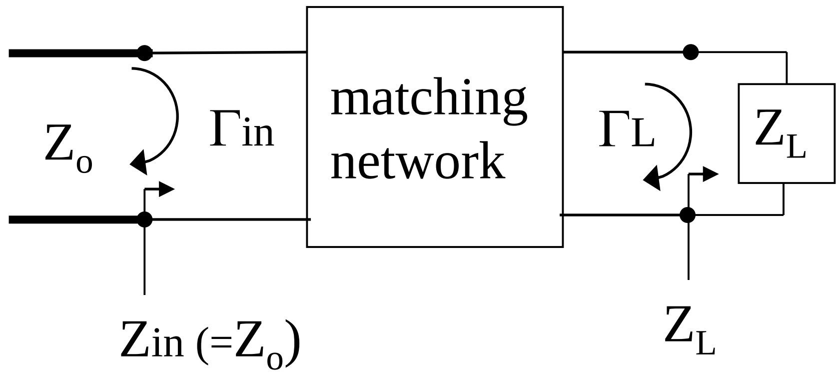
Chapter 5 Impedance matching and tuning

• Impedance matching concept

given Z_L, design a matching network to have Γ_{in}=0 or selected value

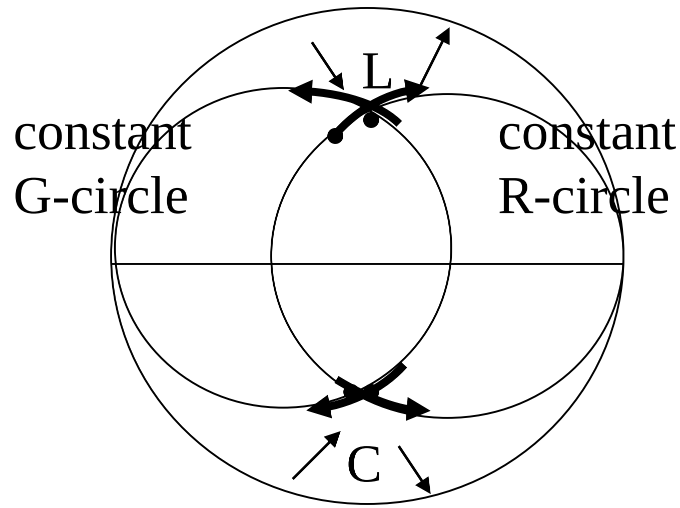


Discussion

1. Matching network usually uses lossless components: L, C, transmission line and transformer.
2. There are ∞ possible solutions for the matching circuit.
3. Properly use Smith chart to find the optimal design.
4. Factors for selecting matching circuit are complexity, bandwidth, implementation and adjustability.

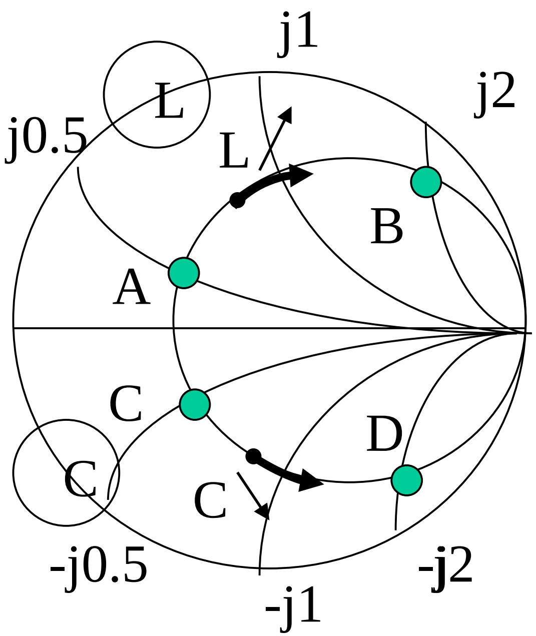
5.1 Matching with lumped elements (2-element L-network)

• Smith chart solution



Z-plane	Y-plane
CW → add series L	CW → add shunt C
(reduce series C)	(reduce shunt L)
CCW → add series C	CCW → add shunt L
(reduce series L)	(reduce shunt C)

(explanation)



constant R-circle → L or C in series

(1) $CW\ A \to B: 1 + j0.5 + jx = 1 + j2 \to jX = j1.5Z_o = j\omega L \to L = \dfrac{1.5Z_o}{\omega}$

: add an L in series

(2) $CCW\ B \to A: 1 + j2 + jx = 1 + j0.5 \to jX = -j1.5Z_o = \dfrac{1}{j\omega C} \to C = \dfrac{1}{1.5\omega Z_o}$

: add a C in series

or $CCW\ B \to A: 1 + j2 - j\Delta x = 1 + j0.5 \to j\Delta X = j1.5Z_o = j\omega\Delta L \to \Delta L = \dfrac{1.5Z_o}{\omega}$

: reduce extra $j\Delta X = j1.5Z_o$ (or reduce series L by $\Delta L = \dfrac{1.5Z_o}{\omega}$)

(3) $CCW\ C \to D: 1 - j0.5 + jx = 1 - j2 \to jX = -j1.5Z_o = \dfrac{1}{j\omega C} \to C = \dfrac{1}{1.5\omega Z_o}$

: add a C in series

(4) $CW\ D \to C: 1 - j2 + jx = 1 - j0.5 \to jX = j1.5Z_o = j\omega L \to L = \dfrac{1.5Z_o}{\omega}$

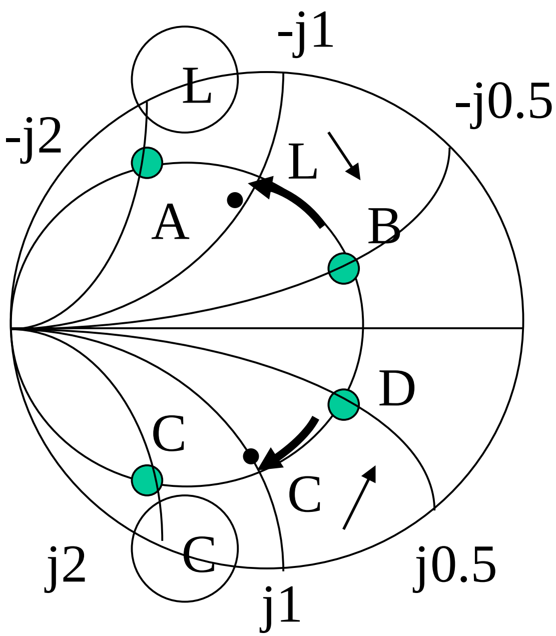
: add an L in series

or $CW\ D \to C: 1 - j2 - j\Delta x = 1 - j0.5 \to j\Delta X = -j1.5Z_o = \dfrac{1}{j\omega\Delta C} \to \Delta C = \dfrac{1}{1.5\omega Z_o}$

reduce extra $j\Delta X = -j1.5Z_o$ (or reduce series C by $\Delta C = \dfrac{1}{1.5\omega Z_o}$)

in Z-plane
CW → add a series L (or reduce series C)
CCW → add a series C (or reduce series L)

constant G-circle → L or C in shunt



(1) $CW\ A \to B: 1 - j2 + jb = 1 - j0.5 \to jB = j1.5Y_o = j\omega C \to C = \frac{1.5Y_o}{\omega}$

: add a C in shunt

or $CW\ A \to B: 1 - j2 - j\Delta b = 1 - j0.5 \to j\Delta B = -j1.5Y_o = \frac{1}{j\omega \Delta L} \to \Delta L = \frac{1}{1.5\omega Y_o}$

: reduce shunt $j\Delta B = -j1.5Y_o$ (or reduce shunt L by $\Delta L = \frac{1}{1.5\omega Y_o}$)

(2) $CCW\ B \to A: 1 - j0.5 + jb = 1 - j2 \to jb = -j1.5 = \frac{1}{j\omega L}$

: add an L in shunt

(3) $CCW\ C \to D: 1 + j2 + jb = 1 + j0.5 \to jb = -j1.5 = \frac{1}{j\omega L}$

: add an L in shunt

or $CCW\ C \to D: 1 + j2 - j\Delta b = 1 + j0.5 \to j\Delta B = j1.5Y_o = j\omega \Delta C \to \Delta C = \frac{1.5Y_o}{\omega}$

reduce shunt $j\Delta B = j1.5Y_o$ (or reduce shunt C by $\Delta C = \frac{1.5Y_o}{\omega}$)

(4) $CW\ D \to C: 1 + j0.5 + jb = 1 + j2 \to jb = j1.5 = j\omega C$

: add a C in shunt

in Y-plane

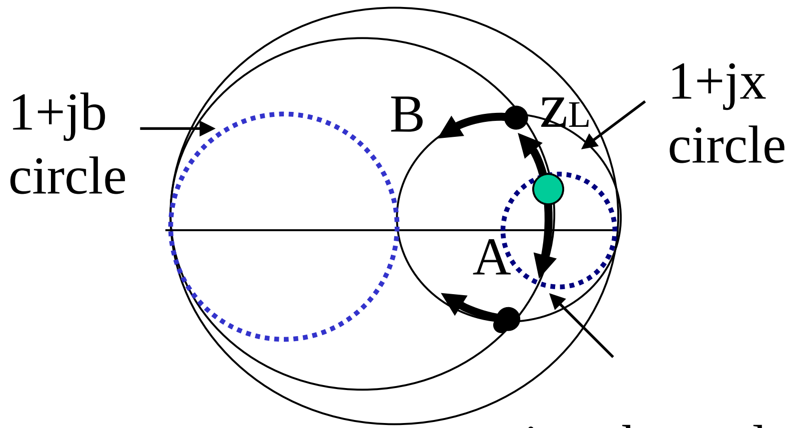
CW → add a shunt C (or reduce shunt L)

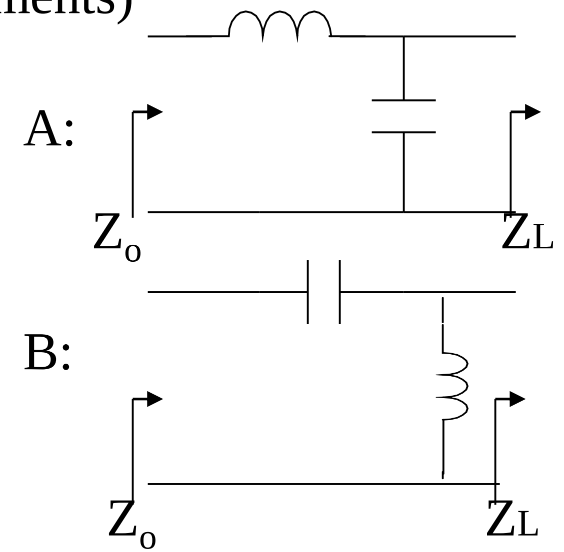
CCW → add a shunt L (or reduce shunt C)

Discussion

1. Z_L inside 1+jx circle, two possible solutions

Smith chart solution (shunt-series elements)

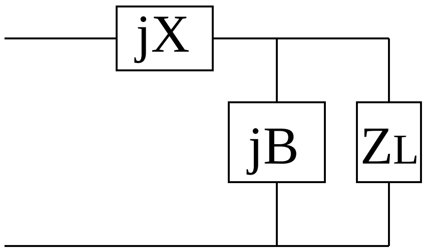




series-shunt elements?

