Chapter 6 Microwave Resonators

6.1 Series and parallel resonant circuits
    series and parallel RLC resonators, quality factor Q

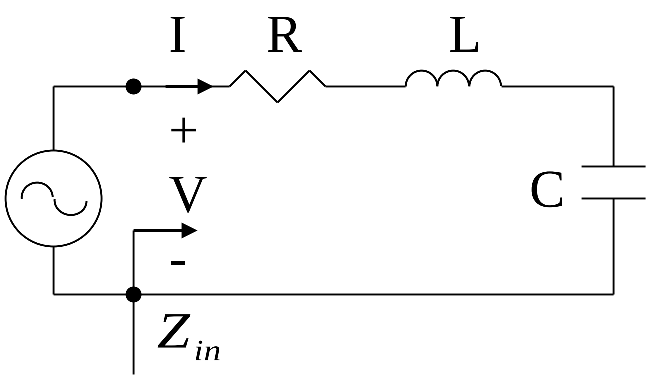
6.2 Transmission line resonators
    $\lambda/2$ and $\lambda/4$ resonators

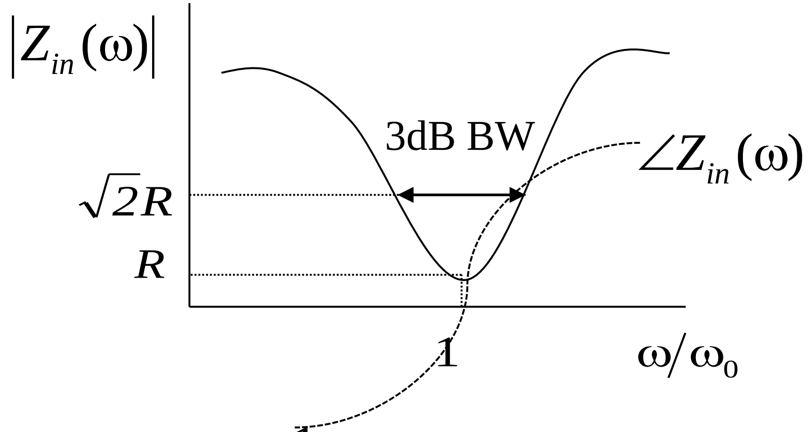
6.5 Dielectric resonator
    concept

6.6 Excitation of resonators
    coefficient of coupling, gap-coupled microstrip resonator, determine Q_u from 2-port measurement

6.1 Series and parallel resonant circuits

- series RLC resonator

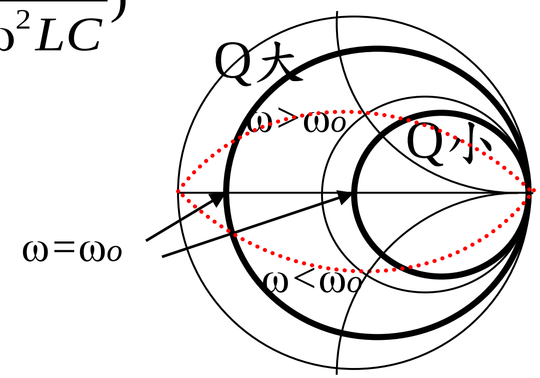




$$Z_{in}(\omega) \equiv \frac{V}{I} = R + j\omega L + \frac{1}{j\omega C} = R + j\omega L\left(1 - \frac{1}{\omega^2 LC}\right)$$

$$= R + j\omega L\left(1 - \frac{\omega_o^2}{\omega^2}\right) = R + j\omega L\,\frac{\omega^2 - \omega_o^2}{\omega^2}$$

$$\equiv \frac{P_{in}}{|I|^2/2} = \frac{P_{loss} + 2j\omega(W_m - W_e)}{|I|^2/2},\ \omega_o = \frac{1}{\sqrt{LC}}$$

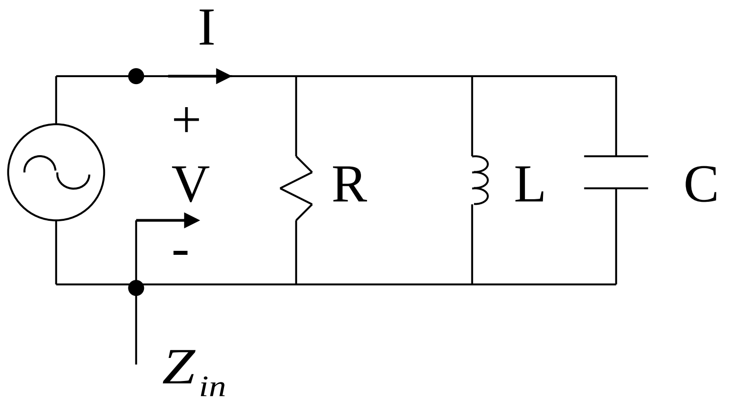


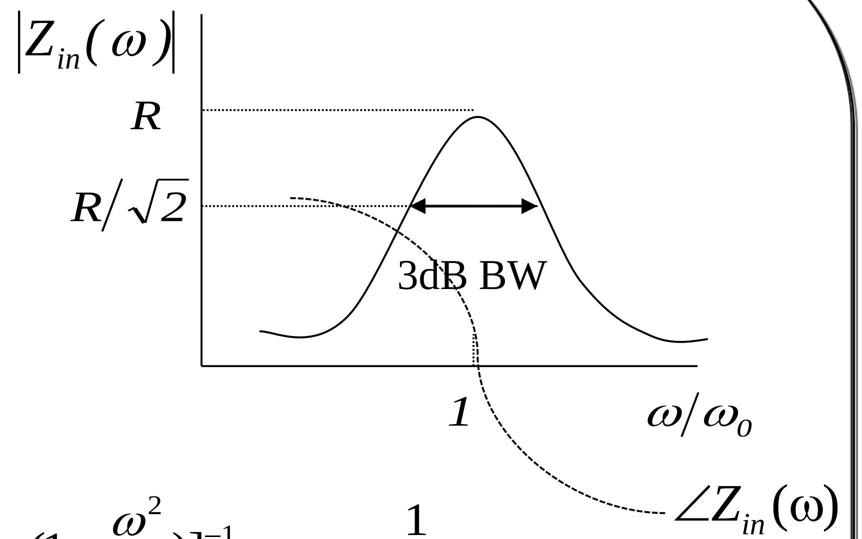
$$P_{loss} = \frac{1}{2}|I|^2 R,\ W_m = \frac{1}{4}|I|^2 L,\ W_e = \frac{1}{4}|V_c|^2 C = \frac{1}{4}|I|^2 \frac{1}{\omega^2 C}$$

