Chapter 7 Power dividers and directional couplers

7.1 Basic properties of dividers and couplers
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7.2 The T-junction power divider
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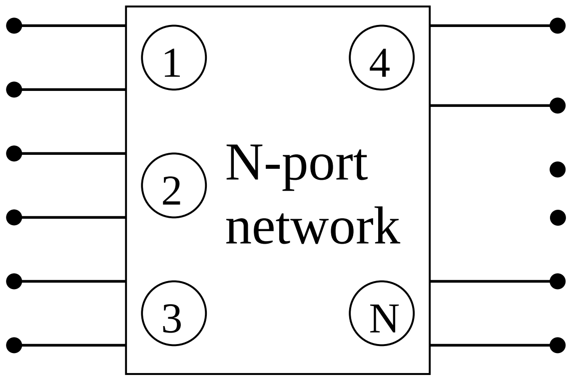
7.1 Basic properties of dividers and couplers

• N-port network



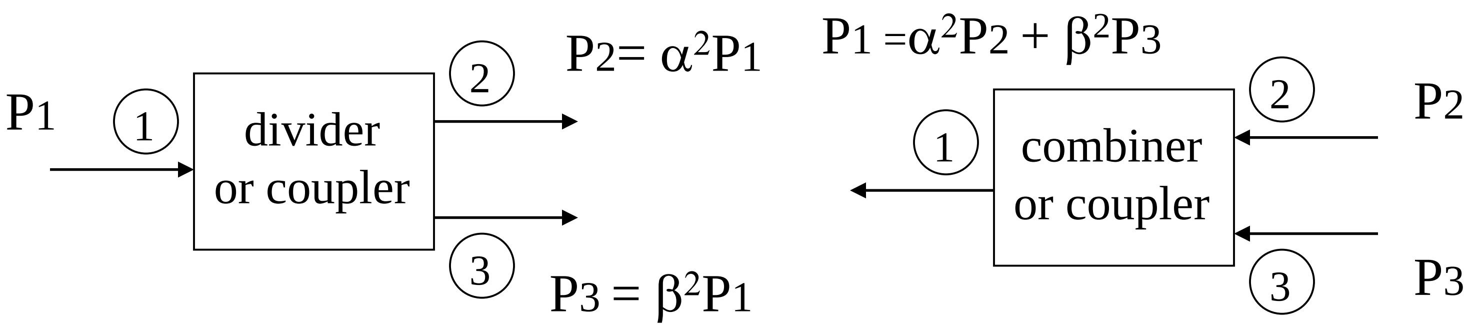
$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & \bullet & S_{1N} \\ S_{21} & S_{22} & S_{23} & \bullet & S_{2N} \\ S_{31} & S_{32} & S_{33} & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ S_{N1} & S_{N2} & \bullet & \bullet & S_{NN} \end{bmatrix} \qquad S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+=0,\,k\neq j}$$

<u>Discussion</u>

1. matched ports → $S_{ii} = 0$
2. reciprocal network → symmetric property $S_{ij} = S_{ji}$
3. lossless network → unitary property

$$\sum_{i=1}^{N} \left|S_{ij}\right|^2 = 1 \quad \forall j, \qquad \sum_{k=1}^{N} S_{ki} S_{kj}^{*} = 0 \quad i \neq j$$

- three-port network (T-junction)

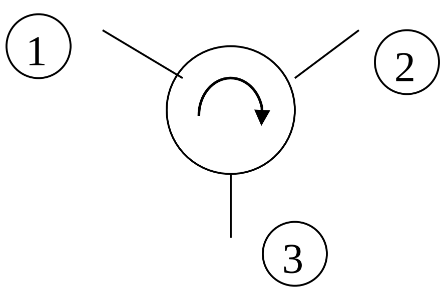


$P_2 = \alpha^2 P_1$, $P_3 = \beta^2 P_1$

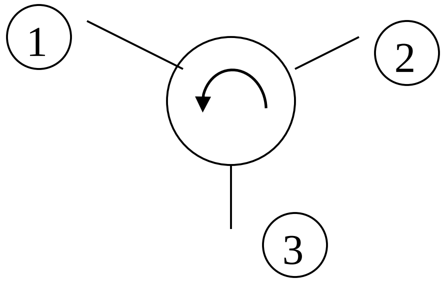
$P_1 = \alpha^2 P_2 + \beta^2 P_3$

<u>Discussion</u>

1. Three-port network cannot be lossless, reciprocal and matched at all ports.
2. A lossless and matched three-port network is nonreciprocal
   → circulator



$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

3. A matched and reciprocal three-port network is lossy $\rightarrow$ resistive divider
4. A matched and lossy three-port network can have ∞ isolation at two output ports ($S_{23}=S_{32}=0$) $\rightarrow$ Wilkinson power divider

(derivation of 1)