"N"

analytical solution



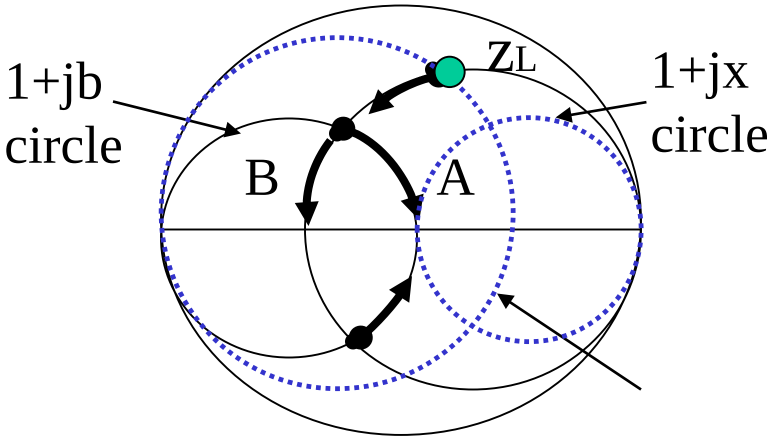
$$Z_o = jX + \cfrac{1}{jB + \cfrac{1}{R_L + jX_L}}$$

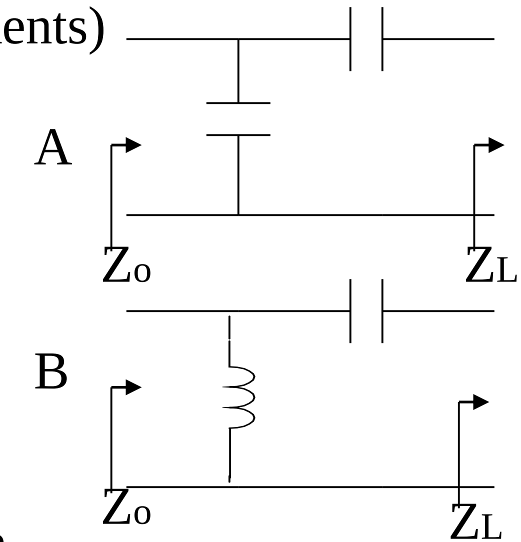
$$\Rightarrow B > 0 \rightarrow C, B < 0 \rightarrow L$$

$$X > 0 \rightarrow L, X < 0 \rightarrow C$$

2. Z_L outside 1+jx circle, two possible solutions

Smith chart solution (series-shunt elements)

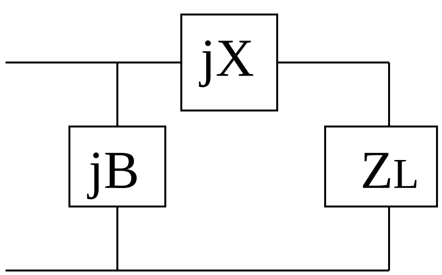




shunt-series elements?

"Y"

analytical solution

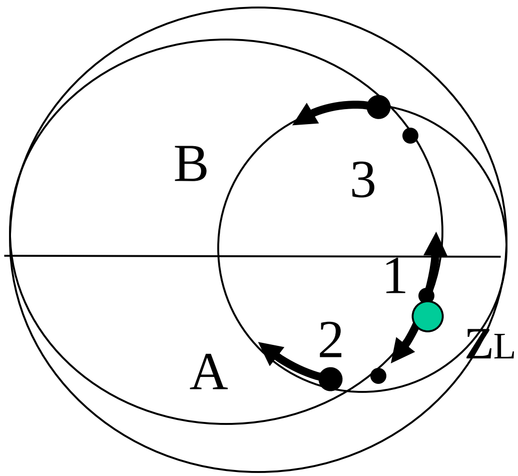


$$\frac{1}{Z_o} = jB + \frac{1}{R_L + j(X + X_L)}$$

$$\Rightarrow B > 0 \rightarrow C, B < 0 \rightarrow L$$

$$X > 0 \rightarrow L, X < 0 \rightarrow C$$

3. Ex. 5.1 Z_L=200-j100, Z_o=100Ω, f=500MHz



1. $z_L=2-j1$, $y_L=0.4+j0.2$

Solution A

2. $y=0.4+j0.5 \rightarrow jb=j0.3 \rightarrow jB=j\omega C=jb/Z_o$

$C=b/Z_o\omega$ =0.92pF

$z=1-j1.2 \rightarrow jx=j1.2 \rightarrow jX=j\,\omega L=jxZ_o$

$L=xZ_o/\omega$ =38.8nH

Solution B

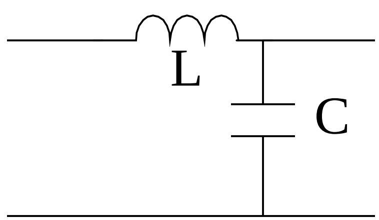
3. $y=0.4-j0.5 \rightarrow jb=-j0.7 \rightarrow -jB=1/j\omega L=-jb/Z_o$

$L=-Z_o/\omega b$=46.1nH

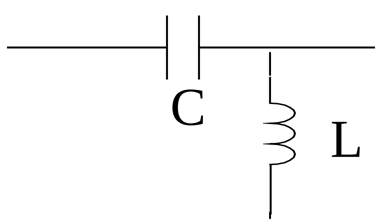
$z=1+j1.2 \rightarrow jx=-j1.2 \rightarrow jX=1/j\omega C=-jxZ_o$

$C=-1/xZ_o\omega$ =2.61pF

A:

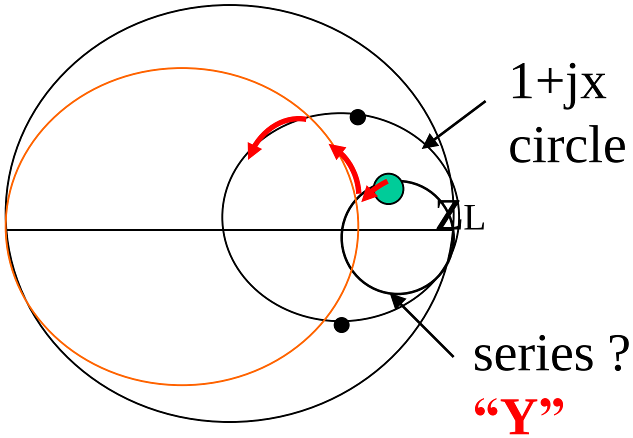


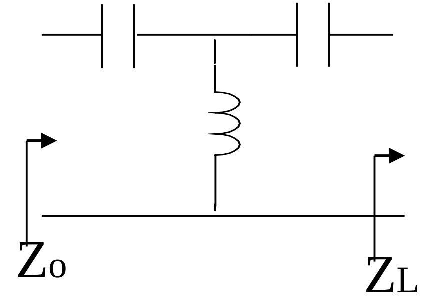
B:

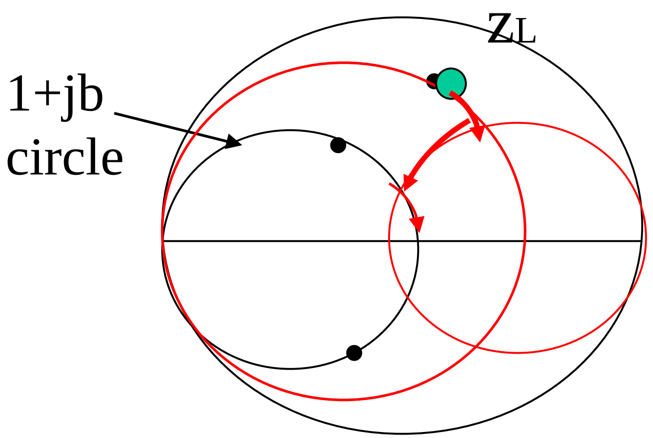


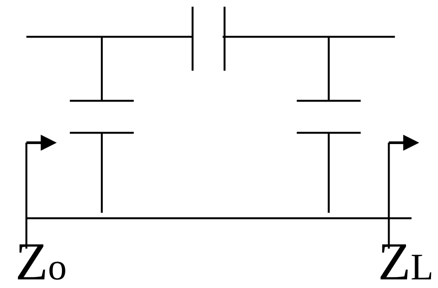
frequency response (p.233, Fig.5.3(c))

4. Possible 3-element L-network



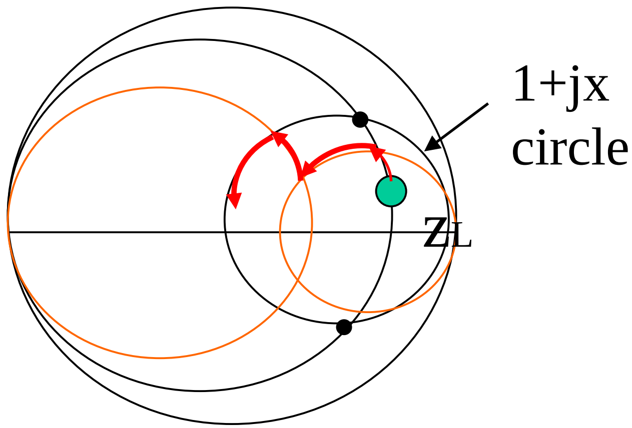


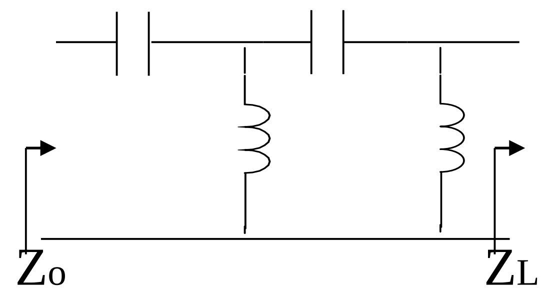


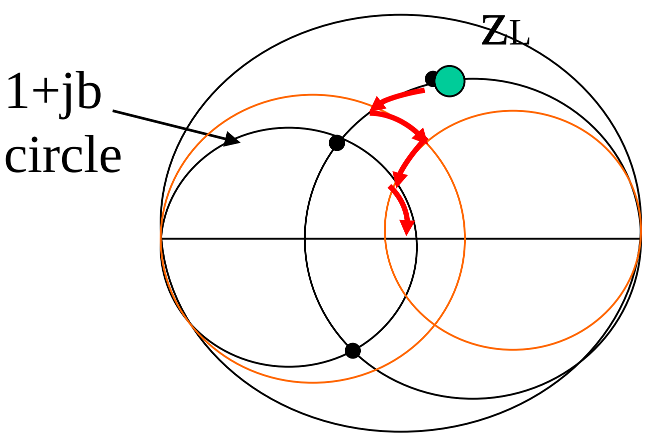


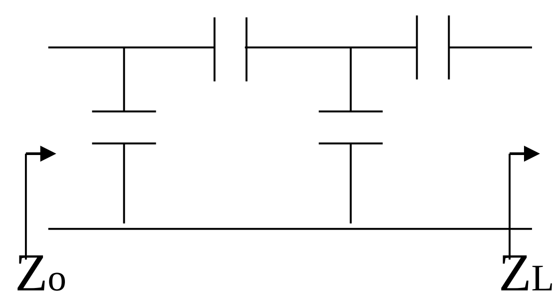
5. Possible 4-element L-network

shorter paths for a wider operational bandwidth









6. Lumped elements (size<λ/10)

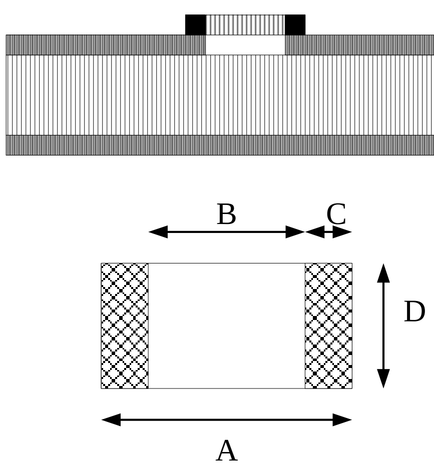
capacitor: chip capacitor, MIM capacitor (<25pF), interdigital
gap capacitor (<0.5pF), open stub(<0.1pF)

inductor: chip inductor, loop inductor, spiral inductor (<10nH)

resistor: chip resistor, planar resistor

All these lumped elements inherently have parasitic elements in the microwave range.