$$\text{quality factor } Q(\omega) \equiv \omega \frac{\text{average energy stored}}{\text{energy loss/second}} = \omega \frac{W_m + W_e}{P_{loss}}$$

$$Q(\omega_o) = \omega_o \frac{2W_m}{P_{loss}} = \omega_o \frac{2\left|I^2\right|L/4}{\left|I^2\right|R/2} = \boxed{\frac{\omega_o L}{R} = \frac{1}{\omega_o RC},}\ R\downarrow (\text{loss } \downarrow) \ \Rightarrow \ Q\uparrow$$

- parallel RLC resonator



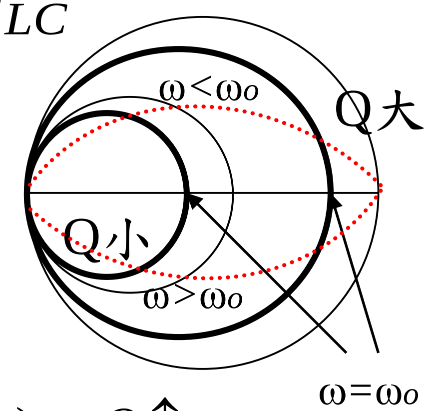


$$Z_{in}(\omega) = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)^{-1} = \left[\frac{1}{R} + \frac{1}{j\omega L}\left(1 - \frac{\omega^2}{\omega_o^2}\right)\right]^{-1}, \omega_o = \frac{1}{\sqrt{LC}}$$

$$\equiv \frac{2P_{in}}{|I|^2} = \frac{P_{loss} + 2j\omega(W_m - W_e)}{|I|^2/2}$$

$$P_{loss} = \frac{1}{2}\frac{|V|^2}{R}, W_m = \frac{1}{4}|I_L|^2 L = \frac{1}{4}|V|^2\frac{1}{\omega^2 L}, W_e = \frac{1}{4}C|V|^2$$

$$Q(\omega_o) = \omega_o \frac{2W_e}{P_{loss}} = \omega_o \frac{2C|V|^2/4}{|V|^2/2R} = \omega_o RC = \frac{R}{\omega_o L}, R\uparrow (\text{loss}\downarrow) \Rightarrow Q\uparrow$$



Discussion

1. At resonance, $W_m=W_e$, $P_{in}=P_{loss}$

2. Near resonant frequency

series resonator: $Z_{in} \approx R + j2L\Delta\omega = R + j\dfrac{2RQ\Delta\omega}{\omega_o}, \because Q(\omega_o) = \dfrac{\omega_o L}{R} = \dfrac{1}{\omega_o RC}$

parallel resonator: $Z_{in} \approx \dfrac{R}{1 + j2\Delta\omega RC} = \dfrac{R}{1 + j2Q\dfrac{\Delta\omega}{\omega_o}}, \because Q(\omega_o) = \omega_o RC = \dfrac{R}{\omega_o L}$

3. Half-power fractional bandwidth

$$P_{in,av} = \frac{1}{2}\mathrm{Re}[VI^*] = \frac{1}{2}\mathrm{Re}[\frac{VV^*}{Z_{in}^*}] = \frac{1}{2}|V|^2\,\mathrm{Re}[\frac{Z_{in}}{Z_{in}Z_{in}^*}] = \frac{1}{2}R_{in}\frac{|V|^2}{|Z_{in}|^2} = \frac{1}{2}\mathrm{Re}[\frac{I}{Y_{in}}I^*] = \frac{1}{2}G_{in}\frac{|I|^2}{|Y_{in}|^2}$$

series resonator: $@\,\omega_{3dB}, R = X, P_{in,av}(\omega_{3dB}) = \dfrac{1}{2}R\dfrac{|V|^2}{|\sqrt{2}R|^2} = \dfrac{1}{2}P_{in,av}(\omega_o) \rightarrow R = X = \dfrac{2RQ\Delta\omega}{\omega_o}$

parallel resonator: $@\,\omega_{3dB}, G = B, P_{in,av}(\omega_{3dB}) = \dfrac{1}{2}G\dfrac{|I|^2}{|\sqrt{2}G|^2} = \dfrac{1}{2}P_{in,av}(\omega_o) \rightarrow G = B = \dfrac{2GQ\Delta\omega}{\omega_o}$

$\Rightarrow Q(\omega_o) = \dfrac{\omega_o}{2\Delta\omega} \equiv \dfrac{1}{BW}$

(derivation of series resonator case)

$$Z_{in}(\omega) = R + j\omega L + \frac{1}{j\omega C} = R + jL(\omega - \frac{1}{\omega LC}) = R + jL(\omega - \frac{\omega_o^2}{\omega})$$

$$= R + jL\frac{\omega^2 - \omega_o^2}{\omega} \overset{\omega=\omega_o+\Delta\omega}{=} R + jL\frac{(\omega_o + \Delta\omega)^2 - \omega_o^2}{\omega} = R + jL\frac{2\Delta\omega\omega_o + \Delta\omega^2}{\omega}$$

$$\overset{\omega\approx\omega_o}{\approx} R + j2L\Delta\omega$$

(derivation of parallel resonator case)

$$Z_{in}(\omega) = [\frac{1}{R} + \frac{1}{j\omega L} + j\omega C]^{-1} \overset{\omega=\omega_o+\Delta\omega}{=} [\frac{1}{R} + \frac{1}{j(\omega_o + \Delta\omega)L} + j(\omega_o + \Delta\omega)C]^{-1}$$

$$\because \frac{1}{\omega_o + \Delta\omega} = \frac{1}{\omega_o(1 + \Delta\omega/\omega_o)} \approx \frac{1}{\omega_o}(1 - \frac{\Delta\omega}{\omega_o}) = \frac{1}{\omega_o} - \frac{\Delta\omega}{\omega_o^2}$$