For a matched, reciprocal three-port network

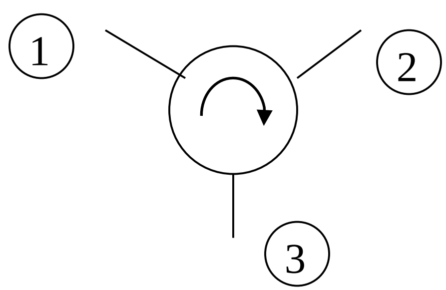
$$\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \xrightarrow{lossless} \begin{matrix} |S_{12}|^2 + |S_{13}|^2 = 1 & S_{13}^* S_{23} = 0 \\ |S_{12}|^2 + |S_{23}|^2 = 1 & , S_{12}^* S_{13} = 0 \\ |S_{13}|^2 + |S_{23}|^2 = 1 & S_{12}^* S_{23} = 0 \end{matrix} \rightarrow \text{if } S_{13} = 0, \begin{matrix} |S_{12}| = 1 \\ |S_{23}| = 0 \\ |S_{13}| = 1 \neq 0 \end{matrix} \rightarrow \text{lossy}$$

(derivation of 2)

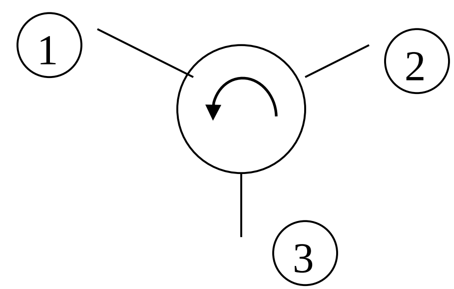
For a matched, lossless, nonreciprocal three-port network

$$\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \xrightarrow{lossless} \begin{matrix} |S_{21}|^2 + |S_{31}|^2 = 1 & S_{31}^* S_{32} = 0 \\ |S_{12}|^2 + |S_{32}|^2 = 1 & , S_{12}^* S_{13} = 0 \\ |S_{13}|^2 + |S_{23}|^2 = 1 & S_{21}^* S_{23} = 0 \end{matrix} \rightarrow \begin{matrix} \text{if } S_{21} = 1 \rightarrow S_{31} = 0, S_{23} = 0, S_{13} = 1, S_{12} = 0, S_{32} = 1 \\ \text{if } S_{21} = 0 \rightarrow S_{31} = 1, S_{32} = 0, S_{12} = 1, S_{13} = 0, S_{23} = 1 \end{matrix}$$

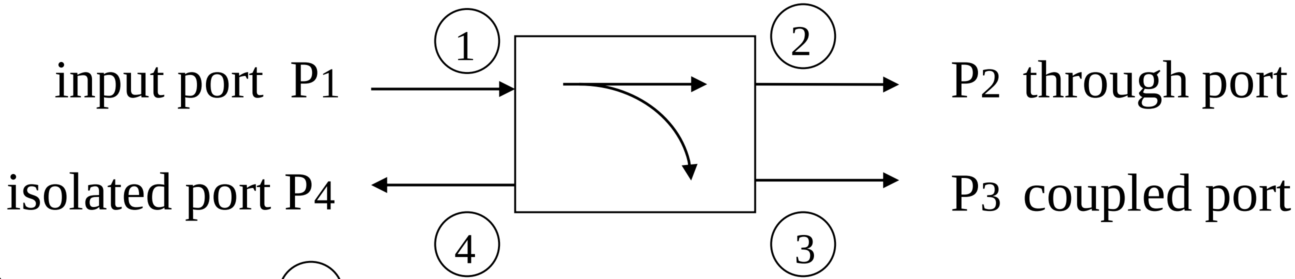
$$\Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

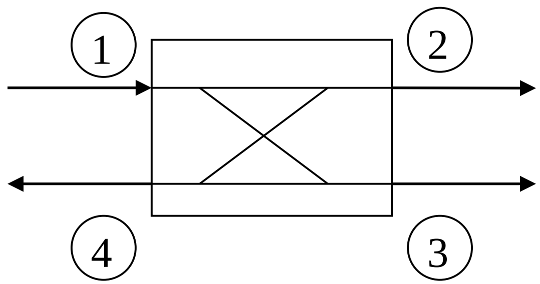


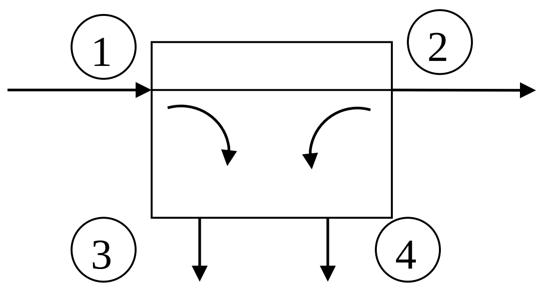
$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



