Chapter 4 Microwave network analysis

4.1 Impedance and equivalent voltages and currents
equivalent transmission line model (β, Zo)
4.2 Impedance and admittance matrices
not applicable in microwave circuits
4.3 The scattering matrix
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4.1 Impedance and equivalent voltages and currents

- Equivalent voltages and currents

Microwave circuit approach

Interest: voltage and current at a set of terminals (ports), power flow through a device, and how to find the response of a network

For a certain mode in the line, the line characteristics are represented by it's global quantities Zo, β, *l*.

Define: equivalent voltage (wave) ∝ transverse electric field
equivalent current (wave) ∝ transverse magnetic field

∍ voltage (wave)/current (wave) = characteristic impedance or wave impedance of the line

and voltage × current = power flow of the mode

→ use transmission line theory to analyze microwave circuit performance at the interested ports

- Impedance

characteristic impedance of the medium $\eta = \sqrt{\dfrac{\mu}{\varepsilon}}$

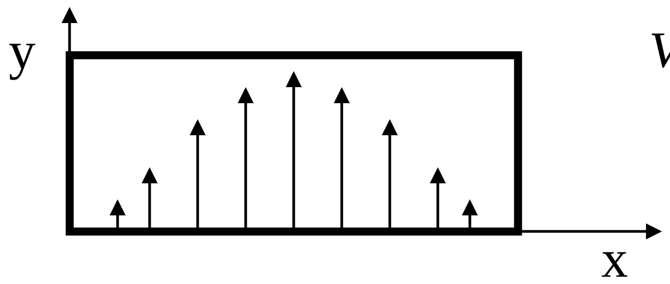
wave impedance of the particular mode of wave $Z_w = \dfrac{E_t^+}{H_t^+}$

characteristic impedance of the line $Z_o = \dfrac{V^+}{I^+}$

input impedance at a port of circuit $Z_{in}(z) = \dfrac{V(z)}{I(z)}$

Discussion

1. Transmission line model for the TE10 mode of a rectangular waveguide



$$V(x,z) \equiv \int \bar{E} \bullet d\bar{l} = \int E_y dy$$

: x – dependent , non - unique value

transverse fields (Table 3.2, p.117)	transmission line model
$E_y = (A^+ e^{-j\beta z} + A^- e^{j\beta z}) \sin \frac{\pi x}{a} \equiv C_1 V \sin \frac{\pi x}{a}$	$V = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$
$H_x = -\frac{1}{Z_{TE_{10}}} (A^+ e^{-j\beta z} - A^- e^{j\beta z}) \sin \frac{\pi x}{a} \equiv C_2 I \sin \frac{\pi x}{a}$	$I = I_o^+ e^{-j\beta z} - I_o^- e^{j\beta z} = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{V_o^-}{Z_o} e^{j\beta z}$
$Z_{TE_{10}} = -\frac{E_y^+}{H_x^+} = \frac{k\eta}{\beta} \equiv Z_o$	$Z_o = \frac{V_o^+}{I_o^+} = \frac{V_o^-}{I_o^-}$
$P^+ = -\frac{1}{2} \int E_y^+ H_x^{+*} dxdy$	$P^+ = \frac{1}{2} V_o^+ I_o^{+*}$

(*derivation of C_1 and C_2*)

$$E_y = (A^+ e^{-j\beta z} + A^- e^{j\beta z})\sin\frac{\pi x}{a} \equiv C_1(V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z})\sin\frac{\pi x}{a}$$

$$\rightarrow V_o^+ = \frac{A^+}{C_1}, V_o^- = \frac{A^-}{C_1}$$

$$H_x = -\frac{1}{Z_{TE_{10}}}(A^+ e^{-j\beta z} - A^- e^{j\beta z})\sin\frac{\pi x}{a} \equiv C_2(I_o^+ e^{-j\beta z} - I_o^- e^{j\beta z})\sin\frac{\pi x}{a}$$

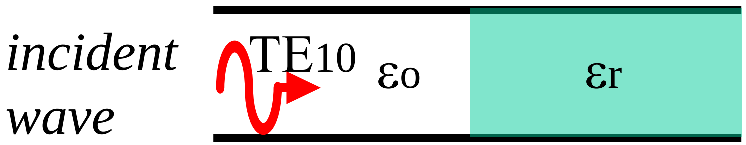
$$\rightarrow I_o^+ = -\frac{A^+}{C_2 Z_{TE_{10}}}, I_o^- = -\frac{A^-}{C_2 Z_{TE_{10}}}$$

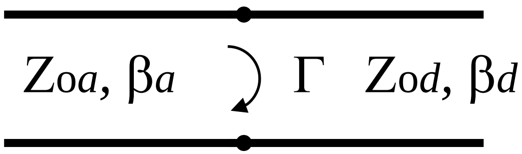
$$P^+ = -\frac{1}{2}\int_0^a\int_0^b E_y^+ H_x^{+*} dxdy = \frac{ab}{4Z_{TE_{10}}}\left|A^+\right|^2 \equiv \frac{1}{2}V_o^+ I_o^{+*} = -\frac{\left|A^+\right|^2}{2C_1C_2^* Z_{TE_{10}}} \rightarrow C_1C_2^* = -\frac{2}{ab}$$

$$Z_{TE_{10}} \equiv Z_o = \frac{V_o^+}{I_o^+} = -\frac{A^+ C_2 Z_{TE_{10}}}{C_1 A^+} \rightarrow \frac{C_2}{C_1} = -1$$

$$\Rightarrow C_1 = \sqrt{\frac{2}{ab}}, \quad C_2 = -\sqrt{\frac{2}{ab}}$$

2. Ex.4.2





$$\Gamma = \frac{Z_{od} - Z_{oa}}{Z_{od} + Z_{oa}}$$

Q: What if the incident wave is from the other direction? **"N"**

$$Z_{oa} = \frac{k_o \eta_o}{\beta_a},\ Z_{od} = \frac{k\eta}{\beta_d},\ k_o\eta_o = k\eta,\ k = \sqrt{\varepsilon_r}\,k_o$$

$$\beta_a^2 + k_c^2 = k_o^2,\ \beta_d^2 + k_c^2 = k^2,\ k_c = \frac{\pi}{a} = \frac{2\pi}{\lambda_c} = \frac{2\pi f_c}{v_p} = \frac{2\pi f_c}{c/\sqrt{\varepsilon_r}}$$

$$X-\text{band: } a = 2.286cm \rightarrow \lambda_c = 2a = 4.472cm \rightarrow k_c = 137m^{-1} \rightarrow f_{c,a} = 6.56GHz,\ f_{c,\varepsilon_r} = 4.17GHz$$

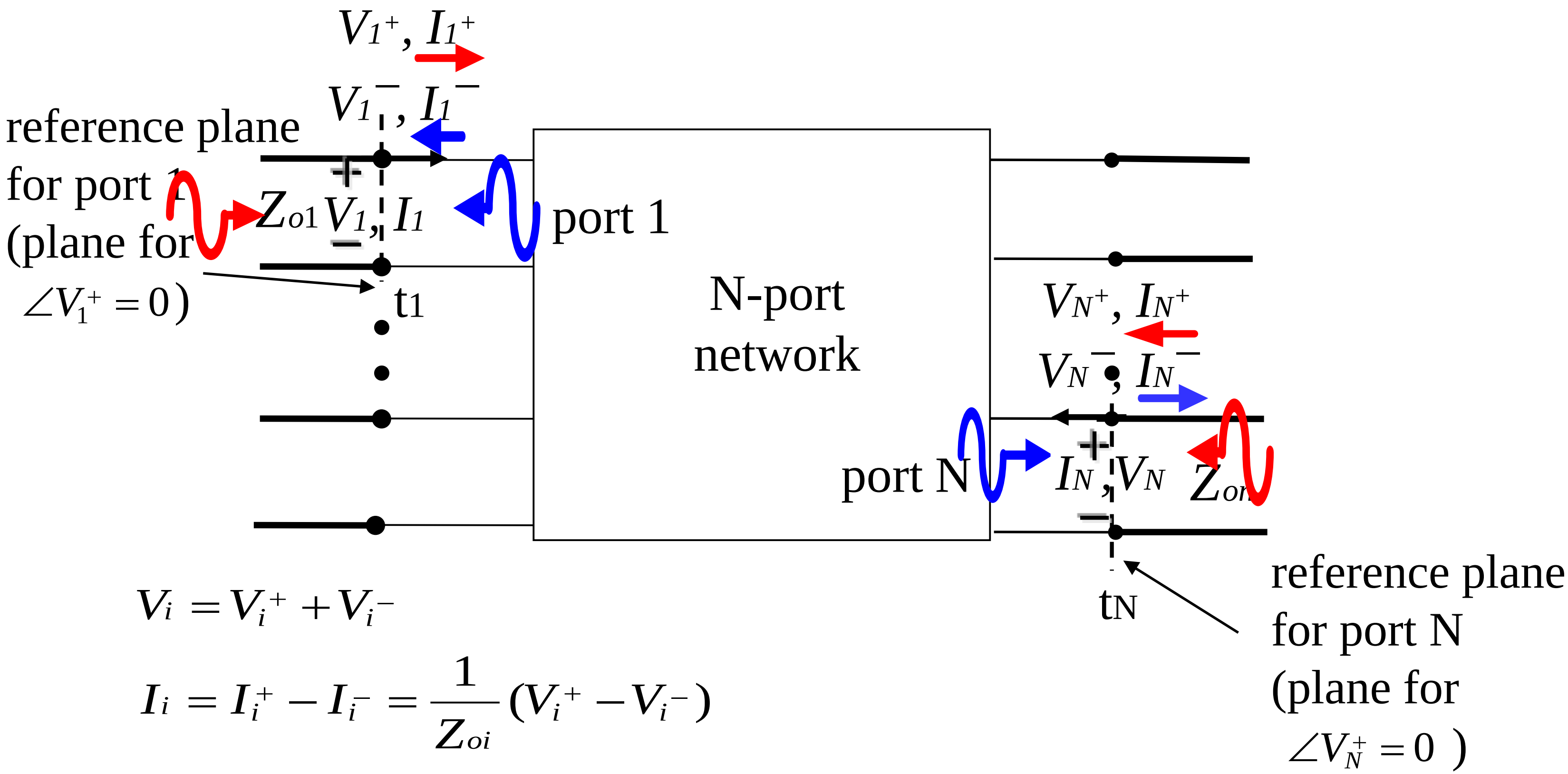
$$\text{if } f = 10GHz \rightarrow k_o = 2\pi f / c = 209m^{-1}, \beta_a = 158m^{-1}, \varepsilon_r = 2.54 \rightarrow k = 333m^{-1}, \beta_d = 304m^{-1}$$

$$Z_{oa} = 500\Omega, Z_{od} = 259.6\Omega, \Gamma = -0.316$$

$$\text{if } f = 6GHz \rightarrow k = \sqrt{\varepsilon_r}\,k_o = 201m^{-1} > k_c \text{ in } \varepsilon_r, \beta_d = 147m^{-1},$$

$$k_o = 126m^{-1} < k_c \text{ in } air, \beta_a = j54m^{-1}, TE_{10}\ "non-exist"$$

4.2 Impedance and admittance matrices



$$V_i = V_i^+ + V_i^-$$

$$I_i = I_i^+ - I_i^- = \frac{1}{Z_{oi}}(V_i^+ - V_i^-)$$

$$Z_{oi} = \frac{V_i^+}{I_i^+} = \frac{V_i^-}{I_i^-}$$

$$P_{inc,i} = \frac{1}{2}\mathrm{Re}\{V_i^+ I_i^{+*}\},\ P_{in,i} = \frac{1}{2}\mathrm{Re}\{V_i I_i^*\}$$

- Impedance matrix

$$[V]=[Z]\ [I]\ ,\ \begin{bmatrix} V_1 \\ V_2 \\ \bullet \\ \bullet \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \bullet & \bullet & Z_{1N} \\ Z_{21} & \bullet & \bullet & \bullet & Z_{2N} \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ Z_{N1} & Z_{N2} & \bullet & \bullet & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \bullet \\ \bullet \\ I_N \end{bmatrix},\ Z_{ij} = \left.\frac{V_i}{I_j}\right|_{I_k=0,k\neq j} = \left.\frac{response_i}{source_j}\right|_{I_k=0,k\neq j}$$

- Admittance matrix

$$[I]=[Y]\ [V],\begin{bmatrix} I_1 \\ I_2 \\ \bullet \\ \bullet \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \bullet & \bullet & Y_{1N} \\ Y_{21} & \bullet & \bullet & \bullet & Y_{2N} \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ Y_{N1} & Y_{N2} & \bullet & \bullet & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \bullet \\ \bullet \\ V_N \end{bmatrix},\ Y_{ij} = \left.\frac{I_i}{V_j}\right|_{V_k=0,k\neq j} = \left.\frac{response_i}{source_j}\right|_{V_k=0,k\neq j}$$