$$Z_{in}(\omega) \approx [\frac{1}{R} + \cancel{\frac{1}{j\omega_o L}} - \frac{\Delta\omega}{j\omega_o^2 L} + \cancel{j\omega_o C} + j\Delta\omega C]^{-1} = [\frac{1}{R} + \frac{j\Delta\omega}{\omega_o^2 L} + j\Delta\omega C]^{-1}$$

$$= [\frac{1}{R} + j2\Delta\omega C]^{-1} = \frac{R}{1 + j2\Delta\omega RC}$$

4. Locus of r= ± x on the Smith chart

(derivation)

$$z = r + jx = \frac{1+\Gamma}{1-\Gamma} = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i} = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} + j\frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$r = \pm x \rightarrow 1-\Gamma_r^2 - \Gamma_i^2 = \pm 2\Gamma_i \rightarrow \Gamma_r^2 + \Gamma_i^2 \pm 2\Gamma_i + 1 = 2$$

$$\Rightarrow \Gamma_r^2 + (\Gamma_i \pm 1)^2 = (\sqrt{2})^2$$

Locus of g = ± b on the Smith chart

(derivation)

$$y = g + jb = \frac{1-\Gamma}{1+\Gamma} = \frac{1-\Gamma_r - j\Gamma_i}{1+\Gamma_r + j\Gamma_i} = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1+\Gamma_r)^2 + \Gamma_i^2} - j\frac{2\Gamma_i}{(1+\Gamma_r)^2 + \Gamma_i^2}$$

$$g = \pm b \rightarrow 1-\Gamma_r^2 - \Gamma_i^2 = \mp 2\Gamma_i \rightarrow \Gamma_r^2 + \Gamma_i^2 \mp 2\Gamma_i + 1 = 2$$

$$\Rightarrow \Gamma_r^2 + (\Gamma_i \mp 1)^2 = (\sqrt{2})^2$$

5. Lossless resonator → lossy resonator: $\omega_o \rightarrow \omega_o(1+\frac{j}{2Q})$
(derivation)

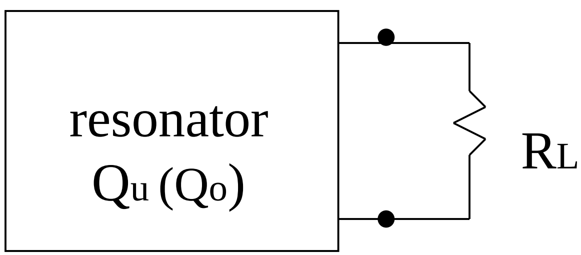
lossless series resonator $Z_{in} = j2\Delta\omega L = j2(\omega-\omega_o)L$

$$\xrightarrow{\omega_o\rightarrow\omega_o(1+\frac{j}{2Q})} Z_{in} = j2[\omega-\omega_o(1+\frac{j}{2Q})]L = j2\Delta\omega L + \frac{\omega_o L}{Q} \overset{Q=\frac{\omega_o L}{R}}{=} R + j2\Delta\omega L$$

lossless parallel resonator $Z_{in} = [j2\Delta\omega C]^{-1} = [j2(\omega-\omega_o)C]^{-1}$

$$\xrightarrow{\omega_o\rightarrow\omega_o(1+\frac{j}{2Q})} Z_{in} = j2[\omega-\omega_o(1+\frac{j}{2Q})]^{-1}C = [j2\Delta\omega C + \frac{\omega_o C}{Q}]^{-1} \overset{Q=\omega_o RC}{=} [j2\Delta\omega C + \frac{1}{R}]^{-1}$$

6. Unloaded Q, Qu, loaded Q, QL, external Q, Qe

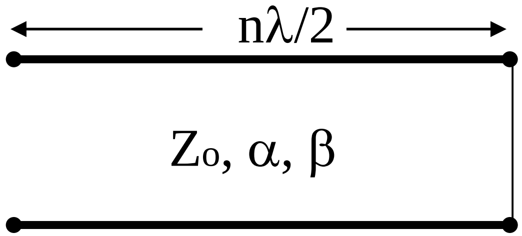


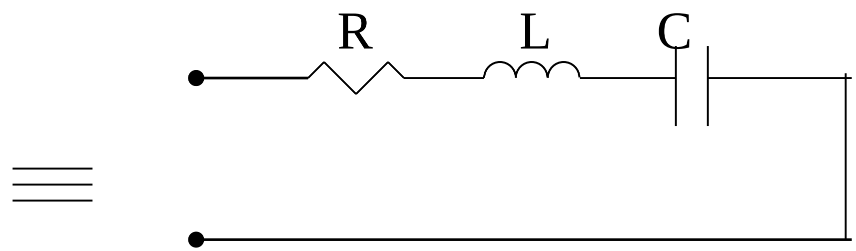
$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_U},$$

$$Q_e = \begin{cases} \dfrac{\omega_o L}{R_L} & \text{for series RLC circuit} \\ \dfrac{R_L}{\omega_o L} & \text{for parallel RLC circuit} \end{cases}$$

6.2 Transmission line resonators

- Short-circuited λ/2 line





$$Z_{in} \approx Z_o(\alpha l + j\frac{\pi\Delta\omega}{\omega_o})$$

$$Z_{in} = R + 2jL\Delta\omega$$

$$R = Z_o\alpha l, L = \frac{Z_o\pi}{2\omega_o}, C = \frac{1}{\omega_o^2 L}$$

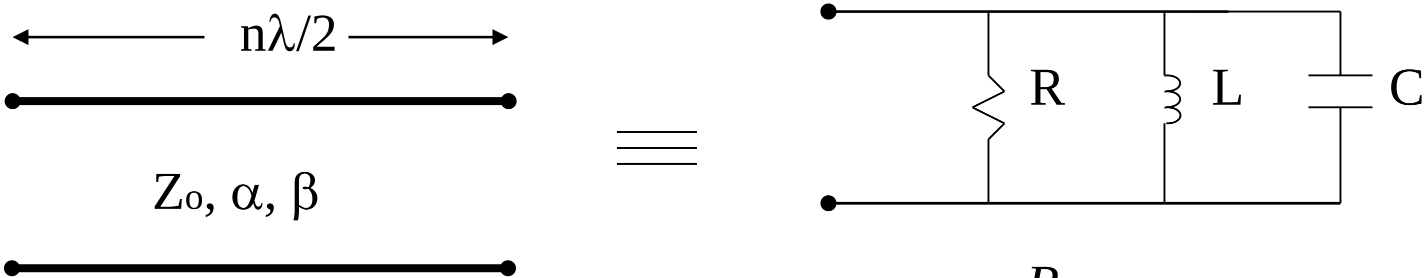
(derivation)

