

Chapter 6 Microwave Resonators

6.1 Series and parallel resonant circuits

series and parallel RLC resonators, quality factor Q

6.2 Transmission line resonators

$\lambda/2$ and $\lambda/4$ resonators

6.5 Dielectric resonator

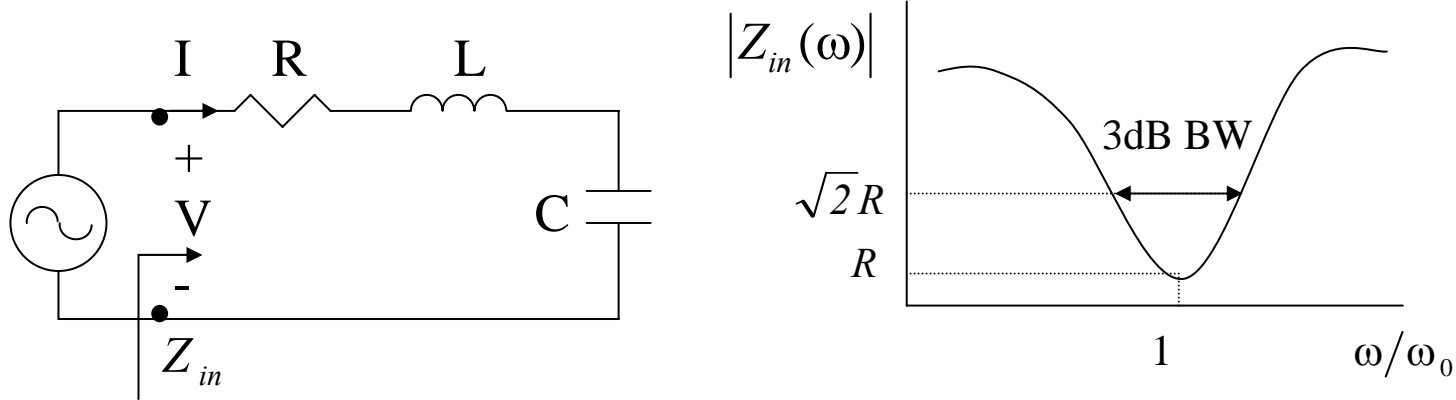
concept

6.6 Excitation of resonators

coefficient of coupling, gap-coupled microstrip resonator

6.1 Series and parallel resonant circuits

- series RLC resonator

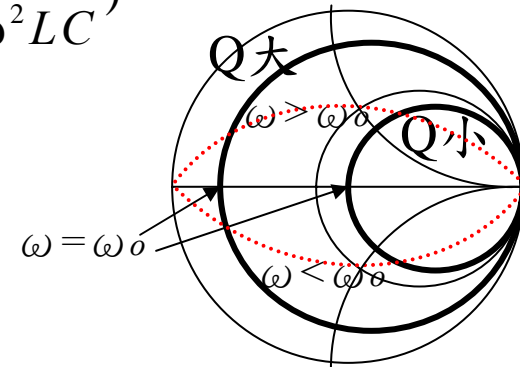


$$Z_{in}(\omega) \equiv \frac{V}{I} = R + j\omega L + \frac{1}{j\omega C} = R + j\omega L \left(1 - \frac{1}{\omega^2 LC}\right)$$

$$= R + j\omega L \left(1 - \frac{\omega_o^2}{\omega^2}\right) = R + j\omega L \frac{\omega^2 - \omega_o^2}{\omega^2}$$

$$\equiv \frac{P_{in}}{|I|^2/2} = \frac{P_{loss} + 2j\omega(W_m - W_e)}{|I|^2/2}, \omega_o = \frac{1}{\sqrt{LC}}$$

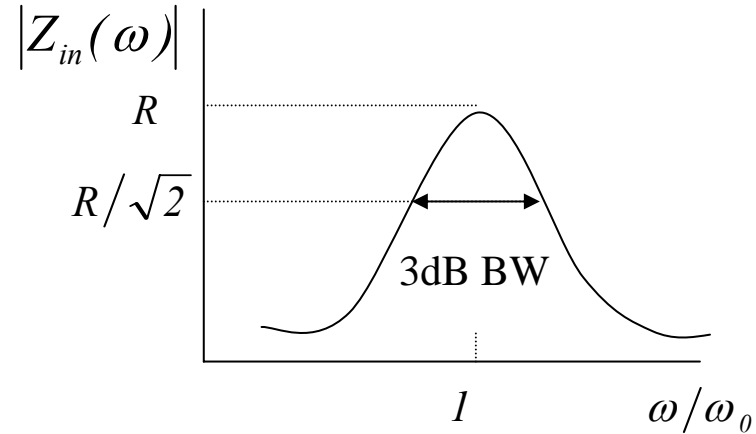
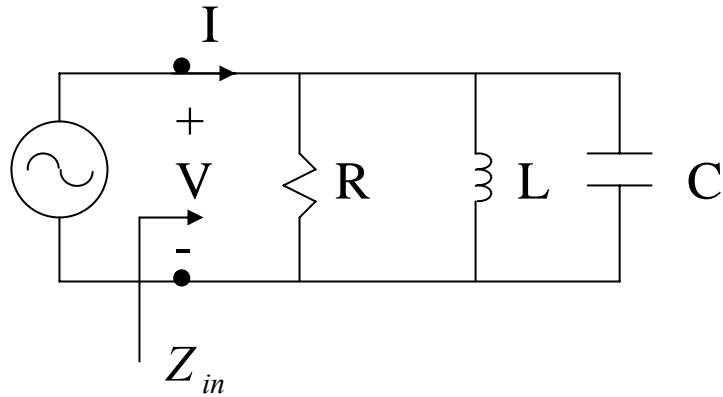
$$P_{loss} = \frac{1}{2}|I|^2 R, W_m = \frac{1}{4}|I|^2 L, W_e = \frac{1}{4}|V_c|^2 C = \frac{1}{4}|I|^2 \frac{1}{\omega^2 C}$$



$$\text{quality factor } Q(\omega) \equiv \omega \frac{\text{average energy stored}}{\text{energy loss/second}} = \omega \frac{W_m + W_e}{P_{loss}}$$

$$Q(\omega_o) = \omega_o \frac{2W_m}{P_{loss}} = \omega_o \frac{2|I^2|L/4}{|I^2|R/2} = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}, R \downarrow (\text{loss } \downarrow) \Rightarrow Q \uparrow$$