Discussion

1. Reciprocal network

$$[Z]=[Z]^t,\quad Z_{ij}=Z_{ji}$$
$$[Y]=[Y]^t,\quad Y_{ij}=Y_{ji}$$
$,[Z]$ and $[\mathrm{Y}]$: symmetric matrix

(*derivation*)

source		port 1	port 2
a	⟶	$V1a, I1a$	$V2a, I2a$
b	⟶	$V1b, I1b$	$V2b, I2b$

reciprocity theorem: $V1aI1b + V2aI2b = V1bI1a + V2bI2a$

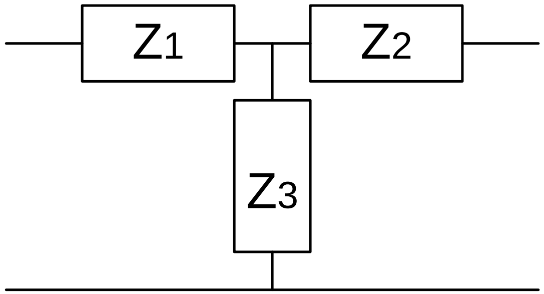
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

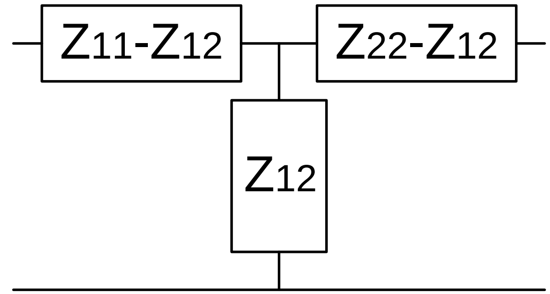
$$(Z_{11}I_{1a}+Z_{12}I_{2a})I_{1b}+(Z_{21}I_{1a}+Z_{22}I_{2a})I_{2b}=(Z_{11}I_{1b}+Z_{12}I_{2b})I_{1a}+(Z_{21}I_{1b}+Z_{22}I_{2b})I_{2a}$$

$$Z_{12}I_{2a}I_{1b}+Z_{21}I_{1a}I_{2b}=Z_{12}I_{2b}I_{1a}+Z_{21}I_{1b}I_{2a}$$

$$(Z_{12}-Z_{21})(I_{2a}I_{1b}-I_{2b}I_{1a})=0 \Rightarrow Z_{12}=Z_{21}$$

2. T and Π networks





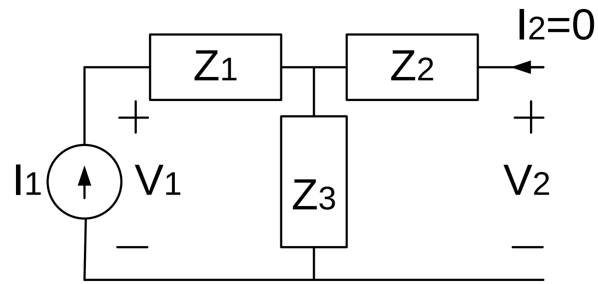
(*derivation*)

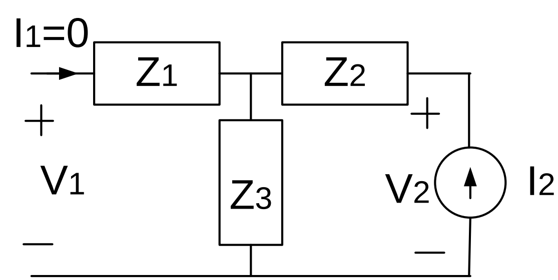
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

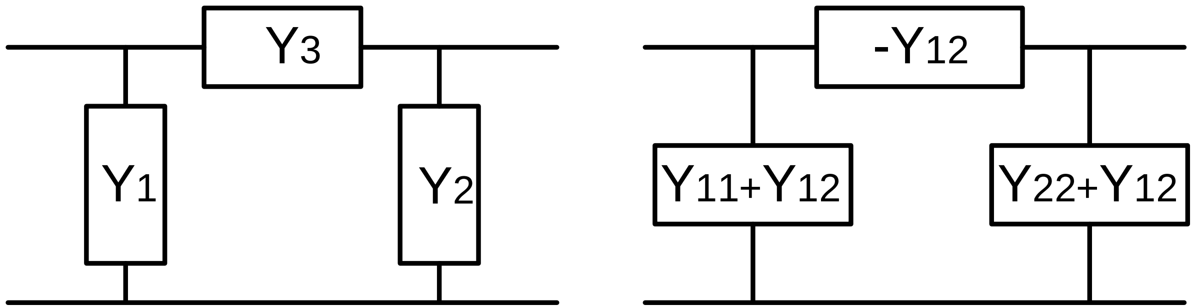
$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = Z_1 + Z_3, Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = Z_3 = Z_{12}$$

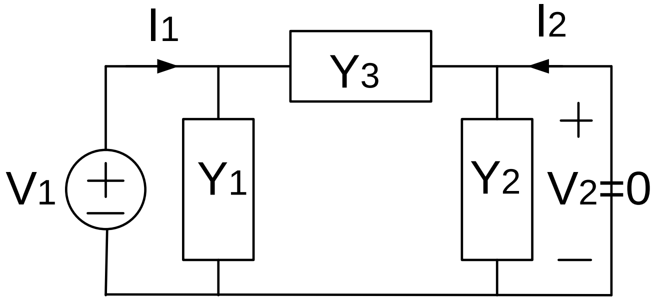
$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = Z_2 + Z_3$$

$$\rightarrow Z_3 = Z_{12}, Z_1 = Z_{11} - Z_{12}, Z_2 = Z_{22} - Z_{12}$$









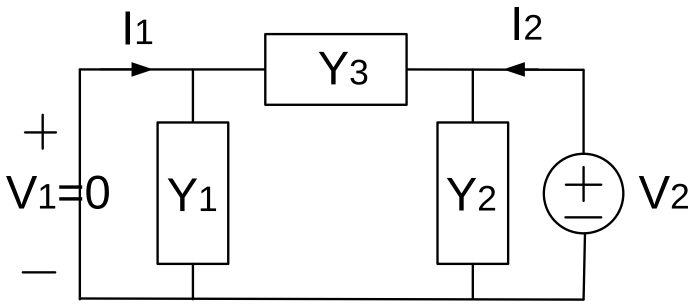
(*derivation*)

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

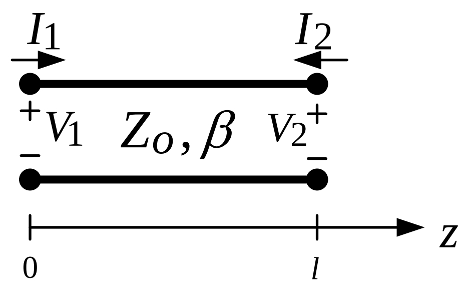
$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_1 + Y_3, Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -Y_3 = Y_{12}$$

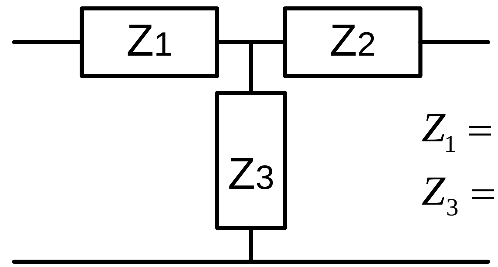
$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_2 + Y_3$$

$$\rightarrow Y_1 = Y_{11} + Y_{12}, Y_3 = -Y_{12}, Y_2 = Y_{22} + Y_{12}$$



3. Z- and Y- matrices of a lossless transmission line section





$$Z_1 = Z_2 = -jZ_o(\cot\beta l - \csc\beta l)$$
$$Z_3 = -jZ_o\csc\beta l$$

(*derivation*)

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z},\ I(z) = Y_o(V_o^+ e^{-j\beta z} - V_o^- e^{j\beta z}),\ \text{B.C. } V_1 = V(0), I_1 = I(0), V_2 = V(l), I_2 = -I(l)$$

$$V_1 = V_o^+ + V_o^-,\ I_1 = Y_o(V_o^+ - V_o^-) \rightarrow V_o^{\pm} = \frac{1}{2}(V_1 \pm Z_o I_1)$$

$$V_2 = V_o^+ e^{-j\beta l} + V_o^- e^{j\beta l} = \frac{1}{2}(V_1 + Z_o I_1)e^{-j\beta l} + \frac{1}{2}(V_1 - Z_o I_1)e^{j\beta l} = V_1\cos\beta l - jZ_o I_1 \sin\beta l \text{.....(1)}$$

$$I_2 = -Y_o(V_o^+ e^{-j\beta l} - V_o^- e^{j\beta l}) = -Y_o[\frac{1}{2}(V_1 + Z_o I_1)e^{-j\beta l} - \frac{1}{2}(V_1 - Z_o I_1)e^{j\beta l}] = jY_o V_1 \sin\beta l - I_1\cos\beta l \text{.....(2)}$$

$$\begin{cases} (2) \rightarrow V_1 = -jZ_o I_1 \cot\beta l - jZ_o I_2 \csc\beta l \text{.....(3)} \\ (1) \overset{(3)}{\rightarrow} V_2 = -jZ_o I_1 \cos\beta l \cot\beta l - jZ_o I_2 \cot\beta l - jZ_o I_1 \sin\beta l \\ \qquad = -jZ_o I_1 \csc\beta l - jZ_o I_2 \cot\beta l \end{cases} \Rightarrow [Z] = -jZ_o \begin{bmatrix} \cot\beta l & \csc\beta l \\ \csc\beta l & \cot\beta l \end{bmatrix}$$

$$Z_1 = Z_{11} - Z_{12} = -jZ_o(\cot\beta l - \csc\beta l) = Z_2,\ Z_3 = Z_{12} = -jZ_o\csc\beta l$$

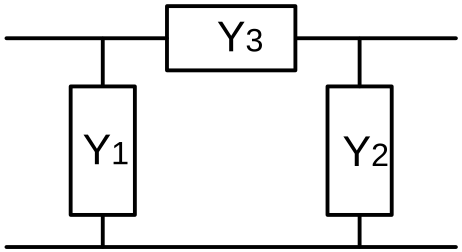
$$(1) V_2 = V_1 \cos\beta l - jZ_o I_1 \sin\beta l \rightarrow I_1 = -jY_o \cot\beta l V_1 + jY_o \csc\beta l V_2 \ldots(3)$$

$$\overset{(3)}{\rightarrow}(2) I_2 = jY_o V_1 \sin\beta l - I_1 \cos\beta l = jY_o V_1 \sin\beta l - (-jY_o \cot\beta l V_1 + jY_o \csc\beta l V_2)\cos\beta l$$

$$= jY_o \csc\beta l V_1 - jY_o \cot\beta l V_2$$

$$[Y] = -jY_o \begin{bmatrix} \cot\beta l & -\csc\beta l \\ -\csc\beta l & \cot\beta l \end{bmatrix}$$

$$\rightarrow Y_1 = Y_{11} + Y_{12} = -jY_o(\cot\beta l - \csc\beta l) = Y_2, Y_3 = jY_o \csc\beta l$$



$$Y_1 = Y_2 = -jY_o(\cot\beta l - \csc\beta l)$$
$$Y_3 = jY_o \csc\beta l$$

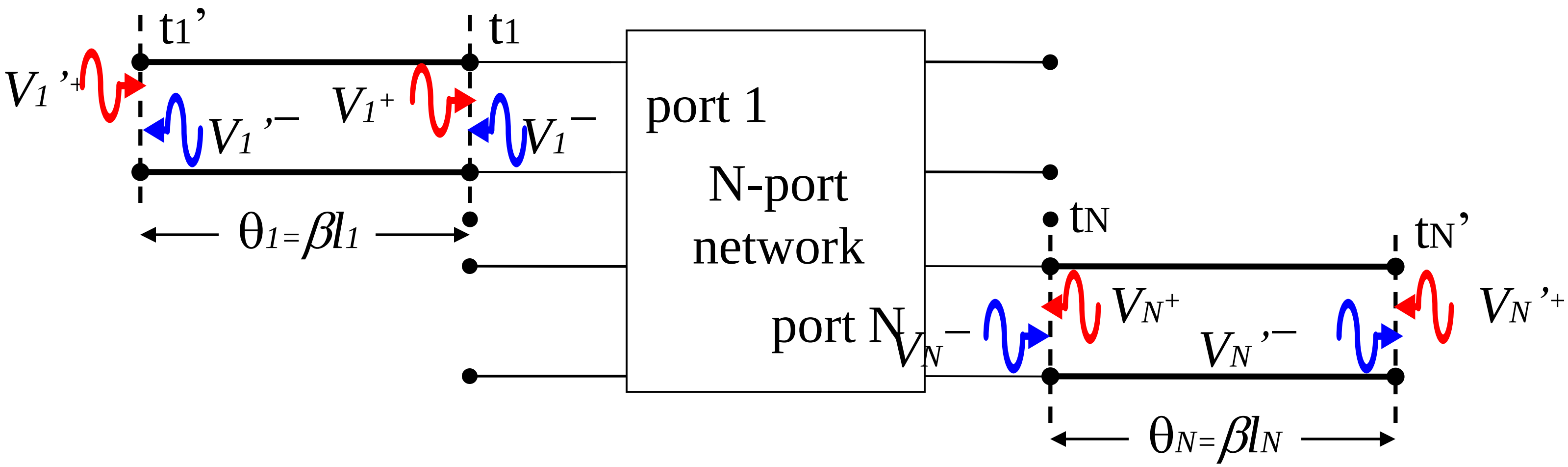
4. Reciprocal lossless network

$$\mathrm{Re}\{Z_{ij}\} = 0$$

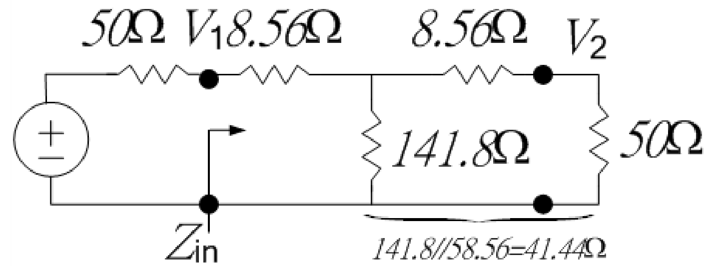
5. Problems to use Z- or Y-matrix in microwave circuits