(p.233, “point of interest”)



Size (mil)	0402	0603	0805	1206
A	39	62	78	125
B	24	38	39	62
C	8	12	20	23
D	20	31	49	62

1mil=0.001in=25um=1/40mm

5.2 Single-stub tuning

- equivalent microstrip elements

a series C	---
a series L	in series with a high impedance microstrip line
a shunt C	in shunt with an open microstrip line, or in series with a low impedance microstrip line
a shunt L	in shunt with a short microstrip line

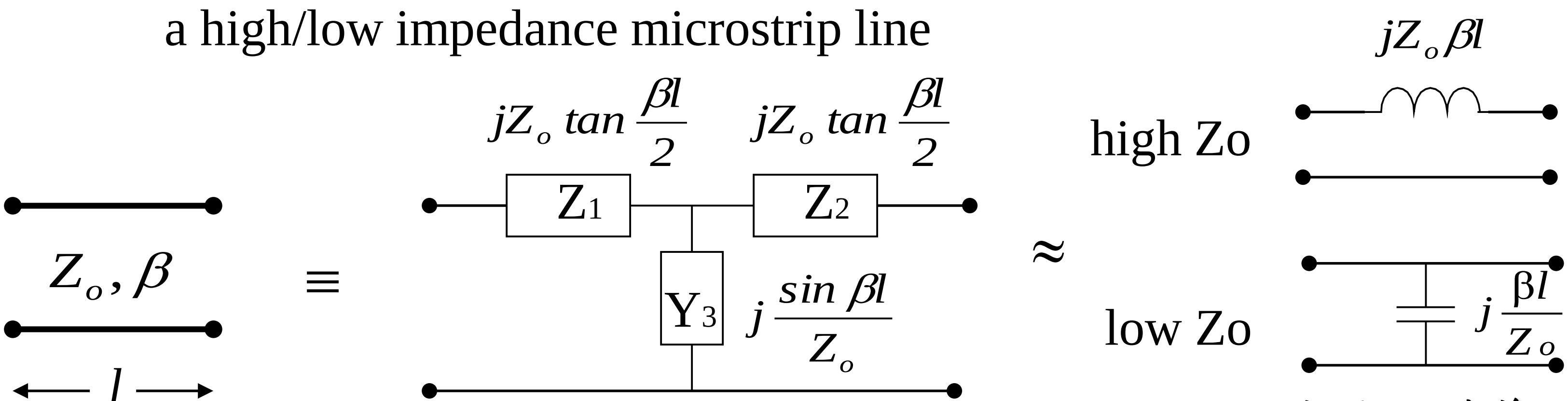
an open-circuited microstrip line

$$Z_{in} = \frac{Z_o}{j\tan\beta l} \equiv \frac{1}{j\omega C}$$

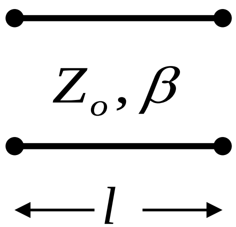
a short-circuited microstrip line

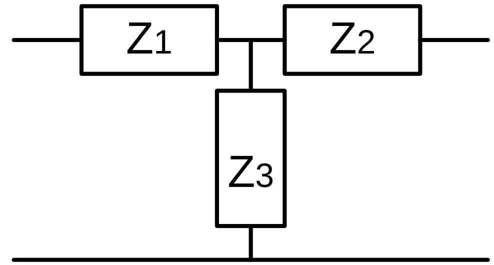
$$Z_{in} = jZ_o\tan\beta l \equiv j\omega L$$

a high/low impedance microstrip line



(*derivation of high/low impedance line*)





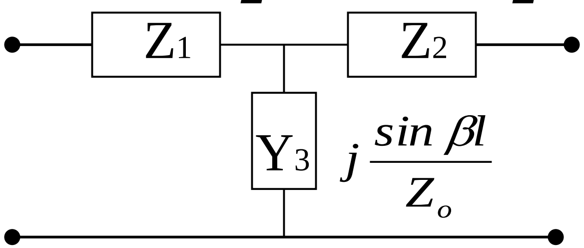
slide (4−12)

$$Z_1 = Z_2 = -jZ_o(\cot\beta l - \csc\beta l)$$

$$Z_3 = -jZ_o \csc\beta l$$

$$Z_3 = -jZ_o \csc\beta l = \frac{Z_o}{j\sin\beta l} \rightarrow Y_3 = \frac{j\sin\beta l}{Z_o}$$

$jZ_o \tan\frac{\beta l}{2}$ $\quad$ $jZ_o \tan\frac{\beta l}{2}$



$j\frac{\sin\beta l}{Z_o}$

$$Z_1 = Z_2 = -jZ_o(\cot\beta l - \csc\beta l) = -jZ_o\frac{\cos\beta l - 1}{\sin\beta l}$$

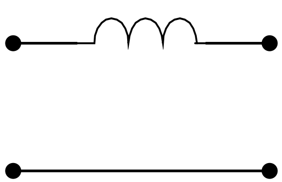
$$\overset{\cos\beta l = 1-2\sin^2\frac{\beta l}{2}}{\underset{\sin\beta l = 2\sin\frac{\beta l}{2}\cos\frac{\beta l}{2}}{=}} -jZ_o\frac{-2\sin^2\frac{\beta l}{2}}{2\sin\frac{\beta l}{2}\cos\frac{\beta l}{2}} = jZ_o\tan\frac{\beta l}{2}$$

high Zo,Y3→0

⇨

$\beta l << \frac{\pi}{2}$

$jZ_o\beta l$

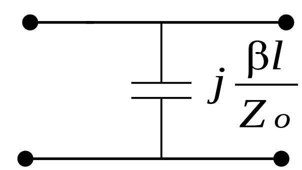


$$jZ_o\beta l = j\omega L \rightarrow L = \frac{Z_o\beta l}{\omega} = \frac{Z_o l}{c}$$

low Zo, Z1, Z2→0

⇨

$\beta l << \frac{\pi}{2}$



$j\frac{\beta l}{Z_o}$

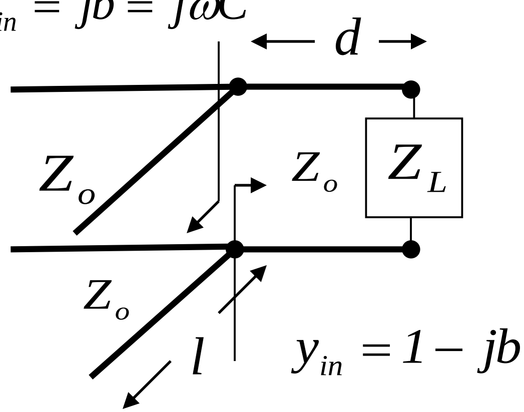
$$j\frac{\beta l}{Z_o} = j\omega C \rightarrow C = \frac{\beta l}{\omega Z_o} = \frac{l}{cZ_o}$$