- parallel RLC resonator

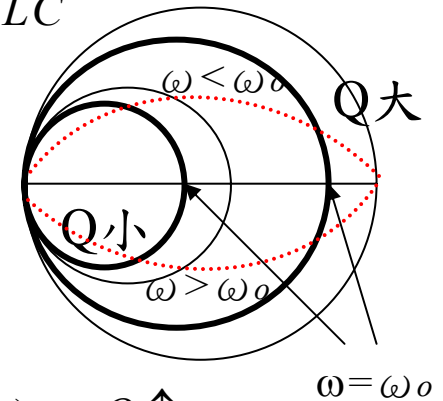


$$Z_{in}(\omega) = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right)^{-1} = \left[\frac{1}{R} + \frac{1}{j\omega L} \left(1 - \frac{\omega^2}{\omega_0^2} \right) \right]^{-1}, \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\equiv \frac{2P_{in}}{|I|^2} = \frac{P_{loss} + 2j\omega(W_m - W_e)}{|I|^2/2}$$

$$P_{loss} = \frac{1}{2} \frac{|V|^2}{R}, W_m = \frac{1}{4} |I_L|^2 L = \frac{1}{4} |V|^2 \frac{1}{\omega^2 L}, W_e = \frac{1}{4} C |V|^2$$

$$Q(\omega_0) = \omega_0 \frac{2W_e}{P_{loss}} = \omega_0 \frac{2C |V|^2 / 4}{|V|^2 / 2R} = \omega_0 RC = \frac{R}{\omega_0 L}, R \uparrow (\text{loss} \downarrow) \Rightarrow Q \uparrow$$



Discussion

1. At resonance, $W_m=W_e$, $P_{in}=P_{loss}$

2. Near resonant frequency

$$\text{series resonator: } Z_{in} \approx R + j2L\Delta\omega = R + j\frac{2RQ\Delta\omega}{\omega_o}, \because Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$$

$$\text{parallel resonator: } Z_{in} \approx \frac{R}{1 + j2\Delta\omega RC} = \frac{R}{1 + j2Q\frac{\Delta\omega}{\omega_o}}, \because Q = \omega_o RC = \frac{R}{\omega_o L}$$

3. Half-power fractional bandwidth

$$P_{in,av} = \frac{1}{2} \text{Re}[VI^*] = \frac{1}{2} \text{Re}\left[\frac{VV^*}{Z_{in}^*}\right] = \frac{1}{2} R_{in} \frac{|V|^2}{|Z_{in}|^2} = \frac{1}{2} \text{Re}\left[\frac{II^*}{Y_{in}}\right] = \frac{1}{2} G_{in} \frac{|I|^2}{|Y_{in}|^2}$$

$$\text{series resonator: @ } \omega_{3dB}, R = X, P_{in,av}(\omega_{3dB}) = \frac{1}{2} R \frac{|V|^2}{|\sqrt{2}R|^2} = \frac{1}{2} P_{in,av}(\omega_o) \rightarrow R = X = \frac{2RQ\Delta\omega}{\omega_o}$$

$$\text{parallel resonator: @ } \omega_{3dB}, G = B, P_{in,av}(\omega_{3dB}) = \frac{1}{2} G \frac{|I|^2}{|\sqrt{2}G|^2} = \frac{1}{2} P_{in,av}(\omega_o) \rightarrow G = B = \frac{2Q\Delta\omega/\omega_o}{R}$$

$$\Rightarrow Q = \frac{\omega_o}{2\Delta\omega} \equiv \frac{1}{BW}$$

(derivation of series resonator case)

$$\begin{aligned}
 Z_{in}(\omega) &= R + j\omega L + \frac{1}{j\omega C} = R + jL\left(\omega - \frac{1}{\omega LC}\right) = R + jL\left(\omega - \frac{\omega_o^2}{\omega}\right) \\
 &= R + jL \frac{\omega^2 - \omega_o^2}{\omega} \stackrel{\omega = \omega_o + \Delta\omega}{=} R + jL \frac{(\omega_o + \Delta\omega)^2 - \omega_o^2}{\omega} = R + jL \frac{2\Delta\omega\omega_o + \Delta\omega^2}{\omega} \\
 &\stackrel{\omega \approx \omega_o}{\approx} R + j2L\Delta\omega
 \end{aligned}$$

(derivation of parallel resonator case)

$$\begin{aligned}
 Z_{in}(\omega) &= \left[\frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right]^{-1} \stackrel{\omega = \omega_o + \Delta\omega}{=} \left[\frac{1}{R} + \frac{1}{j(\omega_o + \Delta\omega)L} + j(\omega_o + \Delta\omega)C \right]^{-1} \\
 \therefore \frac{1}{\omega_o + \Delta\omega} &= \frac{1}{\omega_o(1 + \Delta\omega/\omega_o)} \approx \frac{1}{\omega_o} \left(1 - \frac{\Delta\omega}{\omega_o}\right) = \frac{1}{\omega_o} - \frac{\Delta\omega}{\omega_o^2} \\
 Z_{in}(\omega) &\approx \left[\frac{1}{R} + \frac{1}{j\omega_o L} - \frac{\Delta\omega}{j\omega_o^2 L} + j\omega_o C + j\Delta\omega C \right]^{-1} = \left[\frac{1}{R} + \frac{j\Delta\omega}{\omega_o^2 L} + j\Delta\omega C \right]^{-1} \\
 &= \left[\frac{1}{R} + j2\Delta\omega C \right]^{-1} = \frac{R}{1 + j2\Delta\omega RC}
 \end{aligned}$$

