2 + Z_3$$

$$\rightarrow Z_3 = Z_{12}, Z_1 = Z_{11} - Z_{12}, Z_2 = Z_{22} - Z_{12}$$





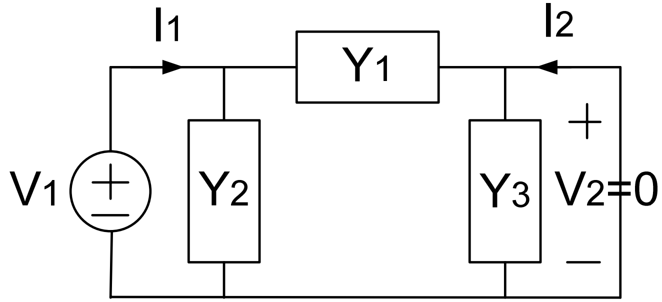
$(derivation)$

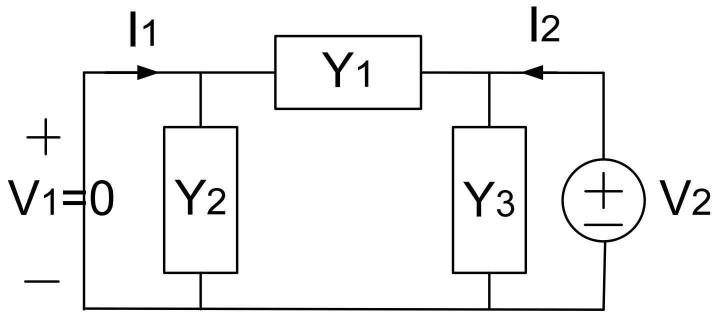
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_1 + Y_2, Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -Y_1 = Y_{12}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_1 + Y_3$$

$$\rightarrow Y_1 = -Y_{12}, Y_2 = Y_{11} + Y_{12}, Y_3 = Y_{22} + Y_{12}$$





3. Reciprocal lossless network

$$\mathrm{Re}\{Z_{ij}\} = 0$$

4. Problems to use Z- or Y-matrix in microwave circuits

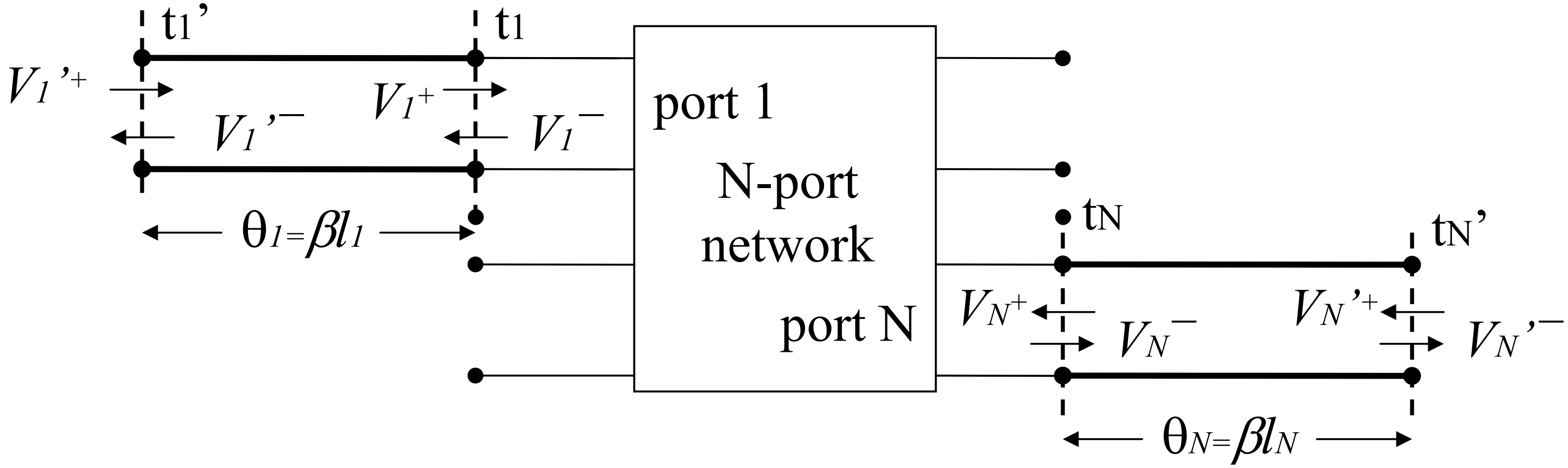
   1) difficult in defining voltage and current for non-TEM lines

   2) no equipment available to measure voltage and current in complex value (eg. sampling scope in microwave range, impedance meter <3GHz)

   3) difficult to make open and short circuits over broadband

   4) active devices not stable as terminated with open or short circuit

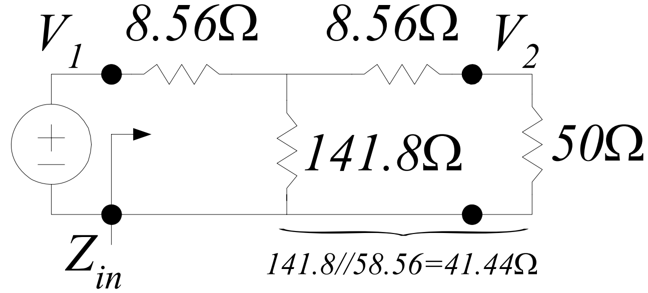
4.3 The scattering matrix



$$[V^-]=[S]\,[V^+],\quad \begin{bmatrix} V_1^- \\ V_2^- \\ \bullet \\ \bullet \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \bullet & \bullet & S_{1N} \\ S_{21} & \bullet & \bullet & \bullet & S_{2N} \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ S_{N1} & S_{N2} & \bullet & \bullet & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \bullet \\ \bullet \\ V_N^+ \end{bmatrix},\quad S_{ij}=\left.\frac{V_i^-}{V_j^+}\right|_{V_k^+=0,k\neq j}=\left.\frac{\text{response}_i}{\text{source}_j}\right|_{V_k^+=0,k\neq j}$$

Discussion

1. Ex 4.4 a 3dB attenuator



$$Z_{in} = 8.56 + 41.44 = 50$$

$$S_{11} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = 0 = S_{22}$$

$$V_2^- = V_2 = V_1 \frac{41.44}{41.44 + 8.56} \frac{50}{50 + 8.56} = 0.707 V_1 = \frac{1}{\sqrt{2}} V_1 = \frac{1}{\sqrt{2}} V_1^+ = S_{21} V_1^+$$

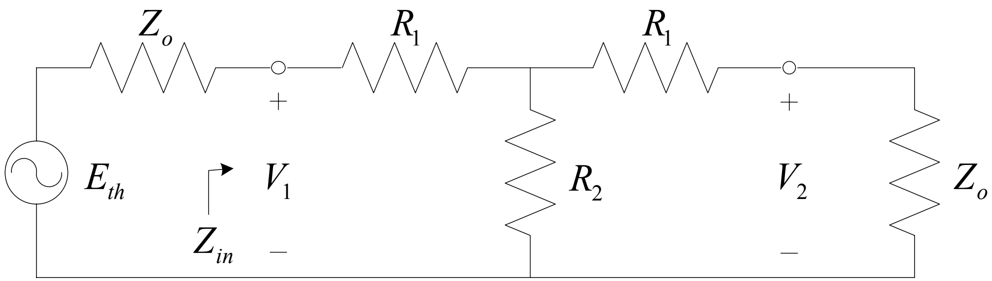
$$\rightarrow [S] = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \text{: lossy, reciprocal, symmetric}$$

incident power to port 1: $\frac{1}{2} \frac{|V_1^+|^2}{Z_o}$

transmitted power from port 2: $\frac{1}{2} \frac{|V_2^-|^2}{Z_o} = \frac{1}{2} \frac{\left|\frac{1}{\sqrt{2}} V_1^+\right|^2}{Z_o} = \frac{1}{2} \times \frac{1}{2} \frac{|V_1^+|^2}{Z_o}$: 3dB attenuation

attenuator design $\left\{ \begin{array}{c} \text{input match} \\ \text{attenuation value} \end{array} \right. \rightarrow R_1, R_2$

2. T-type attenuator design



$$\begin{cases} Z_{in} = \left(R_1 + Z_o\right) // R_2 + R_1 = Z_o \\ S_{21} = \dfrac{V_2}{V_1} = \dfrac{R_2 // \left(R_1 + Z_o\right)}{R_1 + R_2 // \left(R_1 + Z_o\right)} \dfrac{Z_o}{R_1 + Z_o} = \alpha \end{cases} \Rightarrow \begin{cases} R_1 = \dfrac{1-\alpha}{1+\alpha} Z_o \\ R_2 = \dfrac{2\alpha}{1-\alpha^2} Z_o \end{cases}$$

$$3dB \text{ attenuator } \alpha = \frac{1}{\sqrt{2}} \Rightarrow \begin{cases} R_1 = \dfrac{1-\alpha}{1+\alpha} Z_o = \dfrac{\sqrt{2}-1}{\sqrt{2}+1} Z_o \\ R_2 = \dfrac{2\alpha}{1-\alpha^2} Z_o = 2\sqrt{2} Z_o \end{cases}$$

3. Relation of [Z], [Y], and [S]

$$[S]=([Z]+[U])^{-1}([Z]-[U]),\ [Y]=[Z]^{-1}$$

(*derivation*)