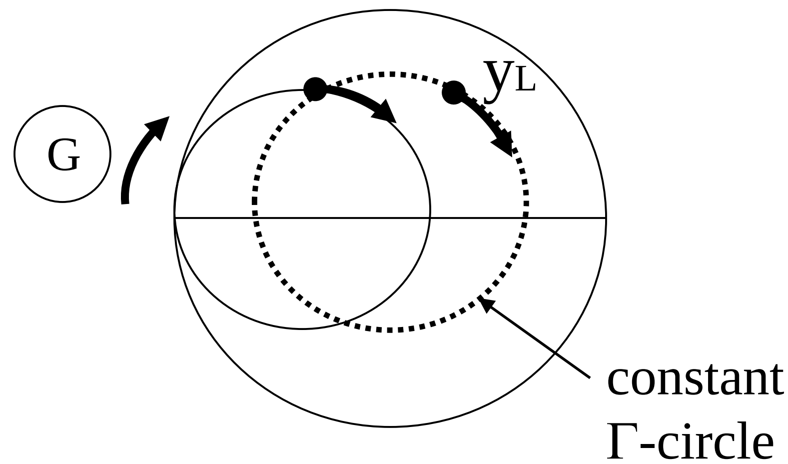
<u>Discussion</u>

1. Shunt stub

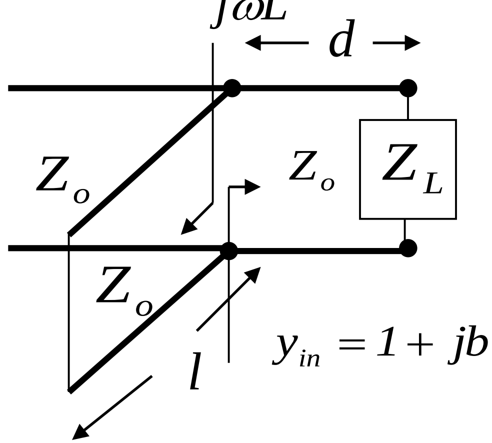
Smith chart solution

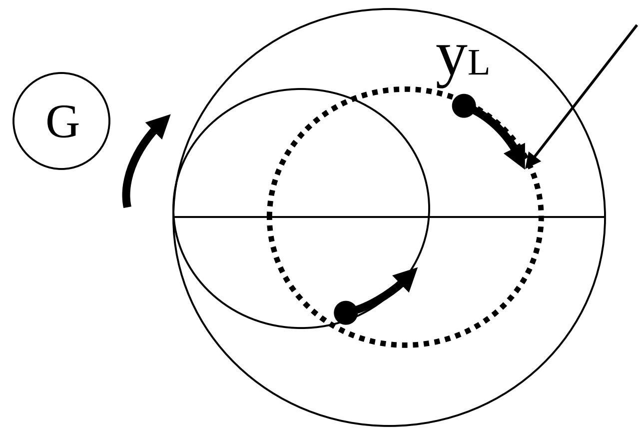
$y_{in} = jb = j\omega C$





$y_{in} = -jb = \frac{1}{j\omega L}$

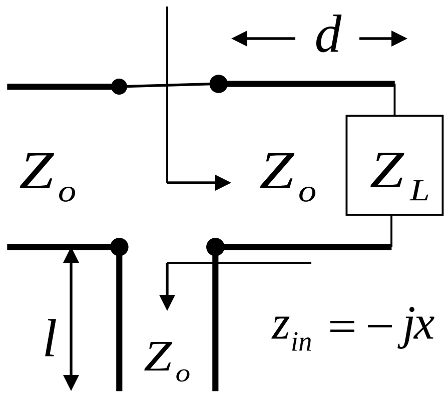




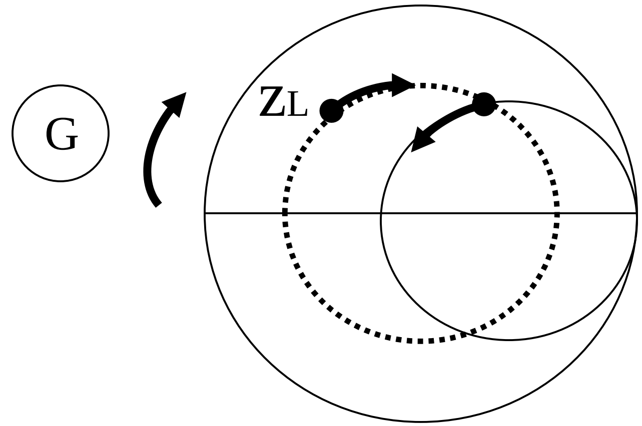
2. Series stub

Smith chart solution

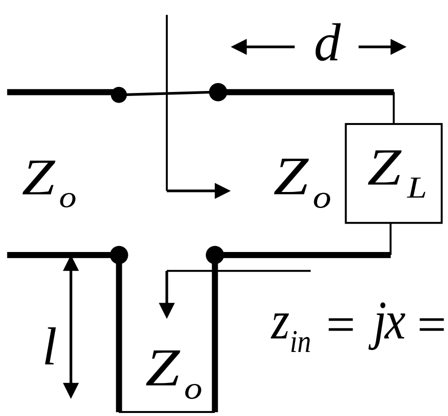
$z_{in} = 1 + jx$



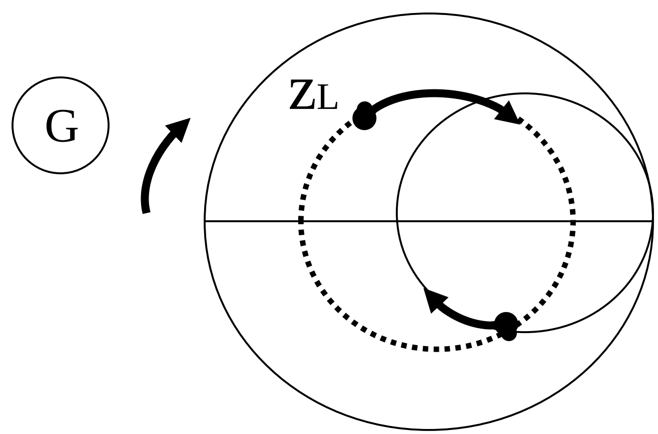
$z_{in} = -jx = \frac{1}{j\omega C}$



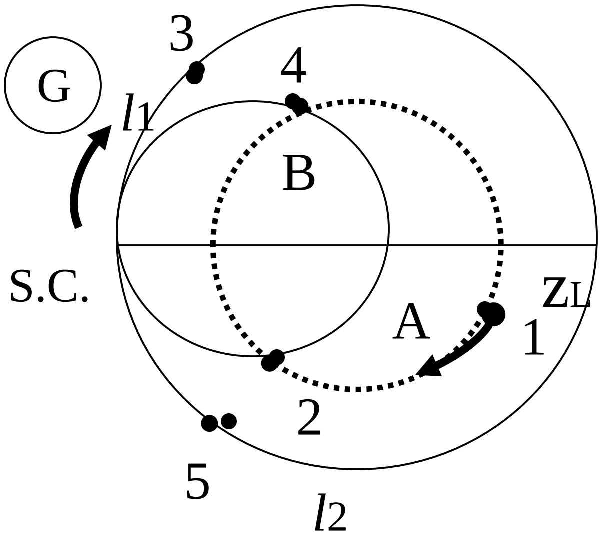
$z_{in} = 1 - jx$



$z_{in} = jx = j\omega L$



3. Ex. 5.2 Z_L=60-j80, Z_o=50Ω, f=2GHz, using a shunt short stub



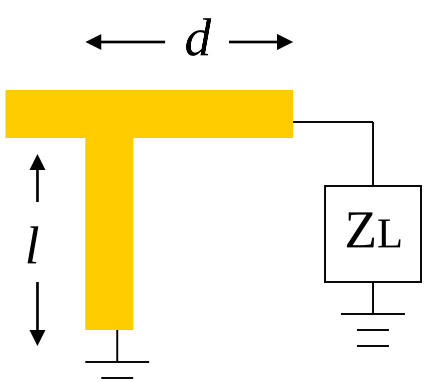
1. z_L=1.2-j1.6, y_L=0.3+j0.4

Solution A

2. y=1+j1.47 → d_1=0.11λ
3. y=-j1.47 → l_1=0.095λ, short stub

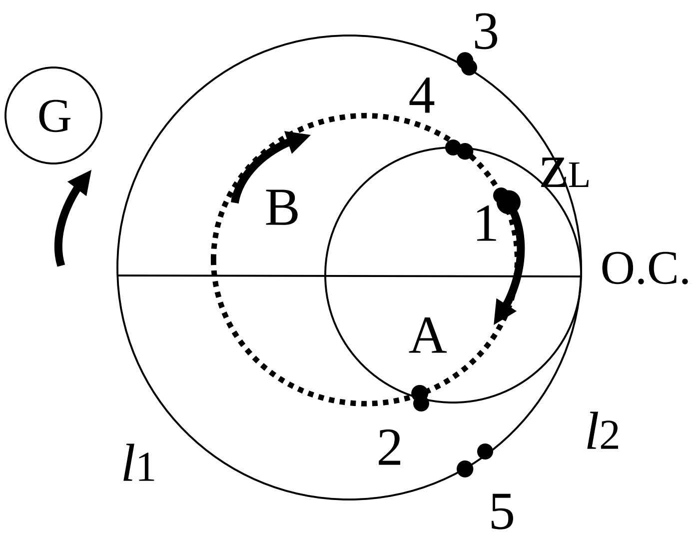
Solution B

4. y=1-j1.47 → d_2=0.26λ
5. y=j1.47 → l_2=0.405λ, short stub



frequency response (p.237, Fig.5.5(c))
Solution A has a wider bandwidth.

4. Ex. 5.3 Z_L=100+j80, Z_o=50Ω, f=2GHz, using series open stub



1. z_L=2+j1.6

Solution A

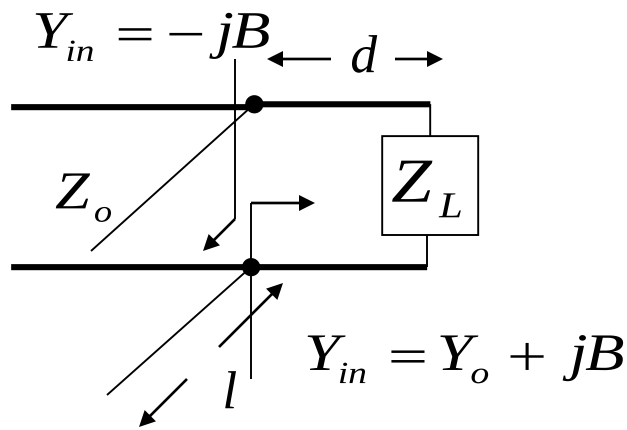
2. z=1-j1.33 → d_1=0.12λ
3. z=j1.33 → l_1=0.397λ, open stub

Solution B

4. z=1+j1.33 → d_2=0.463λ
5. z=-j1.33 → l_2=0.103λ, open stub

frequency response (p.240, Fig.5.6(c))
It can not be implemented in microstrip lines.

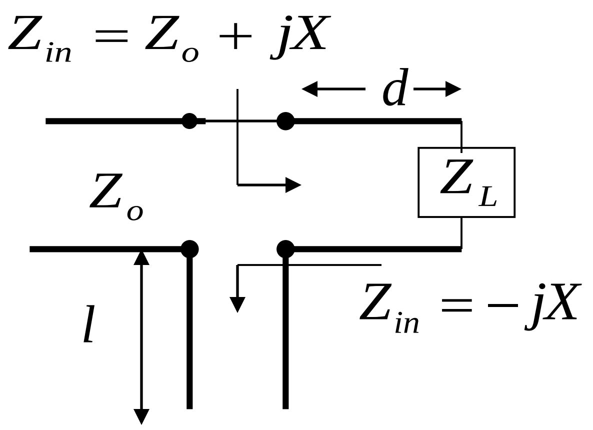
5. Analytical solution for shunt stub



$$Y_o = \mathrm{Re}\left\{\left(Z_o \frac{Z_L + jZ_o \tan\beta d}{Z_o + jZ_L \tan\beta d}\right)^{-1}\right\} \rightarrow d$$

$$-B = -\mathrm{Im}\left\{\left(Z_o \frac{Z_L + jZ_o \tan\beta d}{Z_o + jZ_L \tan\beta d}\right)^{-1}\right\} = \begin{cases} \left(\dfrac{Z_o}{\tan\beta l}\right)^{-1} & \text{open stub} \\ \left(-\dfrac{1}{Z_o \tan\beta l}\right)^{-1} & \text{short stub} \end{cases} \rightarrow l$$

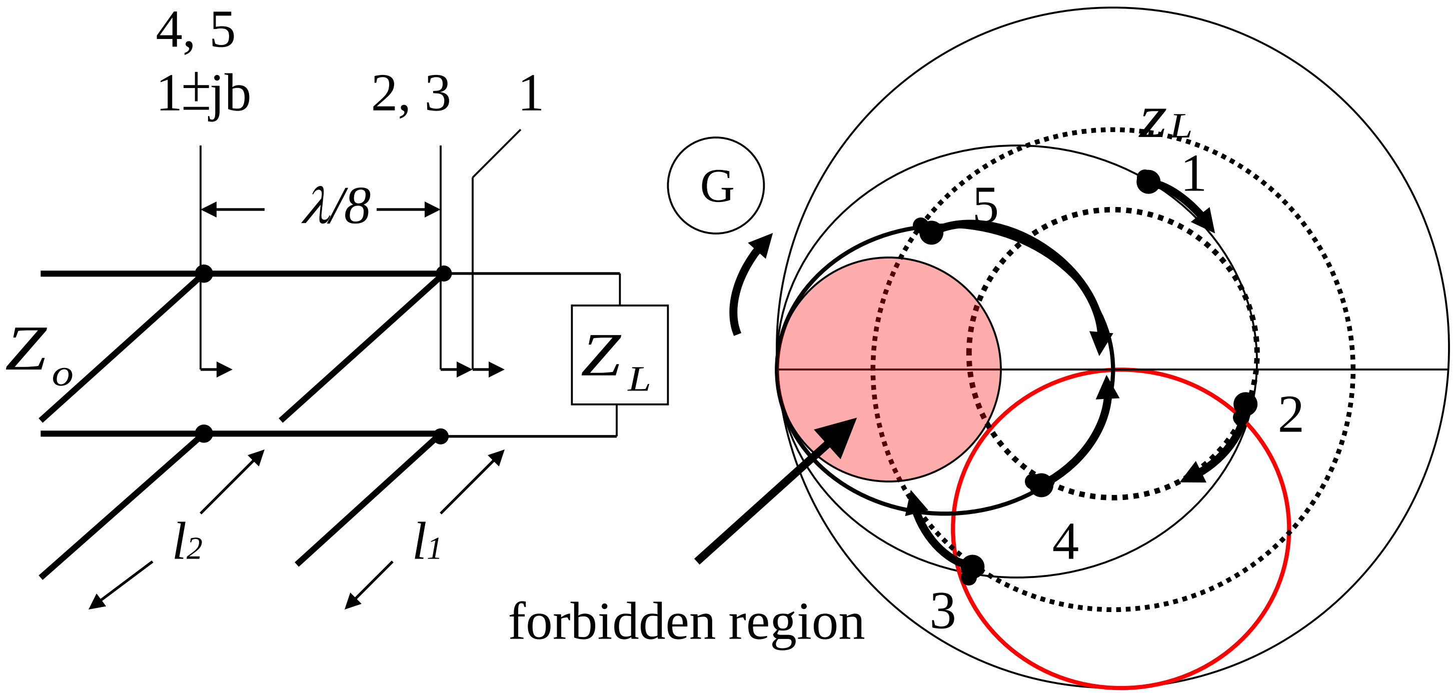
6. Analytical solution for series stub



$$Z_o = \mathrm{Re}\left\{ Z_o \frac{Z_L + jZ_o \tan\beta d}{Z_o + jZ_L \tan\beta d} \right\} \quad \rightarrow \quad d$$

$$-X = -\mathrm{Im}\left\{ Z_o \frac{Z_L + jZ_o \tan\beta d}{Z_o + jZ_L \tan\beta d} \right\} = \begin{cases} -\dfrac{Z_o}{\tan\beta l} & \text{open stub} \\ Z_o \tan\beta l & \text{short stub} \end{cases} \quad \rightarrow \quad l$$