4. Locus of $r=x$ on the Smith chart

(derivation)

$$z = r + jx = \frac{1+\Gamma}{1-\Gamma} = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i} = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} + j \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$r = x \rightarrow 1 - \Gamma_r^2 - \Gamma_i^2 = 2\Gamma_i \rightarrow \Gamma_r^2 + \Gamma_i^2 + 2\Gamma_i + 1 = 2$$

$$\Gamma_r^2 + (\Gamma_i - 1)^2 = (\sqrt{2})^2$$

Locus of $g=b$ on the Smith chart

(derivation)

$$y = g + jb = \frac{1-\Gamma}{1+\Gamma} = \frac{1-\Gamma_r - j\Gamma_i}{1+\Gamma_r + j\Gamma_i} = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1+\Gamma_r)^2 + \Gamma_i^2} - j \frac{2\Gamma_i}{(1+\Gamma_r)^2 + \Gamma_i^2}$$

$$r = x \rightarrow 1 - \Gamma_r^2 - \Gamma_i^2 = -2\Gamma_i \rightarrow \Gamma_r^2 + \Gamma_i^2 - 2\Gamma_i + 1 = 2$$

$$\Gamma_r^2 + (\Gamma_i + 1)^2 = (\sqrt{2})^2$$

5. Lossless resonator \rightarrow lossy resonator: $\omega_o \rightarrow \omega_o(1 + \frac{j}{2Q})$
 (derivation)

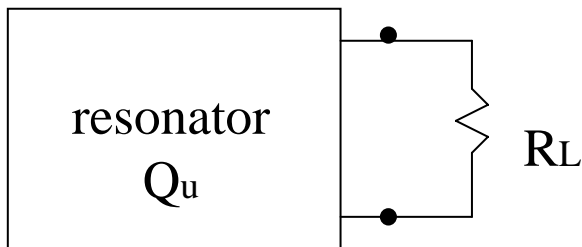
series resonator $Z_{in} = j2\Delta\omega L = j2(\omega - \omega_o)L$

$\omega_o \rightarrow \omega_o(1 + \frac{j}{2Q})$
 $\rightarrow Z_{in} = j2[\omega - \omega_o(1 + \frac{j}{2Q})]L = j2\Delta\omega L + \frac{\omega_o L}{Q} \stackrel{Q = \frac{\omega_o L}{R}}{=} R + j2\Delta\omega L$

parallel resonator $Z_{in} = [j2\Delta\omega C]^{-1} = [j2(\omega - \omega_o)C]^{-1}$

$\rightarrow Z_{in} = j2[\omega - \omega_o(1 + \frac{j}{2Q})]^{-1} C = [j2\Delta\omega C + \frac{\omega_o C}{Q}]^{-1} \stackrel{Q = \omega_o RC}{=} [j2\Delta\omega C + \frac{1}{R}]^{-1}$

6. Unloaded Q, Q_u , loaded Q, Q_L , external Q, Q_e

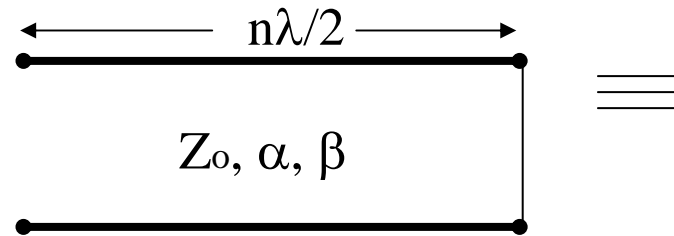


$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_U}$$

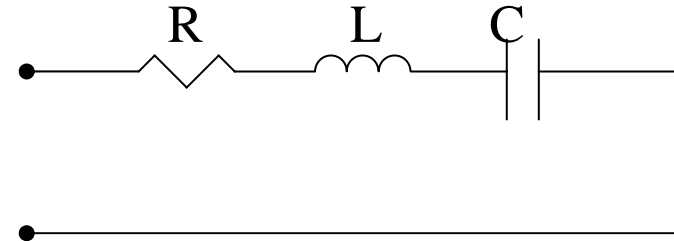
$$Q_e = \begin{cases} \frac{\omega_o L}{R_L} & \text{for series RLC circuit} \\ \frac{R_L}{\omega_o L} & \text{for parallel RLC circuit} \end{cases}$$

6.2 Transmission line resonators

- Short-circuited $\lambda/2$ line



$$Z_{in} \approx Z_o \left(\alpha l + j \frac{\pi \Delta \omega}{\omega_o} \right)$$



$$Z_{in} = R + 2jL\Delta\omega$$

$$R = Z_o \alpha l, L = \frac{Z_o \pi}{2\omega_o}, C = \frac{1}{\omega_o^2 L}$$

(derivation)

$$Z_{in} = Z_o \frac{Z_L + Z_o \tanh \gamma l}{Z_o + Z_L \tanh \gamma l} \Big|_{Z_L=0} = Z_o \tanh(\alpha + j\beta)l = Z_o \frac{\tanh \alpha l + j \tan \beta l}{1 + j \tan \beta l \tanh \alpha l}$$

$$\because \beta l = \frac{\omega}{v_p} \frac{\lambda_o}{2} = \frac{\omega_o + \Delta\omega}{v_p} \frac{v_p}{2f_o} = \frac{\omega_o + \Delta\omega}{2\omega_o / 2\pi} = \pi + \frac{\pi \Delta\omega}{\omega_o}, \tan \beta l = \tan\left(\frac{\pi \Delta\omega}{\omega_o}\right) \approx \frac{\pi \Delta\omega}{\omega_o}, \tanh \alpha l \stackrel{\alpha l \ll 1}{\approx} \alpha l$$

$$Z_{in} \approx Z_o \frac{\alpha l + j \frac{\pi \Delta\omega}{\omega_o}}{1 + j \alpha l \frac{\pi \Delta\omega}{\omega_o}} \approx Z_o \left(\alpha l + j \frac{\pi \Delta\omega}{\omega_o} \right) = R + j2L\Delta\omega, Q = \frac{\omega_o L}{R} = \frac{\omega_o \frac{Z_o \pi}{2\omega_o}}{Z_o \alpha l} = \frac{\pi}{2\alpha l} = \frac{\beta_o l}{2\alpha l} = \frac{\beta_o}{2\alpha}$$