1) difficult in defining voltage and current for non-TEM lines
2) no equipment available to measure voltage and current in complex value (eg. sampling scope in microwave range, impedance meter <3GHz)
3) difficult to make open and short circuits over broadband
4) active devices not stable as terminated with open or short circuit

4.3 The scattering matrix



$$[V^-]=[S][V^+],\quad \begin{bmatrix} V_1^- \\ V_2^- \\ \bullet \\ \bullet \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \bullet & \bullet & S_{1N} \\ S_{21} & \bullet & \bullet & \bullet & S_{2N} \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ S_{N1} & S_{N2} & \bullet & \bullet & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \bullet \\ \bullet \\ V_N^+ \end{bmatrix},\quad S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+=0,k\neq j} = \left.\frac{\text{response}_i}{\text{source}_j}\right|_{V_k^+=0,k\neq j}$$



Discussion

1. Ex 4.4 a 3dB attenuator (Zo=50Ω)

$$Z_{in} = 8.56 + 41.44 = 50$$

$$S_{11} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = 0 (= \frac{V_1^-}{V_1^+}) \rightarrow V_1^- = 0 \rightarrow V_1 = V_1^+ + V_1^- = V_1^+$$

$$V_2^+ = 0, V_2^- = V_2 = V_1 \frac{41.44}{41.44 + 8.56} \frac{50}{50 + 8.56} = 0.707 V_1 = \frac{1}{\sqrt{2}} V_1 = \frac{1}{\sqrt{2}} V_1^+ = S_{21} V_1^+$$

reciprocal $\rightarrow S_{21} = S_{12}$

symmetric $\rightarrow S_{11} = S_{22}$

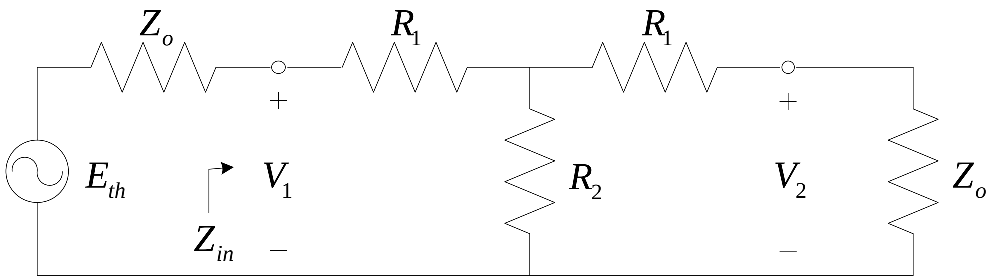
$$\rightarrow [S] = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 \end{bmatrix} : \text{lossy}$$

incident power to port 1: $P_{inc,1} = \frac{1}{2} \frac{|V_1^+|^2}{Z_o}$

transmitted power from port 2: $P_{trans,2} = \frac{1}{2} \frac{|V_2^-|^2}{Z_o} = \frac{1}{2} \frac{\left|\frac{1}{\sqrt{2}} V_1^+\right|^2}{Z_o} = \frac{1}{2} \times \frac{1}{2} \frac{|V_1^+|^2}{Z_o}$: 3dB attenuation

attenuator design $\begin{cases} \text{input match} \\ \text{attenuation value} \end{cases} \rightarrow R_1, R_2$

2. T-type attenuator design



$$\begin{cases} Z_{in} = (R_1 + Z_o) // R_2 + R_1 = Z_o \\ S_{21} = \dfrac{V_2}{V_1} = \dfrac{R_2 // (R_1 + Z_o)}{R_1 + R_2 // (R_1 + Z_o)} \dfrac{Z_o}{R_1 + Z_o} = \alpha \end{cases} \Rightarrow \begin{cases} R_1 = \dfrac{1-\alpha}{1+\alpha} Z_o \\ R_2 = \dfrac{2\alpha}{1-\alpha^2} Z_o \end{cases}$$

$$3dB \text{ attenuator } \alpha = \frac{1}{\sqrt{2}} \Rightarrow \begin{cases} R_1 = \dfrac{1-\alpha}{1+\alpha} Z_o = \dfrac{\sqrt{2}-1}{\sqrt{2}+1} Z_o \\ R_2 = \dfrac{2\alpha}{1-\alpha^2} Z_o = 2\sqrt{2} Z_o \end{cases}$$

3. Relation of [Z], [Y], and [S]

$$[S]=([Z]+[U])^{-1}([Z]-[U]),\ [Y]=[Z]^{-1}$$

(*derivation*)

Let $Z_{on}=1,\ \begin{matrix} V_n=V_n^{+}+V_n^{-} \\ I_n=V_n^{+}-V_n^{-} \end{matrix}$

$$[V]=[Z][I]\rightarrow\left[V^{+}\right]+\left[V^{-}\right]=[Z]\left(\left[V^{+}\right]-\left[V^{-}\right]\right)$$

$$\rightarrow[Z]\left[V^{-}\right]+\left[V^{-}\right]=[Z]\left[V^{+}\right]-\left[V^{+}\right]$$

$$([Z]+[U])\left[V^{-}\right]=([Z]-[U])\left[V^{+}\right]$$

$$\because\left[V^{-}\right]=[S]\left[V^{+}\right]\rightarrow([Z]+[U])[S]\left[V^{+}\right]=([Z]-[U])\left[V^{+}\right]$$

$$[S]=([Z]+[U])^{-1}([Z]-[U])$$

4. Reciprocal network

$$[S]=[S]^t,\quad [S]:\text{symmetric matrix}$$

(*derivation*)

Let $Z_{on}=1,\ \begin{matrix} V_n=V_n^+ + V_n^- \\ I_n=V_n^+ - V_n^- \end{matrix} \rightarrow \begin{matrix} V_n^+=(V_n+I_n)/2 \\ V_n^-=(V_n-I_n)/2 \end{matrix}$

$$\left[V^+\right]=\frac{1}{2}([V]+[I])=\frac{1}{2}([Z]+[U])[I]$$

$$\left[V^-\right]=\frac{1}{2}([V]-[I])=\frac{1}{2}([Z]-[U])[I]$$

$$\left[V^-\right]=([Z]-[U])([Z]+[U])^{-1}\left[V^+\right]\rightarrow[S]=([Z]-[U])([Z]+[U])^{-1}$$

$$[S]^t=(([Z]+[U])^{-1})^t([Z]-[U])^t=([Z]+[U])^{-1}([Z]-[U])\overset{from\ 3}{=}[S]$$

5. Lossless network (unitary property)

$$[S]^t[S]^* = [U], \quad \sum_{k=1}^{N} S_{ki}S_{kj}^* = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

(*derivation*)

Let $Z_{on} = 1$

lossless (incident power=transmitted power) $\rightarrow$ net averaged input power $\sum_i P_{in,i} = 0$

$$P_{in} = \frac{1}{2}\mathrm{Re}([V]^t [I]^*) = \frac{1}{2}\mathrm{Re}[([V^+]+[V^-])^t([V^+]^* - [V^-]^*)]$$

Im

$$= \frac{1}{2}\mathrm{Re}[[V^+]^t[V^+]^* - [V^-]^t[V^-]^* + ([V^-]^t[V^+]^* - [V^+]^t[V^-]^*)] = 0$$

$$[V^+]^t[V^+]^* = [V^-]^t[V^-]^* \overset{[V^-]=[S][V^+]}{=} [V^+]^t[S]^t[S]^*[V^+]^*$$

$$\rightarrow [S]^t[S]^* = [U]$$

6. Lossy network $\quad \sum_{k=1}^{N} S_{ki}S_{ki}^* < 1$

7. Ex.4.5

$$[S]=\begin{bmatrix} 0.15\angle 0^o & 0.85\angle -45^o \\ 0.85\angle 45^0 & 0.2\angle 0^0 \end{bmatrix}$$

$[S]$:not symmetric $\rightarrow$ a non-reciprocal network

$|S_{11}|^2+|S_{21}|^2=0.745\neq 1\rightarrow$ a lossy network

port 1 $RL=-20\log|S_{11}|=16.5dB$

port 2 $RL=-20\log|S_{22}|=14dB$

$IL=-20\log|S_{21}|=1.4dB$

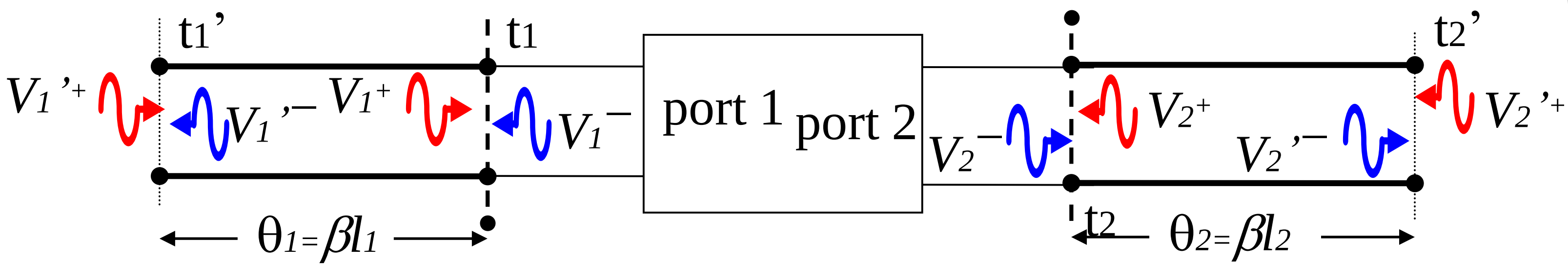
port 2 terminated with a matched load $\Gamma_L=0\rightarrow$

$\Gamma_{in}=|S_{11}|=0.15, RL=-20\log 0.15=16.5dB$

port 2 terminated with a short circuit $\Gamma_L=-1\rightarrow$

$$\Gamma_{in}=S_{11}+\frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}=-0.452, RL=6.9dB$$

8. Shift property

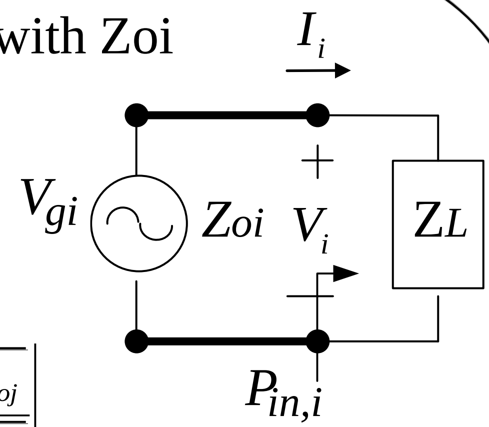


$$S'_{11} = e^{-j2\theta_1} S_{11}, S'_{21} = e^{-j\theta_1} S_{21} e^{-j\theta_2}, S'_{12} = e^{-j\theta_2} S_{12} e^{-j\theta_1}, S'_{22} = e^{-j2\theta_2} S_{22}$$

n-port network: $\left[S'\right] = \begin{bmatrix} e^{-j\theta_1} & 0 & \bullet & 0 \\ 0 & e^{-j\theta_2} & \bullet & 0 \\ \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & e^{-j\theta_N} \end{bmatrix} \left[S\right] \begin{bmatrix} e^{-j\theta_1} & 0 & \bullet & 0 \\ 0 & e^{-j\theta_2} & \bullet & 0 \\ \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & \bullet & e^{-j\theta_N} \end{bmatrix}$

