

Chapter 5 Impedance matching and tuning

5.1 Matching with lumped elements

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5.5 The theory of small reflections

single-section transformer, multi-section transformer

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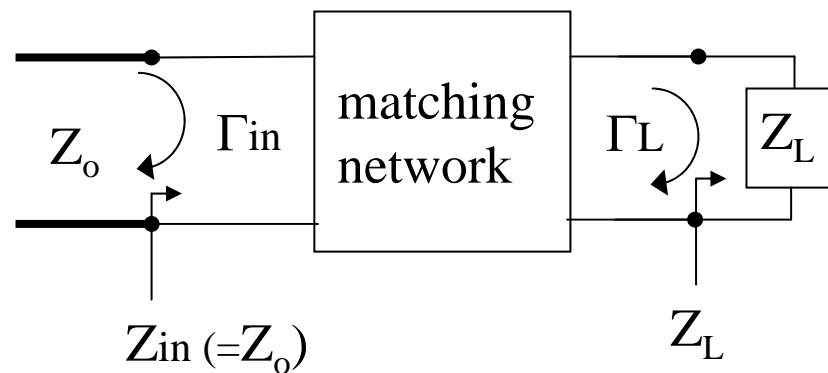
exponential taper, triangular taper

5.9 The Bode-Fano criterion

Γ -Bandwidth

- Impedance matching concept

given Z_L , design a matching network to have $\Gamma_{in}=0$ or selected value

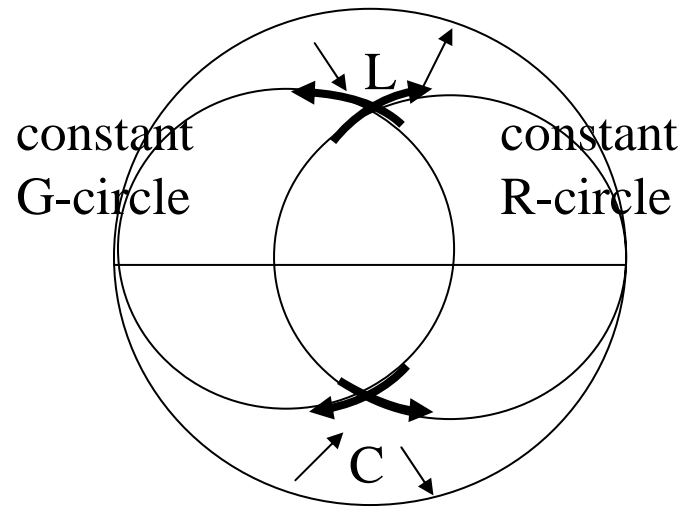


Discussion

1. Matching network usually uses lossless components: L, C, transmission line and transformer.
2. There are ∞ possible solutions for the matching circuit.
3. Properly use Smith chart to find the optimal design.
4. Factors for selecting matching circuit are complexity, bandwidth, implementation and adjustability.

5.1 Matching with lumped elements (2-element L-network)

- Smith chart solution



Z-plane

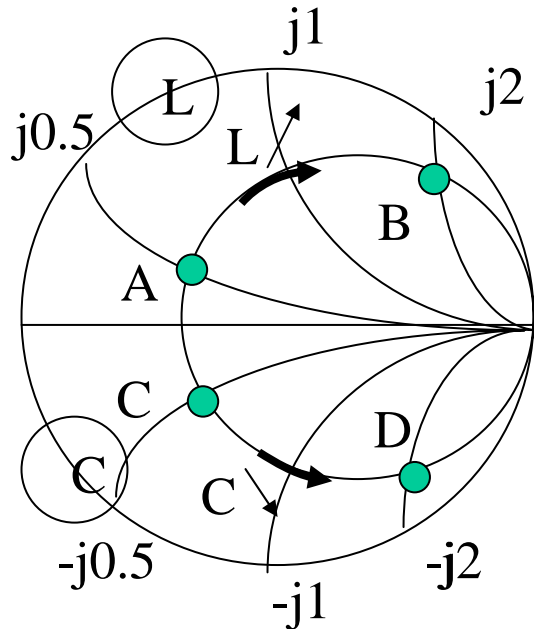
CW \rightarrow add series L
(reduce series C)
CCW \rightarrow add series C
(reduce series L)

Y-plane

CW \rightarrow add shunt C
(reduce shunt L)
CCW \rightarrow add shunt L
(reduce shunt C)

(explanation)

constant R-circle \rightarrow L or C in series



(1) *CW* $A \rightarrow B: 1 + j0.5 + jx = 1 + j2 \rightarrow jx = j1.5 = j\omega L$
: add an L in series

(2) *CCW* $B \rightarrow A: 1 + j2 + jx = 1 + j0.5 \rightarrow jx = -j1.5 = \frac{1}{j\omega C}$
: add a C in series, or reduce extra L

(3) *CCW* $C \rightarrow D: 1 - j0.5 + jx = 1 - j2 \rightarrow jx = -j1.5 = \frac{1}{j\omega C}$
: add a C in series

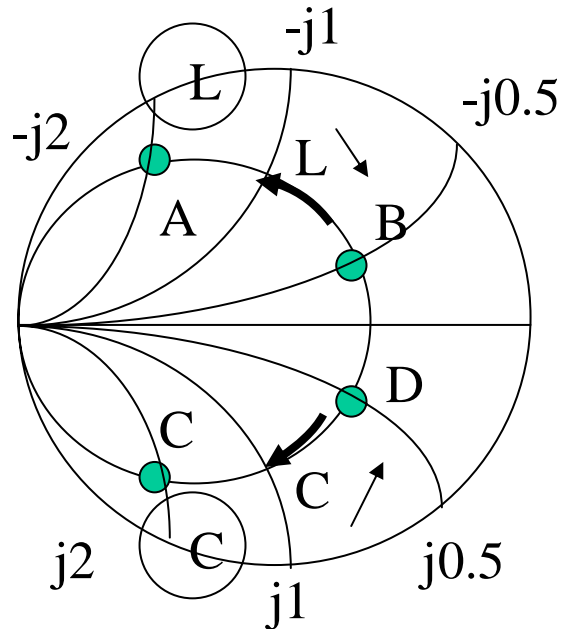
(4) *CW* $D \rightarrow C: 1 - j2 + jx = 1 - j0.5 \rightarrow jx = j1.5 = j\omega L$
: add an L in series or reduce extra C

in Z-plane

CW \rightarrow add a series L (or reduce series C)

CCW \rightarrow add a series C (or reduce series L)

constant G-circle \rightarrow L or C in shunt



(1) CW $A \rightarrow B: 1 - j2 + jb = 1 - j0.5 \rightarrow jb = j1.5 = j\omega C$
: add a C in shunt, or reduce shunt L

(2) CCW $B \rightarrow A: 1 - j0.5 + jb = 1 - j2 \rightarrow jb = -j1.5 = \frac{1}{j\omega L}$
: add an L in shunt

(3) CCW $C \rightarrow D: 1 + j2 + jb = 1 + j0.5 \rightarrow jb = -j1.5 = \frac{1}{j\omega L}$
: add an L in shunt, or reduce shunt C

(4) CW $D \rightarrow C: 1 + j0.5 + jb = 1 + j2 \rightarrow jb = j1.5 = j\omega C$
: add a C in shunt

in Y-plane

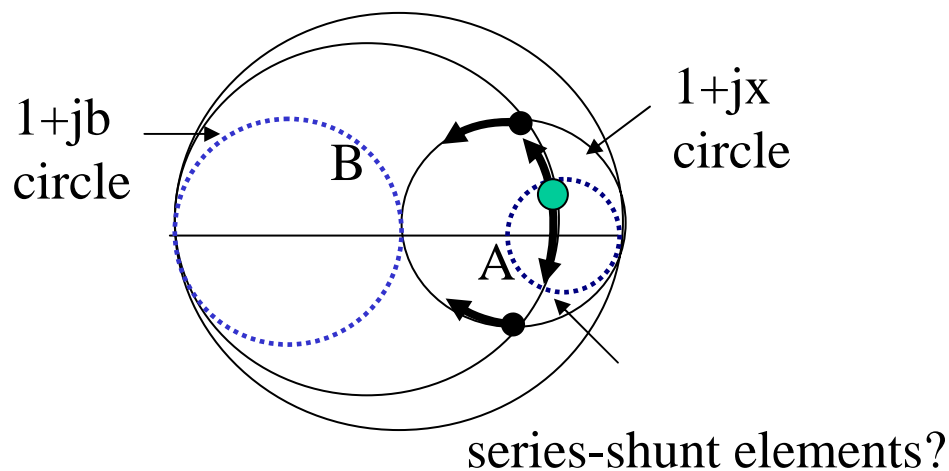
CW \rightarrow add a shunt C (or reduce shunt L)

CCW \rightarrow add a shunt L (or reduce shunt C)

Discussion

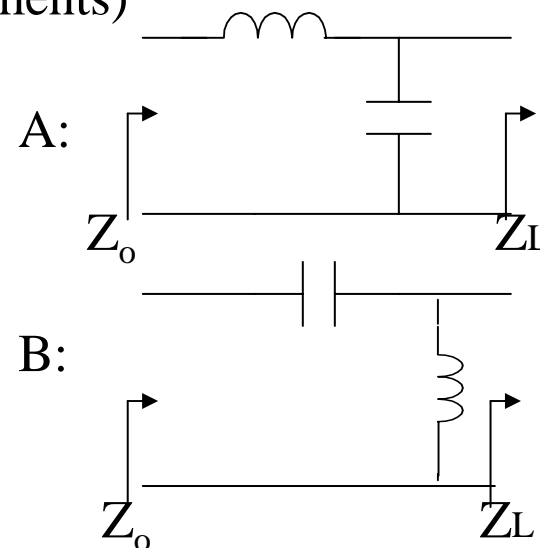
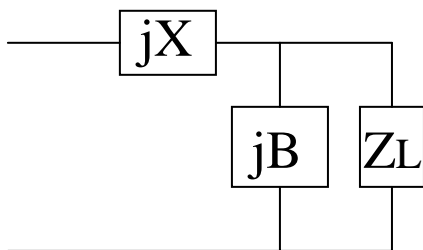
1. Z_L inside $1+jx$ circle, two possible solutions

Smith chart solution (shunt-series elements)