- Open-circuited $\lambda/2$ line

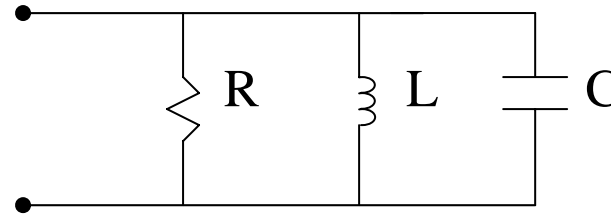
$\longleftarrow n\lambda/2 \longrightarrow$



Z_o, α, β



\equiv



$$Z_{in} \approx \frac{Z_o}{\alpha l + j \frac{\pi \Delta \omega}{\omega_o}}$$

$$Z_{in} = \frac{R}{1 + 2j\Delta\omega RC}$$

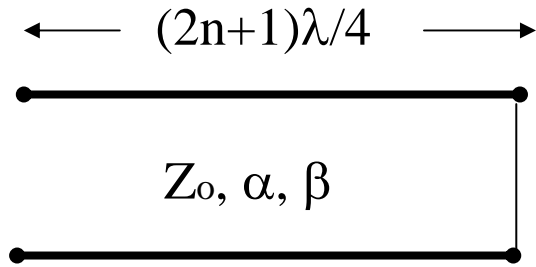
$$R = \frac{Z_o}{\alpha l}, C = \frac{\pi}{2\omega_o Z_o}, L = \frac{1}{\omega_o^2 C}$$

(derivation)

$$Z_{in} = \frac{Z_o}{\tanh(\alpha + j\beta)l} = Z_o \frac{1 + j \tan \beta l \tanh \alpha l}{\tanh \alpha l + j \tan \beta l} \approx Z_o \frac{1 + j \frac{\pi \Delta \omega}{\omega_o} \alpha l}{\alpha l + j \frac{\pi \Delta \omega}{\omega_o}}$$

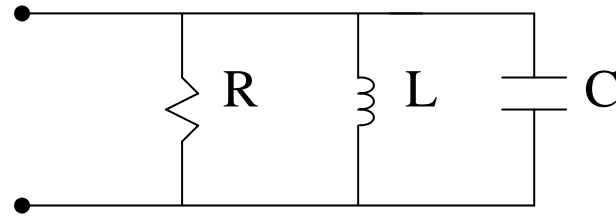
$$\approx \frac{Z_o}{\alpha l + j \frac{\pi \Delta \omega}{\omega_o}} = \frac{R}{1 + j2\Delta\omega RC}, Q = \omega_o RC = \omega_o \frac{Z_o}{\alpha l} \frac{\pi}{2\omega_o Z_o} = \frac{\pi}{2\alpha l} = \frac{\beta_o}{2\alpha}$$

- Short-circuited $\lambda/4$ line



$$Z_{in} \approx \frac{Z_o}{\alpha l + j \frac{\pi \Delta \omega}{2 \omega_o}}$$

≡



$$Z_{in} = \frac{R}{1 + 2j\Delta\omega RC}$$

$$R = \frac{Z_o}{\alpha l}, C = \frac{\pi}{4\omega_o Z_o}, L = \frac{1}{\omega_o^2 C}$$

(derivation)

$$Z_{in} = Z_o \frac{\tanh \alpha l + j \tan \beta l}{1 + j \tan \beta l \tanh \alpha l} \times \frac{j \cot \beta l}{j \cot \beta l} = Z_o \frac{-j \cot \beta l \tanh \alpha l + 1}{-j \cot \beta l + \tanh \alpha l}$$

$$\because \beta l = \frac{\omega}{v_p} \frac{\lambda_o}{4} = \frac{\omega_o + \Delta \omega}{v_p} \frac{v_p}{4 f_o} = \frac{\omega_o + \Delta \omega}{4 \omega_o / 2\pi} = \frac{\pi}{2} + \frac{\pi \Delta \omega}{2 \omega_o}, \cot \beta l = -\tan\left(\frac{\pi \Delta \omega}{2 \omega_o}\right) \approx -\frac{\pi \Delta \omega}{2 \omega_o}$$

$$Z_{in} \approx Z_o \frac{j \frac{\pi \Delta \omega}{2 \omega_o} \alpha l + 1}{j \frac{\pi \Delta \omega}{2 \omega_o} + \alpha l} \approx \frac{Z_o}{\alpha l + j \frac{\pi \Delta \omega}{2 \omega_o}} = \frac{R}{1 + j 2 \Delta \omega RC}, Q = \omega_o RC = \omega_o \frac{Z_o}{\alpha l} \frac{\pi}{4 \omega_o Z_o} = \frac{\pi}{4 \alpha l} = \frac{\beta_o}{2 \alpha}$$

Discussion

1. Transmission line resonator $Q(\omega_o) = \frac{\beta_o}{2\alpha}$

2. Ex. 6.1 $\lambda/4$ coaxial line resonator, $b=4\text{mm}$, $a=1\text{mm}$, $f=5\text{GHz}$,
Teflon $\epsilon_r=2.08$, $\tan\delta=0.0004$, calculate Q_{air} and Q_{Teflon}

$$Q = \frac{\beta_o}{2\alpha}, \quad \beta_o = \frac{2\pi f_o \sqrt{\epsilon_r}}{c} = \begin{cases} 104.7 & \text{air} \\ 104.7\sqrt{2.08} & \text{Teflon} \end{cases}, \quad \alpha = \alpha_c + \alpha_d$$

$$\alpha_c = \frac{R_s}{2\eta \ln \frac{b}{a} \left(\frac{1}{a} + \frac{1}{b}\right)} = 0.022 \text{ Np/m}, \quad R_s = \sqrt{\frac{\omega\mu_o}{2\sigma}} = 1.84 \times 10^{-2} \Omega$$