- four-port network (directional coupler)







coupling $C(dB) \equiv 10\log\dfrac{P_1}{P_3} = -20\log|S_{31}| \ (>0dB)$

directivity $D(dB) \equiv 10\log\dfrac{P_3}{P_4} = 10\log\dfrac{P_3}{P_1}\dfrac{P_1}{P_4} = 20\log\dfrac{|S_{31}|}{|S_{41}|} \ (>0dB)$

isolation $I(dB) \equiv 10\log\dfrac{P_1}{P_4} = C + D = -20\log|S_{41}| \ (>0dB)$

voltage coupling factor $C = 10^{-C(dB)/20} = \left|\dfrac{V_3^-}{V_1^+}\right| < 1$

directivity $D = 10^{D(dB)/20} = \left|\dfrac{V_3^-}{V_4^-}\right| > 1$

i/p $\begin{matrix}\text{port 1}\\ \text{port 2}\end{matrix} \rightarrow$ coupled $\begin{matrix}\text{port 3}\\ \text{port 4}\end{matrix} \Rightarrow$ directivity $\begin{matrix}P_3/P_4\\ P_4/P_3\end{matrix}$: a measure of the coupler's ability to isolate forward and backward waves (or the coupled and isolated ports), the ability to differentiate two input signals from opposite directions

Discussion

1. Matched, reciprocal and lossless four-port network → symmetrical (90°) directional coupler or antisymmetrical (180°) directional coupler

$$\begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

2. C=3dB → 90° hybrid (quadrature hybrid, symmetrical coupler), 180° hybrid (magic-T hybrid, rate-race hybrid)

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

(derivation of 1)

For a matched, reciprocal and lossless four-port network

$$\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix} \rightarrow \begin{array}{l} \text{row } 1^*, 2: S_{13}^* S_{23} + S_{14}^* S_{24} = 0 \ldots (1) \\ \text{row } 3, 4^*: S_{14}^* S_{13} + S_{24}^* S_{23} = 0 \ldots (2) \\ \text{row } 1^*, 3: S_{12}^* S_{23} + S_{14}^* S_{34} = 0 \ldots (3) \\ \text{row } 2, 4^*: S_{14}^* S_{12} + S_{34}^* S_{23} = 0 \ldots (4) \end{array}$$

$$(1)S_{24}^* - (2)S_{13}^* \rightarrow S_{14}^*(|S_{13}|^2 - |S_{24}|^2) = 0$$

$$(3)S_{12} - (4)S_{34} \rightarrow S_{23}(|S_{12}|^2 - |S_{34}|^2) = 0$$

$$\text{case 1}: S_{14} = S_{23} = 0 \rightarrow \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix} \rightarrow \begin{array}{l} |S_{12}|^2 + |S_{13}|^2 = 1 \\ |S_{12}|^2 + |S_{24}|^2 = 1 \\ |S_{13}|^2 + |S_{34}|^2 = 1 \\ |S_{24}|^2 + |S_{34}|^2 = 1 \end{array} \rightarrow \begin{array}{l} |S_{13}| = |S_{24}| \\ |S_{12}| = |S_{34}| \end{array} \xrightarrow{choose} \begin{array}{l} S_{12} = S_{34} = \alpha \\ S_{13} = \beta e^{j\theta} \\ S_{24} = \beta e^{j\phi} \\ \alpha^2 + \beta^2 = 1 \end{array}$$

$$\text{row } 2^*, 3 \rightarrow S_{12}^* S_{13} + S_{24}^* S_{34} = 0 \rightarrow e^{j\theta} + e^{-j\phi} = 0 \rightarrow \theta + \phi = \pm\pi$$

(a) 90^o directional coupler with $\theta = \phi = \pm\dfrac{\pi}{2}$

$$\begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}, \begin{bmatrix} 0 & \alpha & -j\beta & 0 \\ \alpha & 0 & 0 & -j\beta \\ -j\beta & 0 & 0 & \alpha \\ 0 & -j\beta & \alpha & 0 \end{bmatrix}$$

(b) 180^o directional coupler with $\theta = 0, \phi = \pm\pi$

$$\begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix} \rightarrow \begin{array}{l} \text{row } 1^*, 2: S_{13}^* S_{23} + S_{14}^* S_{24} = 0 \ldots (1) \\ \text{row } 3, 4^*: S_{14}^* S_{13} + S_{24}^* S_{23} = 0 \ldots (2) \\ \text{row } 1^*, 3: S_{12}^* S_{23} + S_{14}^* S_{34} = 0 \ldots (3) \\ \text{row } 2, 4^*: S_{14}^* S_{12} + S_{34}^* S_{23} = 0 \ldots (4) \end{array}$$