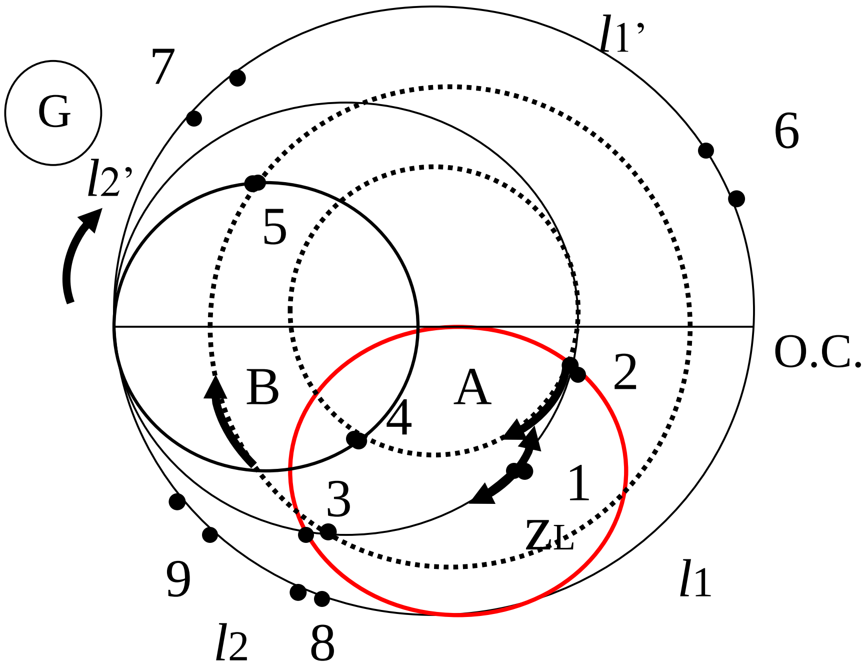
5.3 Double-stub tuning



Discussion

1. There exists a forbidden region for Z_L. It can be tuned out by adding a certain length of line.

2. Ex. 5.4 Z_L=60-j80, Z_o=50Ω, f=2GHz, using double-shunt-open-stubs



1. z_L=1.2-j1.6, y_L=0.3+j0.4

Solution A

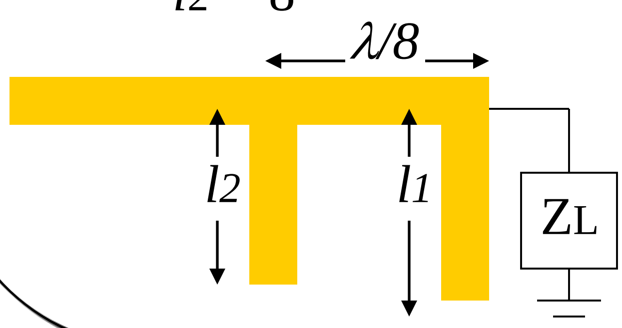
2. y=0.3+j 0.286 → 6. b_1' = - 0.114
→ l_1' = 0.482λ

4. y=1+j1.38 → 7. b_2' = - 1.38
→ l_2' =0.35λ

Solution B

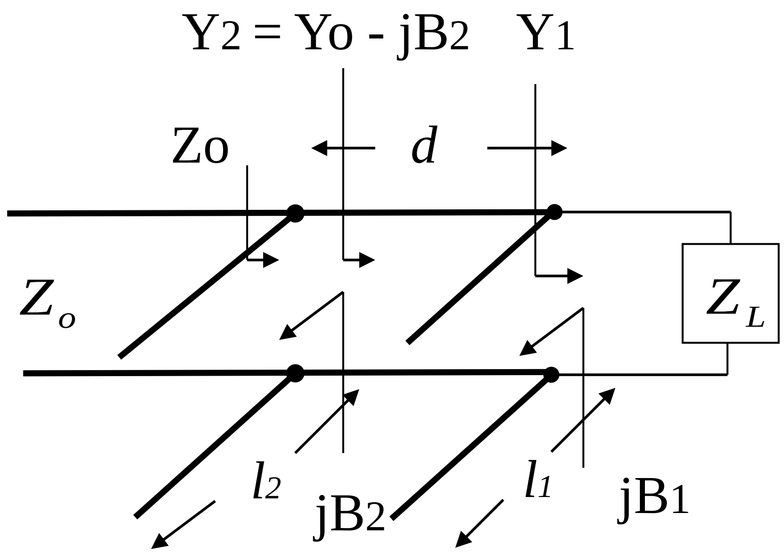
3. y=0.3+j1.714 → 8. b_1 =1.314
→ l_1 =0.146λ

5. y=1-j3.38 → 9. b_2 = 3.38
→ l_2 =0.204λ



frequency response (p.245, Fig.5.9(c))

3. Analytical solution



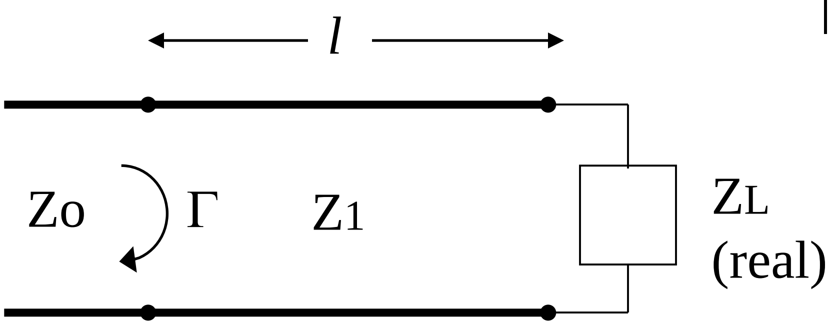
$$Y_1 = Y_L + jB_1, Y_2 = Y_o - jB_2 = \left(Z_o \frac{Z_1 + jZ_o \tan \beta d}{Z_o + jZ_1 \tan \beta d} \right)^{-1}$$

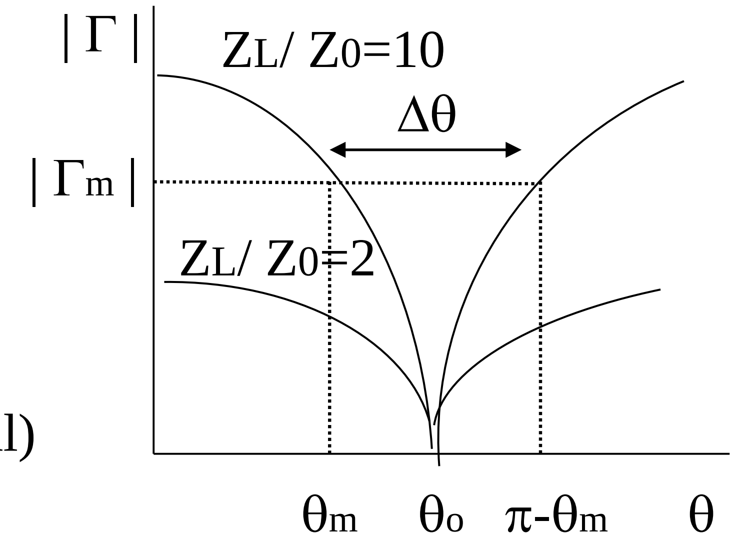
$$\mathrm{Re}\{Y_2\} = Y_o \rightarrow B_1 \rightarrow l_1$$

$$Y_1 \rightarrow \mathrm{Im}\{Y_2\} = -B_2 \rightarrow l_2$$

5.4 The quarter-wave transformer

- frequency response





Γ_m: max. tolerated Γ over the bandwidth

$$Z_1 = \sqrt{Z_o Z_L}$$

$$|\Gamma(\theta)| \approx \frac{|Z_L - Z_o|}{2\sqrt{Z_o Z_L}} |\cos\theta|, \quad \text{for } \theta \text{ near } \theta_o = \frac{\pi}{2}, \ \theta = \beta l$$

$$\frac{\Delta f}{f_o} = 2 - \frac{4}{\pi} \cos^{-1}\left(\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \frac{2\sqrt{Z_o Z_L}}{|Z_L - Z_o|}\right), \quad Z_L \to Z_o, \ \Delta f \text{ increases}$$

(derivation of $\left|\Gamma(\theta)\right| \approx \dfrac{\left|Z_L - Z_o\right|}{2\sqrt{Z_o Z_L}}\left|\cos\theta\right|$)

$$\Gamma(\theta) = \frac{Z_{in}(\theta) - Z_o}{Z_{in}(\theta) + Z_o} = \frac{Z_1 \dfrac{Z_L + jZ_1 \tan\theta}{Z_1 + jZ_L \tan\theta} - Z_o}{Z_1 \dfrac{Z_L + jZ_1 \tan\theta}{Z_1 + jZ_L \tan\theta} + Z_o} = \frac{Z_1 Z_L + jZ_1^2 \tan\theta - Z_o Z_1 - jZ_L Z_o \tan\theta}{Z_1 Z_L + jZ_1^2 \tan\theta + Z_o Z_1 + jZ_L Z_o \tan\theta}$$

$$\overset{Z_1^2 = Z_o Z_L}{=} \frac{Z_1(Z_L - Z_o)}{Z_1(Z_L + Z_o) + j2Z_1^2 \tan\theta} = \frac{Z_L - Z_o}{Z_L + Z_o + j2\sqrt{Z_o Z_L}\tan\theta} = \frac{1}{\dfrac{Z_L + Z_o}{Z_L - Z_o} + \dfrac{j2\sqrt{Z_o Z_L}}{Z_L - Z_o}\tan\theta}$$

$$\overset{\theta \to \theta_o = \frac{\pi}{2}}{\approx} \frac{Z_L - Z_o}{j2\sqrt{Z_o Z_L}}\cos\theta$$

(derivation of $\dfrac{\Delta f}{f_o} = 2 - \dfrac{4}{\pi}\cos^{-1}\left(\dfrac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\dfrac{2\sqrt{Z_o Z_L}}{\left|Z_L - Z_o\right|}\right)$)

$$\left|\Gamma(\theta)\right| = \frac{1}{\left[\left(\dfrac{Z_L + Z_o}{Z_L - Z_o}\right)^2 + \left(\dfrac{2\sqrt{Z_o Z_L}}{Z_L - Z_o}\tan\theta\right)^2\right]^{1/2}} = \frac{1}{\left[\dfrac{(Z_L - Z_o)^2 + 4Z_L Z_o}{(Z_L - Z_o)^2} + \dfrac{4Z_o Z_L}{(Z_L - Z_o)^2}\tan^2\theta\right]^{1/2}}$$

$$\overset{1+\tan^2\theta = \sec^2\theta}{=} \frac{1}{\left[1 + \dfrac{4Z_o Z_L}{(Z_L - Z_o)^2}\sec^2\theta\right]^{1/2}}$$

$$\Gamma(\theta) = \frac{1}{[1 + \frac{4Z_o Z_L}{(Z_L - Z_o)^2} \sec^2 \theta]^{1/2}}$$

$$\Gamma_m^2 = \frac{1}{1 + (\frac{2\sqrt{Z_o Z_L}}{Z_L - Z_o} \frac{1}{\cos\theta_m})^2} \rightarrow \frac{1}{\Gamma_m^2} - 1 = (\frac{2\sqrt{Z_o Z_L}}{Z_L - Z_o})^2 \frac{1}{\cos^2 \theta_m}$$

$$\rightarrow \cos\theta_m = \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_o Z_L}}{|Z_L - Z_o|}$$

$$\text{TEM line: } \theta = \beta l = \frac{2\pi f}{v_p(f)} \frac{\lambda_o}{4} = \frac{2\pi f}{v_p(f)} \frac{v_p(f_o)}{4 f_o} = \frac{\pi f}{2 f_o} \rightarrow \theta_m = \frac{\pi f_m}{2 f_o} \rightarrow \frac{f_m}{f_o} = \frac{2\theta_m}{\pi}$$