$$Z_{in} = Z_o \frac{Z_L + Z_o \tanh\gamma l}{Z_o + Z_L \tanh\gamma l}\bigg|_{Z_L=0} = Z_o \tanh(\alpha + j\beta)l = Z_o \frac{\tanh\alpha l + j\tan\beta l}{1 + j\tan\beta l \tanh\alpha l}$$

$$\because \beta l = \frac{\omega}{v_p}\frac{\lambda_o}{2} = \frac{\omega_o + \Delta\omega}{v_p}\frac{v_p}{2f_o} = \frac{\omega_o + \Delta\omega}{2\omega_o/2\pi} = \pi + \frac{\pi\Delta\omega}{\omega_o}, \tan\beta l = \tan(\frac{\pi\Delta\omega}{\omega_o}) \approx \frac{\pi\Delta\omega}{\omega_o}, \tanh\alpha l \overset{\alpha l<<1}{\approx} \alpha l$$

$$Z_{in} \approx Z_o \frac{\alpha l + j\frac{\pi\Delta\omega}{\omega_o}}{1 + j\alpha l\frac{\pi\Delta\omega}{\omega_o}} \approx Z_o(\alpha l + j\frac{\pi\Delta\omega}{\omega_o}) = R + j2L\Delta\omega, Q_U(\omega_o) = \frac{\omega_o L}{R} = \frac{\omega_o\frac{Z_o\pi}{2\omega_o}}{Z_o\alpha l} = \frac{\pi}{2\alpha l} = \frac{\beta_o l}{2\alpha l} = \frac{\beta_o}{2\alpha}$$

- Open-circuited λ/2 line



$$Z_{in} \approx \frac{Z_o}{\alpha l + j\dfrac{\pi\Delta\omega}{\omega_o}}$$

$$Z_{in} = \frac{R}{1+2j\Delta\omega RC}$$

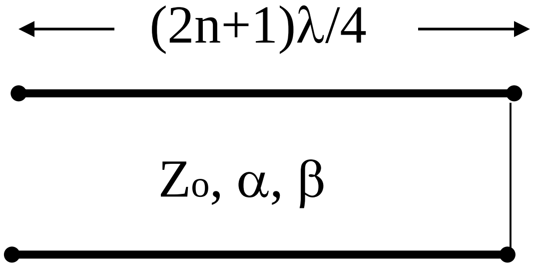
$$R = \frac{Z_o}{\alpha l}, C = \frac{\pi}{2\omega_o Z_o}, L = \frac{1}{\omega_o^2 C}$$

(derivation)

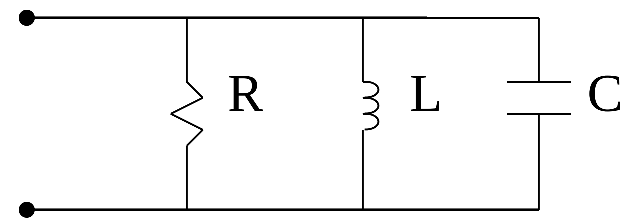
$$Z_{in} = Z_o \left.\frac{Z_L + Z_o \tanh\gamma l}{Z_o + Z_L \tanh\gamma l}\right|_{Z_L=\infty} = \frac{Z_o}{\tanh(\alpha + j\beta)l} = Z_o \frac{1 + j\tan\beta l \tanh\alpha l}{\tanh\alpha l + j\tan\beta l} \approx Z_o \frac{1 + j\dfrac{\pi\Delta\omega}{\omega_o}\alpha l}{\alpha l + j\dfrac{\pi\Delta\omega}{\omega_o}}$$

$$\approx \frac{Z_o}{\alpha l + j\dfrac{\pi\Delta\omega}{\omega_o}} = \frac{Z_o/\alpha l}{1 + j\dfrac{\pi\Delta\omega}{\alpha l\omega_o}} = \frac{R}{1 + j2\Delta\omega RC}, Q_U(\omega_o) = \omega_o RC = \omega_o \frac{Z_o}{\alpha l}\frac{\pi}{2\omega_o Z_o} = \frac{\pi}{2\alpha l} = \frac{\beta_o}{2\alpha}$$

- Short-circuited λ/4 line



≡



$$Z_{in} \approx \frac{Z_o}{\alpha l + j\dfrac{\pi\Delta\omega}{2\omega_o}}$$

$$Z_{in} = \frac{R}{1+2j\Delta\omega RC}$$

$$R = \frac{Z_o}{\alpha l}, C = \frac{\pi}{4\omega_o Z_o}, L = \frac{1}{\omega_o^2 C}$$

(derivation)

$$Z_{in} = Z_o \frac{Z_L + Z_o \tanh\gamma l}{Z_o + Z_L \tanh\gamma l}\bigg|_{Z_L=0} = Z_o \tanh(\alpha + j\beta)l = Z_o \frac{\tanh\alpha l + j\tan\beta l}{1+j\tan\beta l \tanh\alpha l} \times \frac{-j\cot\beta l}{-j\cot\beta l} = Z_o \frac{-j\cot\beta l \tanh\alpha l + 1}{-j\cot\beta l + \tanh\alpha l}$$

$$\because \beta l = \frac{\omega}{v_p}\frac{\lambda_o}{4} = \frac{\omega_o + \Delta\omega}{v_p}\frac{v_p}{4f_o} = \frac{\omega_o + \Delta\omega}{4\omega_o/2\pi} = \frac{\pi}{2} + \frac{\pi\Delta\omega}{2\omega_o}, \cot\beta l = -\tan(\frac{\pi\Delta\omega}{2\omega_o}) \approx -\frac{\pi\Delta\omega}{2\omega_o}$$

$$Z_{in} \approx Z_o \frac{j\dfrac{\pi\Delta\omega}{2\omega_o}\alpha l + 1}{j\dfrac{\pi\Delta\omega}{2\omega_o} + \alpha l} \approx \frac{Z_o}{\alpha l + j\dfrac{\pi\Delta\omega}{2\omega_o}} = \frac{R}{1+j2\Delta\omega RC}, Q_U(\omega_o) = \omega_o RC = \omega_o \frac{Z_o}{\alpha l}\frac{\pi}{4\omega_o Z_o} = \frac{\pi}{4\alpha l} = \frac{\beta_o}{2\alpha}$$