Let $Z_{on}=1,\ \begin{matrix} V_n=V_n^{+}+V_n^{-} \\ I_n=V_n^{+}-V_n^{-} \end{matrix}$

$$[V]=[Z][I]\rightarrow\left[V^{+}\right]+\left[V^{-}\right]=[Z](\left[V^{+}\right]-\left[V^{-}\right])$$

$$\rightarrow[Z]\left[V^{-}\right]+\left[V^{-}\right]=[Z]\left[V^{+}\right]-\left[V^{+}\right]$$

$$([Z]+[U])\left[V^{-}\right]=([Z]-[U])\left[V^{+}\right]$$

$$\because\left[V^{-}\right]=[S]\left[V^{+}\right]\rightarrow([Z]+[U])[S]\left[V^{+}\right]=([Z]-[U])\left[V^{+}\right]$$

$$[S]=([Z]+[U])^{-1}([Z]-[U])$$

4. Reciprocal network

$$[S]=[S]^t,\quad [S]: \text{symmetric matrix}$$

(*derivation*)

$$\text{Let } Z_{on}=1, \begin{matrix} V_n = V_n^+ + V_n^- \\ I_n = V_n^+ - V_n^- \end{matrix} \rightarrow \begin{matrix} V_n^+ = (V_n + I_n)/2 \\ V_n^- = (V_n - I_n)/2 \end{matrix}$$

$$\left[V^+\right]=\frac{1}{2}([V]+[I])=\frac{1}{2}([Z]+[U])[I]$$

$$\left[V^-\right]=\frac{1}{2}([V]-[I])=\frac{1}{2}([Z]-[U])[I]$$

$$\left[V^-\right]=([Z]-[U])([Z]+[U])^{-1}\left[V^+\right]\rightarrow [S]=([Z]-[U])([Z]+[U])^{-1}$$

$$[S]^t=(([Z]+[U])^{-1})^t([Z]-[U])^t=([Z]+[U])^{-1}([Z]-[U])\overset{from\ 3}{=}[S]$$

5. Lossless network (unitary property)

$$[S]^t[S]^* = [U],\quad \sum_{k=1}^{N} S_{ki}S_{kj}^* = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$$

(*derivation*)

Let $Z_{on} = 1$

lossless (incident power=emitted power) $\rightarrow$ net averaged input power $\sum_i P_{in,i} = 0$

$$P_{in} = \frac{1}{2}\mathrm{Re}([V]^t[I]^*) = \frac{1}{2}\mathrm{Re}[([V^+]+[V^-])^t([V^+]^*-[V^-]^*)]$$

Im

$$= \frac{1}{2}\mathrm{Re}[[V^+]^t[V^+]^* - [V^-]^t[V^-]^* + [V^-]^t[V^+]^* - [V^+]^t[V^-]^*)] = 0$$

$$[V^+]^t[V^+]^* = [V^-]^t[V^-]^* \overset{[V^-]=[S][V^+]}{=} [V^+]^t[S]^t[S]^*[V^+]^*$$

$$\rightarrow [S]^t[S]^* = [U]$$

6. Lossy network $\quad \sum_{k=1}^{N} S_{ki}S_{ki}^* < 1$

7. Ex.4.5

$$[S]=\begin{bmatrix} 0.15\angle 0^o & 0.85\angle -45^o \\ 0.85\angle 45^0 & 0.2\angle 0^0 \end{bmatrix}$$

$[S]$:not symmetric $\rightarrow$ a non-reciprocal network

$|S_{11}|^2 + |S_{21}|^2 = 0.745 \neq 1 \rightarrow$ a lossy network

port 1 $RL = -20\log|S_{11}| = 16.5dB$

port 2 $RL = -20\log|S_{22}| = 14dB$

$IL = -20\log|S_{21}| = 1.4dB$

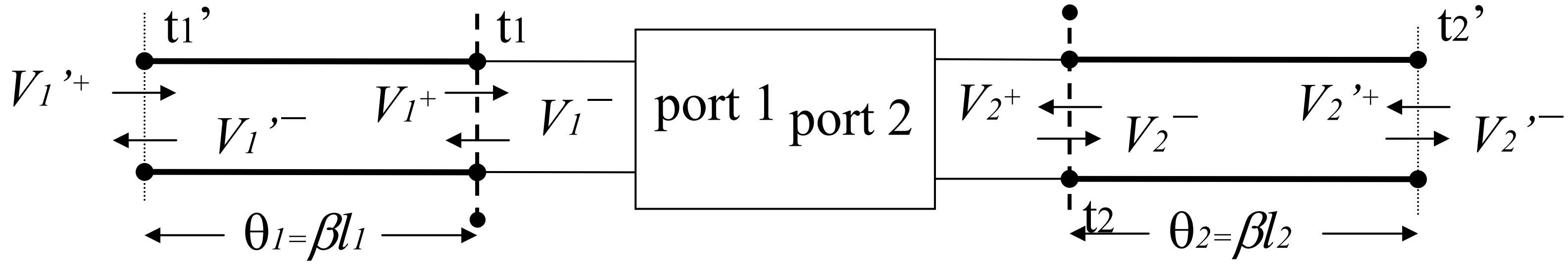
port 2 terminated with a matched load $\Gamma_L = 0 \rightarrow$

$\Gamma_{in} = |S_{11}| = 0.15, RL = -20\log 0.15 = 16.5dB$

port 2 terminated with a short circuit $\Gamma_L = -1 \rightarrow$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = -0.452, RL = 6.9dB$$

8. Shift property



$$S'_{11}=e^{-j2\theta_1}S_{11},S'_{21}=e^{-j\theta_1}S_{21}e^{-j\theta_2},S'_{12}=e^{-j\theta_2}S_{12}e^{-j\theta_1},S'_{22}=e^{-j2\theta_2}S_{22}$$

n-port network: $\left[S'\right]=\begin{bmatrix} e^{-j\theta_1} & 0 & \bullet & 0 \\ 0 & e^{-j\theta_2} & \bullet & 0 \\ \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & e^{-j\theta_N} \end{bmatrix}\left[S\right]\begin{bmatrix} e^{-j\theta_1} & 0 & \bullet & 0 \\ 0 & e^{-j\theta_2} & \bullet & 0 \\ \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & e^{-j\theta_N} \end{bmatrix}$

9. S-matrix is not effected by the network arrangement.

10. Generalized S-matrix

$a_i \equiv \dfrac{V_i^+}{\sqrt{Z_{oi}}}$: incident (power) wave, $b_i \equiv \dfrac{V_i^-}{\sqrt{Z_{oi}}}$: reflected (power) wave

$$\left[V^-\right]=\left[S\right]\left[V^+\right] \Rightarrow \left[b\right]=\left[S\right]\left[a\right],\ S_{ij}=\left.\frac{b_i}{a_j}\right|_{a_k=0,k\neq j}=\left.\frac{V_i^-\sqrt{Z_{oj}}}{V_j^+\sqrt{Z_{oi}}}\right|_{V_k^+=0,k\neq j}$$

$$P_{in,i}=\frac{1}{2}\mathrm{Re}\{V_i I_i^*\}=\frac{1}{2}|a_i|^2-\frac{1}{2}|b_i|^2=P_{inc,i}-P_{ref,i}=P_{inc,i}(1-|S_{ii}|^2)$$

(*derivation*)

$$a_i=\frac{V_i^+}{\sqrt{Z_{oi}}},b_i=\frac{V_i^-}{\sqrt{Z_{oi}}},V_i=V_i^++V_i^-=\sqrt{Z_{oi}}(a_i+b_i),I_i=\frac{V_i^+-V_i^-}{Z_{oi}}=\frac{a_i-b_i}{\sqrt{Z_{oi}}}$$

Im

$$P_{in,i}=\frac{1}{2}\mathrm{Re}\{V_i I_i^*\}=\frac{1}{2}\mathrm{Re}\{(a_i+b_i)(a_i-b_i)^*\}=\frac{1}{2}\mathrm{Re}\{|a_i|^2-|b_i|^2+a_i^*b_i-a_ib_i^*\}$$

$$=\frac{1}{2}|a_i|^2-\frac{1}{2}|b_i|^2$$

11. Two-port device with its S-matrix

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$: reflection coefficient at port 1 with port 2 matched

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$: forward transmission coefficient with port 2 matched

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$: reverse transmission coefficient with port 1 matched

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$: reflection coefficient at port 2 with port 1 matched

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

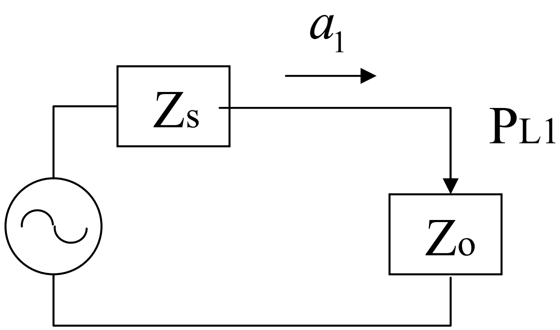
$$b_1 = a_1 S_{11} + a_2 S_{12}$$
$$b_2 = a_1 S_{21} + a_2 S_{22}$$