9. S-matrix is not effected by the network arrangement.

10. Power waves on a lossless transmission line with Zoi



incident (power) wave : $a_i \equiv \dfrac{V_i^+}{\sqrt{Z_{oi}}} = \dfrac{V_i + Z_{oi} I_i}{2\sqrt{Z_{oi}}}$

reflected (power) wave : $b_i \equiv \dfrac{V_i^-}{\sqrt{Z_{oi}}} = \dfrac{V_i - Z_{oi} I_i}{2\sqrt{Z_{oi}}}$

$$\left[V^-\right] = [S]\left[V^+\right] \Rightarrow [b] = [S][a],\ S_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k=0,k\neq j} = \left.\frac{V_i^- \sqrt{Z_{oj}}}{V_j^+ \sqrt{Z_{oi}}}\right|_{V_k^+=0,k\neq j}$$

$$P_{in,i} = \frac{1}{2}\operatorname{Re}\{V_i I_i^*\} = \frac{1}{2}|a_i|^2 - \frac{1}{2}|b_i|^2 = P_{inc,i} - P_{refl,i} = P_{inc,i}(1 - |S_{ii}|^2) = P_L$$

$(derivation)$

$$a_i = \frac{V_i^+}{\sqrt{Z_{oi}}}, b_i = \frac{V_i^-}{\sqrt{Z_{oi}}}, V_i = V_i^+ + V_i^- = \sqrt{Z_{oi}}(a_i + b_i), I_i = \frac{V_i^+ - V_i^-}{Z_{oi}} = \frac{a_i - b_i}{\sqrt{Z_{oi}}}$$

$$P_{in,i} = \frac{1}{2}\operatorname{Re}\{V_i I_i^*\} = \frac{1}{2}\operatorname{Re}\{(a_i + b_i)(a_i - b_i)^*\} = \frac{1}{2}\operatorname{Re}\{|a_i|^2 - |b_i|^2 + a_i^* b_i - a_i b_i^*\}$$

Im

$$= \frac{1}{2}|a_i|^2 - \frac{1}{2}|b_i|^2 = P_{inc,i} - P_{refl,i} = P_{inc,i}(1 - |S_{ii}|^2), S_{ii} = \left.\frac{b_i}{a_i}\right|_{a_{k=0,k\neq i}}$$

$$= \frac{1}{2}\operatorname{Re}\{P_i^+ - P_i^-\} \to P_i^+ = \frac{|V_i^+|^2}{Z_{oi}} = |a_i|^2, P_i^- = \frac{|V_i^-|^2}{Z_{oi}} = |b_i|^2$$

11. Two-port device with its S-matrix

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0}$$: reflection coefficient at port 1 with port 2 matched

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}$$: forward transmission coefficient with port 2 matched

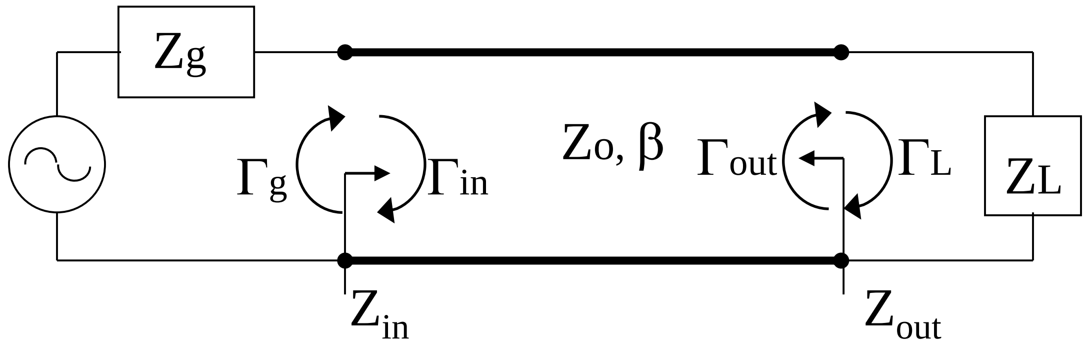
$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$: reverse transmission coefficient with port 1 matched

$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$: reflection coefficient at port 2 with port 1 matched

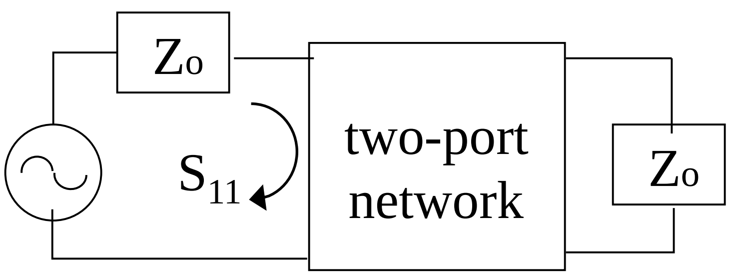
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

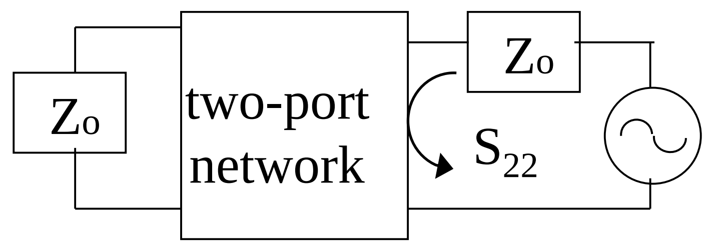
$$b_1 = a_1 S_{11} + a_2 S_{12}$$
$$b_2 = a_1 S_{21} + a_2 S_{22}$$

12. Reflection coefficient and S_{11}, S_{22}



$$\Gamma_{in} = \frac{Z_{in} - Z_g}{Z_{in} + Z_g} = -\Gamma_g, \Gamma_{out} = \frac{Z_{out} - Z_L}{Z_{out} + Z_L} = -\Gamma_L, \text{ if } Z_g = Z_o \text{ then } \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$





$$S_{11} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o}$$

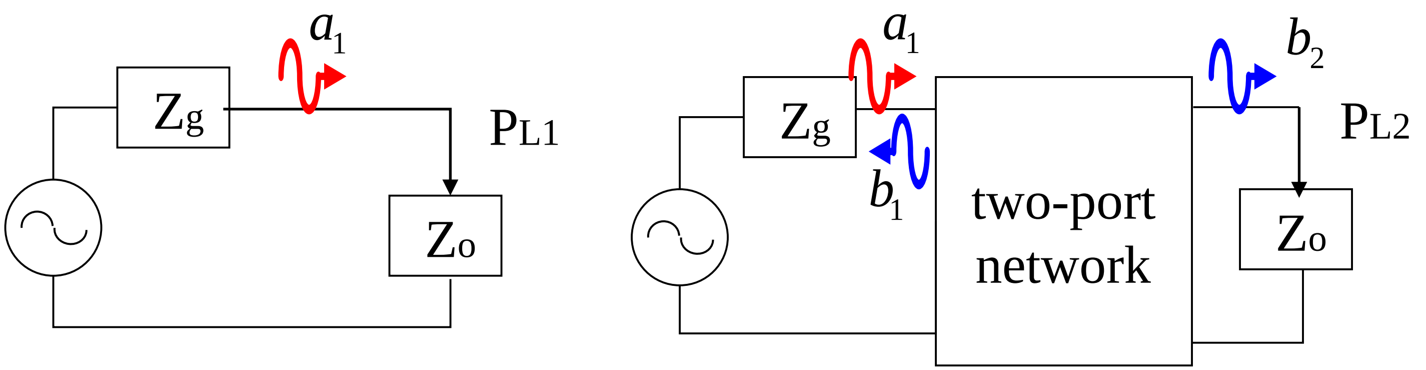
$$S_{22} = \frac{Z_{out} - Z_o}{Z_{out} + Z_o}$$

13. RL and IL

$$RL \text{ at port 1: } -20\log\left|\frac{b_1}{a_1}\right| = -20\log|S_{11}|$$

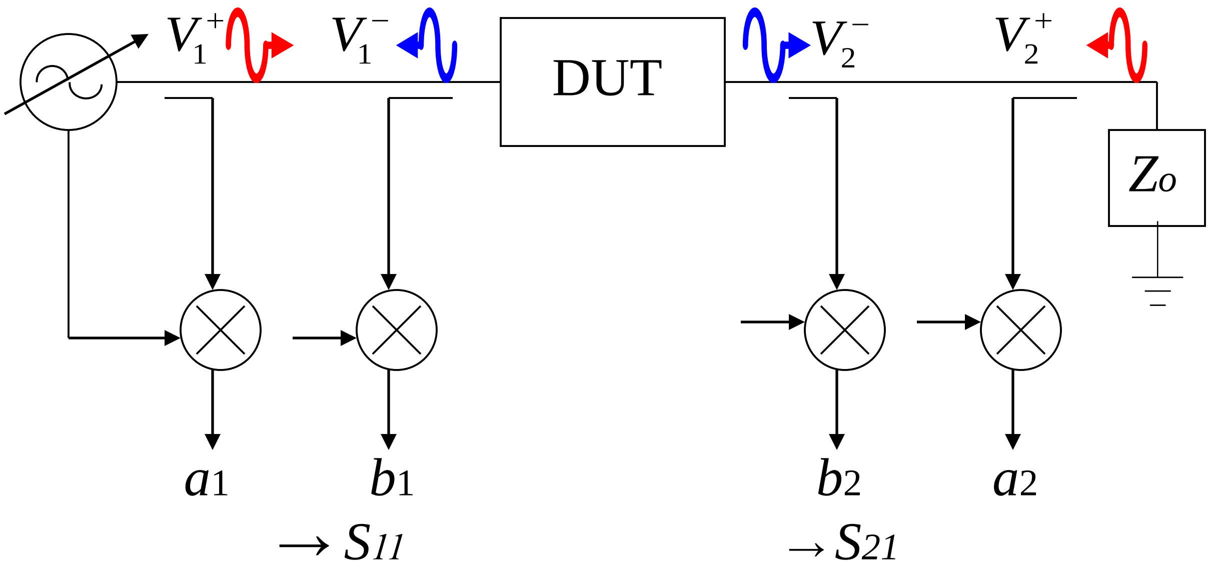
$$IL \text{ from port 1 to port 2: } -20\log\left|\frac{b_2}{a_1}\right| = -20\log|S_{21}|$$

insertion loss $IL(dB) \equiv 10\log\frac{P_{L1}}{P_{L2}}$



usually $Z_g = Z_o$

14. Two-port S-matrix measurement using VNA



15. Advantages to use S-matrix in microwave circuit

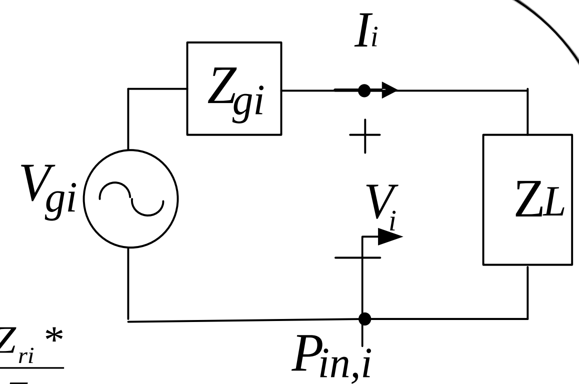
1) matched load available in broadband application

2) measurable quantity in terms of incident, reflected and transmitted waves

3) termination with Z_o causes no oscillation

4) convenient in the use of microwave network analysis

16. Generalized scattering parameters



incident power wave $a_i \equiv \frac{V_i + Z_{ri} I_i}{2\sqrt{|\mathrm{Re}\, Z_{ri}|}}, Z_{ri}$: reference impedance

reflected power wave $b_i \equiv \frac{V_i - Z_{ri}^* I_i}{2\sqrt{|\mathrm{Re}\, Z_{ri}|}}$

$\rightarrow$ power wave reflection coefficient $S_{ii} \equiv \frac{b_i}{a_i} = \frac{V_i - Z_{ri}^* I_i}{V_i + Z_{ri} I_i} = \frac{Z_L - Z_{ri}^*}{Z_L + Z_{ri}}$

power reflection coefficient $|S_{ii}|^2 = \left|\frac{b_i}{a_i}\right|^2 = \left|\frac{Z_L - Z_{ri}^*}{Z_L + Z_{ri}}\right|^2$, conjugate match $Z_L = Z_{ri}^* \rightarrow |S_{ii}|^2 = 0$

$$P_{in,i} = \frac{1}{2}\mathrm{Re}\{V_i I_i^*\} = \frac{1}{2}(|a_i|^2 - |b_i|^2) = P_L$$

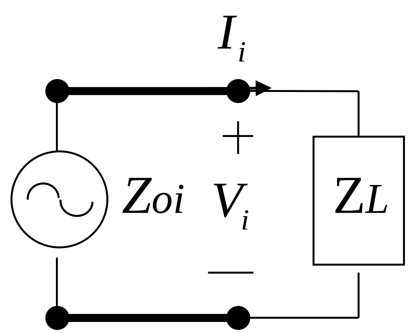
Ref: K.Kurokawa,"Power waves and the scattering matrix", IEEE-MTT, pp.194-201, March 1965