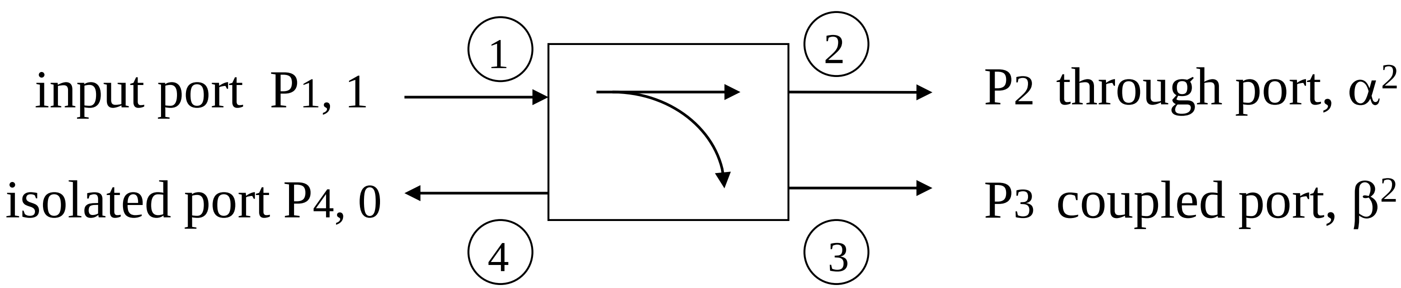
$$\xrightarrow{(1)S_{24}^* - (2)S_{13}^*} S_{14}^*\left(|S_{13}|^2 - |S_{24}|^2\right) = 0$$

$$\xrightarrow{(3)S_{12} - (4)S_{34}} S_{23}\left(|S_{12}|^2 - |S_{34}|^2\right) = 0$$

$$\text{case 2: if } \begin{array}{l} |S_{13}| = |S_{24}| \\ |S_{12}| = |S_{34}| \end{array} \xrightarrow{choose} \begin{array}{l} S_{13} = S_{24} = j\beta \\ S_{12} = S_{34} = \alpha \end{array} \rightarrow \begin{array}{l} (1): j\beta(-S_{23} + S_{14}^*) = 0 \\ (3): \alpha(S_{23} + S_{14}^*) = 0 \end{array}$$

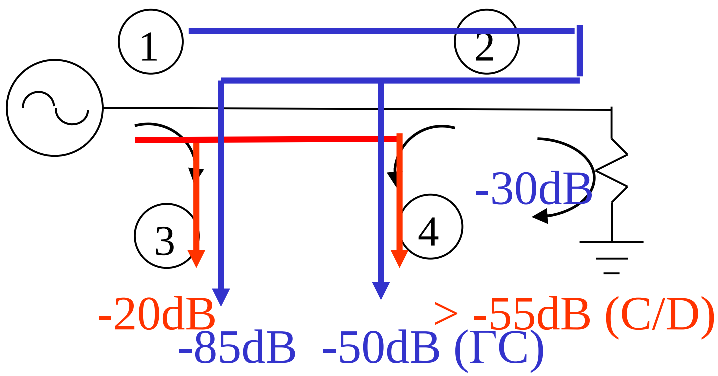
$(a)\ S_{14} = S_{23} = 0 \rightarrow$ directional coupler as case 1

$$(b)\alpha = \beta = 0 \rightarrow \begin{bmatrix} 0 & 0 & 0 & S_{14} \\ 0 & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & 0 \\ S_{14} & 0 & 0 & 0 \end{bmatrix}, \text{two decoupled two-port networks}$$



3. directivity measurement

If C=20dB, D=35dB, RL=30dB (require <D=35dB!)



1→3 (C, -20dB)→4 (C/D, -55dB)

$$V_3^- = CV_1^+,\ D = \frac{V_3^-}{V_4^-} \rightarrow V_4^- = \frac{C}{D}V_1^+$$

1→2→4 (ΓC, -50dB)→3 (ΓC/D, -85dB)

$$V_4^- \approx C\Gamma V_1^+ \left(= C\Gamma\sqrt{1-C^2}V_1^+\right),\ V_3^- \approx \frac{C\Gamma}{D}V_1^+$$

For $|\Gamma| < \frac{1}{D}$

with the use of a sliding load $|\Gamma|e^{j\angle\Gamma}$

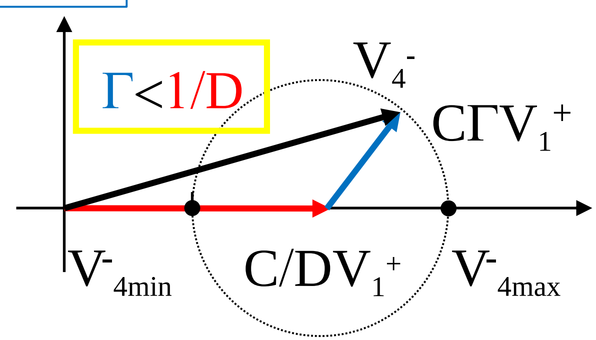
$$P_{4\max} = P_1\left(\frac{C}{D} + C|\Gamma|\right)^2$$

$$P_{4\min} = P_1\left(\frac{C}{D} - C|\Gamma|\right)^2$$

$$P_3 = C^2P_1,\ M = \sqrt{\frac{P_3}{P_{4\max}}} = \frac{D}{1+|\Gamma|D}$$

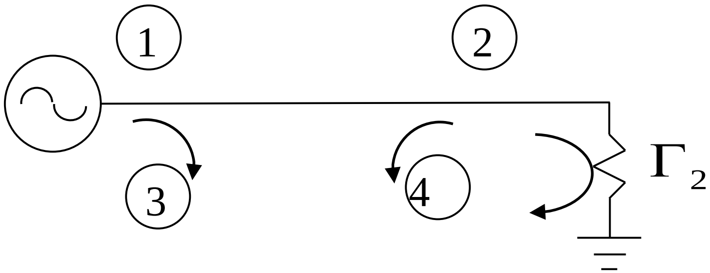
$$m = \sqrt{\frac{P_{4\max}}{P_{4\min}}} = \frac{1+|\Gamma|D}{1-|\Gamma|D}$$