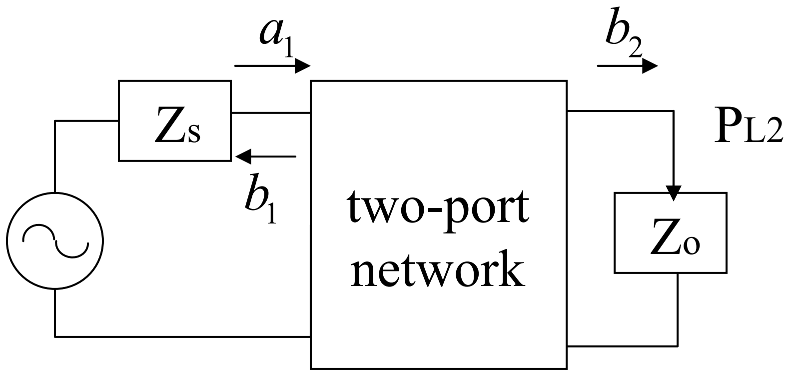
12. RL and IL

RL at port 1: $-20\log\left|\frac{b_1}{a_1}\right| = -20\log|S_{11}|$

IL from port 1 to port 2: $-20\log\left|\frac{b_2}{a_1}\right| = -20\log|S_{21}|$

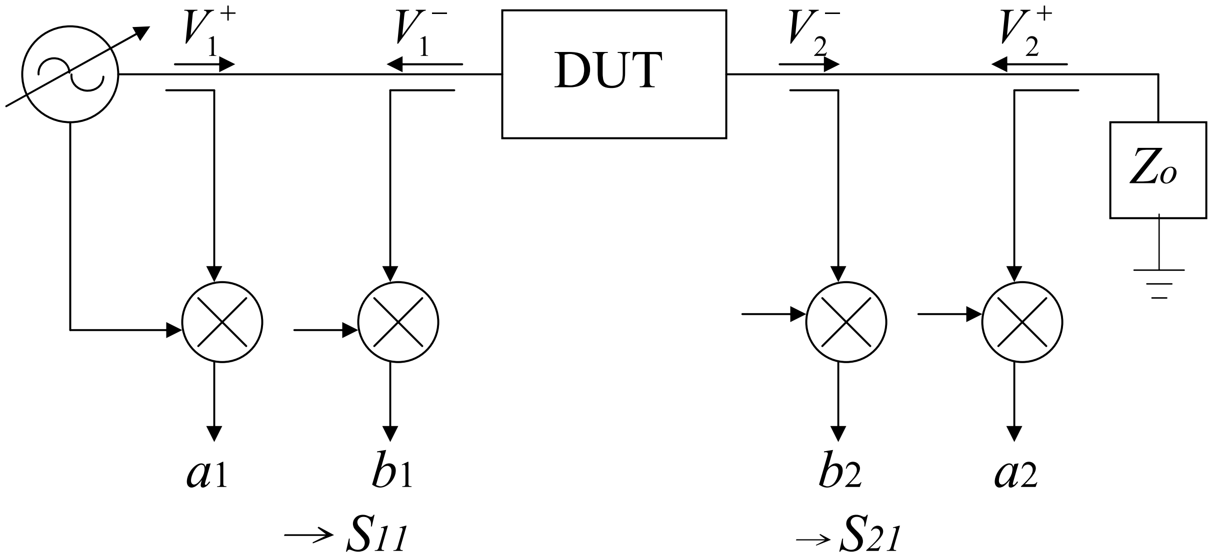
insertion loss $IL(dB) \equiv 10\log\frac{P_{L1}}{P_{L2}}$





usually $Z_s \neq Z_o$

13. Two-port S-matrix measurement using VNA



14. Advantages to use S-matrix in microwave circuit

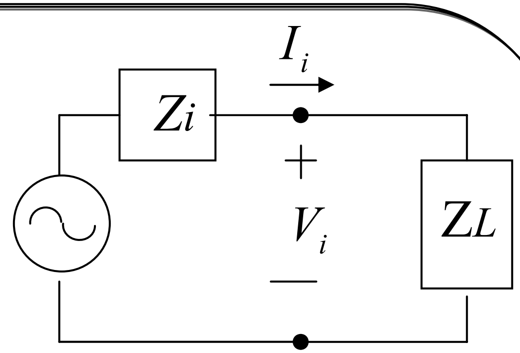
1) matched load available in broadband application

2) measurable quantity in terms of incident, reflected and transmitted waves

3) termination with Z_o causes no oscillation

4) convenient in the use of microwave network analysis

15. ADS S-parameter calculation



incident power wave $a_i \equiv \frac{V_i + Z_i I_i}{\sqrt{|\mathrm{Re}\, Z_i|}}$,

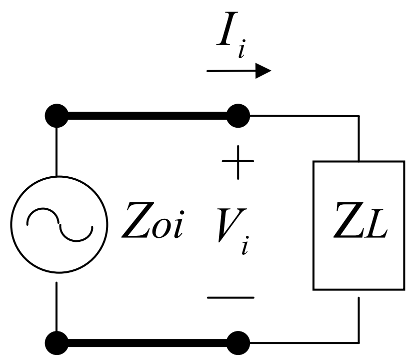
reflected power wave $b_i \equiv \frac{V_i - Z_i{}^* I_i}{\sqrt{|\mathrm{Re}\, Z_i|}}$

$\rightarrow$ power wave reflection coefficient $S_{ii} \equiv \frac{b_i}{a_i} = \frac{V_i - Z_i{}^* I_i}{V_i + Z_i I_i} = \frac{Z_L - Z_i{}^*}{Z_L + Z_i}$

power reflection coefficient $|S_{ii}|^2 = \left|\frac{b_i}{a_i}\right|^2 = \left|\frac{Z_L - Z_i{}^*}{Z_L + Z_i}\right|^2$, conjugate match $Z_L = Z_i{}^* \rightarrow |S_{ii}|^2 = 0$

$P_{in,i} = \frac{1}{2}\mathrm{Re}\{V_i I_i{}^*\} = \frac{1}{2}(|a_i|^2 - |b_i|^2)$

$\Leftrightarrow$ traveling wave along the line with real Z_0



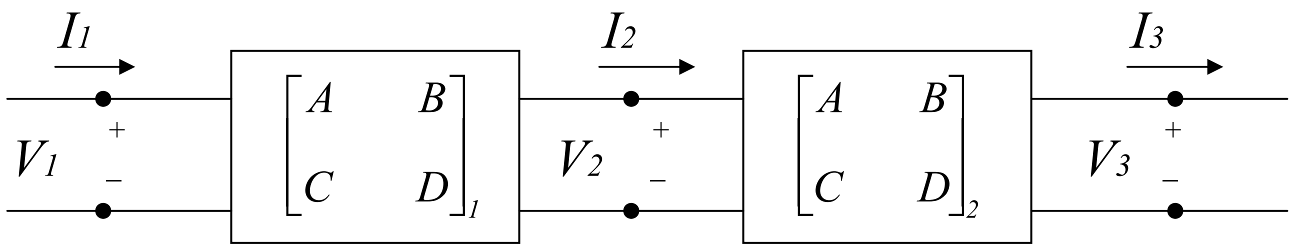
$a_i = \frac{V_i^+}{\sqrt{Z_{oi}}}, b_i = \frac{V_i^-}{\sqrt{Z_{oi}}}, V_i = V_i^+ + V_i^- = \sqrt{Z_{oi}}(a_i + b_i), I_i = \frac{V_i^+ - V_i^-}{Z_{oi}} = \frac{a_i - b_i}{\sqrt{Z_{oi}}}$

$\rightarrow a_i + b_i = \frac{V_i}{\sqrt{Z_{oi}}}, a_i - b_i = I_i\sqrt{Z_{oi}} \rightarrow a_i = \frac{V_i + Z_0 I_i}{\sqrt{Z_0}},\ b_i = \frac{V_i - Z_0 I_i}{\sqrt{Z_0}}$

$S_{ii} = \frac{b_i}{a_i} = \frac{V_i - Z_{oi} I_i}{V_i + Z_{oi} I_i} = \frac{Z_L - Z_{oi}}{Z_L + Z_{oi}}$, impedance match $Z_L = Z_{oi} \rightarrow S_{ii} = 0$

4.4 The transmission (ABCD) matrix

- Cascade network



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_1 \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_1 \begin{bmatrix} A & B \\ C & D \end{bmatrix}_2 \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

Discussion

1. ABCD matrix of two-port circuits (p.185, Table 4.1)
2. Reciprocal network AD-BC=1
3. S-, Z-, Y-, ABCD-matrix relation of 2-port network (p.187, Table 4.2)
4. Ex. 4.6

(*derivation*) Z (ABCD)

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix}, \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{AV_2}{CV_2} = \frac{A}{C}$$

$$Z_{12} = \left.\frac{V_1}{-I_2}\right|_{I_1=0} = \left.\frac{AV_2 + BI_2}{-I_2}\right|_{I_1=0} = -A\left.\frac{V_2}{I_2}\right|_{I_1=0} - B, I_1 = 0 = CV_2 + DI_2 \rightarrow -\frac{V_2}{I_2} = \frac{D}{C}$$

$$= -A(-\frac{D}{C}) - B = \frac{AD - BC}{C}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{CV_2 + DI_2}\right|_{I_2=0} = \frac{1}{C}$$

$$Z_{22} = -\left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{D}{C}, \because I_1 = 0 = CV_2 + DI_2$$

symmetrical network

$$Z_{11} = Z_{22} \rightarrow A = D$$

reciprocal network

$$Z_{12} = Z_{21}$$

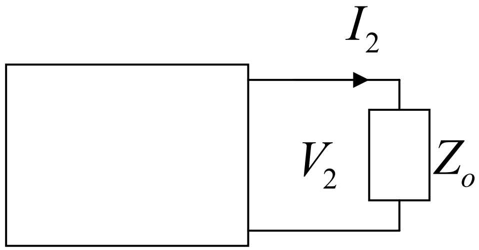
$$\frac{AD - BC}{C} = \frac{1}{C} \rightarrow AD - BC = 1$$