$\Leftrightarrow$ traveling wave along a lossless line with real Z_0

$$a_i = \frac{V_i^+}{\sqrt{Z_{oi}}}, b_i = \frac{V_i^-}{\sqrt{Z_{oi}}}, V_i = V_i^+ + V_i^- = \sqrt{Z_{oi}}(a_i + b_i), I_i = \frac{V_i^+ - V_i^-}{Z_{oi}} = \frac{a_i - b_i}{\sqrt{Z_{oi}}}$$

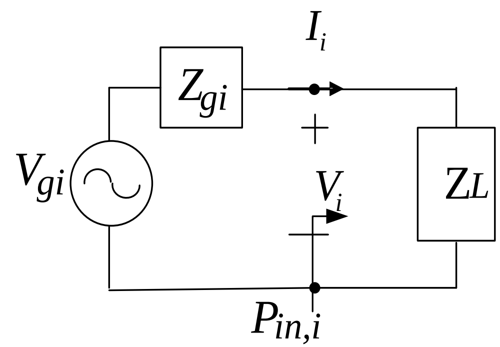
$$\rightarrow a_i + b_i = \frac{V_i}{\sqrt{Z_{oi}}}, a_i - b_i = I_i\sqrt{Z_{oi}} \rightarrow a_i = \frac{V_i + Z_0 I_i}{2\sqrt{Z_0}},\ b_i = \frac{V_i - Z_0 I_i}{2\sqrt{Z_0}}$$

$S_{ii} = \frac{b_i}{a_i} = \frac{V_i - Z_{oi} I_i}{V_i + Z_{oi} I_i} = \frac{Z_L - Z_{oi}}{Z_L + Z_{oi}}$, impedance match $Z_L = Z_{oi} \rightarrow S_{ii} = 0$



$$V_i = V_{gi}\frac{Z_L}{Z_L+Z_{gi}}, I_i = \frac{V_{gi}}{Z_L+Z_{gi}}$$

$$P_L = \frac{1}{2}\operatorname{Re}\{V_i I_i^*\} = \frac{V_{gi}^2}{2}\frac{\operatorname{Re} Z_L}{\left|Z_L+Z_{gi}\right|^2}$$



$$\text{if } Z_L = Z_{ri}^* \rightarrow \quad a_i \equiv \frac{V_i + Z_{ri} I_i}{2\sqrt{\left|\operatorname{Re} Z_{ri}\right|}} = V_{gi}\frac{\frac{Z_L}{Z_L+Z_{gi}} + Z_{ri}\frac{1}{Z_L+Z_{gi}}}{2\sqrt{\left|\operatorname{Re} Z_{ri}\right|}} \overset{Z_L = Z_{ri}^*}{=} V_{gi}\frac{\sqrt{\left|\operatorname{Re} Z_{ri}\right|}}{Z_L+Z_{gi}}$$

$$b_i \equiv \frac{V_i - Z_{ri}^* I_i}{2\sqrt{\left|\operatorname{Re} Z_{ri}\right|}} = V_{gi}\frac{\frac{Z_L}{Z_L+Z_{gi}} - Z_{ri}^*\frac{1}{Z_L+Z_{gi}}}{2\sqrt{\left|\operatorname{Re} Z_{ri}\right|}} = 0 \rightarrow S_{ii} = 0$$

$$\rightarrow P_L = P_{in,i} = \frac{1}{2}\left|a_i\right|^2 = P_{inc,i} = \frac{V_{gi}^2}{2}\frac{\sqrt{\left|\operatorname{Re} Z_{ri}\right|}}{\left|Z_L+Z_{gi}\right|^2}$$

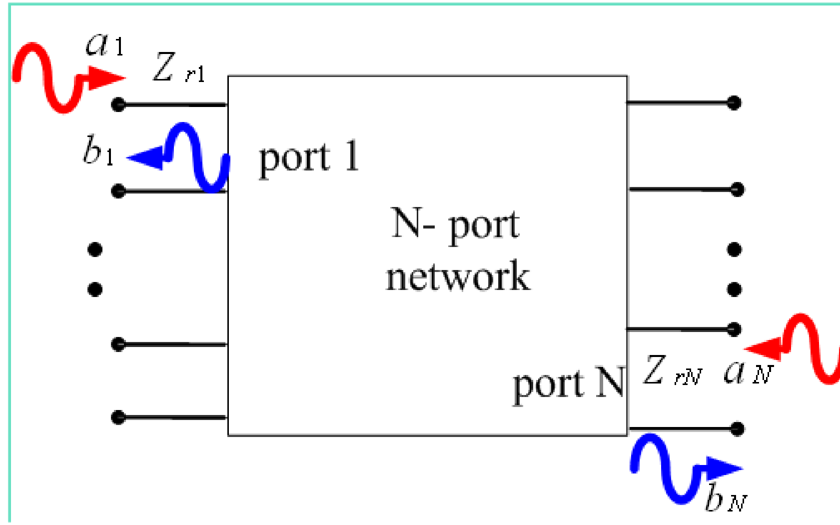
$\text{if } Z_L = Z_g^* \rightarrow P_L = \frac{1}{8}\frac{V_{gi}^2}{\operatorname{Re} Z_L}$:maximum power trasfer from source

$$[a]=[F]([V]+[Z_r][I])$$

$$[b]=[F]([V]-[Z_r]^*[I])$$

$$[Z_r]=\begin{bmatrix} Z_{r1} & 0 & . & 0 \\ 0 & . & 0 & 0 \\ 0 & 0 & . & 0 \\ 0 & 0 & 0 & Z_{rN} \end{bmatrix}$$

$$[F]=\frac{1}{2}\begin{bmatrix} \sqrt{|\mathrm{Re}\,Z_{r1}|} & 0 & . & 0 \\ 0 & . & 0 & 0 \\ 0 & 0 & . & 0 \\ 0 & 0 & 0 & \sqrt{|\mathrm{Re}\,Z_{rN}|} \end{bmatrix},$$

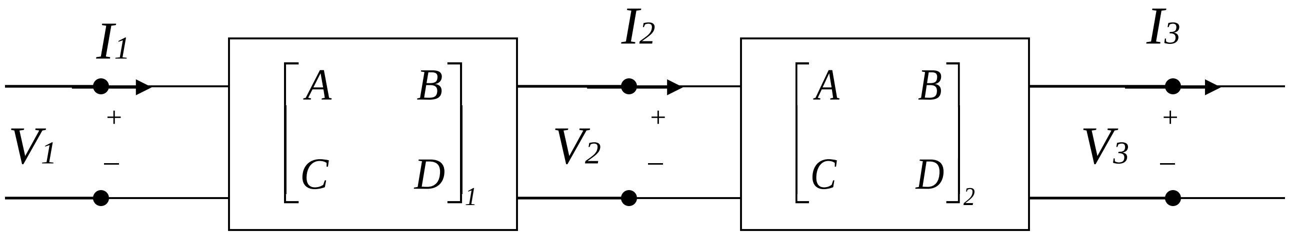


$$[V]=[Z][I]\Rightarrow[b]=[F]([Z]-[Z_r]^*)([Z]+[Z_r])^{-1}[F]^{-1}[a]$$

$$\Rightarrow[S]=[F]([Z]-[Z_r]^*)([Z]+[Z_r])^{-1}[F]^{-1}$$

4.4 The transmission (ABCD) matrix

- Cascade network



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_1 \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_1 \begin{bmatrix} A & B \\ C & D \end{bmatrix}_2 \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

<u>Discussion</u>

1. ABCD matrix of two-port circuits (p.190, Table 4.1)
2. Reciprocal network AD-BC=1
3. S-, Z-, Y-, ABCD-matrix relation of 2-port network (p.192, Table 4.2)
4. Ex. 4.6 ABCD(Z)

(*derivation*) Z (ABCD)

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix}, \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{AV_2}{CV_2} = \frac{A}{C}$$

$$Z_{12} = \left.\frac{V_1}{-I_2}\right|_{I_1=0} = \left.\frac{AV_2 + BI_2}{-I_2}\right|_{I_1=0} = -A\left.\frac{V_2}{I_2}\right|_{I_1=0} - B, I_1 = 0 = CV_2 + DI_2 \rightarrow -\frac{V_2}{I_2} = \frac{D}{C}$$

$$= -A(-\frac{D}{C}) - B = \frac{AD - BC}{C}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{CV_2 + DI_2}\right|_{I_2=0} = \frac{1}{C}$$

$$Z_{22} = -\left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{D}{C}, \because I_1 = 0 = CV_2 + DI_2$$

symmetrical network

$$Z_{11} = Z_{22} \rightarrow A = D$$

reciprocal network

$$Z_{12} = Z_{21}$$

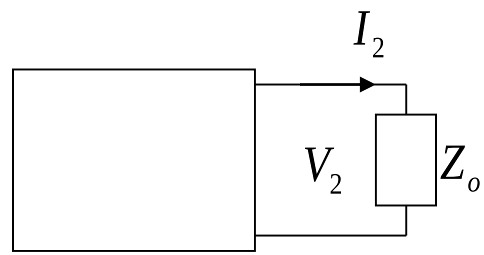
$$\frac{AD - BC}{C} = \frac{1}{C} \rightarrow AD - BC = 1$$

(*derivation*) S (ABCD)

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}, \quad \begin{matrix} V_1 = V_1^+ + V_1^- \\ I_1 = (V_1^+ - V_1^-)/Z_o \end{matrix} \rightarrow \begin{matrix} V_1^+ = (V_1 + Z_o I_1)/2 \\ V_1^- = (V_1 - Z_o I_1)/2 \end{matrix}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \left.\frac{V_1 - Z_o I_1}{V_1 + Z_o I_1}\right|_{V_2^+=0} = \left.\frac{AV_2 + BI_2 - Z_o CV_2 - Z_o DI_2}{AV_2 + BI_2 + Z_o CV_2 + Z_o DI_2}\right|_{V_2^+=0}$$

$$= \frac{A + BY_o - CZ_o - D}{A + BY_o + CZ_o + D}$$



$$V_2^+ = 0$$

$$\rightarrow V_2 = I_2 Z_o, I_2 = V_2 Y_o$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} \overset{V_2 = V_2^-}{=} \frac{V_2}{(V_1 + Z_o I_1)/2} = \frac{2V_2}{AV_2 + BI_2 + Z_o CV_2 + Z_o DI_2}$$

$$\overset{I_2 = V_2 Y_o}{=} \frac{2}{A + BY_o + CZ_o + D}$$

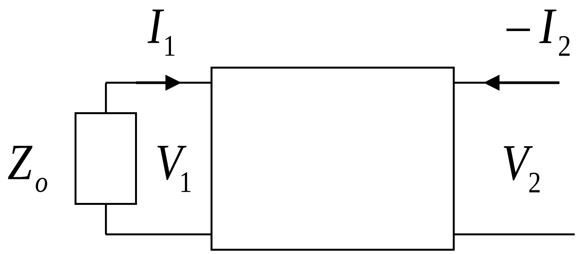
$$S_{12}=\frac{b_1}{a_2}\bigg|_{a_1=0}=\frac{V_1^-}{V_2^+}\bigg|_{V_1^+=0},\quad \begin{matrix} V_2=V_2^++V_2^- \\ -I_2=(V_2^+-V_2^-)/Z_o \end{matrix} \rightarrow \begin{matrix} V_2^+=(V_2-Z_oI_2)/2 \\ V_2^-=(V_2+Z_oI_2)/2 \end{matrix}$$

$$V_1^-=V_1$$

$$=\frac{V_1}{(V_2-Z_oI_2)/2},\begin{bmatrix} V_2 \\ I_2 \end{bmatrix}=\frac{1}{\Delta}\begin{bmatrix} D & -B \\ -C & A \end{bmatrix}\begin{bmatrix} V_1 \\ I_1 \end{bmatrix},\Delta=AD-BC$$

$$=\frac{2V_1}{(DV_1-BI_1+Z_oCV_1-Z_oAI_1)/\Delta}=\frac{2\Delta V_1}{DV_1+BY_oV_1+CZ_oV_1+AV_1}$$

$$=\frac{2\Delta}{A+BY_o+CZ_o+D}$$



$$V_1=-I_1Z_o, I_1=-V_1Y_o$$

$$S_{22}=\frac{b_2}{a_2}\bigg|_{a_1=0}=\frac{V_2^-}{V_2^+}\bigg|_{V_1^+=0}=\frac{V_2+Z_oI_2}{V_2-Z_oI_2}=\frac{DV_1-BI_1+Z_o(-CV_1+AI_1)}{DV_1-BI_1-Z_o(-CV_1+AI_1)}$$

$$\overset{I_1=-V_1Y_o}{=}\frac{DV_1+BY_oV_1+Z_o(-CV_1+AY_oV_1)}{DV_1+BY_oV_1-Z_o(-CV_1+AY_oV_1)}=\frac{-A+BY_o-CZ_o+D}{A+BY_o+CZ_o+D}$$