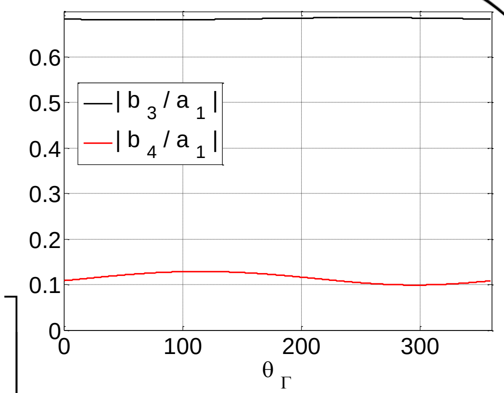
$$\Rightarrow D = M\frac{2m}{m+1}$$



another solution : 3 *different* $\angle\Gamma \rightarrow$ *circle of* V_4^-

$\rightarrow$ *center point* $\frac{C}{D}V_1^+ \rightarrow D$: *directivity*





$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \Rightarrow \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{bmatrix} a_1 \\ \Gamma_2 b_2 \\ 0 \\ 0 \end{bmatrix}$$

$$b_1 = S_{11}a_1 + S_{12}\Gamma_2 b_2$$
$$b_2 = S_{12}a_1 + S_{22}\Gamma_2 b_2$$
$$b_3 = S_{13}a_1 + S_{23}\Gamma_2 b_2$$
$$b_4 = S_{14}a_1 + S_{24}\Gamma_2 b_2$$
$$b_2 = \frac{S_{12}}{1 - S_{22}\Gamma_2} a_1$$

$$\frac{b_3}{a_1} = S_{13} + S_{23}\Gamma_2 \frac{S_{12}}{1 - S_{22}\Gamma_2}$$
$$\frac{b_4}{a_1} = S_{14} + S_{24}\Gamma_2 \frac{S_{12}}{1 - S_{22}\Gamma_2}$$

$$\text{Given} \begin{bmatrix} 0.077\angle 69^o & 0.686\angle 0.8^o & 0.684\angle -89^o & 0.114\angle 24^o \\ 0.686\angle 0.8^o & 0.092\angle 54^o & 0.111\angle 19^o & 0.686\angle -92^o \\ 0.684\angle -89^o & 0.111\angle 19^o & 0.106\angle 72^o & 0.680\angle 0.1^o \\ 0.114\angle 24^o & 0.686\angle -92^o & 0.680\angle 0.1^o & 0.114\angle 64^o \end{bmatrix}$$

$$\Rightarrow |S_{31}| = 0.684, C = 3.3dB, \frac{|S_{31}|}{|S_{41}|} = 6, D = 15.56dB$$

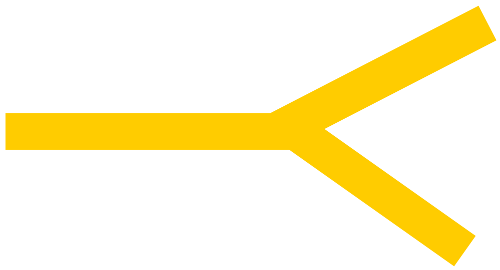
$$RL = 30dB(< D = 15.56dB), \Gamma_2 = 10^{-\frac{30}{20}} e^{j\theta_\Gamma}$$

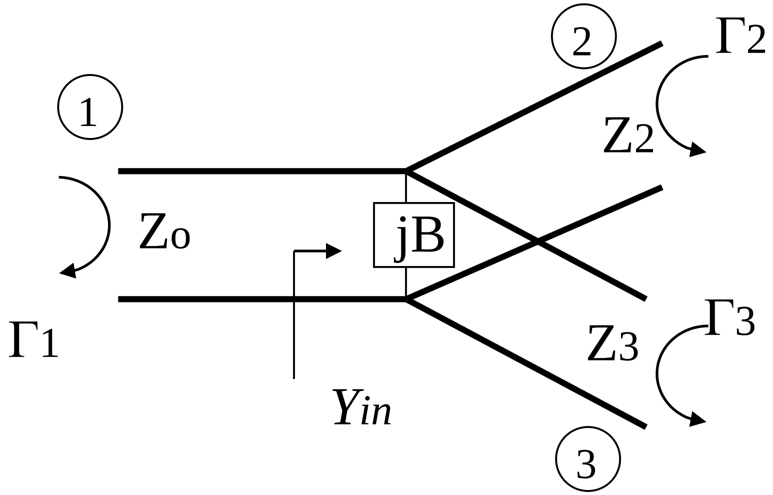
$$M = \frac{\text{mean}\left(\left|\frac{b_3}{a_1}\right|\right)}{\max\left(\left|\frac{b_4}{a_1}\right|\right)} = \frac{0.6840}{0.1288} = 5.3089, m = \frac{\max\left(\left|\frac{b_4}{a_1}\right|\right)}{\min\left(\left|\frac{b_4}{a_1}\right|\right)} = \frac{0.1288}{0.0990} = 1.3004$$

$$D = M\frac{2m}{m+1} = 6.0022,\ 20\log 6 = 15.56\text{dB}$$

7.2 The T-junction power divider

- lossless divider





$$Y_{in} = jB + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{Z_o} \quad \rightarrow \quad B = 0 \quad \text{"not practical"}$$

$\Rightarrow$ A lossless divider has mismatched ports.