“N”

analytical solution



$$Z_o = jX + \frac{1}{jB + \frac{1}{R_L + jX_L}}$$

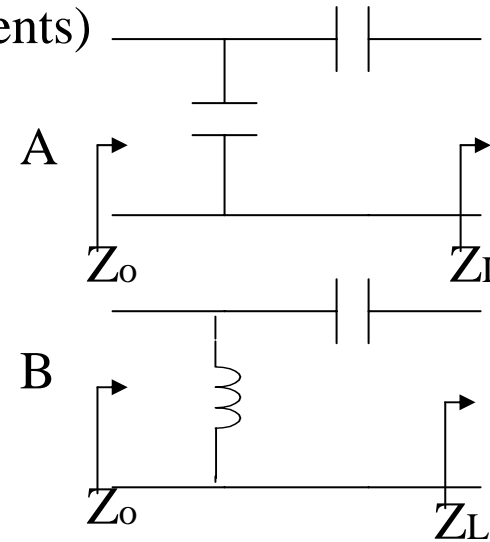
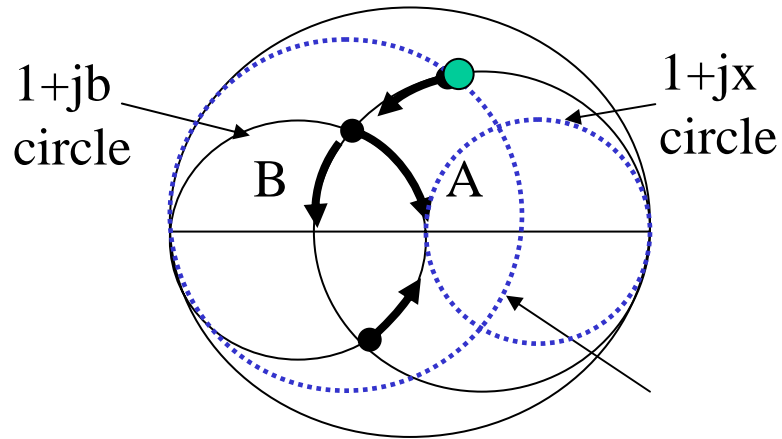
$$\Rightarrow B > 0 \rightarrow C, B < 0 \rightarrow L$$

$$X > 0 \rightarrow L, X < 0 \rightarrow C$$

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2. Z_L outside $1+jx$ circle, two possible solutions

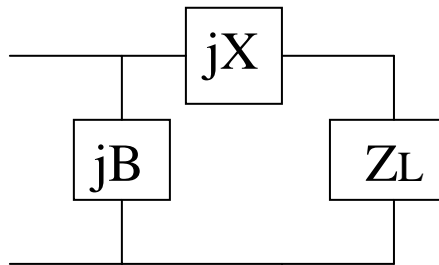
Smith chart solution (series-shunt elements)



shunt-series elements?

“Y”

analytical solution

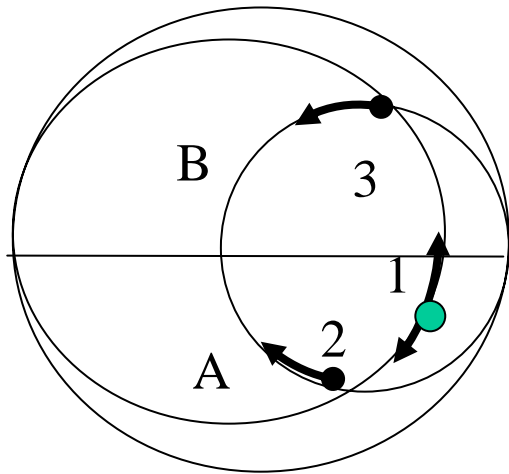


$$\frac{1}{Z_0} = jB + \frac{1}{R_L + j(X + X_L)}$$

$$\Rightarrow B > 0 \rightarrow C, B < 0 \rightarrow L$$

$$X > 0 \rightarrow L, X < 0 \rightarrow C$$

3. Ex. 5.1 $Z_L=200-j100$, $Z_o=100\Omega$, $f=500\text{MHz}$



1. $z_L=2-j1$, $y_L=0.4+j0.2$

Solution A

2. $y=0.4+j0.5 \rightarrow jb=j0.3 \rightarrow jB=j\omega C=jb/Z_o$
 $C=b/Z_o\omega =0.92\text{pF}$

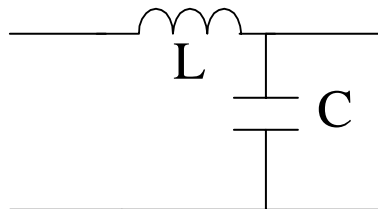
$z=1-j1.2 \rightarrow jx=j1.2 \rightarrow jX=j\omega L=jxZ_o$
 $L=xZ_o/\omega =38.8\text{nH}$

Solution B

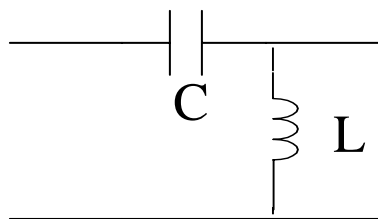
3. $y=0.4-j0.5 \rightarrow jb=-j0.7 \rightarrow -jB=1/j\omega L=-jb/Z_o$
 $L=-Z_o/\omega b=46.1\text{nH}$

$z=1+j1.2 \rightarrow jx=-j1.2 \rightarrow jX=1/j\omega C=-jxZ_o$
 $C=-1/xZ_o\omega =2.61\text{pF}$

A:

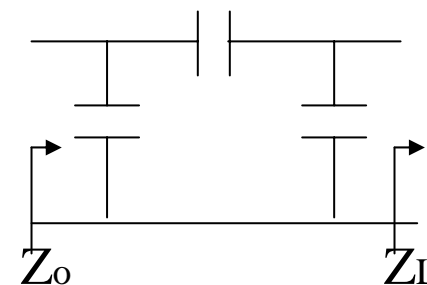
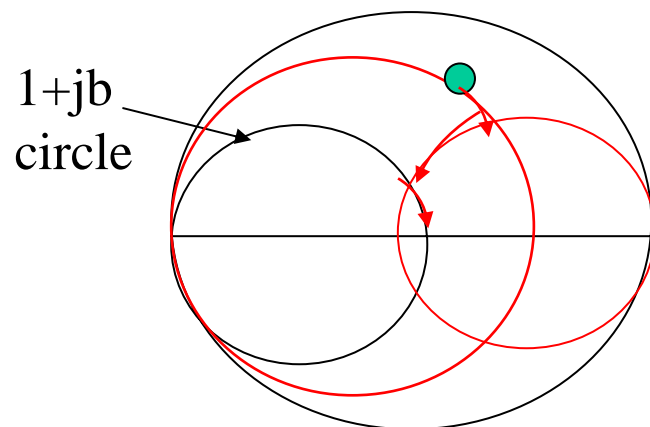
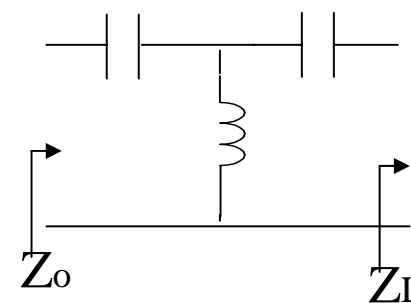
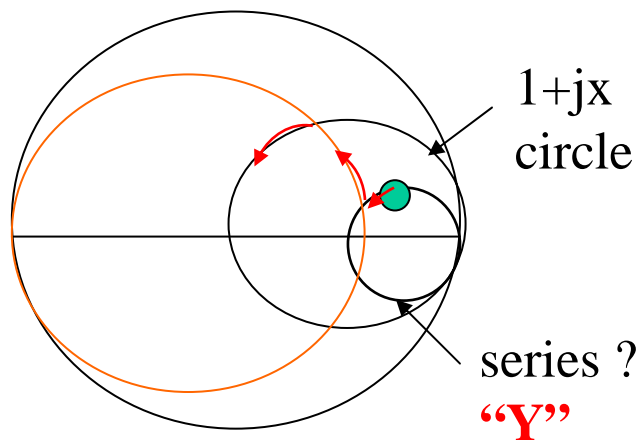


B:



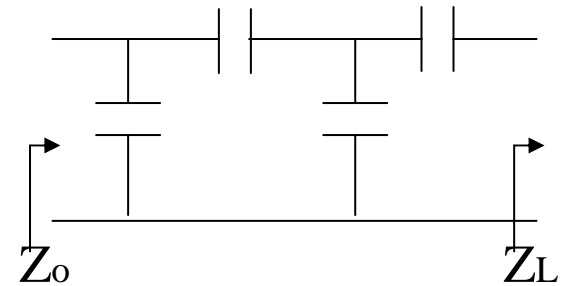
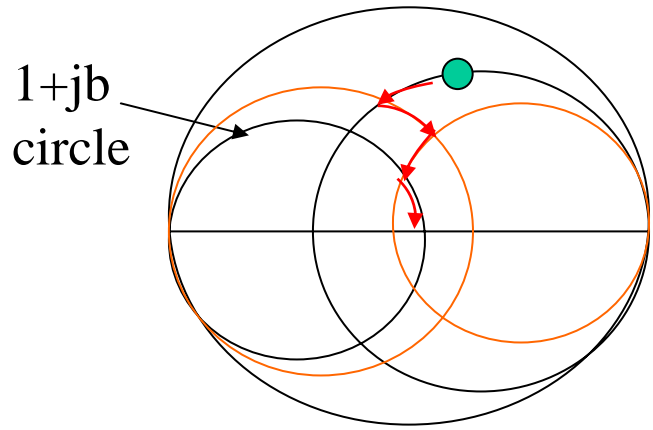
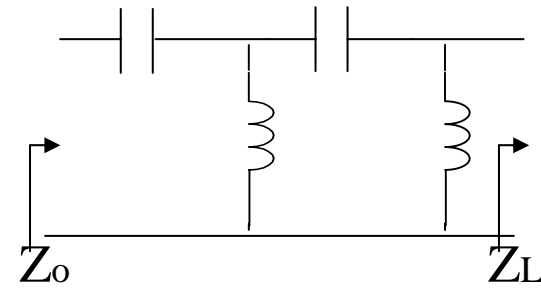
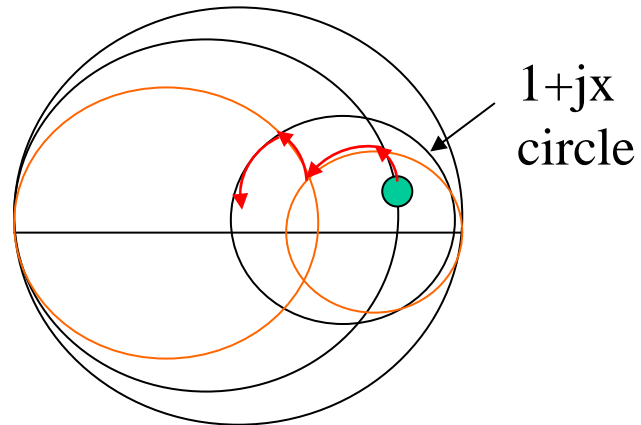
frequency response (p.227, Fig.5.3(c))