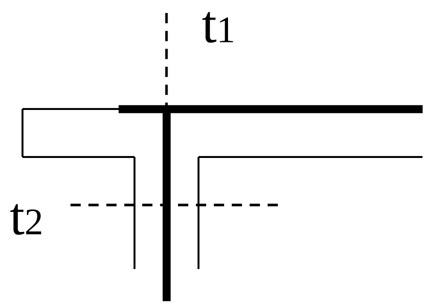
symmetrical

$S_{11}=S_{22}, A=D$

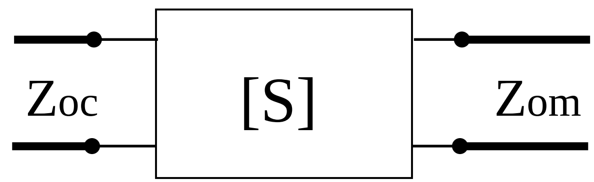
reciprocal

$S_{12}=S_{21}, \Delta=1$

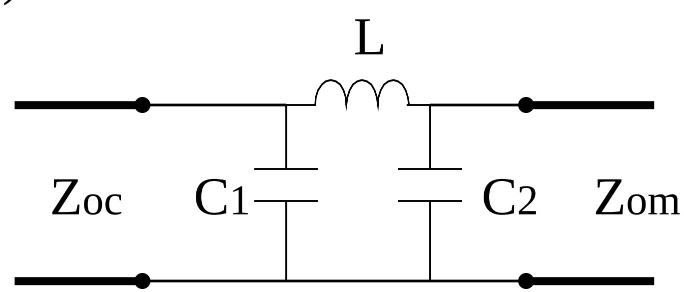
5. Example



coaxial-microstrip transition
(a linear circuit)



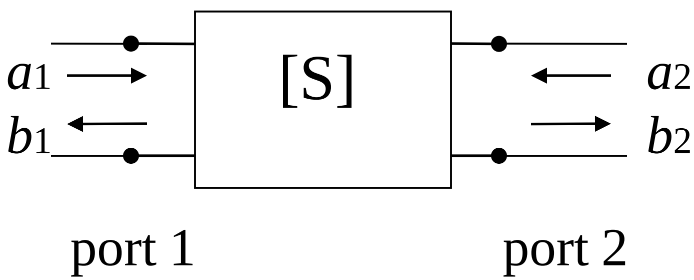
[S] representation can be obtained from measurement or calculation.

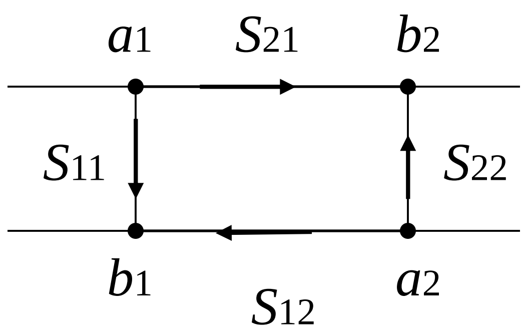


one possible equivalent circuit

4.5 Signal flow graphs

• 2-port representation





$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$b_1 = a_1 S_{11} + a_2 S_{12}$$
$$b_2 = a_1 S_{21} + a_2 S_{22}$$

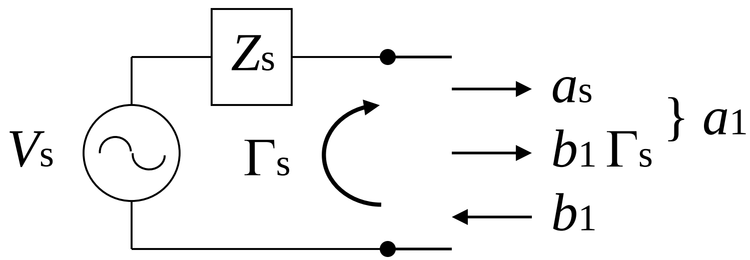
RL at port 1: $-20\log\left|\frac{b_1}{a_1}\right| = -20\log|S_{11}|$

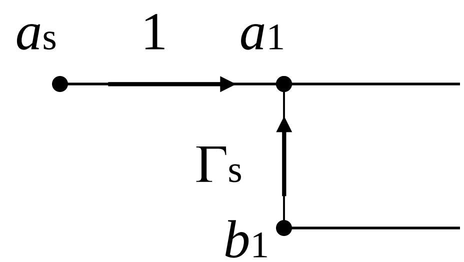
IL from port 1 to port 2: $-20\log\left|\frac{b_2}{a_1}\right| = -20\log|S_{21}|$

Discussion

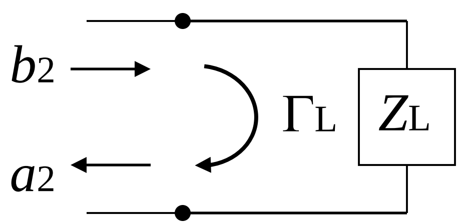
1. Source representation

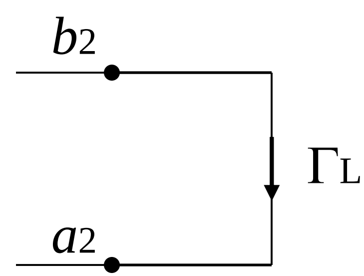
$$e_s = V_s \frac{Z_o}{Z_o + Z_s}, a_s \equiv \frac{e_s}{\sqrt{Z_o}}$$





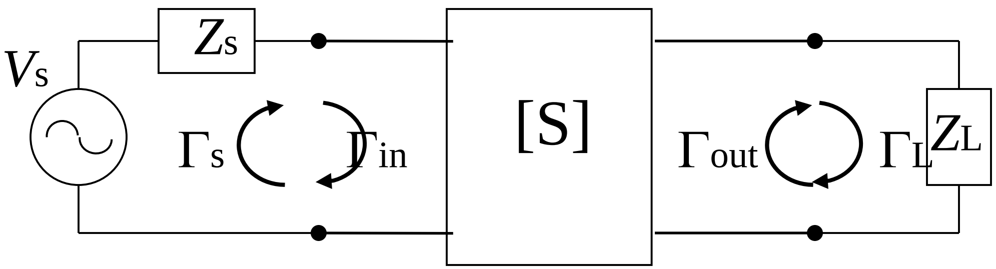
2. Load representation

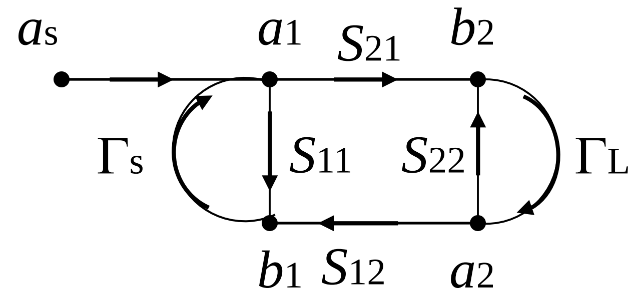




3. Series, parallel, self-loop, splitting rules (p.196, Fig.4.16)

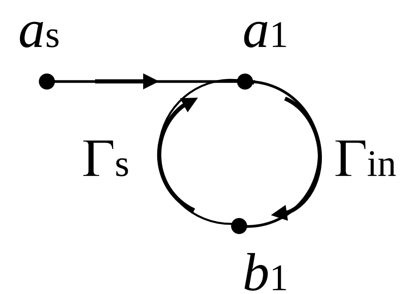
4. 2-port circuit representation





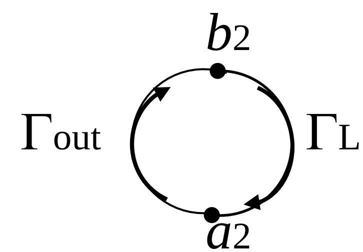
$$b_1 = a_1 S_{11} + a_1 S_{21} \Gamma_L S_{12} (1 + S_{22}\Gamma_L + \ldots) = a_1 S_{11} + a_1 \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\rightarrow \Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L}$$



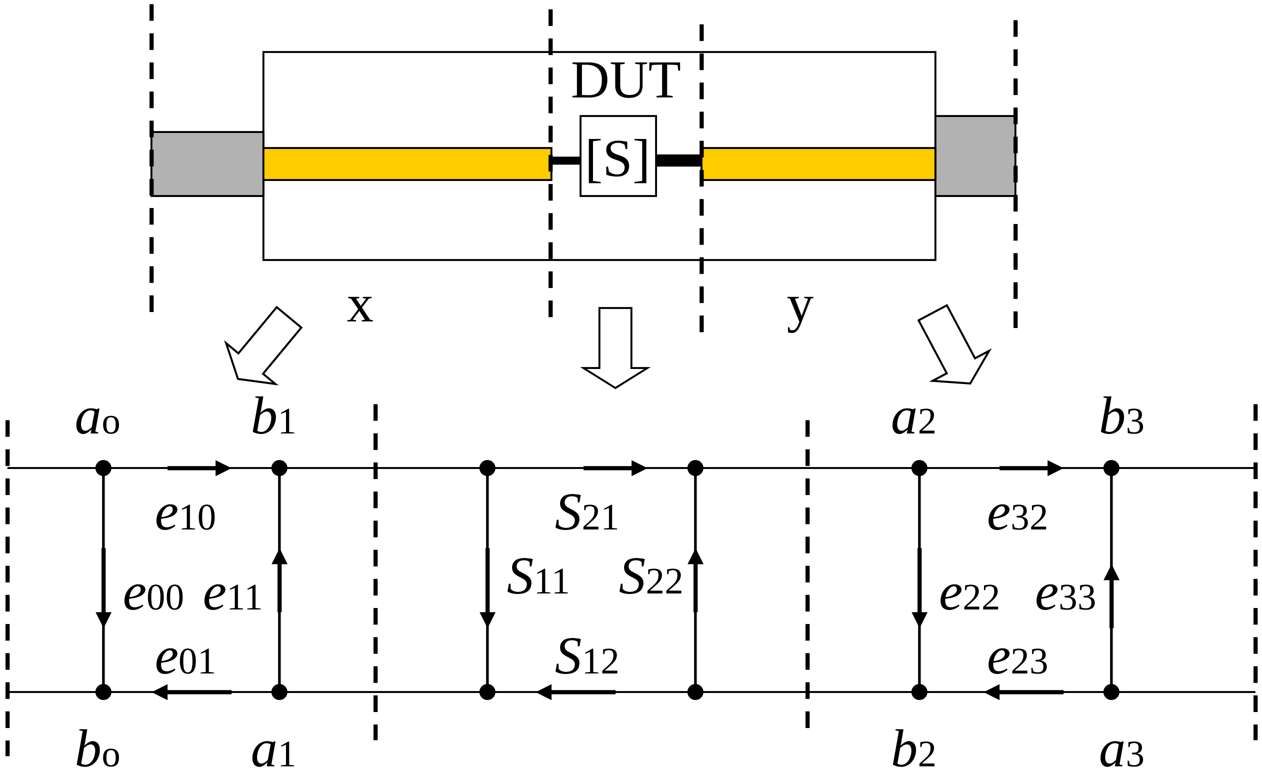
$$b_2 = a_2 S_{22} + a_2 S_{12} \Gamma_S S_{21} (1 + S_{11}\Gamma_S + \ldots) = a_2 S_{22} + a_2 \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S}$$

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S}$$



5. TRL (Thru-Reflect-Line) calibration

Find $[S]_{DUT}$ from 2-port measurement using three calibrators



6 unknowns of $[S]_x$ and $[S]_y$ to be calibrated to acquire $[S]_{DUT}$

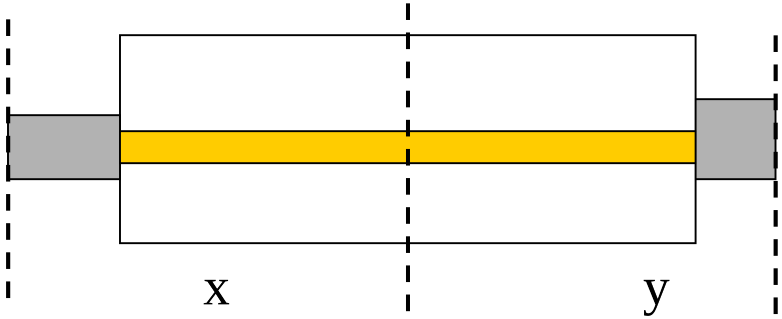
T: Through → 3 eqs., R: Reflection → 2 eqs., L: Line → 3 eqs.