Discussion

1. Ex. 7.1 Z_o=50Ω, P_2:P_3=1:2, calculate Γ_2 and Γ_3.

$$P_1 : P_2 : P_3 = 1 : \frac{1}{3} : \frac{2}{3} = \frac{1}{Z_o} : \frac{1}{Z_2} : \frac{1}{Z_3}$$

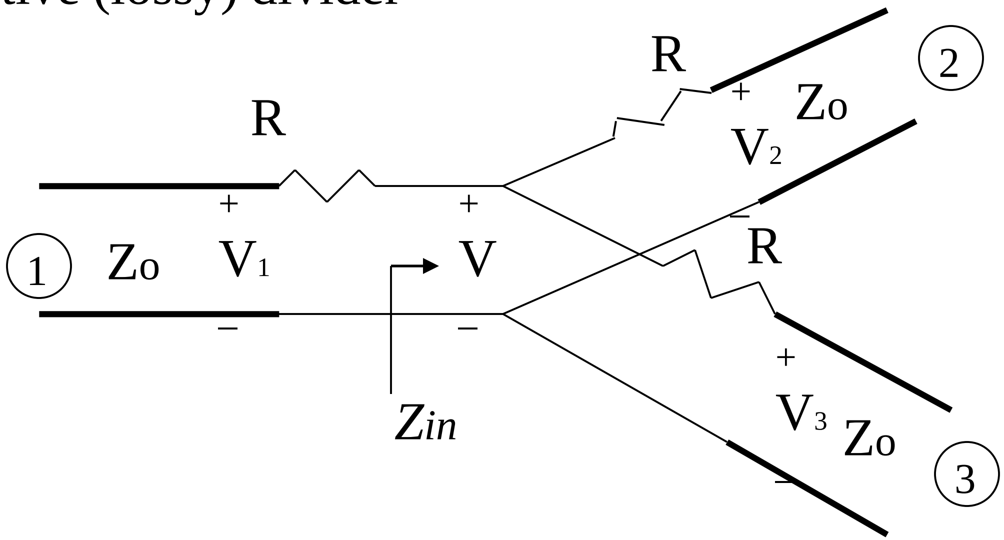
$$\rightarrow Z_2 = 3Z_o = 150\Omega, \quad Z_3 = \frac{3}{2} Z_o = 75\Omega$$

$$Z_{in} = Z_2 // Z_3 = 50\Omega$$

$$\Gamma_2 = \frac{50//75 - Z_2}{50//75 + Z_2} = -0.666, \Gamma_3 = \frac{50//150 - Z_3}{50//150 + Z_3} = -0.333$$

2. It's a lossless and mismatched three-port divider, and not good in isolation.

• resistive (lossy) divider



matched ports $\Rightarrow (R+Z_o)//(R+Z_o)+R=Z_o \quad \rightarrow \quad R=\dfrac{Z_o}{3}$

<u>Discussion</u>

1.

$$[S]=\frac{1}{2}\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}$$

$$\because Z_{in}=\frac{R+Z_o}{2}=\frac{2}{3}Z_o, V=V_1\frac{\frac{2}{3}Z_o}{R+\frac{2}{3}Z_o}=\frac{2}{3}V_1$$

$$V_2,V_3=V\frac{Z_o}{\frac{Z_o}{3}+Z_o}=\frac{3}{4}V=\frac{1}{2}V_1$$

2. $$P_{in}=\frac{1}{2}\frac{V_1^2}{Z_o},\quad P_2=P_3=\frac{1}{2}\frac{V_2^2}{Z_o}=\frac{1}{2}\frac{(\frac{V_1}{2})^2}{Z_o}=\frac{1}{8}\frac{V_1^2}{Z_o}=\frac{P_{in}}{4}$$

$$P_{loss,R@1}=\frac{1}{2}\frac{(V_1-V)^2}{R}=\frac{1}{2}\frac{(V_1-\frac{2}{3}V_1)^2}{\frac{Z_o}{3}}=\frac{1}{6}\frac{V_1^2}{Z_o}=\frac{P_{in}}{3}$$