Discussion

1. Transmission line resonator $Q_U(w_o) = \dfrac{\beta_o}{2\alpha}$

2. Ex. 6.1 λ/2 coaxial line resonator, b=4mm, a=1mm, f=5GHz, Teflon εr=2.08, tanδ=0.0004, calculate $Q_U$air and Q_UTeflon

$$Q_U = \frac{\beta_o}{2\alpha}, \quad \beta_o = \frac{2\pi f_o \sqrt{\varepsilon_r}}{c} = \begin{cases} 104.7 & \text{air} \\ 104.7\sqrt{2.08} & \text{Teflon} \end{cases}, \quad \alpha = \alpha_c + \alpha_d$$

$$\alpha_c = \begin{cases} \dfrac{R_s}{2\eta \ln\frac{b}{a}\left(\frac{1}{a}+\frac{1}{b}\right)} = 0.022 Np/m & \text{air} \\ 0.022\sqrt{2.08} = 0.032 Np/m & \text{Teflon} \end{cases}, \quad R_s = \sqrt{\frac{\omega\mu_o}{2\sigma}} = 1.84\times 10^{-2}\,\Omega$$

$$\sigma = 5.813\times 10^{7}\, S/m \quad \text{copper}$$

$$\text{Teflon} \quad \alpha_d = \frac{k_o\sqrt{\varepsilon_r}}{2}\tan\delta = 0.03 Np/m$$

$$\Rightarrow Q_U(5GHz) = \begin{cases} \dfrac{104.7}{2\times 0.022} = 2380 & \text{air} \\ \dfrac{104.7\sqrt{2.08}}{2\times(0.022+0.03)} = 1218 & \text{Teflon} \end{cases}$$

3. Ex. 6.2 λ/2 open-circuited microstrip resonator, Zo=50Ω, h=1.59mm, Teflon substrate ε_r=2.08, tanδ=0.0004, f=5GHz, calculate resonator length and Q_U.

$$Z_o = 50 \quad \rightarrow \quad W = 5.08mm, \quad \varepsilon_{eff} = 1.8$$

$$l = \frac{\lambda_o}{2} = \frac{c}{2f\sqrt{\varepsilon_{eff}}} = 2.24cm$$

$$\beta_o = \frac{2\pi f_o \sqrt{\varepsilon_{eff}}}{c} = 151 rad/m,$$

$$\alpha_c = \frac{R_s}{Z_o W} = 0.0724 Np/m,$$

$$\alpha_d = \frac{k_o \varepsilon_r (\varepsilon_{eff} - 1)}{2\sqrt{\varepsilon_{eff}}(\varepsilon_r - 1)} \tan\delta = 0.024 Np/m$$

$$\Rightarrow Q_U(5GHz) = \frac{\beta_o}{2\alpha} = \frac{\beta_o}{2(\alpha_c + \alpha_d)} = 783$$

4. For comparison,

Ex. 6.3 rectangular waveguide cavity resonator, ε_r=2.25(polyethylene), $\tan\delta$=0.0004, f=5GHz.

TE_{101} mode d=2.2 cm, Q_c=8403, Q_d=2500, Q_U=1927@5GHz

TE_{102} mode d=4.4 cm, Q_c=11898, Q_d=2500, Q_U=2065@10GHz

Ex. 6.4 Teflon-filled cylindrical cavity, ε_r=2.08, $\tan\delta$=0.0004,f=5GHz. a=2.74cm, d=2a=5.48cm, TE_{011} mode Q_c=29390, Q_d=2500,Q_U=2300 @5GHz

air-filled cylindrical cavity, TE_{011} mode a=3.96cm, d=2a=7.91cm, Q_c=42400@10GHz