$\sigma = 5.813 \times 10^7 \text{ S/m}$ copper

$$\text{Teflon } \alpha_d = \frac{k_o \sqrt{\epsilon_r}}{2} \tan \delta = 0.03 \text{ Np/m}$$

$$\Rightarrow Q = \begin{cases} \frac{104.7}{2 \times 0.022} = 2380 & \text{air} \\ \frac{104.7\sqrt{2.08}}{2 \times (0.022 + 0.03)} = 1218 & \text{Teflon} \end{cases}$$

3. Ex. 6.2 $\lambda/2$ open-circuited microstrip resonator, $Z_o=50\Omega$,
 $h=1.59\text{mm}$, Teflon substrate $\epsilon_r=2.08$, $\tan\delta=0.0004$, $f=5\text{GHz}$,
 calculate resonator length and Q .

$$Z_o = 50 \rightarrow W = 5.08\text{mm}, \quad \epsilon_{eff} = 1.8$$

$$l = \frac{\lambda_o}{2} = \frac{c}{2f\sqrt{\epsilon_{eff}}} = 2.24\text{cm}$$

$$\beta_o = \frac{2\pi f_o\sqrt{\epsilon_{eff}}}{c} = 151\text{rad/m},$$

$$\alpha_c = \frac{R_s}{Z_o W} = 0.0724\text{Np/m},$$

$$\alpha_d = \frac{k_o \epsilon_r (\epsilon_{eff} - 1)}{2\sqrt{\epsilon_{eff}} (\epsilon_r - 1)} \tan \delta = 0.024\text{Np/m}$$

$$\Rightarrow Q = \frac{\beta_o}{2\alpha} = 783$$

4. For comparison,

Ex. 6.3 rectangular waveguide cavity resonator, $\epsilon_r=2.25$ (polyethylene),
 $\tan\delta=0.0004$, $f=5\text{GHz}$.

TE₁₀₁ mode $d=2.2\text{ cm}$, $Q_c=8403$, $Q_d=2500$, $Q=1927$

TE₁₀₂ mode $d=4.4\text{ cm}$, $Q_c=11898$, $Q_d=2500$, $Q=2065$

Ex. 6.4 Teflon-filled cylindrical cavity, $\epsilon_r=2.08$, $\tan\delta=0.0004$, $f=5\text{GHz}$.

$a=2.74\text{cm}$, $d=2a=5.48\text{cm}$, TE₀₁₁ mode $Q_c=29390$, $Q_d=2500$, $Q=2300$

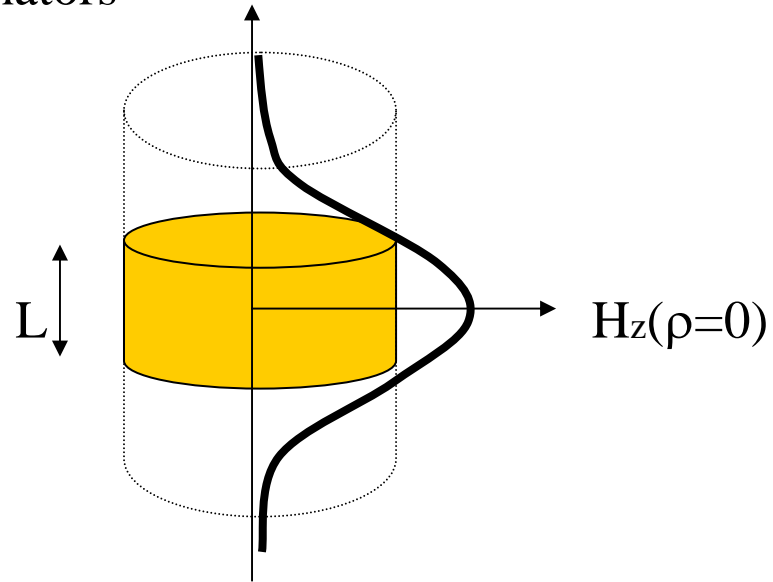
air-filled cylindrical cavity, TE₀₁₁ mode $a=3.96\text{cm}$, $d=2a=7.91\text{cm}$,
 $Q_c=42400$

5. In general

$Q_{\text{spherical}} > Q_{\text{cylindrical}} > Q_{\text{rectangular}} > Q_{\text{coaxial}} > Q_{\text{microstrip}}$

6. Application of resonator: frequency selective component (ω_o , Q)

6.5 Dielectric resonators



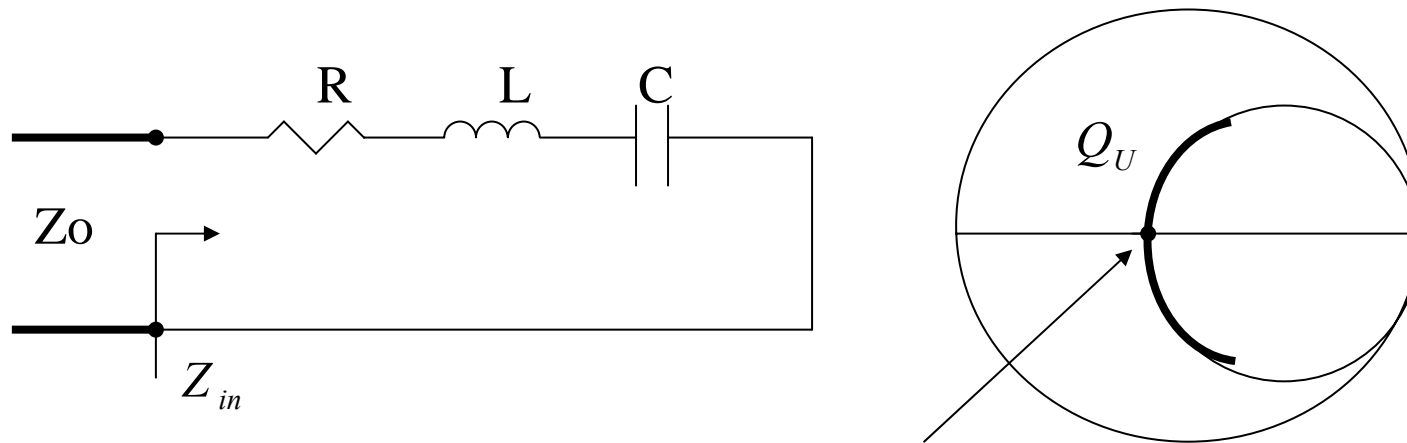
$$10 < \epsilon_r < 100, \quad TE_{01\delta} \text{ mode } \delta = \frac{2L}{\lambda_g} < 1, \quad Q_d \approx \frac{1}{\tan \delta}$$