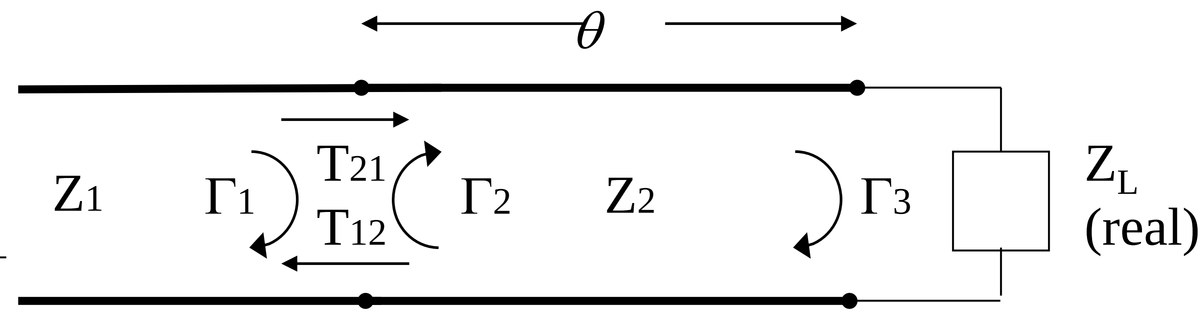
$$\frac{\triangle f}{f_o} = \frac{2(f_o - f_m)}{f_o} = 2 - \frac{2 f_m}{f_o} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi} \cos^{-1} \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_o Z_L}}{|Z_L - Z_o|}$$

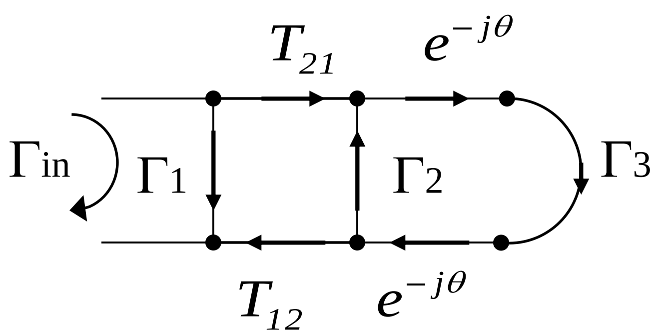
5.5 The theory of small reflections

- single-section transformer

partial reflection coefficients

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2}$$





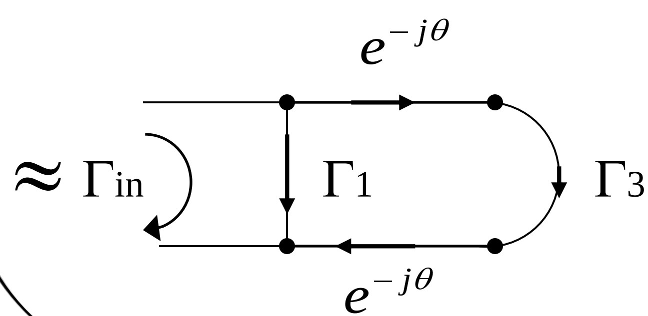
$$\Gamma_2 = -\Gamma_1, \quad T_{21} = 1 + \Gamma_1, \quad T_{12} = 1 + \Gamma_2$$

$$\Gamma_{in} = \Gamma_1 + \frac{T_{12} T_{21} \Gamma_3 e^{-j2\theta}}{1 - \Gamma_2 \Gamma_3 e^{-j2\theta}}$$

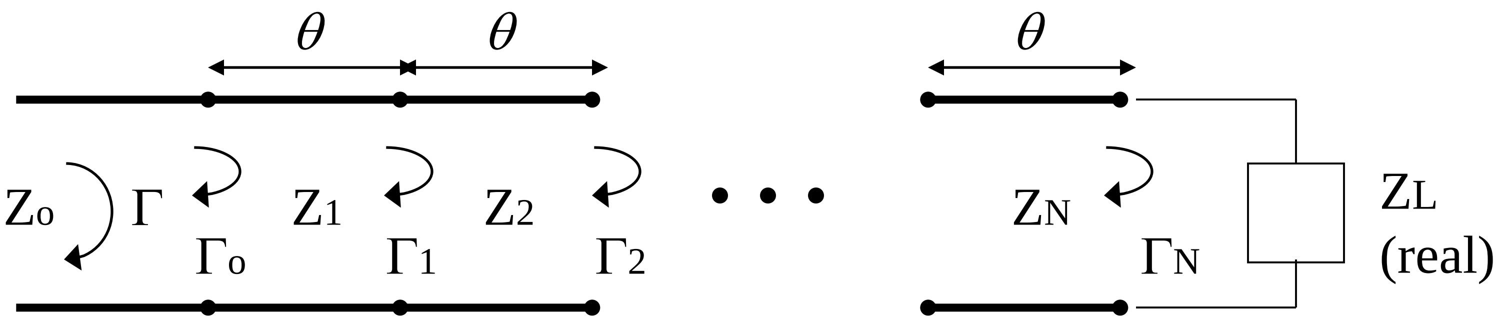
$$= \frac{\Gamma_1 + \Gamma_1^2 \Gamma_3 e^{-j2\theta} + (1 - \Gamma_1)(1 + \Gamma_1) \Gamma_3 e^{-j2\theta}}{1 + \Gamma_1 \Gamma_3 e^{-j2\theta}}$$

$$= \frac{\Gamma_1 + \Gamma_3 e^{-j2\theta}}{1 + \Gamma_1 \Gamma_3 e^{-j2\theta}}$$

$$\approx \Gamma_1 + \Gamma_3 e^{-j2\theta} \quad \text{if } Z_1 \approx Z_2 \approx Z_L$$



• multisection transformer



$$\Gamma(\theta)=\Gamma_o+\Gamma_1 e^{-j2\theta}+\Gamma_2 e^{-j4\theta}+......\Gamma_N e^{-j2N\theta} \quad \text{, if } \Gamma_o=\Gamma_N, \Gamma_1=\Gamma_{N-1}......$$

$$=\begin{cases} e^{-jN\theta}[\Gamma_o(e^{jN\theta}+e^{-jN\theta})+\Gamma_1(e^{j(N-2)\theta}+e^{-j(N-2)\theta})+......\Gamma_{(N-1)/2}(e^{j\theta}+e^{-j\theta})] & N \text{ odd} \\ e^{-jN\theta}[\Gamma_o(e^{jN\theta}+e^{-jN\theta})+\Gamma_1(e^{j(N-2)\theta}+e^{-j(N-2)\theta})+......\Gamma_{N/2}] & N \text{ even} \end{cases}$$

$$=\begin{cases} 2e^{-jN\theta}[\Gamma_o\cos N\theta+\Gamma_1\cos(N-2)\theta+....\Gamma_n\cos(N-2n)\theta+....\frac{1}{2}\Gamma_{(N-1)/2}\cos\theta] & N \text{ odd} \\ 2e^{-jN\theta}[\Gamma_o\cos N\theta+\Gamma_1\cos(N-2)\theta+....\Gamma_n\cos(N-2n)\theta+....\frac{1}{2}\Gamma_{N/2}] & N \text{ even} \end{cases}$$

given $\Gamma(\theta)$, design $Z_1, Z_2, ...Z_n$

5.6 Binomial multisection matching transformer

• maximal flatness response for Γ (θ)

$$\Gamma(\theta) = A(1+e^{-j2\theta})^N = A\sum_{n=0}^{N} C_n^N e^{-j2n\theta} = \Gamma_o + \Gamma_1 e^{-j2\theta} + \Gamma_2 e^{-j4\theta} + ...\Gamma_N e^{-j2N\theta}$$

$$\Rightarrow \Gamma_n = AC_n^N$$

Discussion

1. Maximal flatness response, $\left.\dfrac{d^{N-1}\left|\Gamma(\theta)\right|}{d\theta^{N-1}}\right|_{\theta=\frac{\pi}{2}\ or\ l=\frac{\lambda}{4}} = 0$

2. $\left|\Gamma(0)\right| = \left|A\right|\ 2^N = \left|\dfrac{Z_L - Z_o}{Z_L + Z_o}\right| \rightarrow A = 2^{-N}\left|\dfrac{Z_L - Z_o}{Z_L + Z_o}\right|$

3. $\Gamma_n = \dfrac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \approx \dfrac{1}{2}\ln\dfrac{Z_{n+1}}{Z_n} \qquad (\ln x \approx 2\dfrac{x-1}{x+1})$

$$\rightarrow \ln\frac{Z_{n+1}}{Z_n} \approx 2\Gamma_n = 2AC_n^N = 2\times 2^{-N}\frac{Z_L - Z_o}{Z_L + Z_o}C_n^N \approx 2^{-N}C_n^N\ln\frac{Z_L}{Z_o}$$

(p.254, Table 5.1 for Zn values)

4. $\frac{\Delta f}{f_o}=2-\frac{4}{\pi}\cos^{-1}[\frac{1}{2}(\frac{\Gamma_m}{|A|})^{\frac{1}{N}}]$, $N\uparrow,\Delta f\uparrow,\ \Gamma_m=2^N|A|con^N\theta_m$

$$\because \Gamma(\theta)=A(1+e^{-j2\theta})^N \rightarrow |\Gamma(\theta)|=|A|2^N\cos^N\theta$$

$$\frac{\Delta f}{f_o}=2-\frac{4}{\pi}\theta_m=2-\frac{4}{\pi}\cos^{-1}[\frac{1}{2}(\frac{\Gamma_m}{|A|})^{\frac{1}{N}}]$$

5. Ex.5.6 Z_L=50Ω, Z_o=100Ω, N=3, Γ_m=0.05

$$N=3, A=2^{-N}\left|\frac{Z_L-Z_o}{Z_L+Z_o}\right|=\frac{1}{2^{N+1}}\ln\frac{Z_L}{Z_o}=-0.0433,\quad \frac{\Delta f}{f_o}=70\%$$

$$\ln\frac{Z_{n+1}}{Z_n}=2^{-N}C_n^N\ln\frac{Z_L}{Z_o}\Rightarrow \ln\frac{Z_1}{Z_o}=2^{-3}C_0^3\ln\frac{Z_L}{Z_o}\rightarrow Z_1=91.7\Omega$$

$$\ln\frac{Z_2}{Z_1}=2^{-3}C_1^3\ln\frac{Z_L}{Z_o}\rightarrow Z_2=70.7\Omega,\quad \ln\frac{Z_3}{Z_2}=2^{-3}C_2^3\ln\frac{Z_L}{Z_o}\rightarrow Z_3=54.5\Omega$$

(p.256, Fig.5.15 for frequency response of |Γ|)

5.7 Chebyshev multisection matching transformers

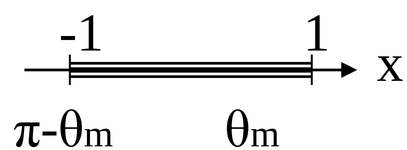
• Equal ripple response for Γ (θ): optimal design

$$\Gamma(\theta) = 2e^{-jN\theta}[\Gamma_o \cos N\theta + \Gamma_1 \cos(N-2)\theta + ...\Gamma_n \cos(N-2n)\theta + ...]$$

$$= Ae^{-jN\theta}T_N(\sec\theta_m \cos\theta) \qquad \Rightarrow \Gamma_n \text{ (p.260, Table 5.2)}$$

$$T_1(x) = x, \quad T_2(x) = 2x^2 - 1, \quad T_3(x) = 4x^3 - 3x, \quad T_4(x) = 8x^4 - 8x^2 + 1,$$

$$T_n(x) = 2xT_{n\text{-}1}(x) - T_{n\text{-}2}(x), \; x \equiv \frac{\cos\theta}{\cos\theta_m}, \; |x| \le 1$$



<u>Discussion</u>

1. $\Gamma(0) = AT_N(\sec\theta_m) = \dfrac{Z_L - Z_o}{Z_L + Z_o} \rightarrow A = \dfrac{Z_L - Z_o}{Z_L + Z_o}\dfrac{1}{T_N(\sec\theta_m)}$

$$\Gamma_m = AT_N(1) = A$$

2. $T_N(\sec\theta_m) = \dfrac{1}{\Gamma_m}\left|\dfrac{Z_L - Z_o}{Z_L + Z_o}\right| \quad \rightarrow \theta_m \quad (5.63)\xrightarrow{\theta_m} \quad \dfrac{\Delta f}{f_o} = 2 - \dfrac{4\theta_m}{\pi}$

3. Optimal design: given Γm, maximal Δf

given Δf , minimal Γm.