(*derivation*) S (ABCD)

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}, \begin{matrix} V_1 = V_1^+ + V_1^- \\ I_1 = (V_1^+ - V_1^-)/Z_o \end{matrix} \rightarrow \begin{matrix} V_1^+ = (V_1 + Z_o I_1)/2 \\ V_1^- = (V_1 - Z_o I_1)/2 \end{matrix}$$

$$S_{11} = \frac{b_1}{a_1}\bigg|_{a_2=0} = \frac{V_1^-}{V_1^+}\bigg|_{V_2^+=0} = \frac{V_1 - Z_o I_1}{V_1 + Z_o I_1}\bigg|_{V_2^+=0} = \frac{AV_2 + BI_2 - Z_o CV_2 - Z_o DI_2}{AV_2 + BI_2 + Z_o CV_2 + Z_o DI_2}\bigg|_{V_2^+=0}$$

$$= \frac{A + BY_o - CZ_o - D}{A + BY_o + CZ_o + D}$$



$$V_2^+ = 0$$

$$\rightarrow V_2 = I_2 Z_o, I_2 = V_2 Y_o$$

$$S_{21} = \frac{b_2}{a_1}\bigg|_{a_2=0} = \frac{V_2^-}{V_1^+}\bigg|_{V_2^+=0} \overset{V_2=V_2^-}{=} \frac{V_2}{(V_1 + Z_o I_1)/2} = \frac{2V_2}{AV_2 + BI_2 + Z_o CV_2 + Z_o DI_2}$$

$$\overset{I_2=V_2 Y_o}{=} \frac{2}{A + BY_o + CZ_o + D}$$

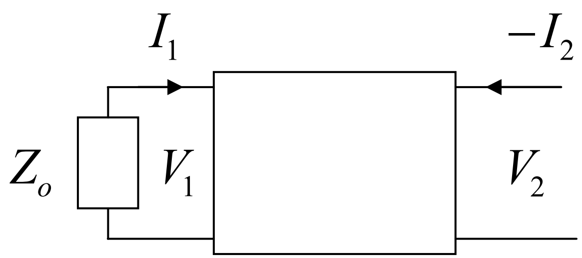
$$S_{12}=\frac{b_1}{a_2}\Big|_{a_1=0}=\frac{V_1^-}{V_2^+}\Big|_{V_1^+=0},\quad \begin{matrix} V_2=V_2^++V_2^- \\ -I_2=(V_2^+-V_2^-)/Z_o \end{matrix} \rightarrow \begin{matrix} V_2^+=(V_2-Z_oI_2)/2 \\ V_2^-=(V_2+Z_oI_2)/2 \end{matrix}$$

$$V_1^-=V_1$$

$$=\frac{V_1}{(V_2-Z_oI_2)/2},\begin{bmatrix} V_2 \\ I_2 \end{bmatrix}=\frac{1}{\Delta}\begin{bmatrix} D & -B \\ -C & A \end{bmatrix}\begin{bmatrix} V_1 \\ I_1 \end{bmatrix},\Delta=AD-BC$$

$$=\frac{2V_1}{(DV_1-BI_1+Z_oCV_1-Z_oAI_1)/\Delta}=\frac{2\Delta V_1}{DV_1+BY_oV_1+CZ_oV_1+AV_1}$$

$$=\frac{2\Delta}{A+BY_o+CZ_o+D}$$



$$V_1=-I_1Z_o, I_1=-V_1Y_o$$

$$S_{22}=\frac{b_2}{a_2}\Big|_{a_1=0}=\frac{V_2^-}{V_2^+}\Big|_{V_1^+=0}=\frac{V_2+Z_oI_2}{V_2-Z_oI_2}=\frac{DV_1-BI_1+Z_o(-CV_1+AI_1)}{DV_1-BI_1-Z_o(-CV_1+AI_1)}$$

$$\overset{I_1=-V_1Y_o}{=}\frac{DV_1+BY_oV_1+Z_o(-CV_1+AY_oV_1)}{DV_1+BY_oV_1-Z_o(-CV_1+AY_oV_1)}=\frac{-A+BY_o-CZ_o+D}{A+BY_o+CZ_o+D}$$

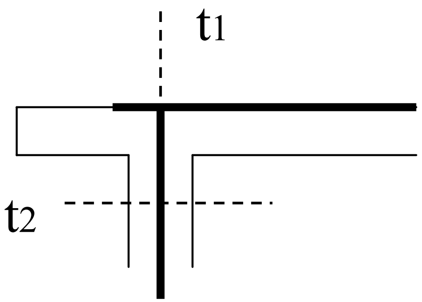
symmetrical

$S_{11}=S_{22}, A=D$

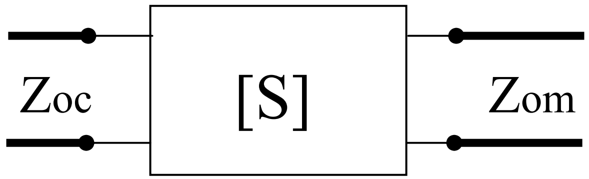
reciprocal

$S_{12}=S_{21}, \Delta=1$

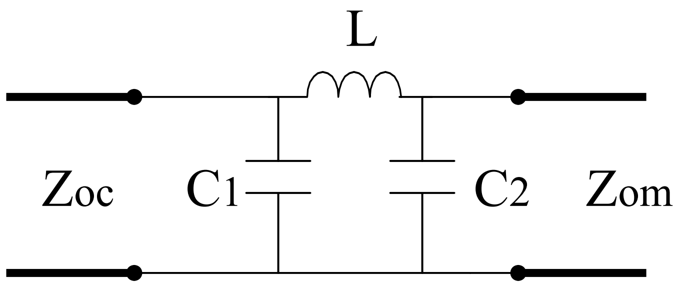
5. Example



coaxial-microstrip transition
(a linear circuit)



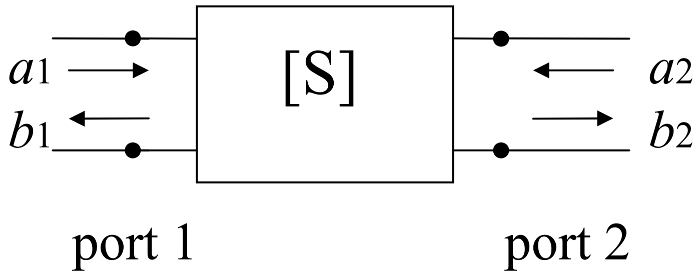
[S] representation can be obtained from measurement or calculation.

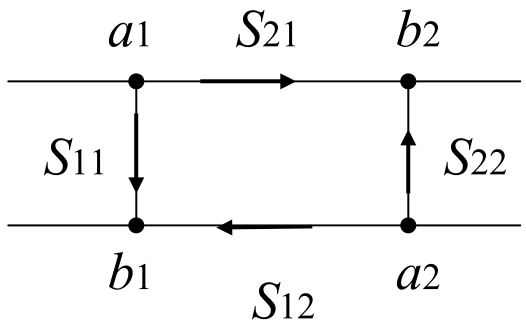


one possible equivalent circuit

4.5 Signal flow graphs

• 2-port representation





$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$b_1 = a_1 S_{11} + a_2 S_{12}$$

$$b_2 = a_1 S_{21} + a_2 S_{22}$$

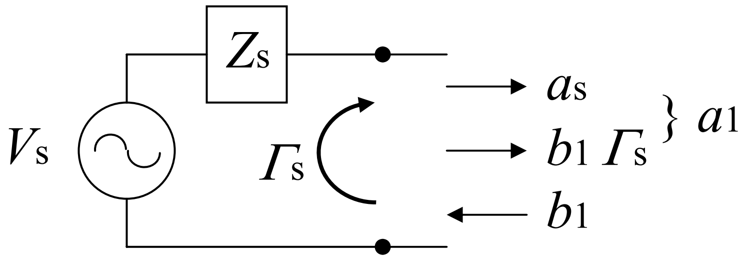
$$RL \text{ at port 1:} -20\log\left|\frac{b_1}{a_1}\right| = -20\log|S_{11}|$$

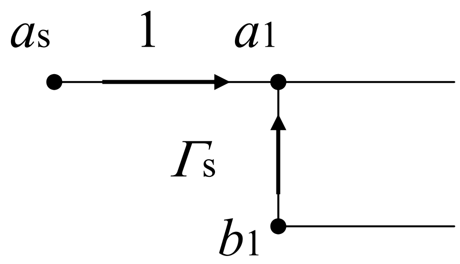
$$IL \text{ from port 1 to port 2:} -20\log\left|\frac{b_2}{a_1}\right| = -20\log|S_{21}|$$