$$\text{Ex. 6.5 } \epsilon_r = 95, \quad \tan \delta = 0.001, \quad a = 0.413 \text{ cm}$$

$$\rightarrow f = 3.4 \text{ GHz}, \quad Q_d = 1000$$

6.6 Excitation of resonators

- Critical coupling $Z_{in}(\omega_o) = Z_o$

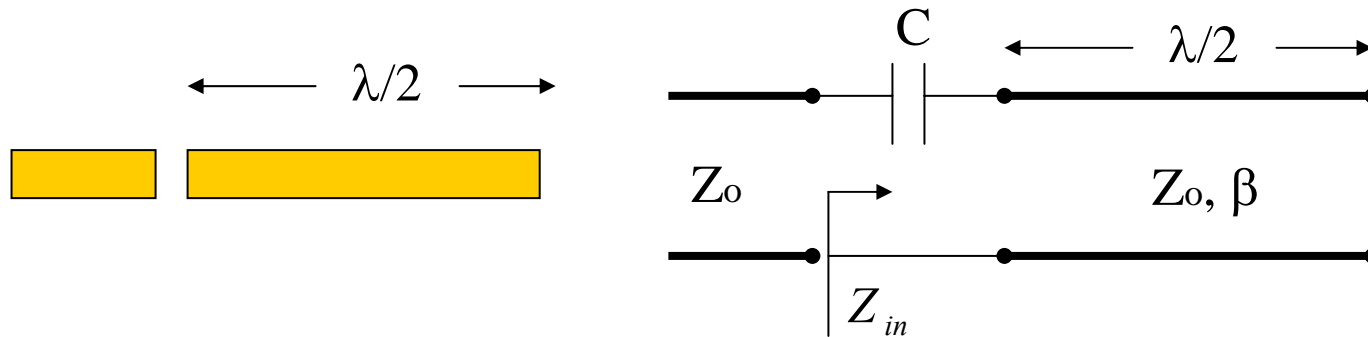


For series RLC resonator, at resonance $R = Z_o$

$$\rightarrow Q_U = Q_e, Q_L = \frac{Q_U}{2} \quad (Q_U(\omega_o) = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}, Q_e(\omega_o) = \frac{\omega_o L}{Z_o} = \frac{1}{\omega_o Z_o C})$$

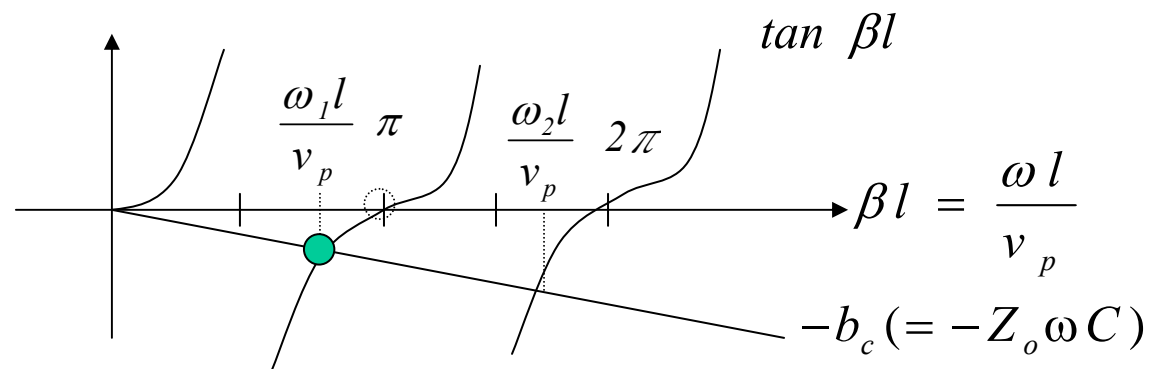
$$\text{coefficient of coupling } g \equiv \frac{Q_U}{Q_e} (= \frac{Z_o}{R} = 1 \text{ for critical coupling})$$

- Gap-coupled $\lambda/2$ open-circuited microstrip resonator



$$Z_{in} \Big|_{\omega=\omega_1} = \frac{Z_{in}}{Z_o} \Big|_{\omega=\omega_1} = \frac{1}{Z_o} \left(\frac{1}{j\omega C} + \frac{Z_o}{j \tan \beta l} \right) \Big|_{\omega=\omega_1} = -j \frac{\tan \beta l + b_c}{b_c \tan \beta l} \Big|_{\omega=\omega_1} = 0, b_c = Z_o \omega C$$

\Rightarrow solve the resonant frequency ω_1



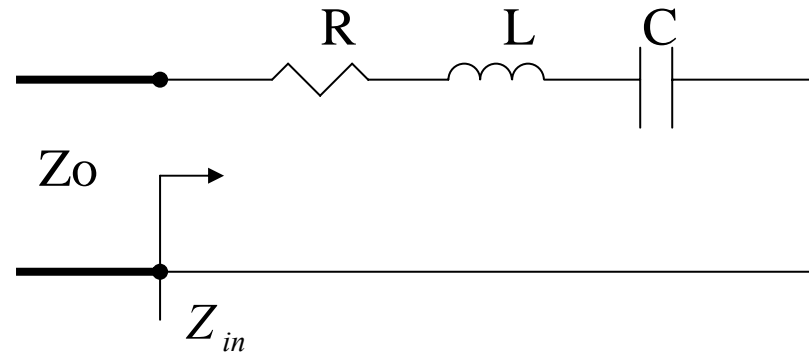
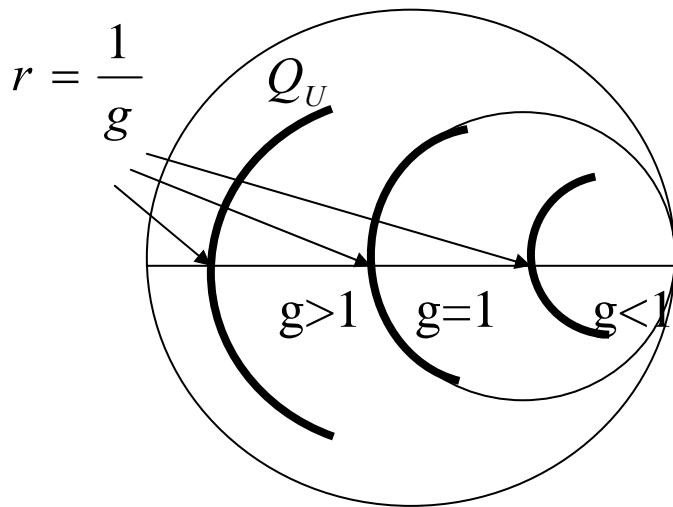
Discussion