4. Possible 3-element L-network



5. Possible 4-element L-network

shorter paths for a wider operational bandwidth



6. Lumped elements (size $< \lambda/10$)

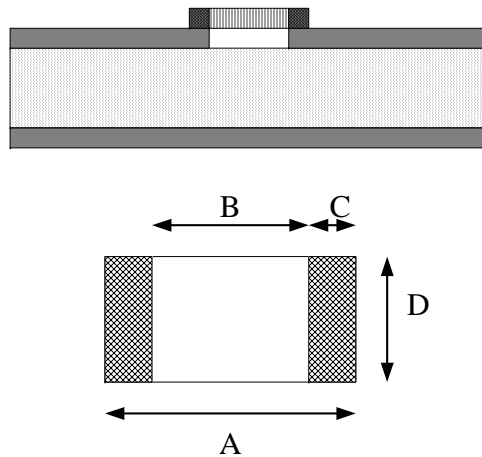
capacitor: chip capacitor, MIM capacitor ($< 25\text{pF}$), interdigital gap capacitor ($< 0.5\text{pF}$), open stub ($< 0.1\text{pF}$)

inductor: chip inductor, loop inductor, spiral inductor ($< 10\text{nH}$)

resistor: chip resistor, planar resistor

All these lumped elements inherently have parasitic elements in the microwave range.

(p.228, “point of interest”)



Size (mil)	0402	0603	0805	1206
A	39	62	78	125
B	24	38	39	62
C	8	12	20	23
D	20	31	49	62

$$1 \text{ mil} = 0.001 \text{ in} = 25 \mu\text{m} = 1/40 \text{ mm}$$

5.2 Single-stub tuning

- equivalent microstrip elements

a series C ---

a series L in series with a high impedance microstrip line

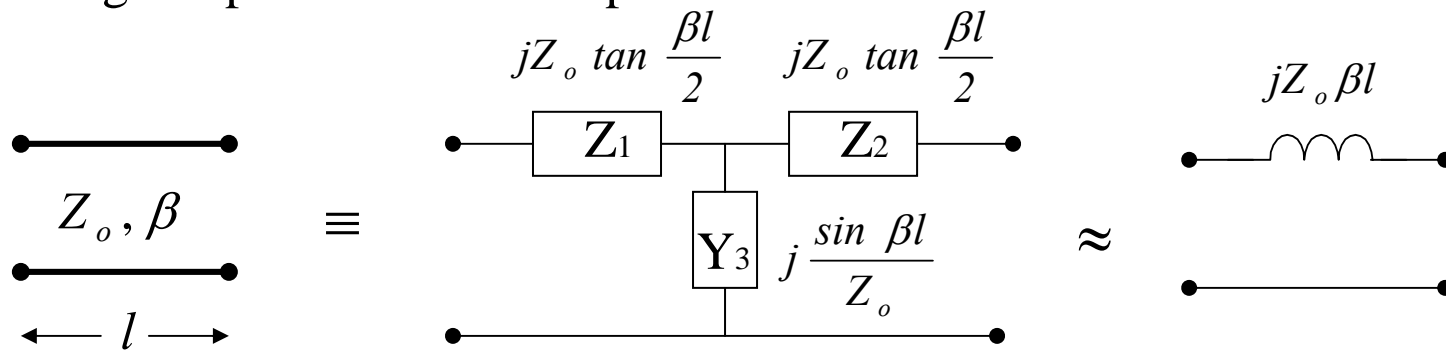
a shunt C in shunt with an open microstrip line

a shunt L in shunt with a short microstrip line

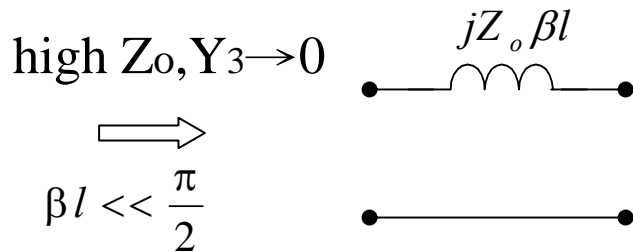
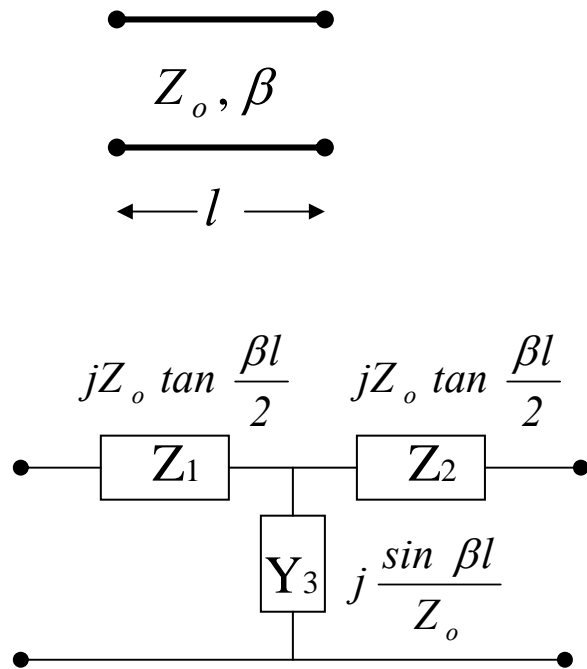
an open-circuited microstrip line $Z_{in} = \frac{Z_o}{j \tan \beta l} \equiv \frac{1}{j \omega C}$

a short-circuited microstrip line $Z_{in} = jZ_o \tan \beta l \equiv j \omega L$

a high impedance microstrip line



(derivation of high impedance line)



From p.185 Table 4.1

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos \beta l & jZ_o \sin \beta l \\ jY_o \sin \beta l & \cos \beta l \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z_1}{Z_3} & Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3} \\ \frac{1}{Z_3} & 1 + \frac{Z_2}{Z_3} \end{bmatrix}$$

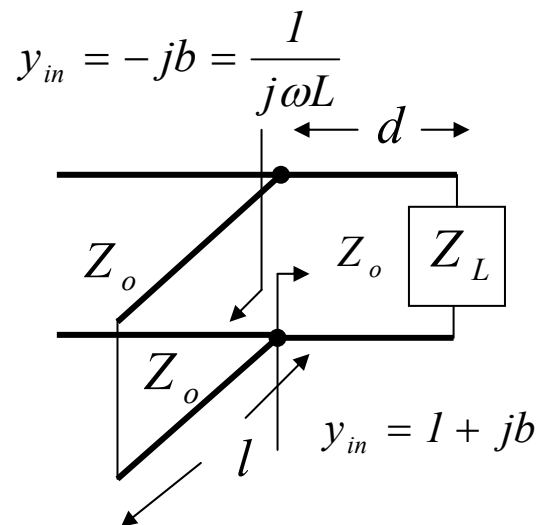
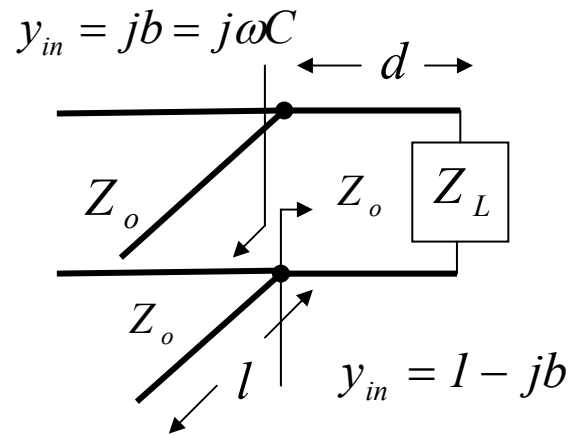
$$Z_3 = \frac{1}{jY_o \sin \beta l} = \frac{Z_o}{j \sin \beta l}, Y_3 = \frac{j \sin \beta l}{Z_o}$$

$$1 + \frac{Z_1}{Z_3} = \cos \beta l, Z_1 = (\cos \beta l - 1)Z_3$$

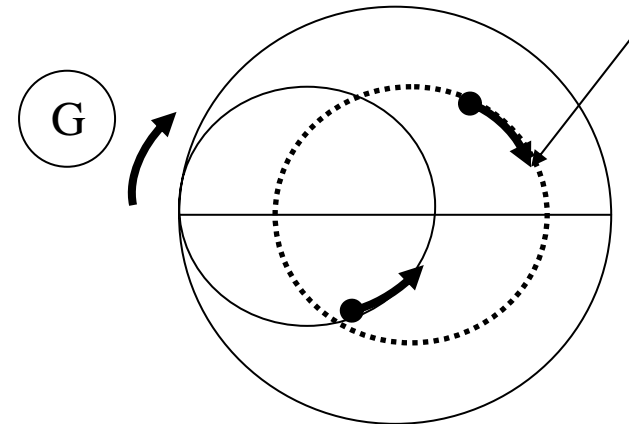
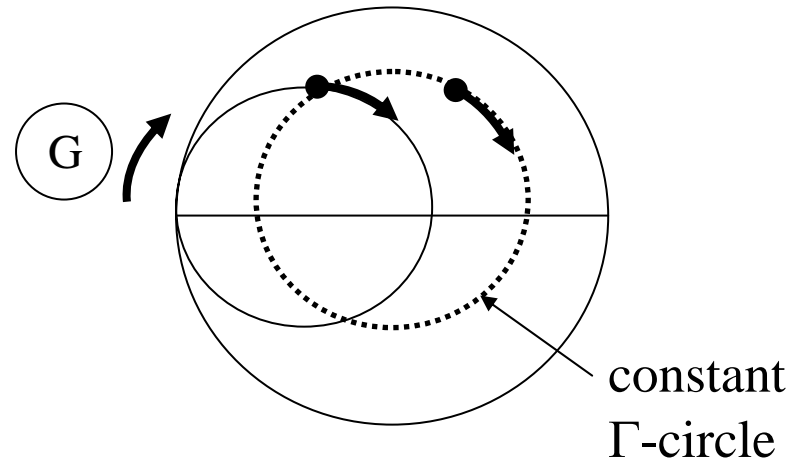
$$= -2 \sin^2 \frac{\beta l}{2} \times -jZ_o \frac{1}{2 \sin \frac{\beta l}{2} \cos \frac{\beta l}{2}} = jZ_o \tan \frac{\beta l}{2}$$