Discussion

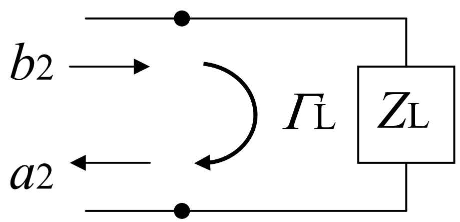
1. Source representation

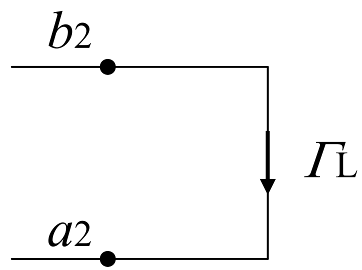
$$e_s = V_s \frac{Z_o}{Z_o + Z_s}, a_s \equiv \frac{e_s}{\sqrt{Z_o}}$$





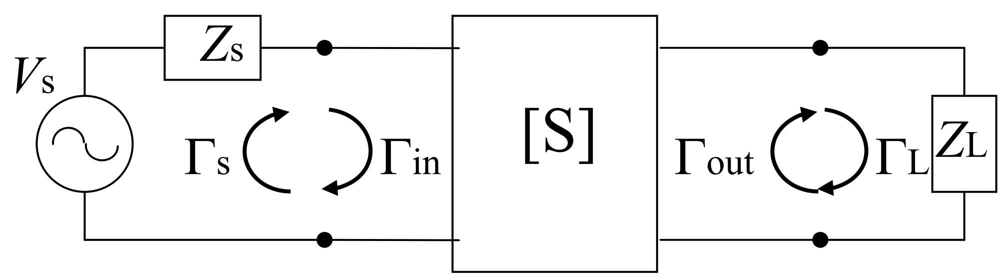
2. Load representation

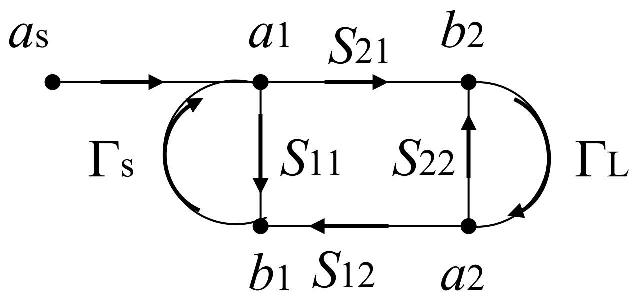




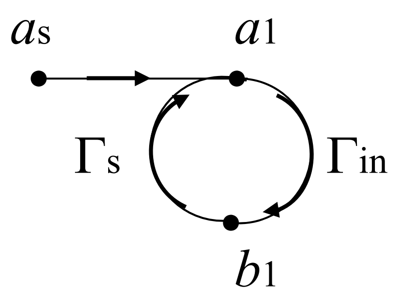
3. Series, parallel, self-loop, splitting rules (p.191, Fig.4.16)

4. 2-port circuit representation





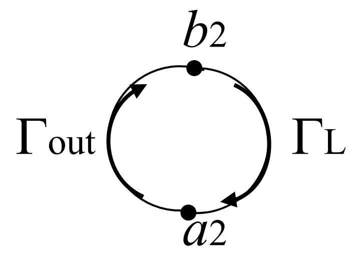
$$b_1 = a_1 S_{11} + a_1 S_{21} \Gamma_L S_{12} (1 + S_{22}\Gamma_L + \ldots) = a_1 S_{11} + a_1 \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L}$$



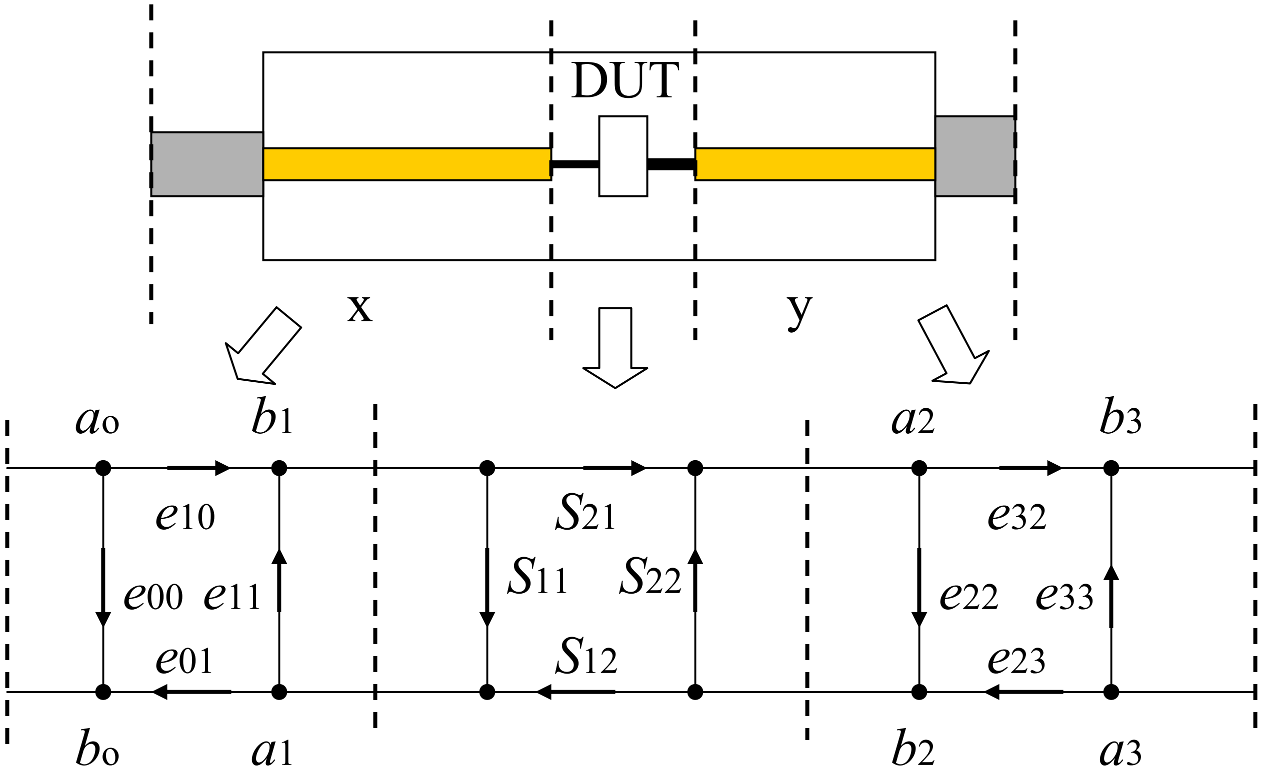
$$\rightarrow \Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L}$$

$$b_2 = a_2 S_{22} + a_2 S_{12} \Gamma_S S_{21} (1 + S_{11}\Gamma_S + \ldots) = a_2 S_{22} + a_2 \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S}$$

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S}$$



5. TRL (Thru-Reflect-Line) calibration



6 unknowns to be solved

T: Through $\rightarrow$ 3 eqs., R: Reflection $\rightarrow$ 2 eqs., L: Line $\rightarrow$ 3 eqs.

$\Rightarrow$ R (Γ) and line length (γl) can be unknown

微波電路講義

Calibrators

T: Through

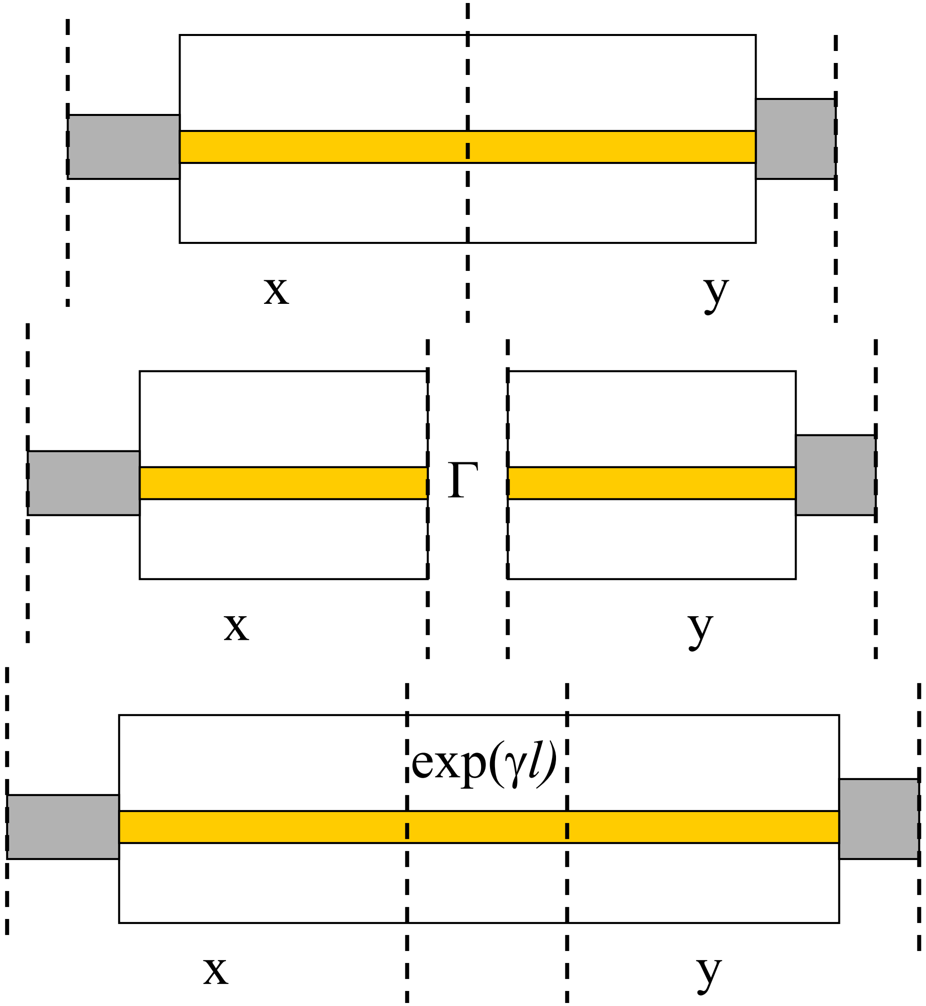
→ 3 eqs.