1.

$$g = \frac{Q_U}{Q_e} = \begin{cases} \frac{Z_o}{R} & \text{series resonator} \\ \frac{R}{Z_o} & \text{parallel resonator} \end{cases}$$

$$Q_L = \frac{Q_U}{1+g}$$

$$g \begin{cases} < 1 & \text{under coupling } Q_L \rightarrow Q_U \\ = 1 & \text{critical coupling } Q_L = Q_U / 2 \\ > 1 & \text{over coupling } Q_L \ll Q_U \end{cases}$$



for series resonator

2. Types of excitation for microwave resonators (p.291, Fig.6.13)

E coupling \Rightarrow C, H coupling \Rightarrow L

3. Gap-coupled $\lambda/2$ open-circuited microstrip resonator

$$z_{in}(\omega) = \frac{Z_{in}}{Z_o} = \begin{cases} \frac{j\pi(\omega - \omega_1)}{\omega_1 b_c^2} & \text{lossless} \\ \frac{\pi}{2Q_U b_c^2} + \frac{j\pi(\omega - \omega_1)}{\omega_1 b_c^2} (= \frac{R + j2L\Delta\omega}{Z_o}) & \text{lossy} \end{cases}$$

$\lambda/2$ open-circuited microstrip resonator (parallel resonator)

\rightarrow gap-coupling \rightarrow series resonator

(derivation) $z_{in}(\omega) \approx z_{in}(\omega_1) + (\omega - \omega_1) \left. \frac{dz_{in}}{d\omega} \right|_{\omega_1} \approx (\omega - \omega_1) \left. \frac{dz_{in}}{d\omega} \right|_{\omega_1}$

$$z_{in} = -j \frac{\tan \beta l + b_c}{b_c \tan \beta l}, \left. \frac{dz_{in}}{d\omega} \right|_{\omega_1} = \frac{dz_{in}}{d\beta l} \left. \frac{d\beta l}{d\omega} \right|_{\omega_1} = j \frac{1 + b_c^2}{b_c^2} \frac{l}{v_p} \Big|_{\omega_1} \approx \frac{j}{b_c^2} \frac{l}{v_p} \Big|_{\omega_1} \approx \frac{j\pi}{\omega_1 b_c^2}$$

$$\therefore l = \frac{\lambda_o}{2} \approx \frac{\lambda_1}{2} = \frac{v_p}{2f_1} = \frac{\pi v_p}{\omega_1} \rightarrow z_{in}(\omega) = \frac{j\pi}{\omega_1 b_c^2} (\omega - \omega_1) : \text{lossless}$$

$$\text{lossy: } \omega_1 \rightarrow \omega_1 \left(1 + \frac{j}{2Q}\right), z_{in}(\omega) = \frac{j\pi}{\omega_1 b_c^2} \left[\omega - \omega_1 \left(1 + \frac{j}{2Q}\right)\right] = \frac{j\pi(\omega - \omega_1)}{\omega_1 b_c^2} + \frac{\pi}{2Q_U b_c^2}$$

4.

$$Z_{in}(\omega_1) = R = \frac{\pi Z_o}{2Q_U b_c^2}, g = \frac{Z_o}{R} = \frac{2Q_U b_c^2}{\pi} \left\{ \begin{array}{l} < 1 \rightarrow b_c < \sqrt{\frac{\pi}{2Q_U}} \quad :C \text{小, gap 大} \\ = 1 \rightarrow b_c = \sqrt{\frac{\pi}{2Q_U}} \\ > 1 \rightarrow b_c > \sqrt{\frac{\pi}{2Q_U}} \quad :C \text{大, gap 小} \end{array} \right.$$

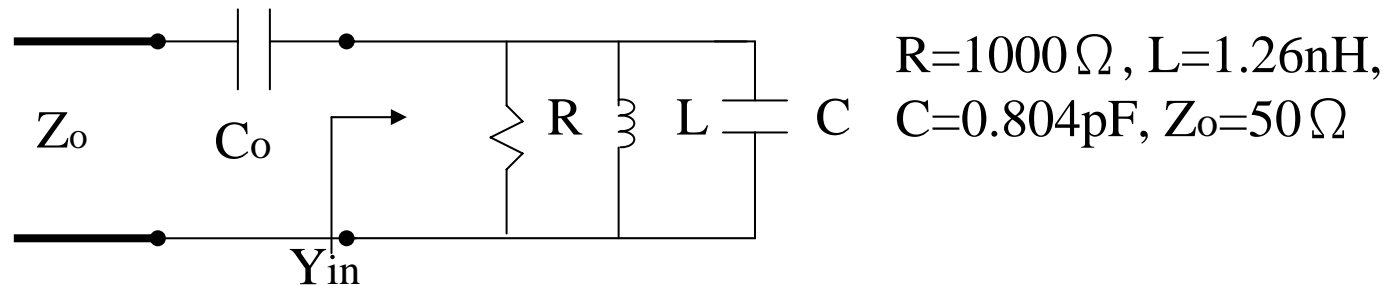
5. Ex. 6.6 $\lambda/2$ open-circuited microstrip resonator, $Z_o=50\Omega$, $l=2.175\text{cm}$, $\epsilon_{\text{eff}}=1.9$, $\alpha=0.01\text{dB/cm}$, calculate C for critical coupling and f_1 .