Discussion

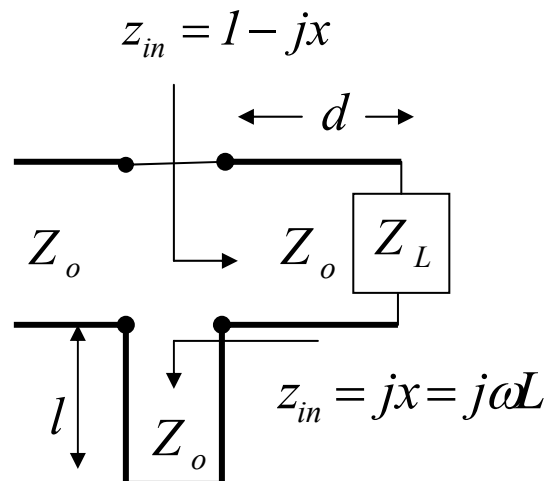
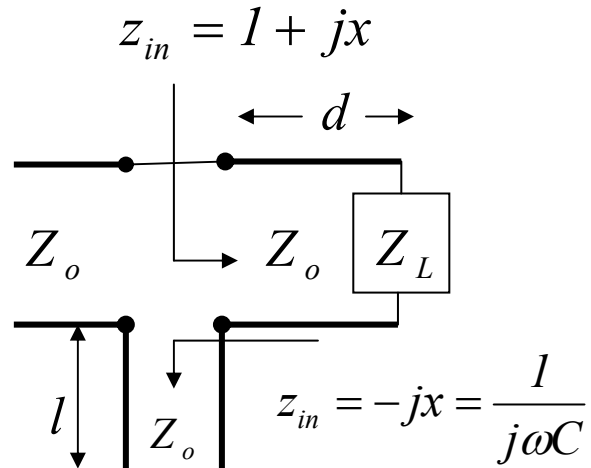
1. Shunt stub



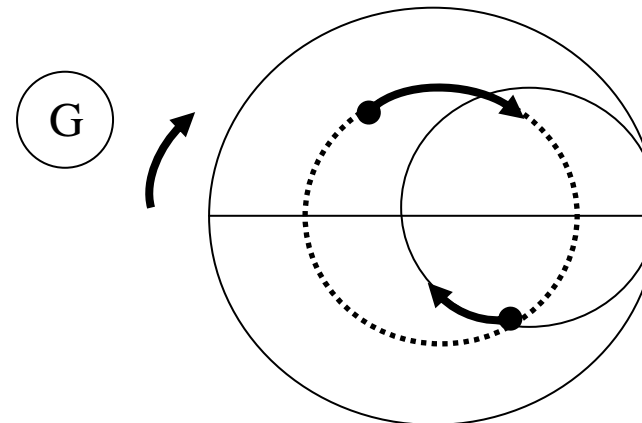
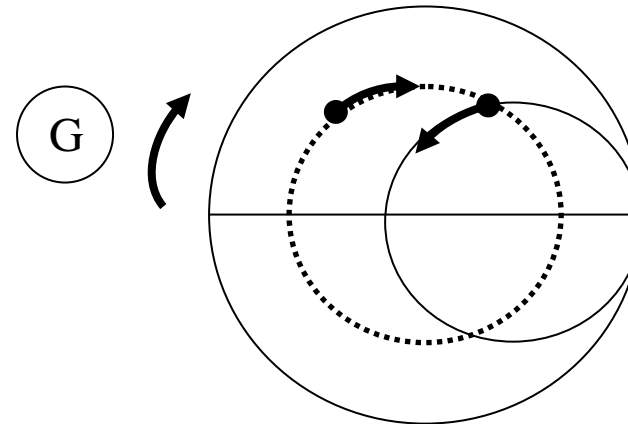
Smith chart solution



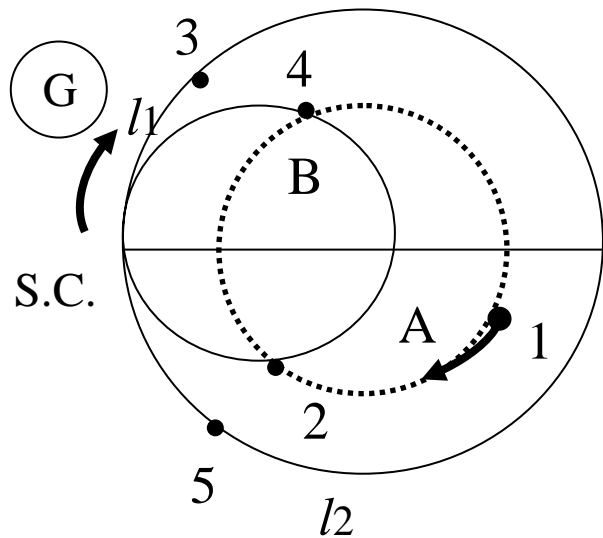
2. Series stub



Smith chart solution



3. Ex. 5.2 $Z_L=60-j80$, $Z_o=50\Omega$, $f=2\text{GHz}$, using a shunt short stub



1. $z_L=1.2-j1.6$, $y_L=0.3+j0.4$

Solution A

2. $y=1+j1.47 \rightarrow d_1=0.11\lambda$

3. $y=-j1.47 \rightarrow l_1=0.095\lambda$, short stub

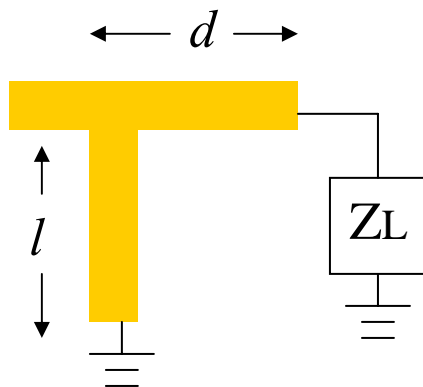
Solution B

4. $y=1-j1.47 \rightarrow d_2=0.26\lambda$

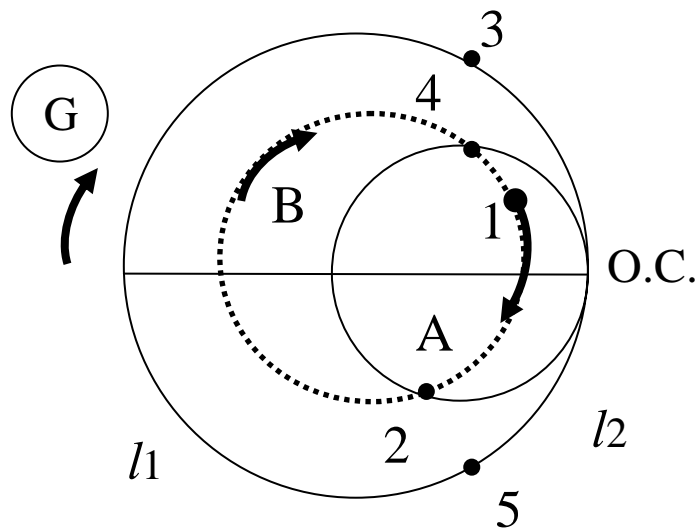
5. $y=j1.47 \rightarrow l_2=0.405\lambda$, short stub

frequency response (p.231, Fig.5.5(c))

Solution A has a wider bandwidth.



4. Ex. 5.3 $Z_L=100+j80$, $Z_o=50\Omega$, $f=2\text{GHz}$, using series open stub



1. $z_L=2+j1.6$

Solution A

2. $z=1-j1.33 \rightarrow d_1=0.12\lambda$

3. $z=j1.33 \rightarrow l_1=0.397\lambda$, open stub

Solution B

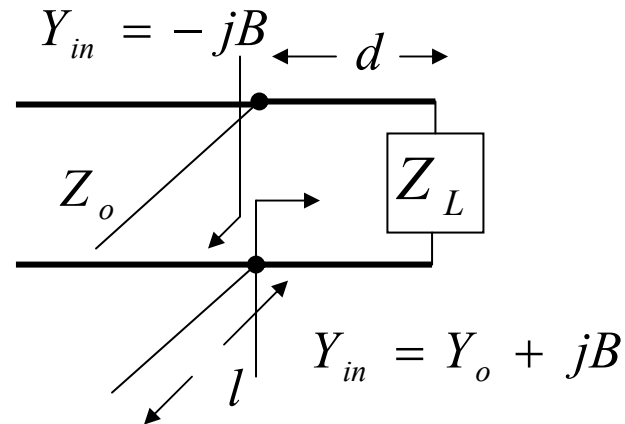
4. $z=1+j1.33 \rightarrow d_2=0.463\lambda$

5. $z=-j1.33 \rightarrow l_2=0.103\lambda$, open stub

frequency response (p.234, Fig.5.6(c))

It can not be implemented in
microstrip lines.

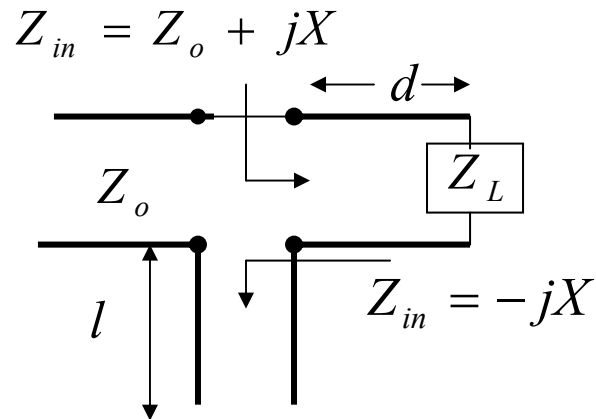
5. Analytical solution for shunt stub



$$Y_o = \text{Re} \left\{ \left(Z_o \frac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} \right)^{-1} \right\} \rightarrow d$$

$$-B = -\text{Im} \left\{ \left(Z_o \frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l} \right)^{-1} \right\} = \begin{cases} \left(\frac{Z_o}{\tan \beta l} \right)^{-1} & \text{open stub} \\ \left(-\frac{1}{Z_o \tan \beta l} \right)^{-1} & \text{short stub} \end{cases} \rightarrow l$$