4. Ex.5.7 Z_L=100Ω, Z_o=50Ω, N=3, Γ_m=0.05

$$N=3, \Gamma(\theta)=2e^{-j3\theta}(\Gamma_o\cos 3\theta+\Gamma_1\cos\theta)=Ae^{-j3\theta}T_3(\sec\theta_m\cos\theta)$$

$$=Ae^{-j3\theta}(4\sec^3\theta_m\cos^3\theta-3\sec\theta_m\cos\theta)$$

$$=Ae^{-j3\theta}[\sec^3\theta_m(\cos 3\theta+3\cos\theta)-3\sec\theta_m\cos\theta]$$

$$A=\frac{Z_L-Z_o}{Z_L+Z_o}\frac{1}{T_3(\sec\theta_m)}=\Gamma_m \rightarrow \sec\theta_m=1.408 \rightarrow \theta_m=44.7^o \rightarrow \quad \frac{\Delta f}{f_o}=101\%$$

$$\Rightarrow 2\Gamma_o=A\sec^3\theta_m \quad \rightarrow \Gamma_o=0.0698=\Gamma_3$$

$$2\Gamma_1=A(3\sec^3\theta_m-3\sec\theta_m) \quad \rightarrow \Gamma_1=0.1037=\Gamma_2$$

$$\Gamma_o=\frac{Z_1-Z_o}{Z_1+Z_o} \quad \rightarrow Z_1=57.5\Omega$$

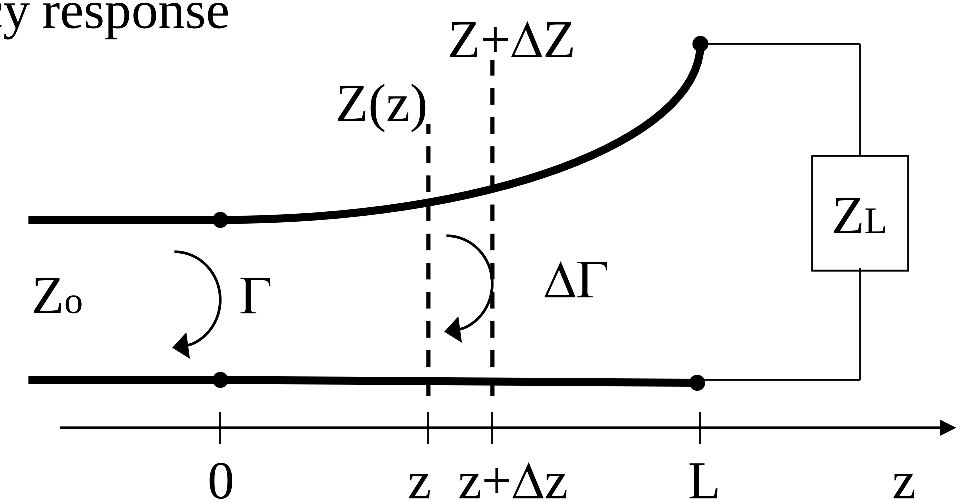
$$\Gamma_1=\frac{Z_2-Z_1}{Z_2+Z_1} \quad \rightarrow Z_2=70.7\Omega$$

$$\Gamma_o=\Gamma_3=\frac{Z_3-Z_2}{Z_3+Z_2} \quad \rightarrow Z_3=87\Omega$$

frequency response (p.261, Fig.5.17)

5.8 Tapered lines

- Frequency response



$$\Delta\Gamma = \frac{Z+\Delta Z - Z}{Z+\Delta Z + Z} \approx \frac{\Delta Z}{2Z}$$

$$\rightarrow d\Gamma = \frac{1}{2}\frac{dZ}{Z} = \frac{1}{2}\frac{d\ln Z/Z_o}{dz}dz$$

$$\rightarrow \Gamma(\theta) = \frac{1}{2}\int_0^L e^{-j2\beta z}\frac{d}{dz}(\ln\frac{Z}{Z_o})dz$$

Discussion

1. Exponential taper

$$Z(z) = Z_o e^{az} \qquad 0 < z < L$$

$$Z(L) = Z_L = Z_o e^{aL} \quad \rightarrow a = \frac{1}{L} \ln \frac{Z_L}{Z_o}$$

$$\rightarrow \ \Gamma(\theta) = \frac{1}{2} \int_0^L e^{-j2\beta z} \frac{d}{dz} (\ln e^{az}) dz = \frac{1}{2} \ln \frac{Z_L}{Z_o} e^{-j\beta L} \frac{\sin \beta L}{\beta L}$$

$L \uparrow, \ \Gamma(\theta) \downarrow$ (p. 263, Fig.5.19)

2. Triangular taper

$$Z(z) = \begin{cases} Z_o e^{2(z/L)^2 \ln Z_L/Z_o} & 0 < z < L/2 \\ Z_o e^{(4z/L - 2z^2/L^2 - 1)^2 \ln Z_L/Z_o} & L/2 < z < L \end{cases}$$

$$\Gamma(\theta) = \frac{1}{2} \ln \frac{Z_L}{Z_o} e^{-j\beta L} [\frac{\sin(\beta L/2)}{\beta L/2}]^2$$

first null at 2π (p. 264, Fig.5.20)

3. Klopfenstein taper

$Z(z)$ (5.74), (5.75), $\Gamma(\theta)$ (5.76), optimal taper

4. Ex.5.8 Z_L=100Ω, Z_o=50Ω, Γ_m=0.02

exponential taper: $Z(z) = Z_o e^{az}, \quad a = \frac{1}{L}\ln\frac{Z_L}{Z_o} = 0.693/L$

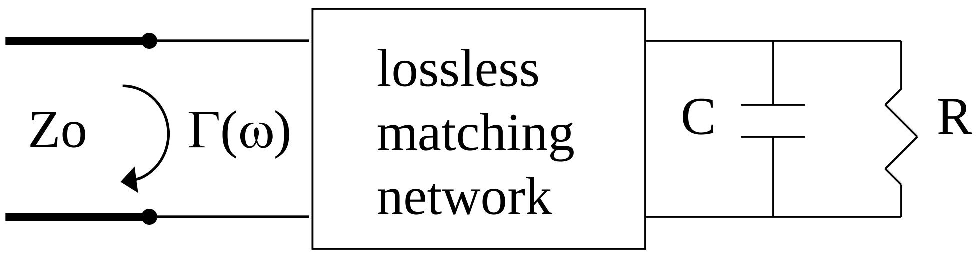
$$\left|\Gamma(\theta)\right| = \frac{1}{2}\ln\frac{Z_L}{Z_o}\frac{\sin\beta L}{\beta L}$$

triangular taper: $\left|\Gamma(\theta)\right| = \frac{1}{2}\ln\frac{Z_L}{Z_o}[\frac{\sin\beta L/2}{\beta L/2}]^2$

Klopfenstein taper: $\left|\Gamma(\theta)\right| = \Gamma_o\frac{\cos\sqrt{(\beta L)^2 - A^2}}{\cosh A}, \quad A = 3.543, \quad \Gamma_o = 0.346$

frequency response (p. 266, Fig.5.21)

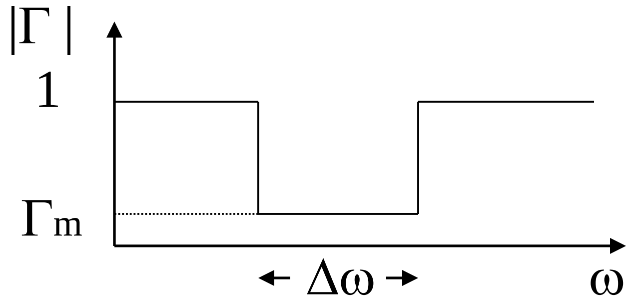
5.9 The Bode-Fano criterion

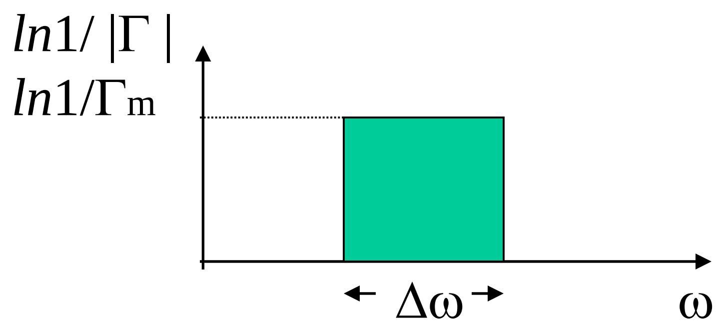


$$\int_0^{\infty} \ln \frac{1}{|\Gamma(\omega)|} d\omega \le \frac{\pi}{RC}$$

Discussion

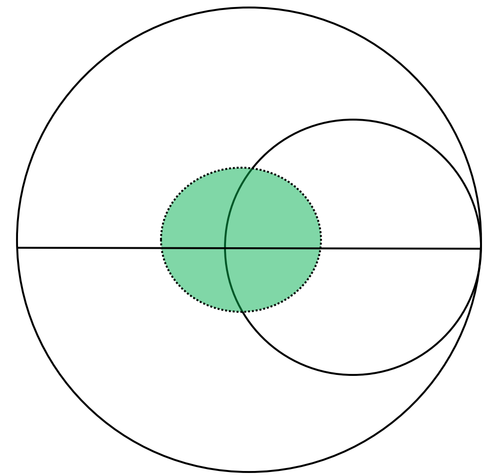
1.





$$\int_0^\infty \ln \frac{1}{|\Gamma(\omega)|} d\omega = \int_{\Delta\omega} \ln \frac{1}{\Gamma_m} d\omega$$

$$= \Delta\omega \ln \frac{1}{\Gamma_m} \le \frac{\pi}{RC} : \text{constant}$$



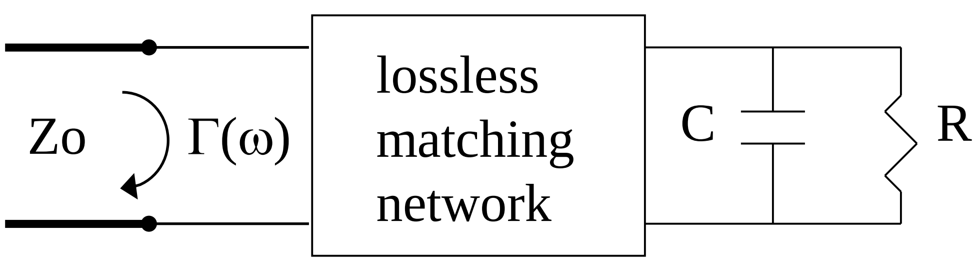
(1) given RC $\rightarrow$ $\Delta\omega\uparrow \rightarrow \Gamma_m\uparrow$

(2) $\Gamma_m \neq 0$, unless $\Delta\omega=0$

i.e., $\Gamma_m=0$ only at a finite number of frequencies

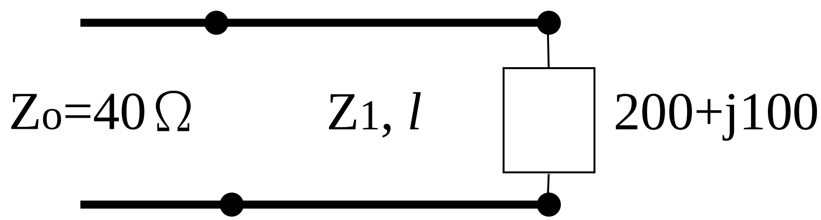
(3) R and/or C $\uparrow \rightarrow$ $\Delta\omega \downarrow$ and/or $\Gamma_m\uparrow$