$$\text{uncoupled line: } l = n \frac{\lambda_g}{2} = \frac{c}{2f \sqrt{\epsilon_{\text{eff}}}} \rightarrow f_o = 5\text{GHz}$$

$$Q_U = \frac{\beta}{2\alpha} = \frac{2\pi}{2\lambda_g \alpha} = \frac{\pi}{2l\alpha} = 628, b_c = \sqrt{\frac{\pi}{2Q_U}} = 0.05 \rightarrow C = \frac{b_c}{\omega Z_o} = 0.032\text{pF}$$

$$\tan \beta l + b_c = 0 \rightarrow f_1 = 4.918\text{GHz}$$

Solved problems: Prob. 6.22 A parallel resonator, calculate C_o for critical coupling and f_o .



$$Y_{in}(\omega) \approx \frac{1}{R} + j \frac{2Q_u \Delta\omega}{R\omega_o} = 10^{-3} + j25.2 \times 10^{-3} \frac{\Delta\omega}{\omega_o}, Q_u = \frac{R}{\omega_o L} = 25.3, \omega_o = \frac{1}{\sqrt{LC}} = 31.4 \times 10^9$$

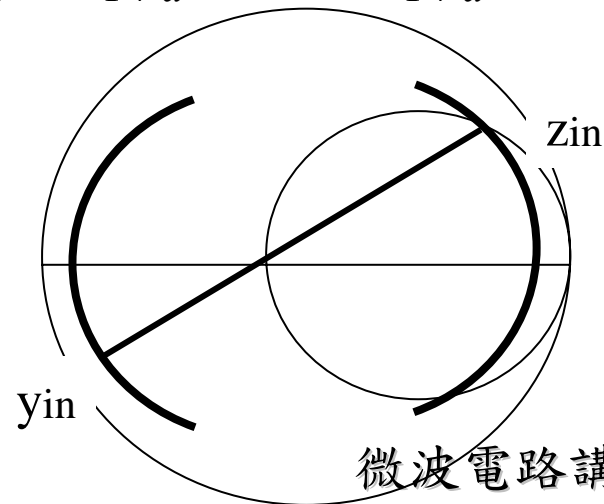
$$\rightarrow y_{in} = 50 \times Y_{in} = 0.05 + j2.53 \frac{\Delta\omega}{\omega_o} = \frac{1}{1+ja} = \frac{1}{1+a^2} - j \frac{a}{1+a^2} \rightarrow 0.05 = \frac{1}{1+a^2} \rightarrow a = 4.36$$

$$\because z_{in} + \frac{1}{j\omega CZ_o} = 1 \rightarrow z_{in} = 1 + j \frac{1}{\omega CZ_o} = 1 + ja$$

$$-\frac{a}{1+a^2} = -\frac{4.36}{20} = 2.53 \frac{\Delta\omega}{\omega_o} \rightarrow \frac{\Delta\omega}{\omega_o} = -0.086$$

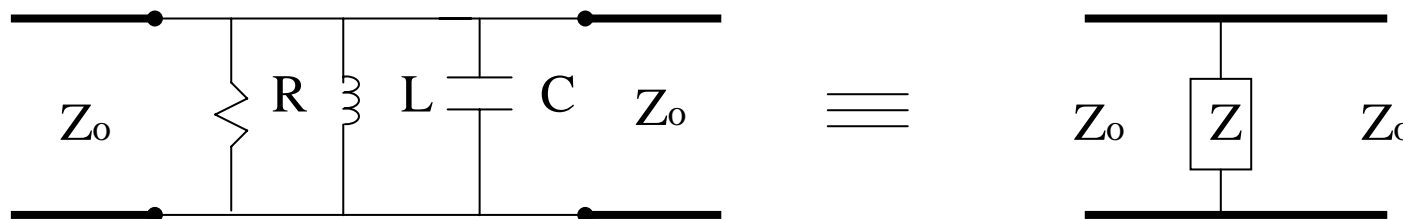
$$\rightarrow f_r = f_o - 0.086 f_o = 4.57\text{GHz}$$

$$\frac{1}{\omega_o CZ_o} = 4.36 \rightarrow C = 0.146\text{pF}$$

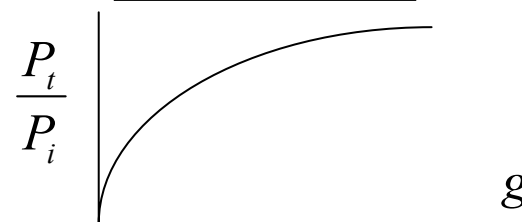


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Prob. 6.25 A two-port transmission resonator (Q, ω_o, g), find P_t/P_i in terms of g



$$Z \approx \frac{R}{1 + j \frac{2Q\Delta\omega}{\omega_o}}, \quad \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$$



$$\frac{P_t}{P_i} = |T|^2 = \left| \frac{2}{A + B/Z_o + CZ_o + D} \right|^2 = \left| \frac{2}{2 + Z/Z_o} \right|^2, \quad \therefore \frac{Z_o}{Z} = \frac{Z_o}{R} + j \frac{2Q\Delta\omega Z_o}{R\omega_o} \rightarrow \frac{Z_o}{Z} = \frac{1}{g} + j \frac{2Q\Delta\omega}{g\omega_o}$$

$$g = \frac{R}{Z_o}$$

$$= \frac{4}{\left(2 + \frac{1}{g}\right)^2 + \left(\frac{2Q\Delta\omega}{g\omega_o}\right)^2} = \frac{4g^2}{(1 + 2g)^2 + \left(\frac{2Q\Delta\omega}{\omega_o}\right)^2}, \quad \frac{P_t}{P_i}(\omega_o) = \left(\frac{2g}{1 + 2g}\right)^2$$

Suggested homework (due 2 weeks): 4, 6, 8

ADS examples: Ch6_prj