⇒ R (Γ) and line length (γl) can be unknown

Calibrators

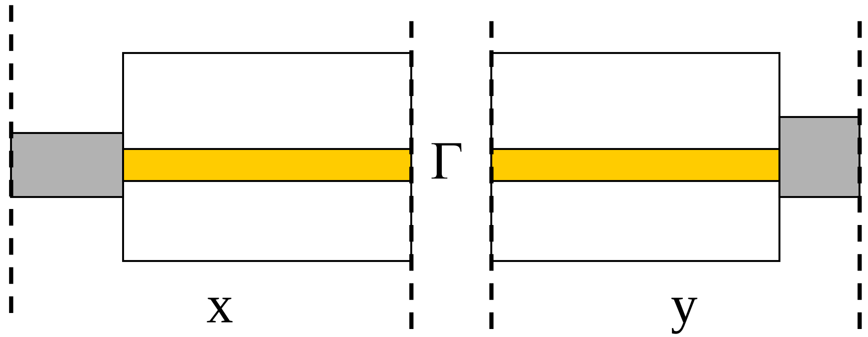
T: Through

→ 3 eqs.



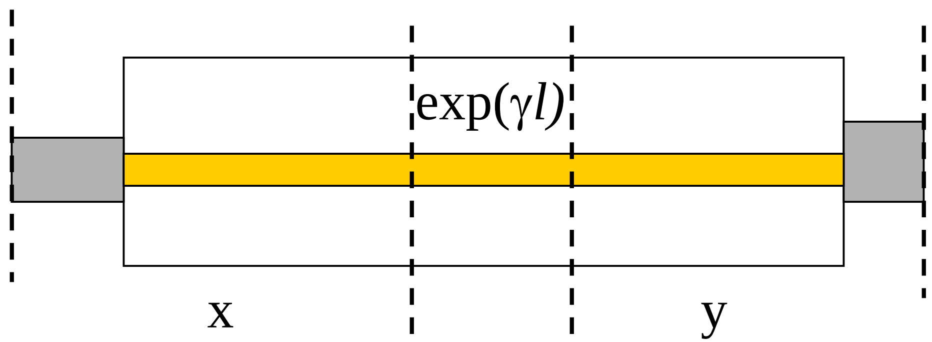
R: Reflection

→2 eqs.



L: Line

→ 3 eqs.



Requirement: connectors and line have same characteristics for 3 calibrators

Limitation: operation bandwidth $20^\circ \leqq \beta l \leqq 160^\circ$

(*brief derivation*)

R – matrix (wave cascade matrix)

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$

$$[R] = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = \frac{1}{S_{21}} \begin{bmatrix} S_{12}S_{21} - S_{11}S_{22} & S_{11} \\ -S_{22} & 1 \end{bmatrix}$$

error matrices $[R_x] = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$, $[R_y] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

Through: $[S_T] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow [R_T] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Line: $[S_L] = \begin{bmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{bmatrix} \rightarrow [R_L] = \begin{bmatrix} e^{-\gamma l} & 0 \\ 0 & 1/e^{-\gamma l} \end{bmatrix}$

Reflection: Γ

Thru measurement: $[R_{mT}] = [R_x]\,[R_T]\,[R_y] = [R_x]\,[R_y]$

Line measurement: $[R_{mL}] = [R_x]\,[R_L]\,[R_y]$

$$\Rightarrow [M][R_x]=[R_x][R_L], \quad [M]\equiv[R_{mL}][R_{mT}]^{-1} \quad \Rightarrow \quad e_{00}, \frac{e_{01}e_{10}}{e_{11}}$$

$$[R_y][N]=[R_L][R_y], \quad [N]\equiv[R_{mT}]^{-1}[R_{mL}] \quad \Rightarrow \quad e_{33}, \frac{e_{23}e_{32}}{e_{22}}$$

reflection measurement at port 1 $\Gamma_{mx} = e_{00} + \dfrac{e_{10}e_{01}\Gamma}{1-e_{11}\Gamma}$

reflection measurement at port 2 $\Gamma_{my} = e_{33} + \dfrac{e_{23}e_{32}\Gamma}{1-e_{22}\Gamma}$

$$\left.\begin{matrix}\varGamma_{mx}\\ \varGamma_{my}\\ \varGamma_{mT}\end{matrix}\right\} \Rightarrow \quad e_{11},\ e_{22}, e_{10}e_{01}, e_{23}e_{32} \qquad \left.\begin{matrix}S_{21mT}\\ S_{12mT}\end{matrix}\right\} \Rightarrow \quad e_{10}e_{32}, e_{23}e_{01}$$

$$\Rightarrow e_{10}, e_{01}, e_{23}, e_{32}$$

$$\Rightarrow \quad \varGamma,\ e^{-\gamma l}$$

(detailed derivation)

$$[M][R_x]=[R_x][R_L]\rightarrow\begin{bmatrix}m_{11}&m_{12}\\m_{21}&m_{22}\end{bmatrix}\begin{bmatrix}x_{11}&x_{12}\\x_{21}&x_{22}\end{bmatrix}=\begin{bmatrix}x_{11}&x_{12}\\x_{21}&x_{22}\end{bmatrix}\begin{bmatrix}e^{-\gamma l}&0\\0&1/e^{-\gamma l}\end{bmatrix}$$

$$\rightarrow\begin{cases}m_{11}x_{11}+m_{12}x_{21}=x_{11}e^{-\gamma l}\\m_{21}x_{11}+m_{22}x_{21}=x_{21}e^{-\gamma l}\\m_{11}x_{12}+m_{12}x_{22}=x_{12}/e^{-\gamma l}\\m_{21}x_{12}+m_{22}x_{22}=x_{22}/e^{-\gamma l}\end{cases}\begin{matrix}\rightarrow m_{21}(\frac{x_{11}}{x_{21}})^2+(m_{22}-m_{11})\frac{x_{11}}{x_{21}}-m_{12}=0\\\rightarrow m_{21}(\frac{x_{12}}{x_{22}})^2+(m_{22}-m_{11})\frac{x_{12}}{x_{22}}-m_{12}=0\end{matrix}\rightarrow\begin{matrix}a=\frac{x_{11}}{x_{21}}=e_{00}-\frac{e_{10}e_{01}}{e_{11}}\\b=\frac{x_{12}}{x_{22}}=e_{00}\end{matrix}$$

$$root\ choice: e_{10}e_{01}\neq 0\rightarrow a\neq b, |e_{00}|\approx|e_{11}|\ll 1\rightarrow |a|>|b|$$

$$[R_y][N]=[R_L][R_y]\rightarrow\begin{bmatrix}y_{11}&y_{12}\\y_{21}&y_{22}\end{bmatrix}\begin{bmatrix}n_{11}&n_{12}\\n_{21}&n_{22}\end{bmatrix}=\begin{bmatrix}e^{-\gamma l}&0\\0&1/e^{-\gamma l}\end{bmatrix}\begin{bmatrix}y_{11}&y_{12}\\y_{21}&y_{22}\end{bmatrix}$$

$$\rightarrow\begin{cases}y_{11}n_{11}+y_{12}n_{21}=y_{11}e^{-\gamma l}\\y_{21}n_{11}+y_{22}n_{21}=y_{12}e^{-\gamma l}\\y_{11}n_{12}+y_{12}n_{22}=y_{12}/e^{-\gamma l}\\y_{21}n_{12}+y_{22}n_{22}=y_{22}/e^{-\gamma l}\end{cases}\begin{matrix}\rightarrow n_{12}(\frac{y_{11}}{y_{12}})^2+(n_{22}-n_{11})\frac{y_{11}}{y_{12}}-n_{21}=0\\\rightarrow n_{12}(\frac{y_{21}}{y_{22}})^2+(n_{22}-n_{11})\frac{y_{21}}{y_{22}}-n_{21}=0\end{matrix}\rightarrow\begin{matrix}c=\frac{y_{11}}{y_{12}}=-e_{33}+\frac{e_{23}e_{32}}{e_{22}}\\d=\frac{y_{21}}{y_{22}}=-e_{33}\end{matrix}$$

$$root\ choice: e_{23}e_{32}\neq 0\rightarrow c\neq d, |e_{22}|\approx|e_{33}|\ll 1\rightarrow |c|>|d|$$

$$\begin{cases} \Gamma_{mx} = e_{00} + \dfrac{e_{10}e_{01}\Gamma}{1-e_{11}\Gamma} \rightarrow \Gamma = \dfrac{1}{e_{11}}\dfrac{b-\Gamma_{mx}}{a-\Gamma_{mx}} \\ \Gamma_{my} = e_{33} + \dfrac{e_{23}e_{32}\Gamma}{1-e_{22}\Gamma} \rightarrow \Gamma = \dfrac{1}{e_{22}}\dfrac{d+\Gamma_{my}}{c+\Gamma_{my}} \\ \Gamma_{mT} = e_{00} + \dfrac{e_{10}e_{01}e_{22}}{1-e_{11}e_{22}} \rightarrow e_{11} = \dfrac{1}{e_{22}}\dfrac{b-\Gamma_{mT}}{a-\Gamma_{mT}} \end{cases}$$

$$\Rightarrow e_{11}^2 = \frac{b-\Gamma_{mx}}{a-\Gamma_{mx}}\frac{c+\Gamma_{my}}{d+\Gamma_{my}}\frac{b-\Gamma_{mT}}{a-\Gamma_{mT}}, e_{22} = \frac{1}{e_{11}}\frac{b-\Gamma_{mT}}{a-\Gamma_{mT}}, e_{10}e_{01} = (b-a)e_{11}, e_{23}e_{32} = (c-d)e_{22}$$

$$\begin{cases} S_{21mT} = \dfrac{e_{10}e_{32}}{1-e_{11}e_{22}} \\ S_{12mT} = \dfrac{e_{23}e_{01}}{1-e_{11}e_{22}} \end{cases} \Rightarrow e_{10}e_{32} = S_{21mT}(1-e_{11}e_{22}), e_{23}e_{01} = S_{12mT}(1-e_{11}e_{22}) \Rightarrow e_{10}, e_{01}, e_{23}, e_{32}$$

$$\Rightarrow \Gamma = \frac{1}{e_{11}}\frac{b-\Gamma_{mx}}{a-\Gamma_{mx}} \text{...(also for } e_{11} \text{ selection)}, \; e^{-\gamma l} = m_{11} + \frac{m_{12}}{a}$$

4.6 Discontinuities and modal analysis

- equivalent circuit components

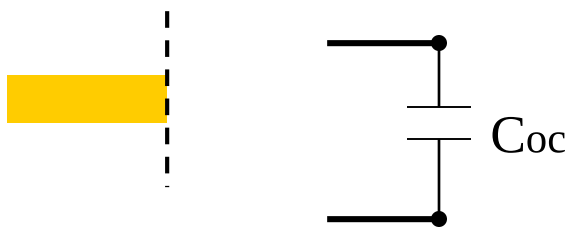
  $\Delta E \Rightarrow C, \quad \Delta H \Rightarrow L$

  constant E (V) $\Rightarrow$ parallel connection

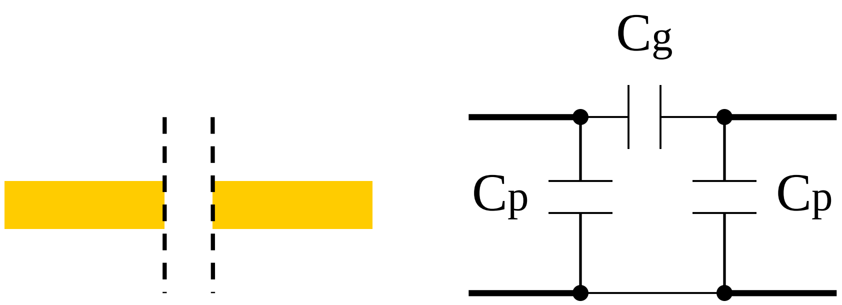
  constant H (I) $\Rightarrow$ serial connection