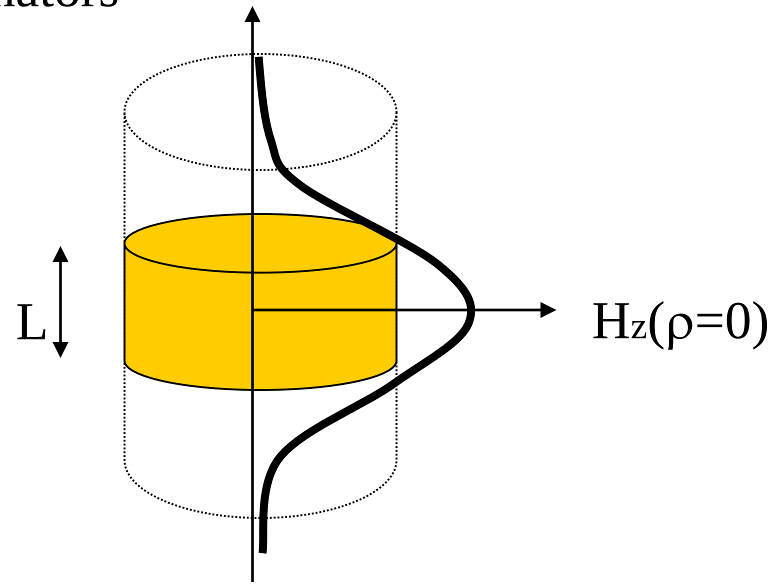
5. In general for Q_U

Qspherical > Qcylindrical > Qrectangular > Qcoaxial > Qmicrostrip

6. Application of resonator: frequency selective component (ω_o, Q_U)

eg., frequency meter, oscillator, filter, matching circuits

6.5 Dielectric resonators



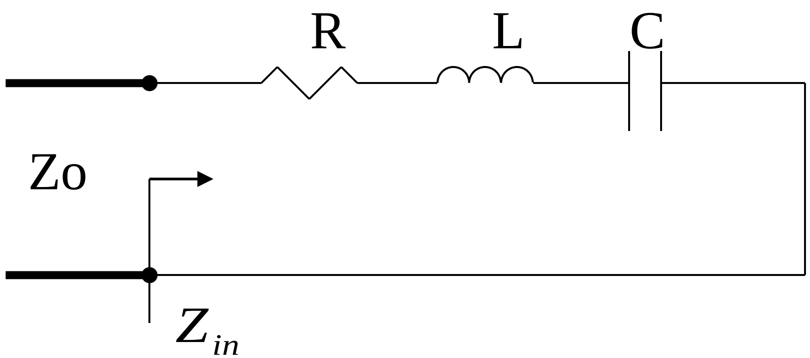
$$10 < \varepsilon_r < 100, \quad TE_{01\delta} \text{ mode } \delta = \frac{2L}{\lambda_g} < 1, \quad Q_d \approx \frac{1}{\tan\delta}$$

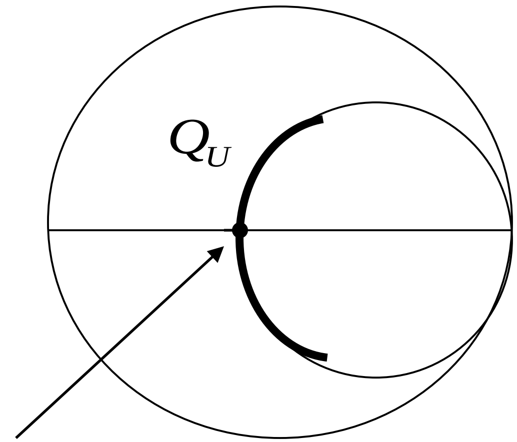
Ex.6.5 $\varepsilon_r = 95,\ \tan\delta = 0.001,\ a = 0.413cm$

$\rightarrow\ f = 3.4GHz,\ \ Q_d = 1000$

6.6 Excitation of resonators

• Critical coupling $Z_{in}(\omega_o) = Z_o$



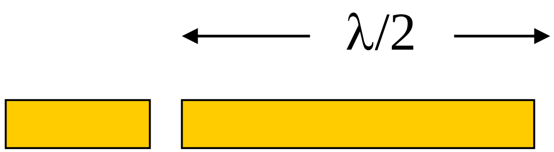


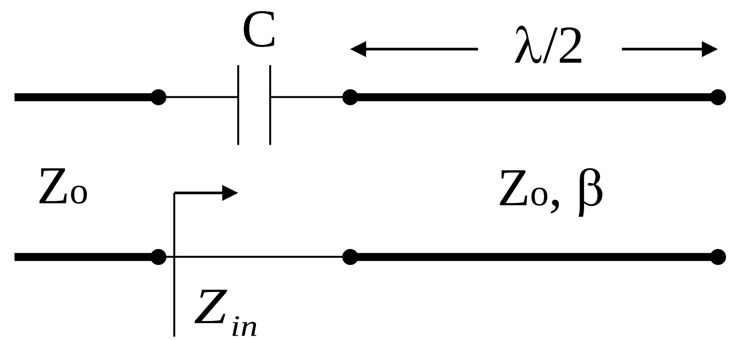
A series RLC resonator is given to match with the feedline, i.e., $R = Z_o$ at resonance.

$$\rightarrow \; Q_U = Q_e ,\; Q_L = \frac{Q_U}{2} \qquad (Q_U(\omega_o) = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}, Q_e(\omega_o) = \frac{\omega_o L}{Z_o} = \frac{1}{\omega_o Z_o C})$$

coefficient of coupling $g \equiv \frac{Q_U}{Q_e} (= \frac{Z_o}{R} = 1$ for critical coupling)

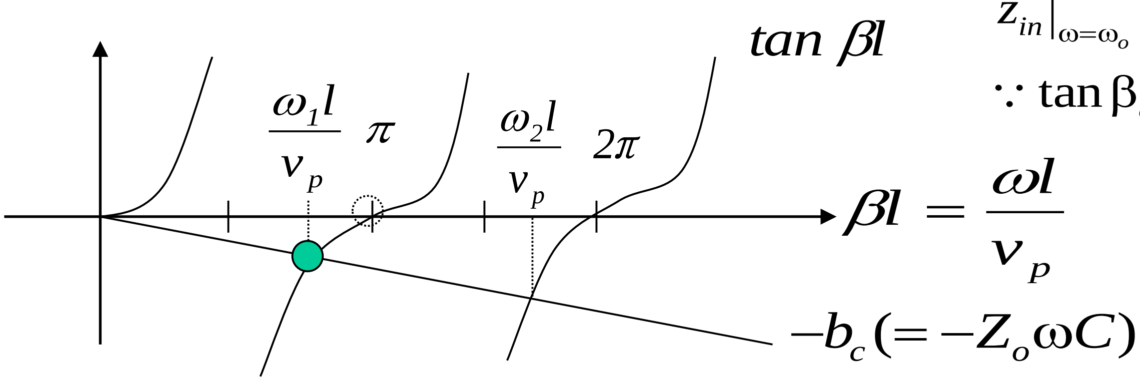
- Gap-coupled $\lambda/2$ open-circuited microstrip resonator





$$z_{in}\Big|_{\omega=\omega_1} = \frac{Z_{in}}{Z_o}\Big|_{\omega=\omega_1} = \frac{1}{Z_o}\left(\frac{1}{j\omega C} + \frac{Z_o}{j\tan\beta l}\right)\Big|_{\omega=\omega_1} = -j\,\frac{\tan\beta l + b_c}{b_c \tan\beta l}\Big|_{\omega=\omega_1} = 0,\ \ b_c = Z_o\omega C$$

$\Rightarrow$ solve the resonant frequency ω_1



$z_{in}\Big|_{\omega=\omega_o} = \infty$ resonance or open?