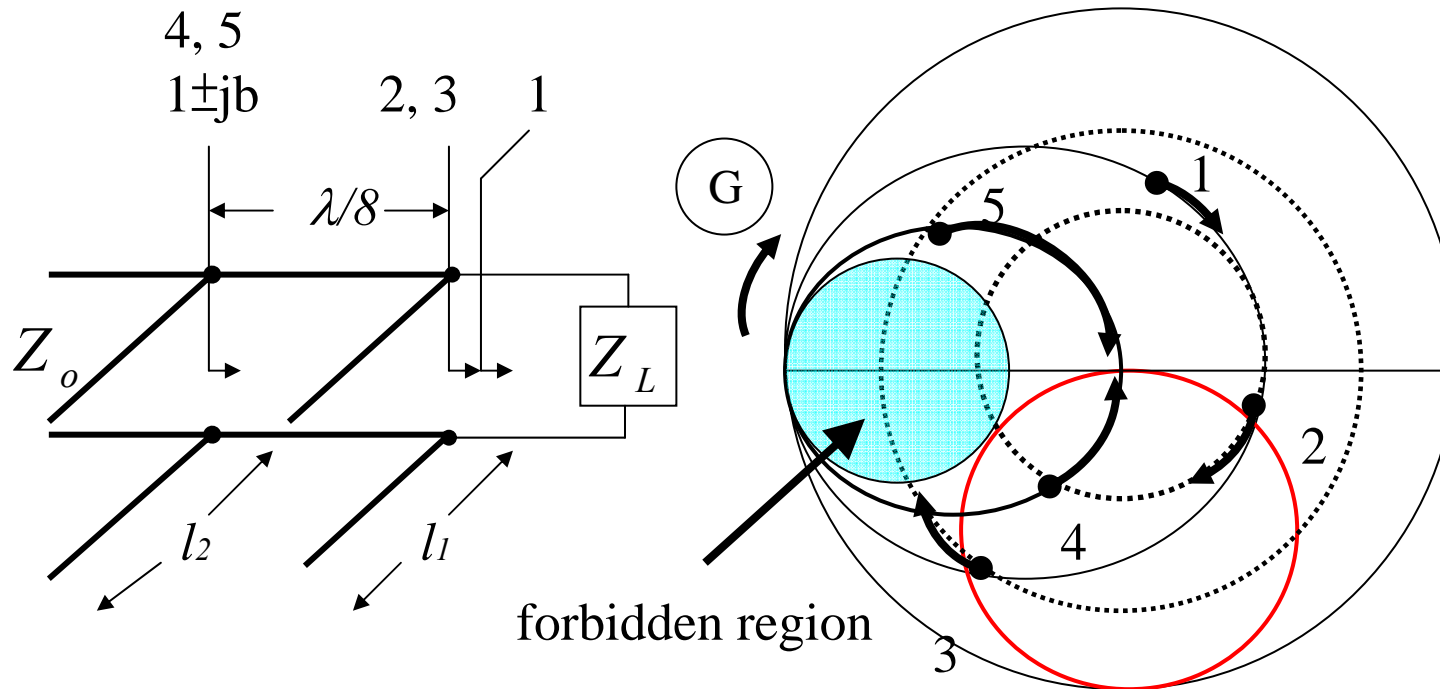
6. Analytical solution for series stub



$$Z_o = \operatorname{Re} \left\{ Z_o \frac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} \right\} \rightarrow d$$

$$-X = -\operatorname{Im} \left\{ Z_o \frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l} \right\} = \begin{cases} -\frac{Z_o}{\tan \beta l} & \text{open stub} \\ Z_o \tan \beta l & \text{short stub} \end{cases} \rightarrow l$$

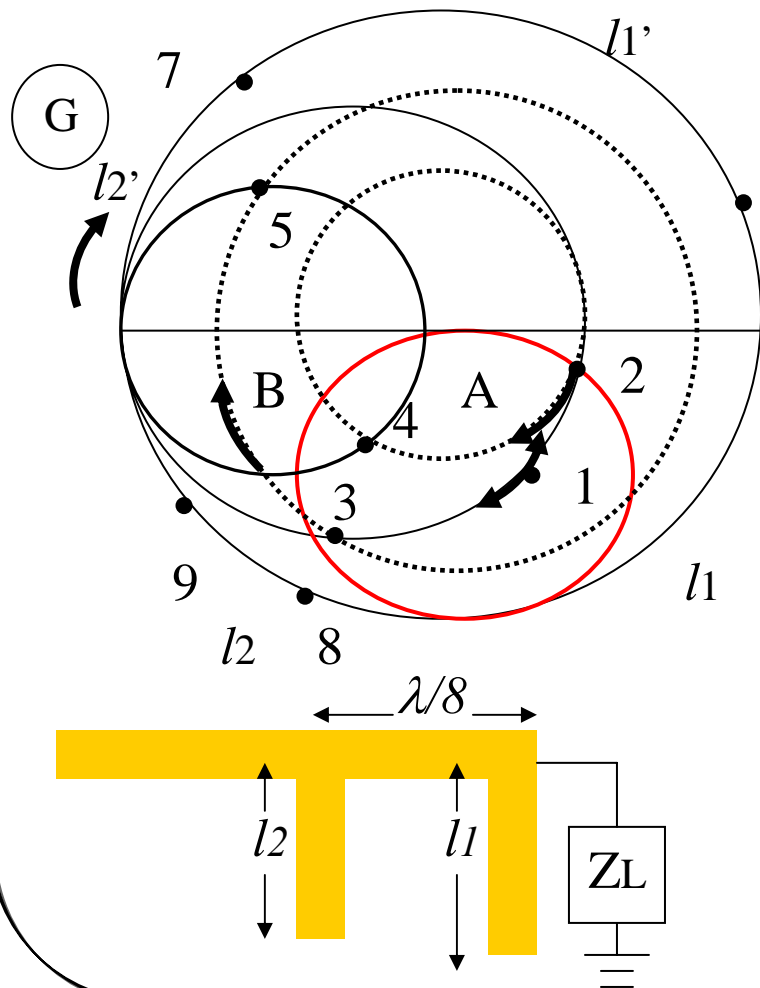
5.3 Double-stub tuning



Discussion

1. There exists a forbidden region for Z_L . It can be tuned out by adding a certain length of line.

2. Ex. 5.4 $Z_L=60-j80$, $Z_o=50\Omega$, $f=2\text{GHz}$, using double-shunt-open-stubs



1. $z_L=1.2-j1.6$, $y_L=0.3+j0.4$

Solution A

2. $y=0.3+j0.286 \rightarrow$ 6. $b1' = -0.114$
 $\rightarrow l1' = 0.482\lambda$

4. $y=1+j1.38 \rightarrow$ 7. $b2' = -1.38$
 $\rightarrow l2' = 0.35\lambda$

O.C.

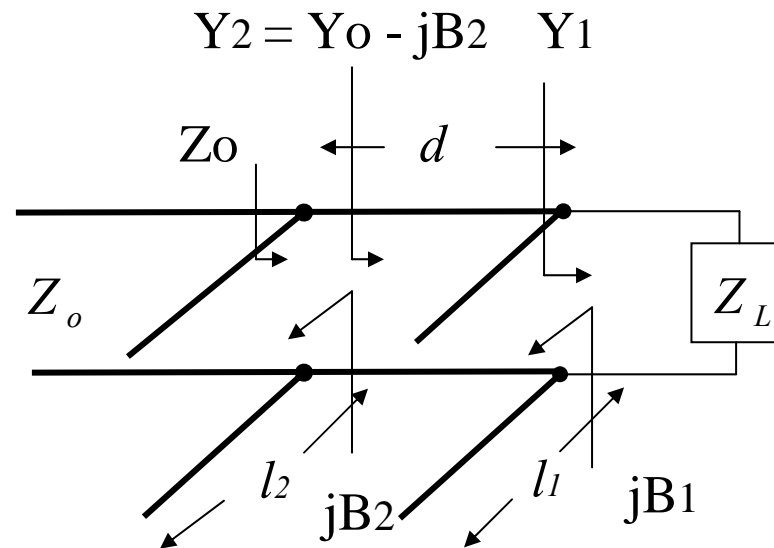
Solution B

3. $y=0.3+j1.714 \rightarrow$ 8. $b1 = 1.314$
 $\rightarrow l1 = 0.146\lambda$

5. $y=1-j3.38 \rightarrow$ 9. $b2 = 3.38$
 $\rightarrow l2 = 0.204\lambda$

frequency response (p.239, Fig.5.9(c))

3. Analytical solution



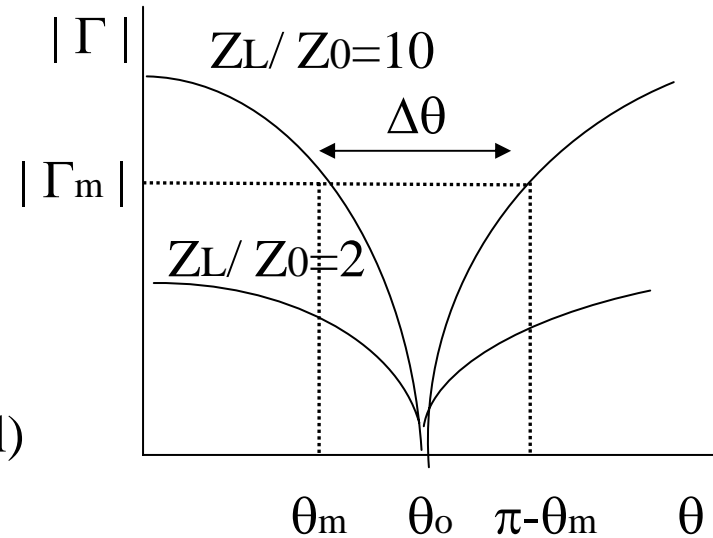
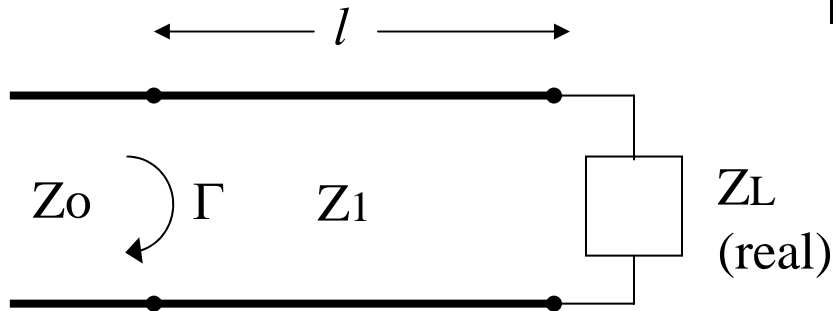
$$Y_1 = Y_L + jB_1, Y_2 = Y_o - jB_2 = \left(Z_o \frac{Z_1 + jZ_o \tan \beta d}{Z_o + jZ_1 \tan \beta d} \right)^{-1}$$

$$\text{Re}\{Y_2\} = Y_o \rightarrow B_1 \rightarrow l_1$$

$$Y_1 \rightarrow \text{Im}\{Y_2\} = -B_2 \rightarrow l_2$$

5.4 The quarter-wave transformer

- frequency response



Γ_m : max. tolerated Γ over the bandwidth

$$Z_1 = \sqrt{Z_o Z_L}$$

$$|\Gamma(\theta)| \approx \frac{|Z_L - Z_o|}{2\sqrt{Z_o Z_L}} |\cos \theta|, \text{ for } \theta \text{ near } \theta_o = \frac{\pi}{2}, \theta = \beta l$$

$$\frac{\Delta f}{f_o} = 2 - \frac{4}{\pi} \cos^{-1} \left(\frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_o Z_L}}{|Z_L - Z_o|} \right), \quad Z_L \rightarrow Z_o, \Delta f \text{ increases}$$