R: Reflection

→2 eqs.

L: Line

→ 3 eqs.



Requirement: connectors and line have same characteristics for 3 calibrators

Limitation: operation bandwidth $20^\circ \leqq \beta l \leqq 340^\circ$

R – matrix (wave cascade matrix)

$$\begin{bmatrix} \mathrm{b}_1 \\ \mathrm{a}_1 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} \mathrm{a}_2 \\ \mathrm{b}_2 \end{bmatrix}$$

$$[R] = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = \frac{1}{S_{21}} \begin{bmatrix} S_{12}S_{21} - S_{11}S_{22} & S_{11} \\ -S_{22} & 1 \end{bmatrix}$$

error matrices $[R_x] = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$, $[R_y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

Through: $[S_T] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow [R_T] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Line: $[S_L] = \begin{bmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{bmatrix} \rightarrow [R_L] = \begin{bmatrix} e^{-\gamma l} & 0 \\ 0 & 1/e^{-\gamma l} \end{bmatrix}$ Reflection: Γ

Thru measurement: $[R_{mT}] = [R_x]\,[R_T]\,[R_y] = [R_x]\,[R_y]$

Line measurement: $[R_{mL}] = [R_x]\,[R_L]\,[R_y]$

$$\Rightarrow \left[M\right]\left[R_x\right]=\left[R_x\right]\left[R_L\right], \quad \left[M\right]\equiv\left[R_{mL}\right]\left[R_{mT}\right]^{-1} \quad \Rightarrow \quad e_{00}, \frac{e_{01}e_{10}}{e_{11}}$$

$$\left[R_y\right]\left[N\right]=\left[R_L\right]\left[R_y\right], \quad \left[N\right]\equiv\left[R_{mT}\right]^{-1}\left[R_{mL}\right] \quad \Rightarrow \quad e_{33}, \frac{e_{23}e_{32}}{e_{22}}$$

reflection measurement at port 1 $\Gamma_{mx} = e_{00} + \dfrac{e_{10}e_{01}\Gamma}{1-e_{11}\Gamma}$

reflection measurement at port 2 $\Gamma_{my} = e_{33} + \dfrac{e_{23}e_{32}\Gamma}{1-e_{22}\Gamma}$

$$\left.\begin{array}{l}\Gamma_{mx}\\ \Gamma_{my}\\ \Gamma_{mT}\end{array}\right\} \Rightarrow \quad e_{11},\ e_{22}, e_{10}e_{01}, e_{23}e_{32} \qquad \left.\begin{array}{l}S_{21mT}\\ S_{12mT}\end{array}\right\} \Rightarrow \quad e_{10}e_{32}, e_{23}e_{01}$$

$\Rightarrow e_{10}, e_{01}, e_{23}, e_{32}$

$\Rightarrow \quad \Gamma, \ e^{-\gamma l}$

(detailed derivation)

$$[M][R_x]=[R_x][R_L]\rightarrow\begin{bmatrix}m_{11}&m_{12}\\m_{21}&m_{22}\end{bmatrix}\begin{bmatrix}x_{11}&x_{12}\\x_{21}&x_{22}\end{bmatrix}=\begin{bmatrix}x_{11}&x_{12}\\x_{21}&x_{22}\end{bmatrix}\begin{bmatrix}e^{-\gamma l}&0\\0&1/e^{-\gamma l}\end{bmatrix}$$

$$\rightarrow\begin{cases}m_{11}x_{11}+m_{12}x_{21}=x_{11}e^{-\gamma l}\\m_{21}x_{11}+m_{22}x_{21}=x_{21}e^{-\gamma l}\\m_{11}x_{12}+m_{12}x_{22}=x_{12}/e^{-\gamma l}\\m_{21}x_{12}+m_{22}x_{22}=x_{22}/e^{-\gamma l}\end{cases}\begin{matrix}\rightarrow m_{21}(\frac{x_{11}}{x_{21}})^2+(m_{22}-m_{11})\frac{x_{11}}{x_{21}}-m_{12}=0\\\rightarrow m_{21}(\frac{x_{12}}{x_{22}})^2+(m_{22}-m_{11})\frac{x_{12}}{x_{22}}-m_{12}=0\end{matrix}\rightarrow\begin{matrix}\frac{x_{11}}{x_{21}}=e_{00}-\frac{e_{10}e_{01}}{e_{11}}\\\frac{x_{12}}{x_{22}}=e_{00}\end{matrix}$$

$$[R_y][N]=[R_L][R_y]\rightarrow\begin{bmatrix}y_{11}&y_{12}\\y_{21}&y_{22}\end{bmatrix}\begin{bmatrix}n_{11}&n_{12}\\n_{21}&n_{22}\end{bmatrix}=\begin{bmatrix}e^{-\gamma l}&0\\0&1/e^{-\gamma l}\end{bmatrix}\begin{bmatrix}y_{11}&y_{12}\\y_{21}&y_{22}\end{bmatrix}$$

$$\rightarrow\begin{cases}y_{11}n_{11}+y_{12}n_{21}=y_{11}e^{-\gamma l}\\y_{21}n_{11}+y_{22}n_{21}=y_{21}e^{-\gamma l}\\y_{11}n_{12}+y_{12}n_{22}=y_{12}/e^{-\gamma l}\\y_{21}n_{12}+y_{22}n_{22}=y_{22}/e^{-\gamma l}\end{cases}\begin{matrix}\rightarrow n_{21}(\frac{y_{11}}{y_{12}})^2+(n_{22}-n_{11})\frac{y_{11}}{y_{12}}-n_{12}=0\\\rightarrow n_{21}(\frac{y_{21}}{y_{22}})^2+(n_{22}-n_{11})\frac{y_{21}}{y_{22}}-n_{21}=0\end{matrix}\rightarrow\begin{matrix}\frac{y_{11}}{y_{12}}=-e_{33}+\frac{e_{23}e_{32}}{e_{22}}\\\frac{y_{21}}{y_{22}}=-e_{33}\end{matrix}$$

$$\begin{cases} \Gamma_{mx} = e_{00} + \dfrac{e_{10}e_{01}\Gamma}{1-e_{11}\Gamma} \rightarrow \Gamma = \dfrac{1}{e_{11}}\dfrac{b-\Gamma_{mx}}{a-\Gamma_{mx}}, (b = e_{00}, a = e_{00} - \dfrac{e_{10}e_{01}}{e_{11}}) \\ \Gamma_{my} = e_{33} + \dfrac{e_{23}e_{32}\Gamma}{1-e_{22}\Gamma} \rightarrow \Gamma = \dfrac{1}{e_{22}}\dfrac{d+\Gamma_{mx}}{c+\Gamma_{mx}}, (d = -e_{33}, c = -e_{33} + \dfrac{e_{23}e_{32}}{e_{22}}) \\ \Gamma_{mT} = e_{00} + \dfrac{e_{10}e_{01}e_{22}}{1-e_{11}e_{22}} \rightarrow e_{11} = \dfrac{1}{e_{22}}\dfrac{b-\Gamma_{mx}}{a-\Gamma_{mx}} \end{cases}$$

$$\Rightarrow e_{11}^2 = \frac{b-\Gamma_{mx}}{a-\Gamma_{mx}}\frac{c+\Gamma_{mx}}{d+\Gamma_{mx}}\frac{b-\Gamma_{mx}}{a-\Gamma_{mx}}, e_{22} = \frac{1}{e_{11}}\frac{b-\Gamma_{mT}}{a-\Gamma_{mT}}, e_{10}e_{01} = (b-a)e_{11}, e_{23}e_{32} = (c-d)e_{11}$$

$$\begin{cases} S_{21mT} = \dfrac{e_{10}e_{32}}{1-e_{11}e_{22}} \\ S_{12mT} = \dfrac{e_{23}e_{01}}{1-e_{11}e_{22}} \end{cases} \Rightarrow e_{10}e_{32} = S_{21mT}(1-e_{11}e_{22}), e_{23}e_{01} = S_{12mT}(1-e_{11}e_{22}) \Rightarrow e_{10}, e_{01}, e_{23}, e_{32}$$

$$\Rightarrow \Gamma = \frac{1}{e_{11}}\frac{b-\Gamma_{mx}}{a-\Gamma_{mx}} \text{...(also for } e_{11} \text{ selection)}, e^{-\gamma l} = m_{11} + \frac{m_{12}}{a}$$