$\because \tan\beta_o l = 0$

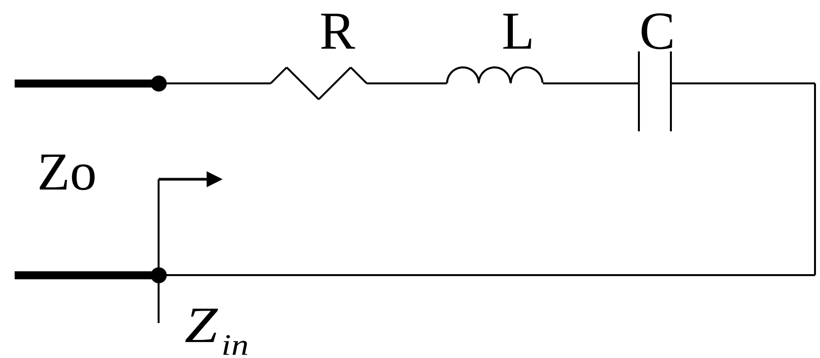
Discussion

1.

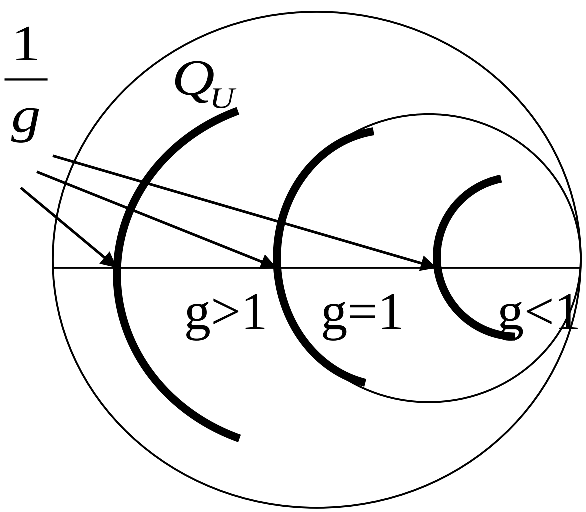
$$g = \frac{Q_U}{Q_e} = \begin{cases} \dfrac{Z_o}{R} & \text{series resonator} \\ \dfrac{R}{Z_o} & \text{parallel resonator} \end{cases}$$

$$Q_L = \frac{Q_U}{1+g}$$

$$g \begin{cases} <1 & \text{under coupling} \quad Q_L \to Q_U \\ =1 & \text{critical coupling} \quad Q_L = Q_U/2 \\ >1 & \text{over coupling} \quad Q_L << Q_U \end{cases}$$



$$r = \frac{1}{g}$$



for series resonator

2. Types of excitation for microwave resonators (p.291, Fig.6.13)

E coupling $\Rightarrow$ C, H coupling $\Rightarrow$ L

3. Gap-coupled $\lambda/2$ open-circuited microstrip resonator

$$z_{in}(\omega)=\frac{Z_{in}}{Z_o}=\begin{cases}\dfrac{j\pi(\omega-\omega_1)}{\omega_1 b_c^2} & \text{lossless}\\ \dfrac{\pi}{2Q_U b_c^2}+\dfrac{j\pi(\omega-\omega_1)}{\omega_1 b_c^2}=\dfrac{R}{Z_0}+\dfrac{j2L\Delta\omega}{Z_0} & \text{lossy}\end{cases}$$

$\lambda/2$ open-circuited microstrip resonator (parallel resonator)

$\rightarrow$ gap-coupling $\rightarrow$ series resonator

(derivation) $z_{in}(\omega)\approx z_{in}(\omega_1)+(\omega-\omega_1)\left.\dfrac{dz_{in}}{d\omega}\right|_{\omega_1}\approx(\omega-\omega_1)\left.\dfrac{dz_{in}}{d\omega}\right|_{\omega_1}$

$$z_{in}=-j\frac{\tan\beta l+b_c}{b_c\tan\beta l},\ \left.\frac{dz_{in}}{d\omega}\right|_{\omega_1}=\left.\frac{dz_{in}}{d\beta l}\frac{d\beta l}{d\omega}\right|_{\omega_1}=j\left.\frac{1+b_c^2}{b_c^2}\frac{l}{v_p}\right|_{\omega_1}\approx\left.\frac{j}{b_c^2}\frac{l}{v_p}\right|_{\omega_1}\approx\frac{j\pi}{\omega_1 b_c^2}$$

$$\because l=\frac{\lambda_o}{2}\approx\frac{\lambda_1}{2}=\frac{v_p}{2f_1}=\frac{\pi v_p}{\omega_1}\rightarrow z_{in}(\omega)=\frac{j\pi}{\omega_1 b_c^2}(\omega-\omega_1):\text{lossless}$$

$$\text{lossy}:\omega_1\rightarrow\omega_1(1+\frac{j}{2Q}),\ z_{in}(\omega)=\frac{j\pi}{\omega_1 b_c^2}[\omega-\omega_1(1+\frac{j}{2Q})]=\frac{j\pi(\omega-\omega_1)}{\omega_1 b_c^2}+\frac{\pi}{2Q_U b_c^2}$$

4. Series resonator @w_1

$$Z_{in}(\omega_1) = R = \frac{\pi Z_o}{2Q_U b_c^2}, g = \frac{Z_o}{R} = \frac{2Q_U b_c^2}{\pi} \begin{cases} <1 \rightarrow b_c < \sqrt{\frac{\pi}{2Q_U}} & \text{:small C, large gap} \\ =1 \rightarrow b_c = \sqrt{\frac{\pi}{2Q_U}} \\ >1 \rightarrow b_c > \sqrt{\frac{\pi}{2Q_U}} & \text{: large C, small gap} \end{cases}$$

5. Ex. 6.6 λ/2 open-circuited microstrip resonator, Zo=50Ω, l=2.175cm, ε_{eff} =1.9, α=0.01dB/cm, calculate C for critical coupling and f_1.