(derivation of $|\Gamma(\theta)| \approx \frac{|Z_L - Z_o|}{2\sqrt{Z_o Z_L}} |\cos \theta|$)

$$\Gamma(\theta) = \frac{Z_{in}(\theta) - Z_o}{Z_{in}(\theta) + Z_o} = \frac{Z_1 \frac{Z_L + jZ_1 \tan \theta}{Z_1 + jZ_L \tan \theta} - Z_o}{Z_1 \frac{Z_L + jZ_1 \tan \theta}{Z_1 + jZ_L \tan \theta} + Z_o} = \frac{Z_1 Z_L + jZ_1^2 \tan \theta - Z_o Z_1 - jZ_L Z_o \tan \theta}{Z_1 Z_L + jZ_1^2 \tan \theta + Z_o Z_1 + jZ_L Z_o \tan \theta}$$

$$\stackrel{Z_1^2 = Z_o Z_L}{=} \frac{Z_1(Z_L - Z_o)}{Z_1(Z_L + Z_o) + j2Z_1^2 \tan \theta} = \frac{Z_L - Z_o}{Z_L + Z_o + j2\sqrt{Z_o Z_L} \tan \theta} = \frac{1}{\frac{Z_L + Z_o}{Z_L - Z_o} + \frac{j2\sqrt{Z_o Z_L}}{Z_L - Z_o} \tan \theta}$$

$$\stackrel{\theta \rightarrow \theta_o = \frac{\pi}{2}}{\approx} \frac{Z_L - Z_o}{j2\sqrt{Z_o Z_L}} \cos \theta$$

(derivation of $\frac{\Delta f}{f_o} = 2 - \frac{4}{\pi} \cos^{-1} \left(\frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_o Z_L}}{|Z_L - Z_o|} \right)$)

$$|\Gamma(\theta)| = \frac{1}{\left[\left(\frac{Z_L + Z_o}{Z_L - Z_o} \right)^2 + \left(\frac{2\sqrt{Z_o Z_L}}{Z_L - Z_o} \tan \theta \right)^2 \right]^{1/2}} = \frac{1}{\left[\frac{(Z_L - Z_o)^2 + 4Z_L Z_o}{(Z_L - Z_o)^2} + \frac{4Z_o Z_L}{(Z_L - Z_o)^2} \tan^2 \theta \right]^{1/2}}$$

$$\stackrel{1 + \tan^2 \theta = \sec^2 \theta}{=} \frac{1}{\left[1 + \frac{4Z_o Z_L}{(Z_L - Z_o)^2} \sec^2 \theta \right]^{1/2}}$$

$$\Gamma(\theta) = \frac{1}{\left[1 + \frac{4Z_o Z_L}{(Z_L - Z_o)^2} \sec^2 \theta\right]^{1/2}}$$

$$\Gamma_m^2 = \frac{1}{1 + \left(\frac{2\sqrt{Z_o Z_L}}{Z_L - Z_o} \frac{1}{\cos \theta_m}\right)^2} \rightarrow \frac{1}{\Gamma_m^2} - 1 = \left(\frac{2\sqrt{Z_o Z_L}}{Z_L - Z_o}\right)^2 \frac{1}{\cos^2 \theta_m}$$

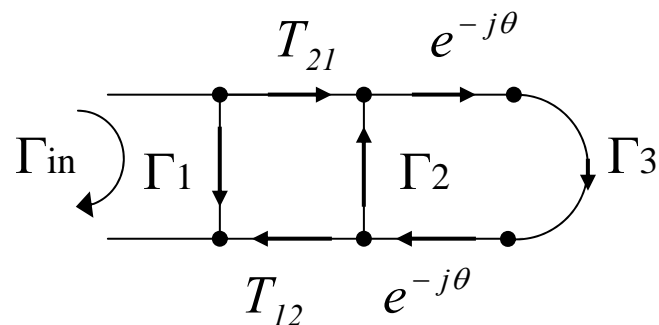
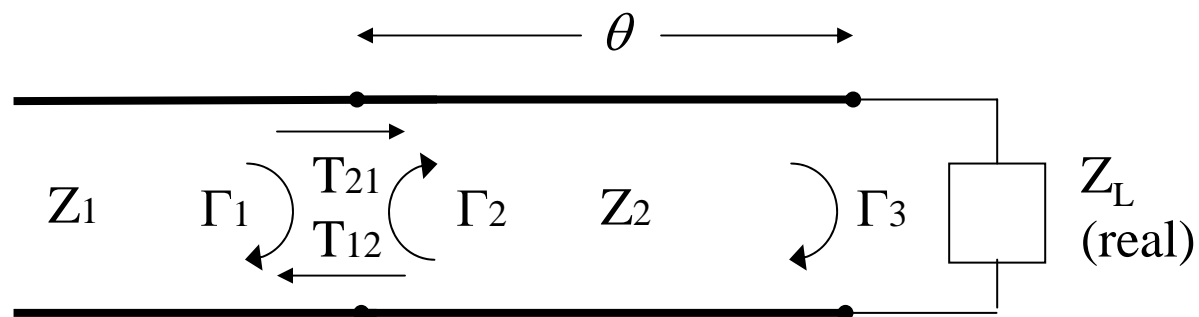
$$\rightarrow \cos \theta_m = \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_o Z_L}}{|Z_L - Z_o|}$$

$$\text{TEM line: } \theta = \beta l = \frac{2\pi f}{v_p(f)} \frac{\lambda_o}{4} = \frac{2\pi f}{v_p(f)} \frac{v_p(f_o)}{4f_o} = \frac{\pi f}{2f_o} \rightarrow \theta_m = \frac{\pi f_m}{2f_o} \rightarrow \frac{f_m}{f_o} = \frac{2\theta_m}{\pi}$$

$$\frac{\Delta f}{f_o} = \frac{2(f_o - f_m)}{f_o} = 2 - \frac{2f_m}{f_o} = 2 - \frac{4}{\pi} \cos^{-1} \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_o Z_L}}{|Z_L - Z_o|}$$

5.5 The theory of small reflections

- single-section transformer



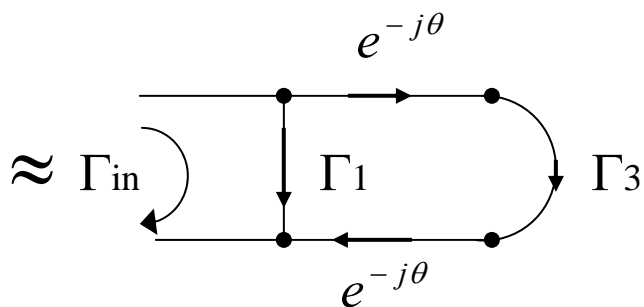
$$\Gamma_2 = -\Gamma_1, \quad T_{21} = 1 + \Gamma_1, \quad T_{12} = 1 + \Gamma_2$$

$$\Gamma_{in} = \Gamma_1 + \frac{T_{12}T_{21}\Gamma_3 e^{-j2\theta}}{1 - \Gamma_2\Gamma_3 e^{-j2\theta}}$$

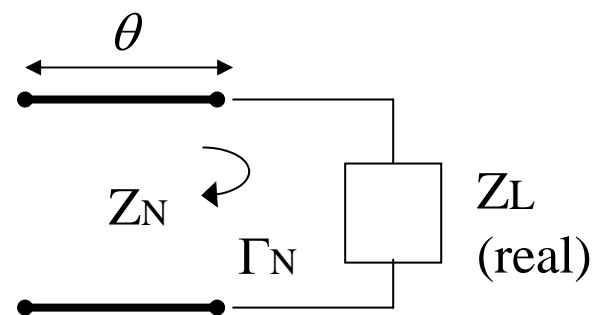
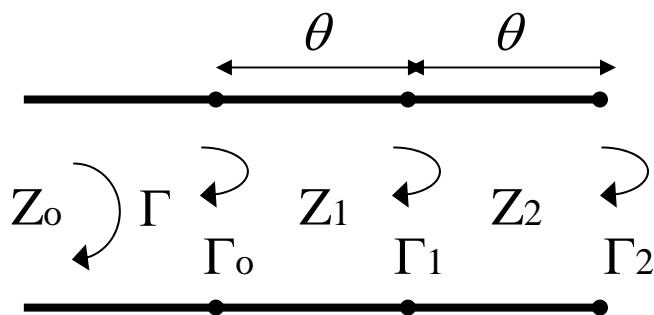
$$= \frac{\Gamma_1 + \Gamma_1^2\Gamma_3 e^{-j2\theta} + (1 - \Gamma_1)(1 + \Gamma_1)\Gamma_3 e^{-j2\theta}}{1 + \Gamma_1\Gamma_3 e^{-j2\theta}}$$

$$= \frac{\Gamma_1 + \Gamma_3 e^{-j2\theta}}{1 + \Gamma_1\Gamma_3 e^{-j2\theta}}$$

$$\approx \Gamma_1 + \Gamma_3 e^{-j2\theta} \quad \text{if } Z_1 \approx Z_2 \approx Z_L$$