4.6 Discontinuities and modal analysis

- equivalent circuit components

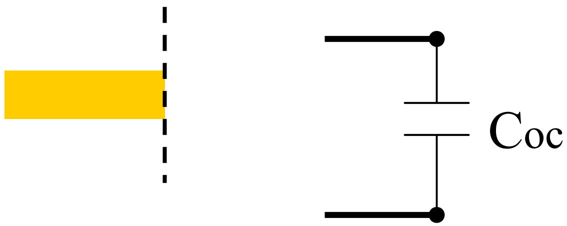
  $\Delta E \Rightarrow C, \quad \Delta H \Rightarrow L$

  constant E (V) $\Rightarrow$ parallel connection

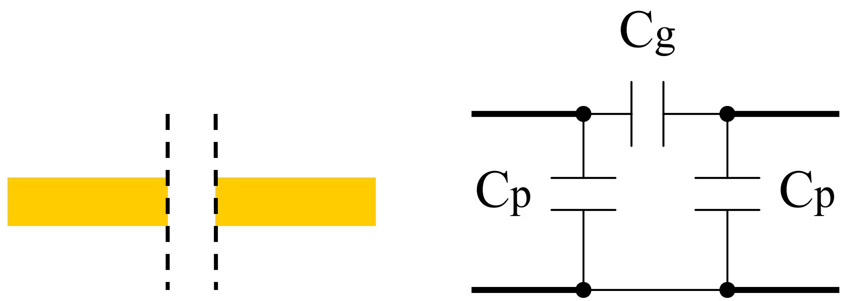
  constant H (I) $\Rightarrow$ serial connection

Discussion

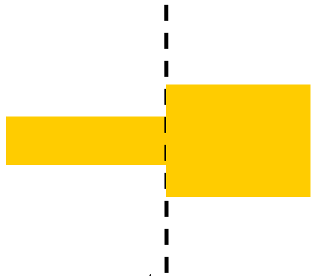
1. Microstrip discontinuities



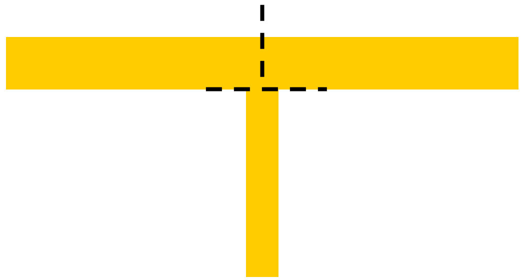
open-end



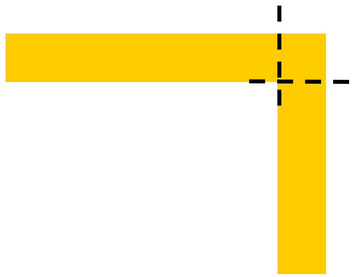
gap



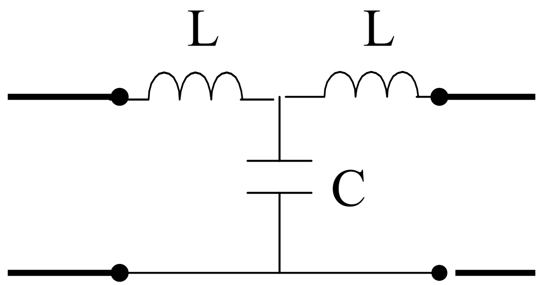
step

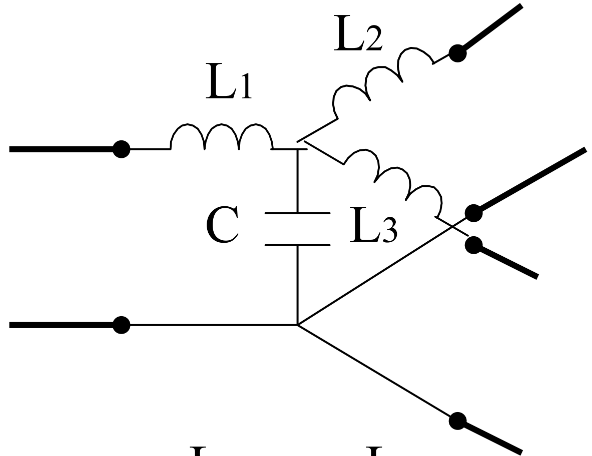


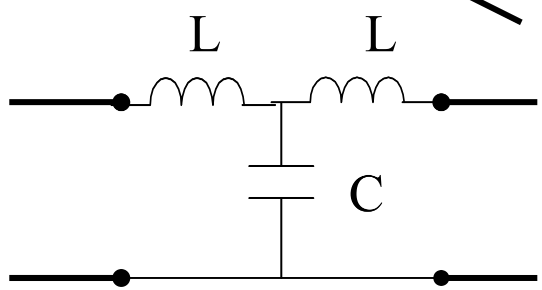
T-junction



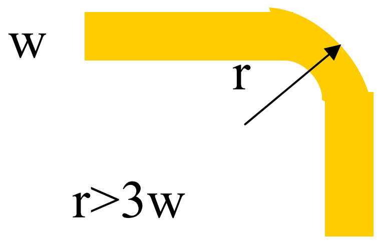
bend



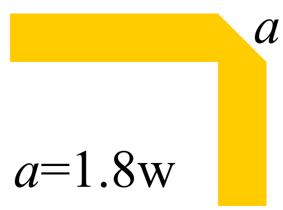




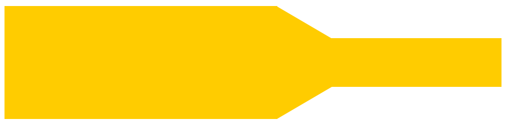
2. Microstrip discontinuity compensation



swept bend



mitered bends



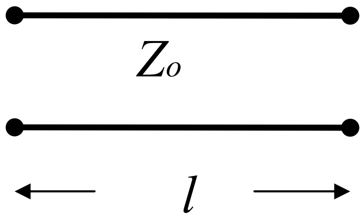
mitered step

mitered T-junction

Solved Problems

Prob. 4.10 Find [S] relative to Z_0

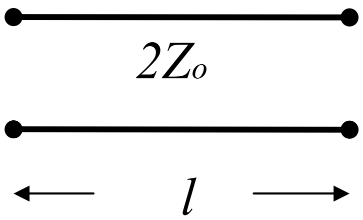
(a)



$$S_{11} = S_{22} = 0$$

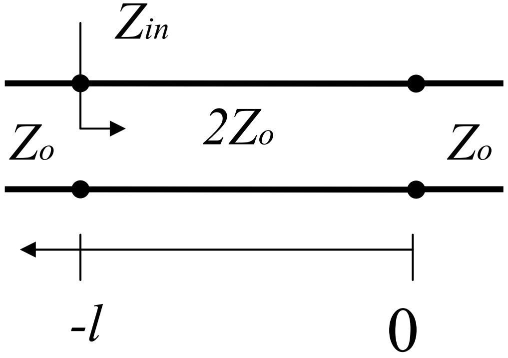
$$S_{12} = S_{21} = e^{-j\beta l}$$

(b)

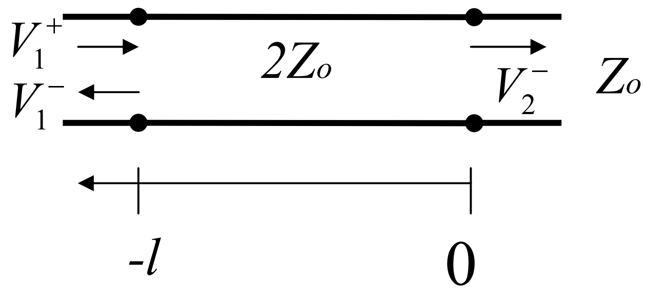


$$Z_{in} = \frac{V_1}{I_1} = \frac{V_o^+ e^{j\beta l} + V_o^- e^{-j\beta l}}{\frac{1}{2Z_o}(V_o^+ e^{j\beta l} - V_o^- e^{-j\beta l})} = 2Z_0 \frac{1+\Gamma(0)e^{-j2\beta l}}{1-\Gamma(0)e^{-j2\beta l}}$$

$$\Gamma(0) = \frac{Z_o - 2Z_o}{Z_o + 2Z_o} = -\frac{1}{3}$$



$$S_{11} = S_{22} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = \frac{2\dfrac{1-\frac{1}{3}e^{-j2\beta l}}{1+\frac{1}{3}e^{-j2\beta l}} - 1}{2\dfrac{1-\frac{1}{3}e^{-j2\beta l}}{1+\frac{1}{3}e^{-j2\beta l}} + 1} = \frac{1 - e^{-j2\beta l}}{3 - \frac{1}{3}e^{-j2\beta l}}$$



$$V_1 = V_o^+ e^{j\beta l} + V_o^- e^{-j\beta l}, \frac{V_o^-}{V_o^+} = \Gamma(0) = -\frac{1}{3}$$