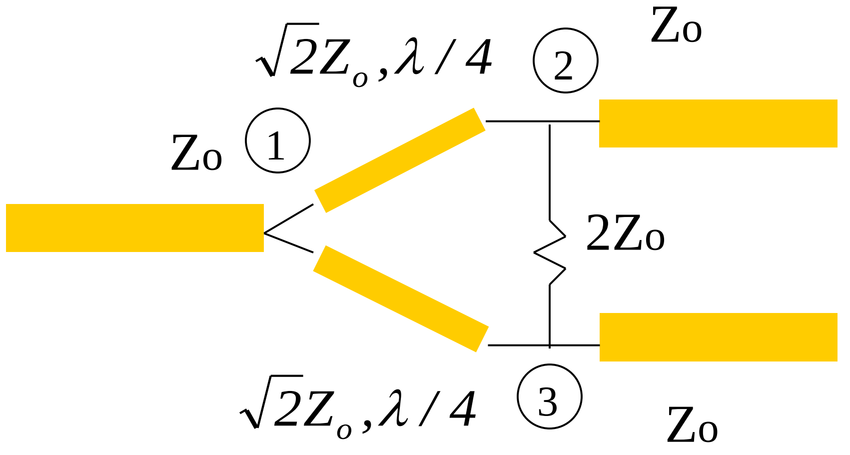
$$P_{loss,R@2,3}=\frac{1}{2}\frac{(V-V_2)^2}{R}=\frac{1}{2}\frac{(\frac{2}{3}V_1-\frac{1}{2}V_1)^2}{\frac{Z_o}{3}}=\frac{1}{24}\frac{V_1^2}{Z_o}=\frac{P_{in}}{12}$$

$\rightarrow$ lossy divider

3. It’s a lossy and matched three-port divider, and not good in isolation.

7.3 The Wilkinson power divider

- basic concept



$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -j & -j \\ -j & 0 & 0 \\ -j & 0 & 0 \end{bmatrix}$$

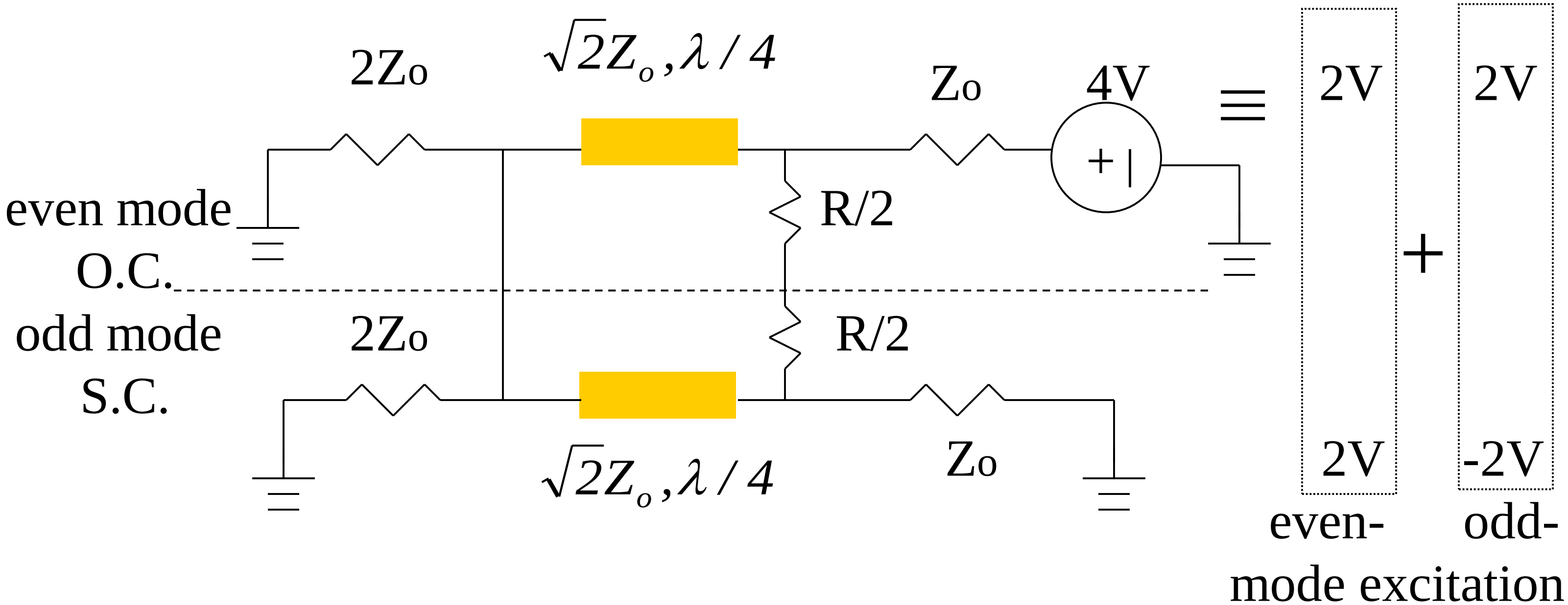
Input port 1 is matched→ Port 2 and port 3 have equal voltage and power.