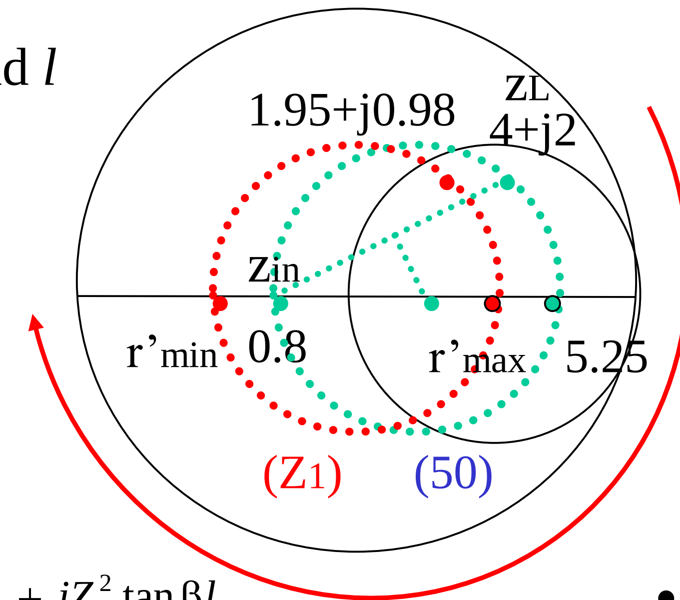
$\Rightarrow$ high Q load is harder to match



parallel resonator Q= $\omega_o RC$

Solved problems: Prob.5.7 find Z1 and *l*





$$40 = Z_1 \frac{200 + j100 + jZ_1 \tan\beta l}{Z_1 + j(200 + j100)\tan\beta l}$$

$$\rightarrow 40Z_1 + j8000\tan\beta l - 4000\tan\beta l = 200Z_1 + j100Z_1 + jZ_1^2 \tan\beta l$$

$$\rightarrow \begin{cases} 40Z_1 - 4000\tan\beta l = 200Z_1 \\ j8000\tan\beta l = j100Z_1 + jZ_1^2\tan\beta l \end{cases}$$

$$\rightarrow \begin{cases} Z_1 = -25\tan\beta l \\ j8000 = -j2500 + j625\tan^2\beta l \rightarrow \tan\beta l = -4.1 \end{cases}$$

$$\rightarrow \begin{cases} Z_1 = 102.5 \\ l = 0.288\lambda = 103.68^\circ \end{cases}$$

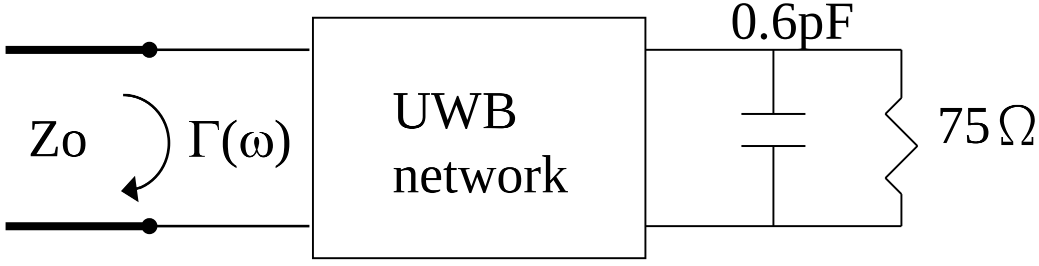
$$k = \frac{Z_1}{50}, \quad \begin{matrix} kr'_{min} = \frac{Z_1}{50}\frac{R'_{min}}{Z_1} = \frac{R'_{min}}{50} = 0.8 \\ kr'_{max} = \frac{Z_1}{50}\frac{R'_{max}}{Z_1} = \frac{R'_{max}}{50} = 5.25 \end{matrix}$$

$$\rightarrow k^2 r'_{min}\, r'_{max} \overset{r'_{min}\, r'_{max} = 1}{=} k^2 = 4.2$$

$$\rightarrow k = 2.05$$

$$Z_1 = 50 \times 2.05 = 102.5$$

Prob.5.24 find the best *RL* over operating range of 3.1~10.6GHz

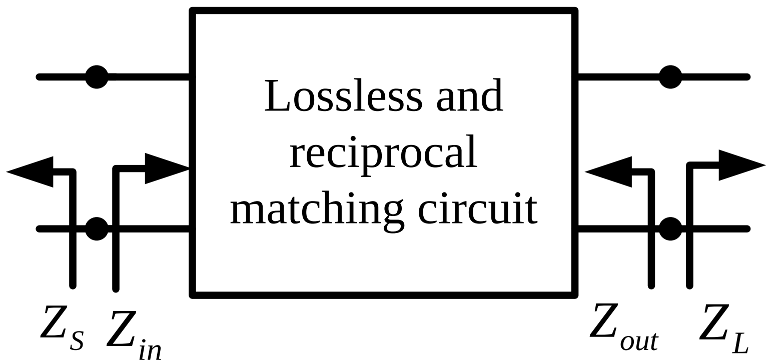


$$\int_0^\infty \ln \frac{1}{|\Gamma(\omega)|} d\omega \le \frac{\pi}{RC}$$

$$\rightarrow \ln \frac{1}{\Gamma_m} \le \frac{\pi}{2\pi(10.6-3.1)\times 10^9 \times 75 \times 0.6 \times 10^{-12}}$$

$$\rightarrow \ln \frac{1}{\Gamma_m} \le 1.48 \rightarrow \Gamma_m \ge 0.228, RL \le 6.4dB$$

Prob. For a lossless and reciprocal two-port matching network, if $Z_s = Z_{in}^*$ then $Z_{out} = Z_L^*$.



(1) if $Z_S = Z_{in}^*$

$$\Gamma_S = \frac{Z_S - Z_{in}}{Z_S + Z_{in}} = \frac{Z_{in}^* - Z_{in}}{Z_{in}^* + Z_{in}}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_S}{Z_S + Z_{in}} = \frac{Z_{in} - Z_{in}^*}{Z_{in}^* + Z_{in}}$$

$$\Rightarrow \Gamma_S^* = \left(\frac{Z_{in}^* - Z_{in}}{Z_{in}^* + Z_{in}}\right)^* = \frac{Z_{in} - Z_{in}^*}{Z_{in}^* + Z_{in}} = \Gamma_{in}$$

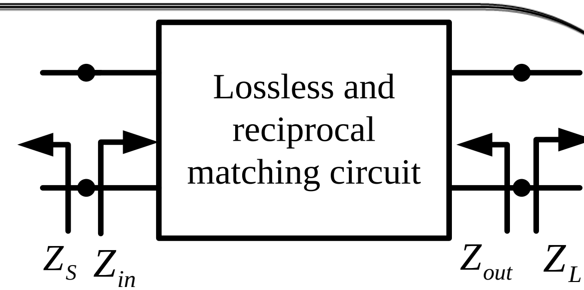
if $Z_L = Z_{out}^*$

$$\Gamma_L = \frac{Z_L - Z_{out}}{Z_L + Z_{out}} = \frac{Z_{out}^* - Z_{out}}{Z_{out}^* + Z_{out}}$$

$$\Gamma_{out} = \frac{Z_{out} - Z_L}{Z_{out} + Z_L} = \frac{Z_{out} - Z_{out}^*}{Z_{out} + Z_{out}^*}$$

$$\Rightarrow \Gamma_L^* = \left(\frac{Z_{out}^* - Z_{out}}{Z_{out}^* + Z_{out}}\right)^* = \frac{Z_{out} - Z_{out}^*}{Z_{out} + Z_{out}^*} = \Gamma_{out}$$

$$(2)\begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \xrightarrow{\text{lossless}} \begin{array}{l} S_{11}^* S_{12} + S_{12}^* S_{22} = 0 \ldots (i) \\ S_{11} S_{12}^* + S_{12} S_{22}^* = 0 \ldots (ii) \\ |S_{11}|^2 + |S_{12}|^2 = 1 \ldots (iii) \\ |S_{12}|^2 + |S_{22}|^2 = 1 \ldots (iv) \end{array}$$



$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L} \overset{(i)}{=} S_{11} + \frac{S_{12} S_{12} \Gamma_L}{1 + \frac{S_{12} S_{11}^*}{S_{12}^*}\Gamma_L} = S_{11} + \frac{S_{12}|S_{12}|^2 \Gamma_L}{S_{12}^* + S_{12} S_{11}^* \Gamma_L} = \frac{S_{11} S_{12}^* + S_{12}(|S_{11}|^2 + |S_{12}|^2)\Gamma_L}{S_{12}^* + S_{12} S_{11}^* \Gamma_L}$$

$$\overset{(iii)}{=} \frac{S_{11} S_{12}^* + S_{12}\Gamma_L}{S_{12}^* + S_{12} S_{11}^* \Gamma_L} \rightarrow \Gamma_{in}^* = \frac{S_{11}^* S_{12} + S_{12}^* \Gamma_L^*}{S_{12} + S_{12}^* S_{11} \Gamma_L^*} \ldots (v)$$

if $\Gamma_S = \Gamma_{in}^*$

$$\Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S} \overset{(i)}{=} S_{22} + \frac{S_{12} S_{12} \Gamma_S}{1 + \frac{S_{12} S_{22}^*}{S_{12}^*}\Gamma_S} = S_{22} + \frac{S_{21}|S_{12}|^2 \Gamma_S}{S_{12}^* + S_{12} S_{22}^* \Gamma_S} = \frac{S_{22} S_{12}^* + S_{12}(|S_{22}|^2 + |S_{12}|^2)\Gamma_S}{S_{12}^* + S_{12} S_{22}^* \Gamma_S}$$

$$\overset{(iv)}{=} \frac{S_{22} S_{12}^* + S_{12}\Gamma_S}{S_{12}^* + S_{12} S_{22}^* \Gamma_S} \overset{(i),(ii)}{=} \frac{-S_{12} S_{11}^* + S_{12}\Gamma_S}{S_{12}^* - S_{11} S_{12}^* \Gamma_S} = \frac{S_{12}(\Gamma_{in}^* - S_{11}^*)}{S_{12}^*(1 - S_{11}\Gamma_{in}^*)} \overset{(v)}{=} \frac{S_{12}\left(\frac{S_{11}^* S_{12} + S_{12}^* \Gamma_L^*}{S_{12} + S_{12}^* S_{11} \Gamma_L^*} - S_{11}^*\right)}{S_{12}^*\left(1 - S_{11}\frac{S_{11}^* S_{12} + S_{12}^* \Gamma_L^*}{S_{12} + S_{12}^* S_{11} \Gamma_L^*}\right)}$$

$$= \frac{S_{12} S_{11}^* S_{12} + S_{12} S_{12}^* \Gamma_L^* - S_{12} S_{12} S_{11}^* - S_{12}^* S_{12} S_{11}^* S_{11} \Gamma_L^*}{S_{12}^* S_{12} + S_{12}^* S_{12}^* S_{11} \Gamma_L^* - S_{12}^* S_{11} S_{11}^* S_{12} - S_{11} S_{12}^* S_{12}^* \Gamma_L^*} = \frac{S_{12} S_{12}^* \Gamma_L^* - S_{12}^* S_{12} S_{11}^* S_{11} \Gamma_L^*}{S_{12}^* S_{12} - S_{12}^* S_{11} S_{11}^* S_{12}} = \Gamma_L^*$$