$$= V_1^+ + V_1^- = V_1^+(1+S_{11})$$

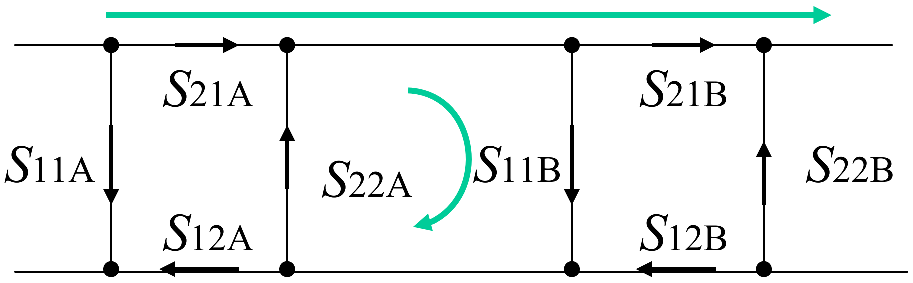
$$V_2 = V_o^+ + V_o^- = V_o^+(1+\Gamma(0))$$

$$= V_2^-$$

$$S_{21} = \frac{V_2^-}{V_1^+} = \frac{V_o^+(1-\frac{1}{3})}{\frac{V_o^+(e^{j\beta l} - \frac{1}{3}e^{-j\beta l})}{1+S_{11}}} = \frac{\frac{2}{3}(1+\frac{1-e^{-j2\beta l}}{3-\frac{1}{3}e^{-j2\beta l}})}{e^{j\beta l} - \frac{1}{3}e^{-j\beta l}} = \frac{\frac{2}{3}\frac{4-\frac{4}{3}e^{-j2\beta l}}{3-\frac{1}{3}e^{-j2\beta l}}}{e^{j\beta l} - \frac{1}{3}e^{-j\beta l}}$$

$$= \frac{8}{3}\frac{1-\frac{1}{3}e^{-j2\beta l}}{3-\frac{1}{3}e^{-j2\beta l}}\frac{1}{e^{j\beta l} - \frac{1}{3}e^{-j\beta l}} = \frac{8}{3}\frac{e^{j\beta l}}{3-\frac{1}{3}e^{-j2\beta l}}$$

Prob. 4.11 Find S_{21} of $[S_A]$ in cascade of $[S_B]$



$$S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{22}^A S_{11}^B}$$

Prob. 4.16

$$\begin{bmatrix} 0.1\angle 90^0 & 0.8\angle -45^0 & 0.3\angle -45^0 & 0 \\ 0.8\angle -45^0 & 0 & 0 & 0.4\angle 45^0 \\ 0.3\angle -45^0 & 0 & 0 & 0.6\angle -45^0 \\ 0 & 0.4\angle 45^0 & 0.6\angle -45^0 & 0 \end{bmatrix}$$

$(1)|S_{12}|^2 + |S_{42}|^2 = 0.8 \neq 1 \rightarrow lossy$

$(2)[S]$ symmetrical $\rightarrow reciprocal$

(3)return loss at port $1 = -20\log|S_{11}| = -20\log 0.1 = 20dB$

(4)insertion loss between port 2 and port $4 = -20\log|S_{24}| = -20\log 0.4 = 8dB$

phase delay between port 2 and port $4 = 45^0$

(5) Reflection at port 1 as port 3 is connected to a short circuit

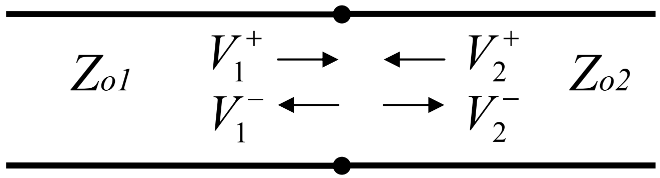
$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0.1\angle 90^0 & 0.8\angle -45^0 & 0.3\angle -45^0 & 0 \\ 0.8\angle -45^0 & 0 & 0 & 0.4\angle 45^0 \\ 0.3\angle -45^0 & 0 & 0 & 0.6\angle -45^0 \\ 0 & 0.4\angle 45^0 & 0.6\angle -45^0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ -b_3 \\ 0 \end{bmatrix}$$

$$b_3 = 0.3\angle -45^0 a_1$$

$$b_1 = 0.1\angle 90^0 a_1 - 0.3\angle -45^0 b_3 = 0.1\angle 90^0 a_1 - 0.09\angle -90^0 a_1$$

$$S_{11} = \frac{b_1}{a_1} = 0.09j + 0.1j = 0.19j$$

Prob. 4.18 find [S] of the junction



$$S_{11}=\frac{V_1^-/\sqrt{Z_{o1}}}{V_1^+/\sqrt{Z_{o1}}}=\frac{Z_{o2}-Z_{o1}}{Z_{o2}+Z_{o1}},\ S_{22}=\frac{V_2^-/\sqrt{Z_{o2}}}{V_2^+/\sqrt{Z_{o2}}}=\frac{Z_{o1}-Z_{o2}}{Z_{o1}+Z_{o2}}$$

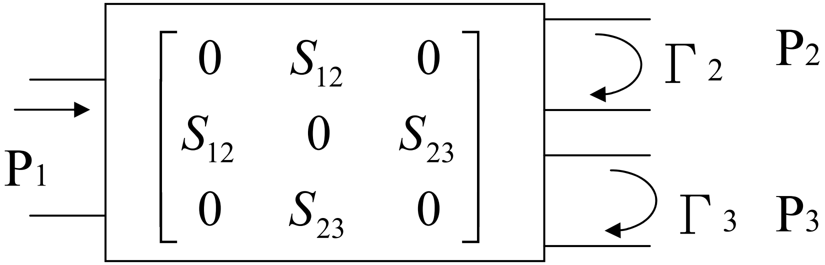
$$S_{21}=\frac{V_2^-/\sqrt{Z_{o2}}}{V_1^+/\sqrt{Z_{o1}}}=\frac{2Z_{o2}}{Z_{o1}+Z_{o2}}\frac{\sqrt{Z_{o1}}}{\sqrt{Z_{o2}}}=\frac{2\sqrt{Z_{o1}Z_{o2}}}{Z_{o1}+Z_{o2}}\quad \text{(p.63 (2.51)?)}$$

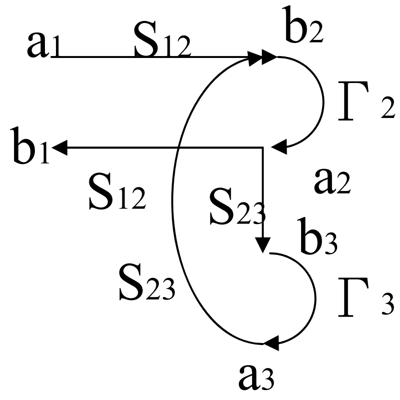
$$\because V_1^+ + V_1^- = V_2^- \rightarrow \frac{V_2^-}{V_1^+}=1+S_{11}=1+\frac{Z_{o2}-Z_{o1}}{Z_{o2}+Z_{o1}}=\frac{2Z_{o2}}{Z_{o1}+Z_{o2}}$$

$$S_{12}=\frac{V_1^-/\sqrt{Z_{o1}}}{V_2^+/\sqrt{Z_{o2}}}=\frac{2Z_{o1}}{Z_{o1}+Z_{o2}}\frac{\sqrt{Z_{o2}}}{\sqrt{Z_{o1}}}=\frac{2\sqrt{Z_{o1}Z_{o2}}}{Z_{o1}+Z_{o2}}=S_{21}$$

$$\because V_2^+ + V_2^- = V_1^- \rightarrow \frac{V_1^-}{V_2^+}=1+S_{22}=1+\frac{Z_{o1}-Z_{o2}}{Z_{o1}+Z_{o2}}=\frac{2Z_{o1}}{Z_{o1}+Z_{o2}}$$

Prob. 4.30 find P_2/P_1 and P_3/P_1





$$b_1 = a_1 \frac{S_{12}^2 \Gamma_2}{1-\Gamma_2 \Gamma_3 S_{23}^2} = a_1 \Gamma_{in}, b_2 = a_1 \frac{S_{12}}{1-\Gamma_2 \Gamma_3 S_{23}^2}, b_3 = a_1 \frac{S_{12}\Gamma_2 S_{23}}{1-\Gamma_2 \Gamma_3 S_{23}^2}$$

$$\frac{P_2}{P_1} = \frac{|b_2|^2 - |a_2|^2}{|a_1|^2 - |b_1|^2} = \frac{|b_2|^2 (1-|\Gamma_2|^2)}{|a_1|^2 (1-|\Gamma_{in}|^2)} = \frac{|S_{12}|^2 (1-|\Gamma_2|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2 (1-\frac{|S_{12}^2\Gamma_2|^2}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2})} = \frac{|S_{12}|^2 (1-|\Gamma_2|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2 - |S_{12}^2\Gamma_2|^2}$$

$$\frac{P_3}{P_1} = \frac{|b_3|^2 - |a_3|^2}{|a_1|^2 - |b_1|^2} = \frac{|b_3|^2 (1-|\Gamma_3|^2)}{|a_1|^2 (1-|\Gamma_{in}|^2)} = \frac{|S_{12}|^2 |\Gamma_2|^2 |S_{23}|^2 (1-|\Gamma_3|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2 (1-\frac{|S_{12}^2\Gamma_2|^2}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2})} = \frac{|S_{12}|^2 |\Gamma_2|^2 |S_{23}|^2 (1-|\Gamma_2|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2 - |S_{12}^2\Gamma_2|^2}$$

Suggested homework (due 2 weeks): 9, 21, 25, 28, 29

ADS examples: Ch4_prj