• multisection transformer



$$\Gamma(\theta) = \Gamma_o + \Gamma_1 e^{-j2\theta} + \Gamma_2 e^{-j4\theta} + \dots + \Gamma_N e^{-j2N\theta} \quad , \text{ if } \Gamma_o = \Gamma_N, \Gamma_1 = \Gamma_{N-1}, \dots$$

$$= \begin{cases} e^{-jN\theta} [\Gamma_o (e^{jN\theta} + e^{-jN\theta}) + \Gamma_1 (e^{j(N-2)\theta} + e^{-j(N-2)\theta}) + \dots + \Gamma_{(N-1)/2} (e^{j\theta} + e^{-j\theta})] & N \text{ odd} \\ e^{-jN\theta} [\Gamma_o (e^{jN\theta} + e^{-jN\theta}) + \Gamma_1 (e^{j(N-2)\theta} + e^{-j(N-2)\theta}) + \dots + \Gamma_{N/2}] & N \text{ even} \end{cases}$$

$$= \begin{cases} 2e^{-jN\theta} [\Gamma_o \cos N\theta + \Gamma_1 \cos(N-2)\theta + \dots + \Gamma_n \cos(N-2n)\theta + \dots + \frac{1}{2} \Gamma_{(N-1)/2} \cos \theta] & N \text{ odd} \\ 2e^{-jN\theta} [\Gamma_o \cos N\theta + \Gamma_1 \cos(N-2)\theta + \dots + \Gamma_n \cos(N-2n)\theta + \dots + \frac{1}{2} \Gamma_{N/2}] & N \text{ even} \end{cases}$$

given $\Gamma(\theta)$, design Z_1, Z_2, \dots, Z_n

5.6 Binomial multisection matching transformer

- maximal flatness response for $\Gamma(\theta)$

$$\Gamma(\theta) = A(1 + e^{-j2\theta})^N = A \sum_{n=0}^N C_n^N e^{-j2n\theta} = \Gamma_0 + \Gamma_1 e^{-j2\theta} + \Gamma_2 e^{-j4\theta} + \dots + \Gamma_N e^{-j2N\theta}$$

$$\Rightarrow \Gamma_n = AC_n^N$$

Discussion

1. Maximal flatness response,

$$\left. \frac{d^{N-1} |\Gamma(\theta)|}{d\theta^{N-1}} \right|_{\theta=\frac{\pi}{2} \text{ or } l=\frac{\lambda}{4}} = 0$$

2. $|\Gamma(0)| = |A| 2^N = \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| \rightarrow A = 2^{-N} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|$

3. $\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \approx \frac{1}{2} \ln \frac{Z_{n+1}}{Z_n} \quad (\ln x \approx 2 \frac{x-1}{x+1})$

$$\rightarrow \ln \frac{Z_{n+1}}{Z_n} \approx 2\Gamma_n = 2AC_n^N = 2 \times 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0} C_n^N \approx 2^{-N} C_n^N \ln \frac{Z_L}{Z_0}$$

(p.249, Table 5.1 for Z_n values)

$$4. \quad \frac{\Delta f}{f_o} = 2 - \frac{4}{\pi} \cos^{-1} \left[\frac{1}{2} \left(\frac{\Gamma_m}{|A|} \right)^{\frac{1}{N}} \right], \quad N \uparrow, \Delta f \uparrow, \quad \Gamma_m = 2^N |A| \cos^N \theta_m$$

$$\because \Gamma(\theta) = A(1 + e^{-j2\theta})^N \rightarrow |\Gamma(\theta)| = |A| 2^N \cos^N \theta$$

$$\frac{\Delta f}{f_o} = 2 - \frac{4}{\pi} \theta_m = 2 - \frac{4}{\pi} \cos^{-1} \left[\frac{1}{2} \left(\frac{\Gamma_m}{|A|} \right)^{\frac{1}{N}} \right]$$

5. Ex.5.6 $Z_L=50\Omega$, $Z_o=100\Omega$, $N=3$, $\Gamma_m=0.05$

$$N = 3, \quad A = 2^{-N} \left| \frac{Z_L - Z_o}{Z_L + Z_o} \right| = \frac{1}{2^{N+1}} \ln \frac{Z_L}{Z_o} = -0.0433, \quad \frac{\Delta f}{f_o} = 70\%$$

$$\ln \frac{Z_{n+1}}{Z_n} = 2^{-N} C_n^N \ln \frac{Z_L}{Z_o} \Rightarrow \ln \frac{Z_1}{Z_o} = 2^{-3} C_0^3 \ln \frac{Z_L}{Z_o} \rightarrow Z_1 = 91.7\Omega$$

$$\ln \frac{Z_2}{Z_1} = 2^{-3} C_1^3 \ln \frac{Z_L}{Z_o} \rightarrow Z_2 = 70.7\Omega, \quad \ln \frac{Z_3}{Z_2} = 2^{-3} C_2^3 \ln \frac{Z_L}{Z_o} \rightarrow Z_3 = 54.5\Omega$$

(p.250, Fig.5.15 for frequency response of $|\Gamma|$)

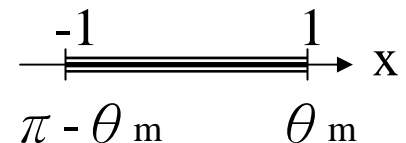
5.7 Chebyshev multisection matching transformers

- Equal ripple response for $\Gamma(\theta)$: optimal design

$$\Gamma(\theta) = 2e^{-jN\theta} [\Gamma_o \cos N\theta + \Gamma_1 \cos(N-2)\theta + \dots \Gamma_n \cos(N-2n)\theta + \dots]$$

$$= Ae^{-jN\theta} T_N(\sec \theta_m \cos \theta) \Rightarrow \Gamma_n \text{ (p.254, Table 5.2)}$$

$$T_1(x) = x, \quad T_2(x) = 2x^2 - 1, \quad T_3(x) = 4x^3 - 3x, \quad T_4(x) = 8x^4 - 8x^2 + 1,$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x), \quad x \equiv \frac{\cos \theta}{\cos \theta_m}, \quad |x| \leq 1$$


Discussion

$$1. \quad \Gamma(0) = AT_N(\sec \theta_m) = \frac{Z_L - Z_o}{Z_L + Z_o} \rightarrow \Gamma_m = A = \frac{Z_L - Z_o}{Z_L + Z_o} \frac{1}{T_N(\sec \theta_m)}$$

$$2. \quad T_N(\sec \theta_m) = \frac{1}{\Gamma_m} \left| \frac{Z_L - Z_o}{Z_L + Z_o} \right| \rightarrow \theta_m \quad (5.63) \rightarrow \frac{\Delta f}{f_o} = 2 - \frac{4\theta_m}{\pi}$$

3. Optimal design: given Γ_m , maximal Δf
given Δf , minimal Γ_m .

4. Ex.5.7 $Z_L=100\Omega$, $Z_o=50\Omega$, $N=3$, $\Gamma_m=0.05$

$$N = 3, \Gamma(\theta) = 2e^{-j3\theta} (\Gamma_o \cos 3\theta + \Gamma_1 \cos \theta) = Ae^{-j3\theta} T_3(\sec \theta_m \cos \theta)$$

$$= Ae^{-j3\theta} (4 \sec^3 \theta_m \cos^3 \theta - 3 \sec \theta_m \cos \theta)$$

$$= Ae^{-j3\theta} [\sec^3 \theta_m (\cos 3\theta + 3 \cos \theta) - 3 \sec \theta_m \cos \theta]$$

$$A = \frac{Z_L - Z_o}{Z_L + Z_o} \frac{1}{T_3(\sec \theta_m)} = \Gamma_m \rightarrow \sec \theta_m = 1.408 \rightarrow \theta_m = 44.7^\circ \rightarrow \frac{\Delta f}{f_o} = 101\%$$

$$\Rightarrow 2\Gamma_o = A \sec^3 \theta_m \rightarrow \Gamma_o = 0.0698 = \Gamma_3$$

$$2\Gamma_1 = A(3 \sec^3 \theta_m - 3 \sec \theta_m) \rightarrow \Gamma_1 = 0.1037 = \Gamma_2$$

$$\Gamma_o = \frac{Z_1 - Z_o}{Z_1 + Z_o} \rightarrow Z_1 = 57.5\Omega$$

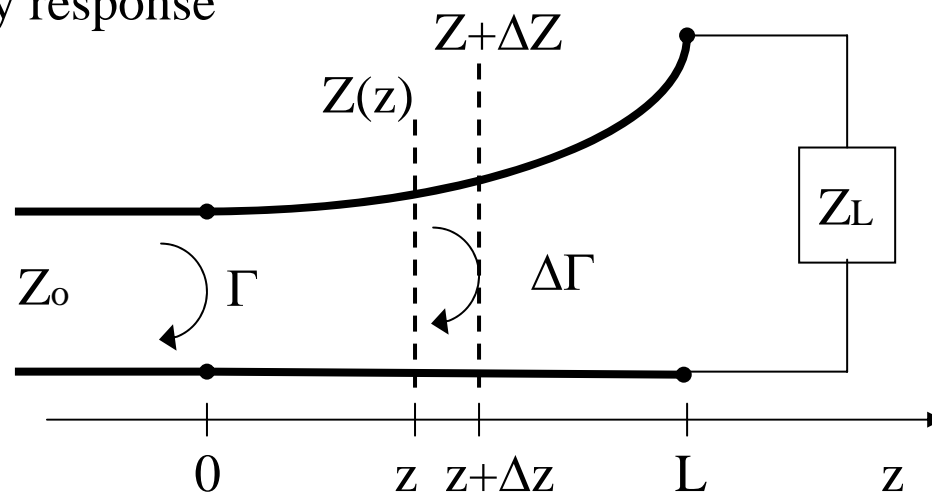
$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} \rightarrow Z_2 = 70.7\Omega$$

$$\Gamma_o = \frac{Z_3 - Z_2}{Z_3 + Z_2} \rightarrow Z_3 = 87\Omega$$

frequency response (p.255, Fig.5.17)

5.8 Tapered lines

- Frequency response



$$\Delta\Gamma = \frac{Z + \Delta Z - Z}{Z + \Delta Z + Z} \approx \frac{\Delta Z}{2Z}$$

$$\rightarrow d\Gamma = \frac{1}{2} \frac{dZ}{Z} = \frac{1}{2} \frac{d \ln Z / Z_0}{dz} dz$$

$$\rightarrow \Gamma(\theta) = \frac{1}{2} \int_0^L e^{-j2\beta z} \frac{d}{dz} \left(\ln \frac{Z}{Z_0} \right) dz$$

Discussion

1. Exponential taper

$$Z(z) = Z_o e^{az} \quad 0 < z < L$$

$$Z(L) = Z_L = Z_o e^{aL} \rightarrow a = \frac{1}{L} \ln \frac{Z_L}{Z_o}$$

$$\rightarrow \Gamma(\theta) = \frac{1}{2} \int_0^L e^{-j2\beta z} \frac{d}{dz} (\ln e^{az}) dz = \frac{1}{2} \ln \frac{Z_L}{Z_o} e^{-j\beta L} \frac{\sin \beta L}{\beta L}$$

$L \uparrow, \Gamma(\theta) \downarrow$ (p. 257, Fig.5.19)