ω_1

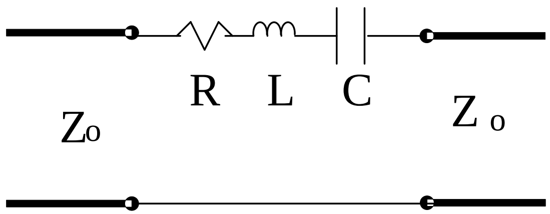
$$\text{uncoupled line: } l = n\frac{\lambda_g}{2} = \frac{c}{2f\sqrt{\varepsilon_{eff}}} \rightarrow \quad f_o = 5GHz$$

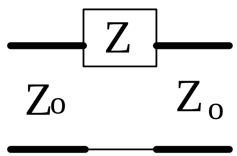
$$Q_U = \frac{\beta_o}{2\alpha} = \frac{2\pi}{\lambda_g 2\alpha} = \frac{\pi}{2l\alpha} = 628, b_c = \sqrt{\frac{\pi}{2Q_U}} = 0.05 \xrightarrow{g=1} C = \frac{b_c}{\omega_1 Z_o} = 0.032\,pF$$

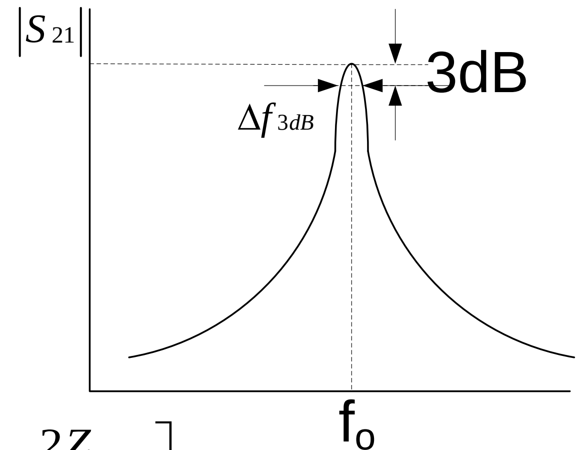
$$\tan\beta l + b_c = 0 \rightarrow \ f_1 = 4.918GHz < f_o = 5GHz$$

$$Q_L = \frac{Q_U}{2} = 314 < Q_U = 628$$

6.Determin Q_U from 2-port measurement







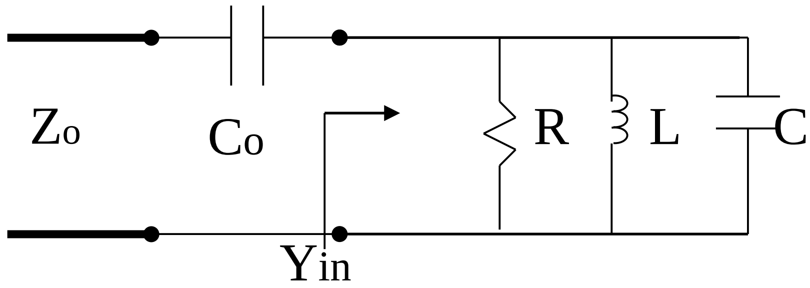
$$@\ f_o : [Z] = \begin{bmatrix} R & R \\ R & R \end{bmatrix}, [S] = \begin{bmatrix} \dfrac{R}{R+2Z_o} & \dfrac{2Z_o}{R+2Z_o} \\ \dfrac{2Z_o}{R+2Z_o} & \dfrac{R}{R+2Z_o} \end{bmatrix} \text{.....prob 4.11}$$

$$Q_U = \frac{\omega_o L}{R},\ Q_e = \frac{\omega_o L}{R_L} = \frac{\omega_o L}{2Z_o} \rightarrow g = \frac{Q_U}{Q_e} = \frac{2Z_o}{R}$$

$$S_{21}(f_o) = \frac{2Z_o}{R+2Z_o} = \frac{g}{1+g} \rightarrow g = \frac{S_{21}(f_o)}{1-S_{21}(f_o)}$$

$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_U} = \frac{1}{Q_U}(1+g), \text{measure } Q_L = \frac{f_o}{\Delta f_{3sB}} \rightarrow Q_U = Q_L(1+g)$$

Solved problems: Prob. 6.22 A parallel resonator, calculate Co for critical coupling and fr.



R=1000Ω, L=1.26nH, C=0.804pF, Zo=50Ω

$$Y_{in}(\omega) \approx \frac{1}{R} + j\frac{2Q_u\Delta\omega}{R\omega_o} = 10^{-3} + j50.5\times10^{-3}\frac{\Delta\omega}{\omega_o}, Q_U = \frac{R}{\omega_o L} = 25.3, \omega_o = \frac{1}{\sqrt{\mathrm{LC}}} = 31.4\times10^9$$

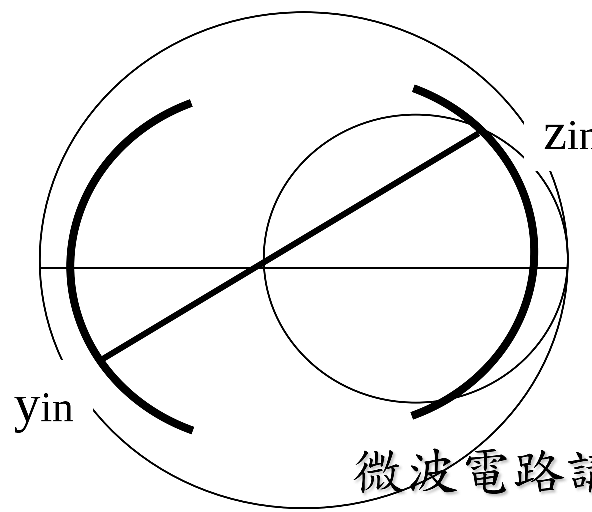
$$\to y_{in} = 50\times Y_{in} = 0.05 + j2.53\frac{\Delta\omega}{\omega_o} = \frac{1}{1+ja} = \frac{1}{1+a^2} - j\frac{a}{1+a^2} \to 0.05 = \frac{1}{1+a^2} \to a = 4.36$$

$$\because z_{in} + \frac{1}{j\omega C_o Z_o} = 1 \to z_{in} = 1 + j\frac{1}{\omega C_o Z_o} = 1 + ja$$

$$-\frac{a}{1+a^2} = -\frac{4.36}{20} = 2.53\frac{\Delta\omega}{\omega_o} \to \frac{\Delta\omega}{\omega_o} = -0.086$$

$$\to f_r = f_o - 0.086 f_o = 4.57GHz$$

$$\frac{1}{\omega_r C_o Z_o} = 4.36 \to C_o = 0.16pF$$



ADS examples: Ch6_prj