<u>Discussion</u>

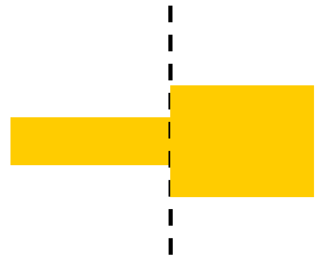
1. Microstrip discontinuities



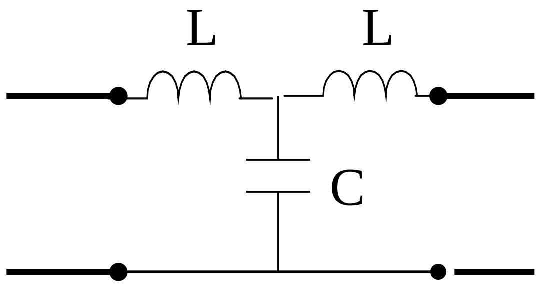
open-end

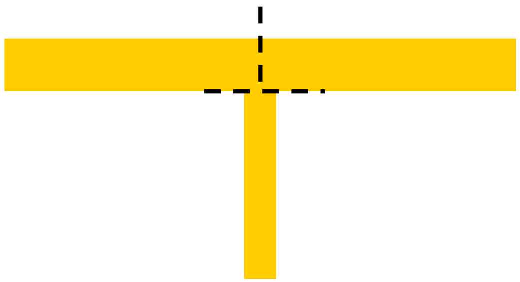


gap

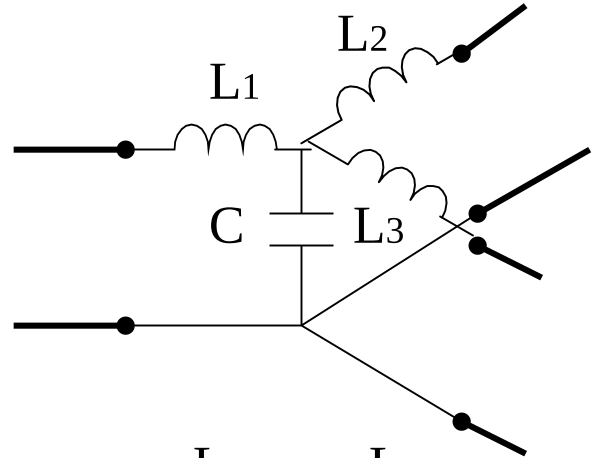


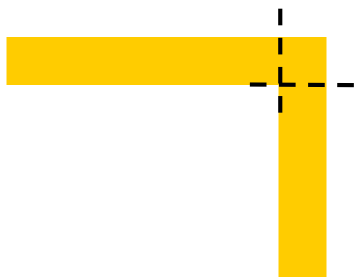
step



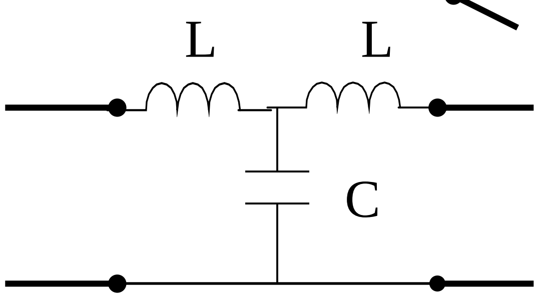


T-junction

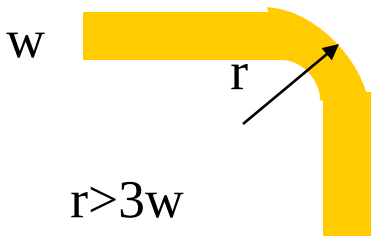




bend

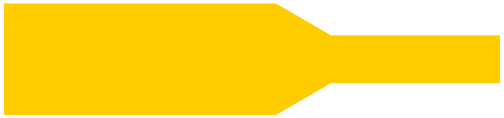


2. Microstrip discontinuity compensation



swept bend

mitered bends

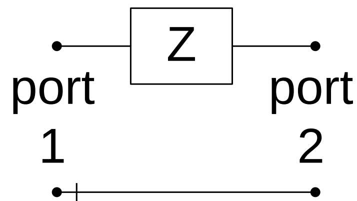


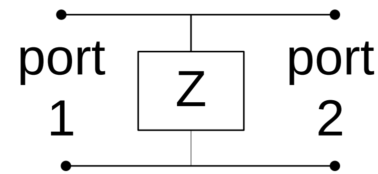
mitered step

mitered T-junction

<u>Solved Problems</u>

Prob. 4.11 Find [S] relative to Z_0





$$[Y]=\begin{bmatrix} 1/Z & -1/Z \\ -1/Z & 1/Z \end{bmatrix}, DY=(Y_o+Y_{11})(Y_o+Y_{22})+Y_{12}Y_{21}=\frac{Z+2Z_o}{ZZ_o^2}$$

$$[S]=\begin{bmatrix} \frac{(Y_o-Y_{11})(Y_o+Y_{22})+Y_{12}Y_{21}}{DY} & \frac{-2Y_{12}Y_o}{DY} \\ \frac{-2Y_{21}Y_o}{DY} & \frac{(Y_o+Y_{11})(Y_o-Y_{22})+Y_{12}Y_{21}}{DY} \end{bmatrix}$$

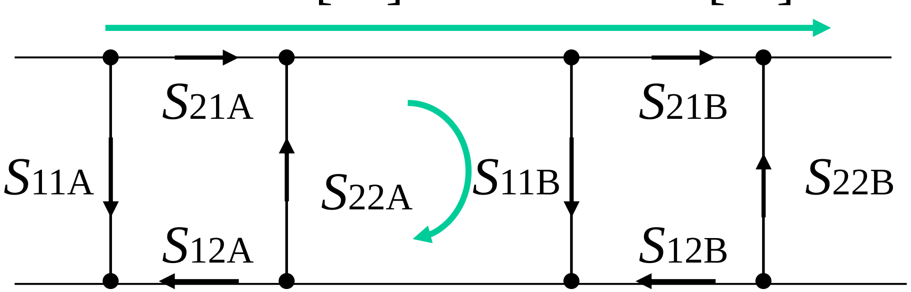
$$=\begin{bmatrix} \frac{1/Z_o^2}{DY} & \frac{2/ZZ_o}{DY} \\ \frac{2/ZZ_o}{DY} & \frac{1/Z_o^2}{DY} \end{bmatrix}=\begin{bmatrix} \frac{Z}{Z+2Z_o} & \frac{2Z_o}{Z+2Z_o} \\ \frac{2Z_o}{Z+2Z_o} & \frac{Z}{Z+2Z_o} \end{bmatrix}, 1-S_{11}=S_{21}$$

$$[Z]=\begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}, \Delta Z=(Z_o+Z_{11})(Z_o+Z_{22})-Z_{12}Z_{21}=2Z_oZ+Z_o^2$$

$$[S]=\begin{bmatrix} \frac{(Z_{11}-Z_o)(Z_{22}+Z_o)-Z_{12}Z_{21}}{\Delta Z} & \frac{2Z_{12}Z_o}{\Delta Z} \\ \frac{2Z_{21}Z_o}{\Delta Z} & \frac{(Z_{11}+Z_o)(Z_{22}-Z_o)-Z_{12}Z_{21}}{\Delta Z} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{-Z_o^2}{\Delta Z} & \frac{2ZZ_o}{\Delta Z} \\ \frac{2ZZ_o}{\Delta Z} & \frac{-Z_o^2}{\Delta Z} \end{bmatrix}=\begin{bmatrix} \frac{-Z_o}{Z_o+2Z} & \frac{2Z}{Z_o+2Z} \\ \frac{2Z}{Z_o+2Z} & \frac{-Z_o}{Z_o+2Z} \end{bmatrix}, 1+S_{11}=S_{21}$$

Prob. 4.12 Find S_{21} of $[S_A]$ in cascade of $[S_B]$



$$S_{21}=\frac{S_{21A}S_{21B}}{1-S_{22A}S_{11B}}$$

Prob. 4.14

$$\begin{bmatrix} 0.178\angle 90^0 & 0.6\angle 45^0 & 0.4\angle 45^0 & 0 \\ 0.6\angle 45^0 & 0 & 0 & 0.3\angle -45^0 \\ 0.4\angle 45^0 & 0 & 0 & 0.5\angle -45^0 \\ 0 & 0.3\angle -45^0 & 0.5\angle -45^0 & 0 \end{bmatrix}$$

(1) $|S_{12}|^2 + |S_{42}|^2 = 0.8 \neq 1 \rightarrow lossy$

(2) $[S]$ symmetrical $\rightarrow reciprocal$

(3) return loss at port1 $= -20\log|S_{11}| = -20\log 0.1 = 20dB$

(4) insertion loss between port 2 and port 4 $= -20\log|S_{24}| = -20\log 0.4 = 8dB$

phase delay between port 2 and port 4 $= 45^0$

(5) reflection at port 1 as port 3 is connected to a short circuit

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0.178\angle 90^0 & 0.6\angle 45^0 & 0.4\angle 45^0 & 0 \\ 0.6\angle 45^0 & 0 & 0 & 0.3\angle -45^0 \\ 0.4\angle 45^0 & 0 & 0 & 0.5\angle -45^0 \\ 0 & 0.3\angle -45^0 & 0.5\angle -45^0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ -b_3 \\ 0 \end{bmatrix}$$

$$b_3 = 0.4\angle 45^0 a_1$$

$$b_1 = 0.178\angle 90^0 a_1 - 0.4\angle 45^0 b_3 = 0.178\angle 90^0 a_1 - 0.16\angle 90^0 a_1 = 0.018\angle 90^0 a_1$$

$$S_{11} = \frac{b_1}{a_1} = 0.018j$$

Prob. 4.19 given [S_{ij}] of a two-port network normalized to Zo, find its generalized [S'_{ij}] in terms of Z_{o1} and Z_{o2}

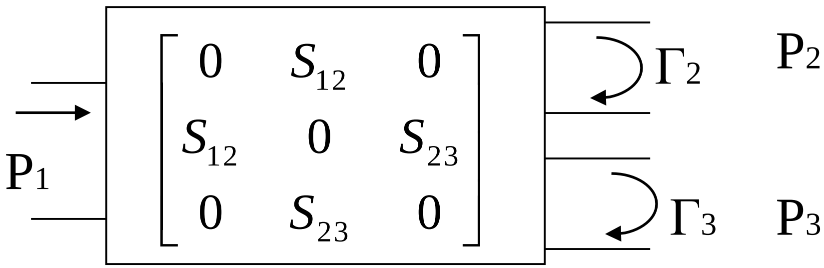
incident (power) wave : $a_i \equiv \dfrac{V_i^+}{\sqrt{Z_{oi}}} = \dfrac{V_i + Z_{oi} I_i}{2\sqrt{Z_{oi}}}$

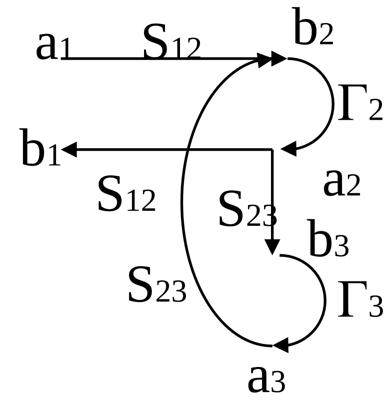
reflected (power) wave : $b_i \equiv \dfrac{V_i^-}{\sqrt{Z_{oi}}} = \dfrac{V_i - Z_{oi} I_i}{2\sqrt{Z_{oi}}}$

$$\left[V^-\right] = [S]\left[V^+\right] \Rightarrow \ [b] = [S][a],\ S_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k=0, k\neq j} = \left.\frac{V_i^- \sqrt{Z_{oj}}}{V_j^+ \sqrt{Z_{oi}}}\right|_{V_k^+=0, k\neq j}$$

$$\Rightarrow S'_{11} = S_{11}, S'_{12} = S_{12}\frac{\sqrt{Z_{o2}}}{\sqrt{Z_{o1}}}, S'_{21} = S_{21}\frac{\sqrt{Z_{o1}}}{\sqrt{Z_{o2}}}, S'_{22} = S_{22}$$

Prob. 4.28 find P_2/P_1 and P_3/P_1





$$b_1 = a_1 \frac{S_{12}^2 \Gamma_2}{1-\Gamma_2\Gamma_3 S_{23}^2} = a_1 \Gamma_{in}, b_2 = a_1 \frac{S_{12}}{1-\Gamma_2\Gamma_3 S_{23}^2}, b_3 = a_1 \frac{S_{12}\Gamma_2 S_{23}}{1-\Gamma_2\Gamma_3 S_{23}^2}$$

$$\frac{P_2}{P_1} = \frac{|b_2|^2 - |a_2|^2}{|a_1|^2 - |b_1|^2} = \frac{|b_2|^2 (1-|\Gamma_2|^2)}{|a_1|^2 (1-|\Gamma_{in}|^2)} = \frac{|S_{12}|^2 (1-|\Gamma_2|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2 \left(1-\frac{|S_{12}^2\Gamma_2|^2}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2}\right)} = \frac{|S_{12}|^2 (1-|\Gamma_2|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2 - |S_{12}^2\Gamma_2|^2}$$

$$\frac{P_3}{P_1} = \frac{|b_3|^2 - |a_3|^2}{|a_1|^2 - |b_1|^2} = \frac{|b_3|^2 (1-|\Gamma_3|^2)}{|a_1|^2 (1-|\Gamma_{in}|^2)} = \frac{|S_{12}|^2 |\Gamma_2|^2 |S_{23}|^2 (1-|\Gamma_3|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2 \left(1-\frac{|S_{12}^2\Gamma_2|^2}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2}\right)} = \frac{|S_{12}|^2 |\Gamma_2|^2 |S_{23}|^2 (1-|\Gamma_2|^2)}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2 - |S_{12}^2\Gamma_2|^2}$$

ADS examples: Ch4_prj