2. Triangular taper

$$Z(z) = \begin{cases} Z_o e^{2(z/L)^2 \ln Z_L/Z_o} & 0 < z < L/2 \\ Z_o e^{(4z/L - 2z^2/L^2 - 1)^2 \ln Z_L/Z_o} & L/2 < z < L \end{cases}$$

$$\Gamma(\theta) = \frac{1}{2} \ln \frac{Z_L}{Z_o} e^{-j\beta L} \left[\frac{\sin(\beta L/2)}{\beta L/2} \right]^2$$

first null at 2π (p. 258, Fig.5.20)

3. Klopfenstein taper

$Z(z)$ (5.74), (5.75), $\Gamma(\theta)$ (5.76), optimal taper

4. Ex.5.8 $Z_L=100\Omega$, $Z_o=50\Omega$, $\Gamma_m=0.02$

exponential taper: $Z(z) = Z_o e^{az}$, $a = \frac{1}{L} \ln \frac{Z_L}{Z_o} = 0.693/L$

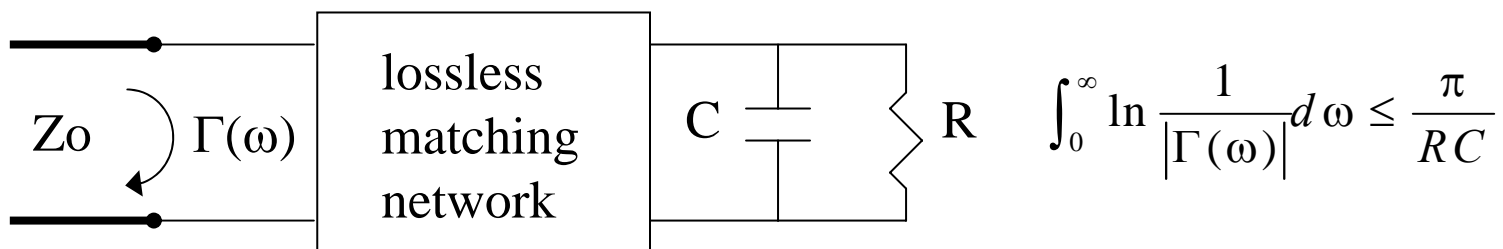
$$|\Gamma(\theta)| = \frac{1}{2} \ln \frac{Z_L}{Z_o} \frac{\sin \beta L}{\beta L}$$

triangular taper: $|\Gamma(\theta)| = \frac{1}{2} \ln \frac{Z_L}{Z_o} \left[\frac{\sin \beta L/2}{\beta L/2} \right]^2$

Klopfenstein taper: $|\Gamma(\theta)| = \Gamma_o \frac{\cos \sqrt{(\beta L)^2 - A^2}}{\cosh A}$, $A = 3.543$, $\Gamma_o = 0.346$

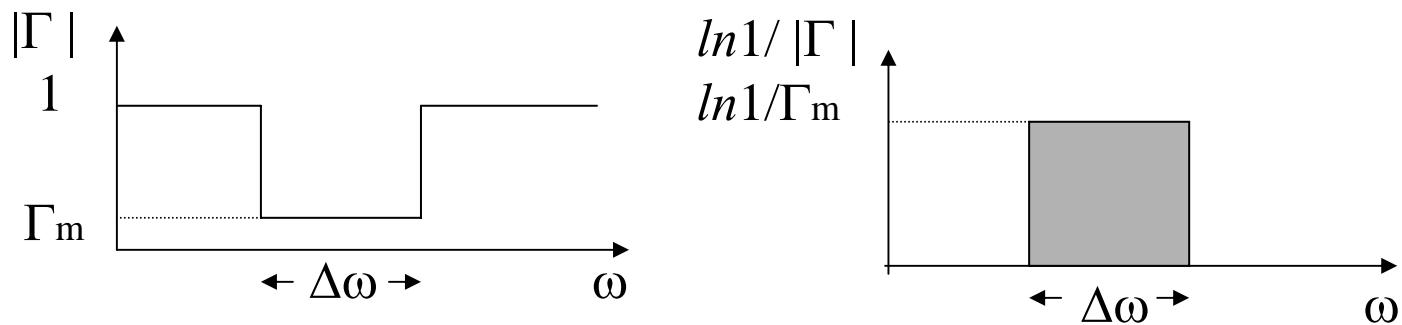
frequency response (p. 260, Fig.5.21)

5.9 The Bode-Fano criterion



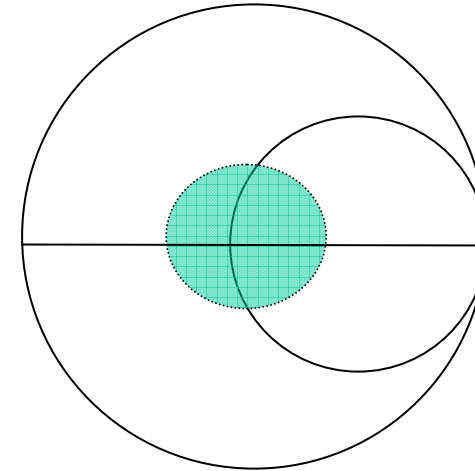
Discussion

1.



$$\int_0^\infty \ln \frac{1}{|\Gamma(\omega)|} d\omega = \int_{\Delta\omega} \ln \frac{1}{\Gamma_m} d\omega$$

$$= \Delta\omega \ln \frac{1}{\Gamma_m} \leq \frac{\pi}{RC} : \text{constant}$$

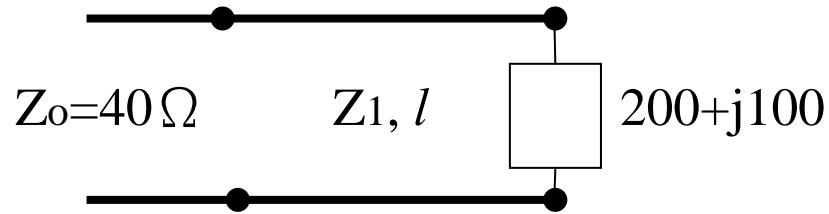


- (1) given $RC \rightarrow \Delta\omega \uparrow \rightarrow \Gamma_m \uparrow$
- (2) $\Gamma_m \neq 0$, unless $\Delta\omega = 0$
i.e., $\Gamma_m = 0$ only at a finite number of frequencies
- (3) R and/or $C \uparrow \rightarrow \Delta\omega \downarrow$ and/or $\Gamma_m \uparrow$
 \Rightarrow high Q load is harder to match



parallel resonator $Q = \omega_0 RC$

Solved problems: Prob.5.7 find Z_1 and l



$$40 = Z_1 \frac{200 + j100 + jZ_1 \tan \beta l}{Z_1 + j(200 + j100) \tan \beta l}$$

$$\rightarrow 40Z_1 + j8000 \tan \beta l - 4000 \tan \beta l = 200Z_1 + j100Z_1 + jZ_1^2 \tan \beta l$$

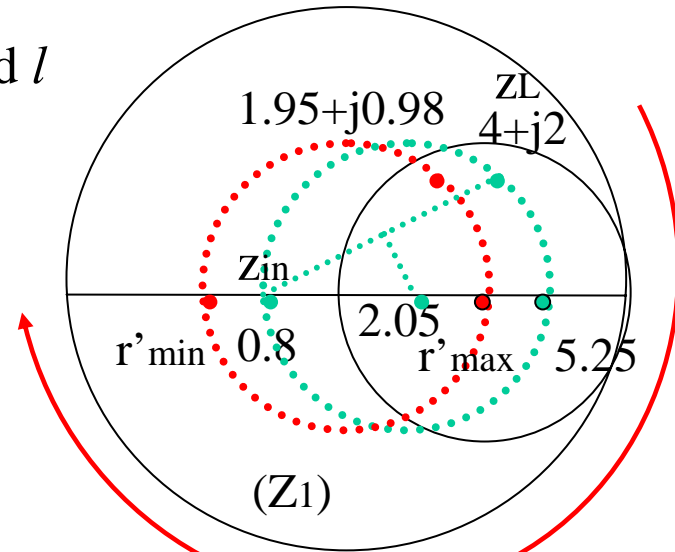
$$\rightarrow 40Z_1 - 4000 \tan \beta l = 200Z_1 \rightarrow Z_1 = -25 \tan \beta l$$

$$\rightarrow j8000 \tan \beta l = j100Z_1 + jZ_1^2 \tan \beta l$$

$$\rightarrow j8000 = -j2500 + j625 \tan^2 \beta l \rightarrow \tan \beta l = -4.1$$

$$\rightarrow Z_1 = 102.5$$

$$\rightarrow l = 0.288\lambda$$



$$k = \frac{Z_1}{50}, \quad kr'_{\min} = 0.8$$

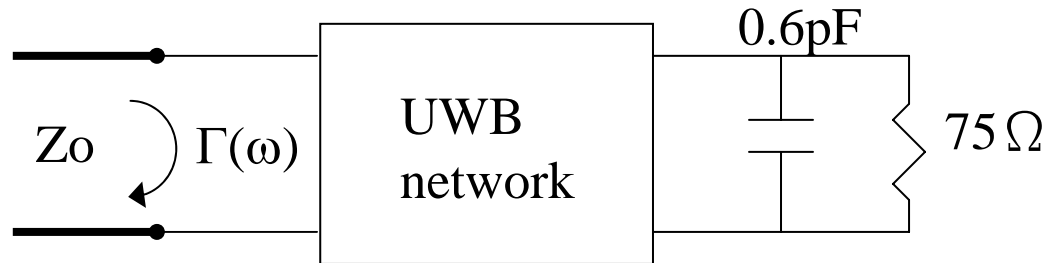
$$50, \quad kr'_{\max} = 5.25$$

$$\rightarrow k^2 r'_{\min} r'_{\max} = r'_{\min} r'_{\max} = 1 \quad k^2 = 4.2$$

$$\rightarrow k = 2.05$$

$$Z_1 = 50 \times 2.05 = 102.5$$

Prob.5.25 find the best RL over operating range of 3.1~10.6GHz



$$\int_0^{\infty} \ln \frac{1}{|\Gamma(\omega)|} d\omega \leq \frac{\pi}{RC}$$

$$\rightarrow \ln \frac{1}{\Gamma_m} \leq \frac{\pi}{2\pi(10.6 - 3.1) \times 10^9 \times 75 \times 0.6 \times 10^{-12}}$$

$$\rightarrow \ln \frac{1}{\Gamma_m} \leq 1.48 \rightarrow \Gamma_m \geq 0.228, RL \leq 6.4 dB$$

Suggested homework (due 2 weeks): 5, 11, 13, 22

ADS examples: Ch5_prj