→ $\sqrt{2}Z_o, \lambda/4$

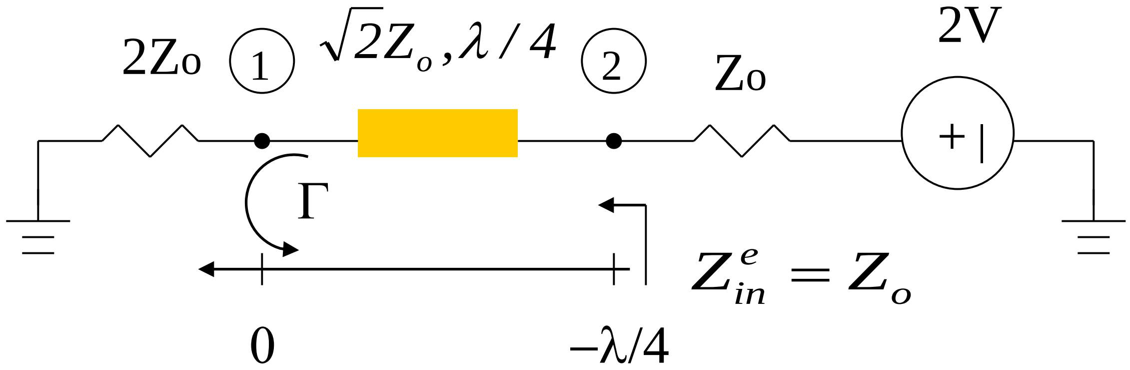
input port 2 or 3 → Port 2 and port 3 have perfect isolation.

=> a lossy, matched and good isolation (equal phase) three-port divider

- even-odd mode analysis
  A linear, symmetric network



even-mode



ports 2, 3 matched $\rightarrow S_{22}^e = 0,\ S_{33}^e = 0$

$\rightarrow V_2^e = V$, symmetry of ports 2 and 3 $\rightarrow V_3^e = V$

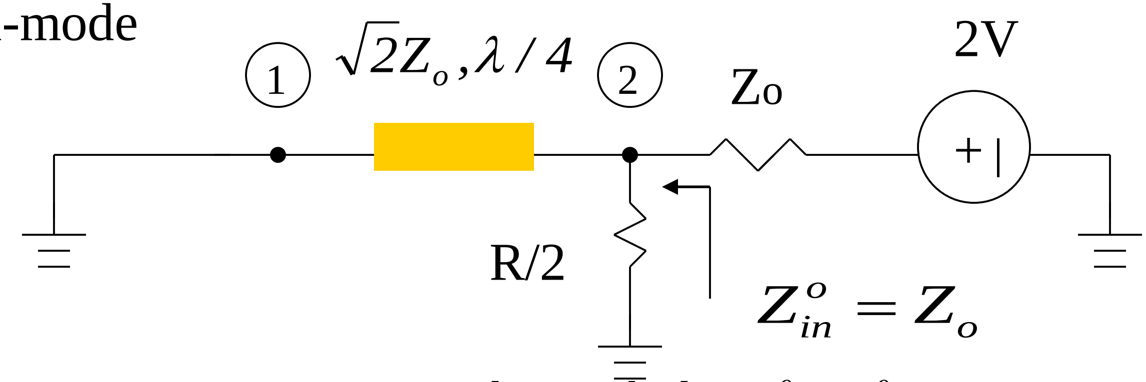
$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}} \rightarrow V_1^e = jV\frac{\Gamma+1}{\Gamma-1} = -j\sqrt{2}V$$

(derivation of V_{1e}) $V(z) = V^+e^{-j\beta z} + V^-e^{j\beta z}$

$$V_2^e = V^+e^{-j\frac{2\pi}{\lambda}\frac{-\lambda}{4}} + V^-e^{j\frac{2\pi}{\lambda}\frac{-\lambda}{4}} = V^+e^{j\frac{\pi}{2}} + V^-e^{-j\frac{\pi}{2}} = jV^+(1-\Gamma) \equiv V$$

$$V_1^e = V^+ + V^- = V^+(1+\Gamma) = -j\frac{1+\Gamma}{1-\Gamma}V = -j\frac{1+\frac{2-\sqrt{2}}{2+\sqrt{2}}}{1-\frac{2-\sqrt{2}}{2+\sqrt{2}}}V = -j\frac{4}{2\sqrt{2}}V = -j\sqrt{2}V$$

odd-mode



$$\frac{R}{2} = Z_o \rightarrow R = 2Z_o \rightarrow \text{ports 2 and 3 matched} \rightarrow S_{22}^o = S_{33}^o = 0$$

$$\rightarrow V_2^o = V, V_3^o = -V, V_1^o = 0$$

$$\left.\begin{array}{l} V_2^e = V \\ V_2^o = V \end{array}\right\} \rightarrow V_2 = V_2^e + V_2^o = 2V$$

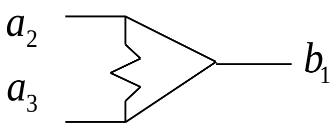
$$\left.\begin{array}{l} V_1^e = -j\sqrt{2}V \\ V_1^o = 0 \end{array}\right\} \rightarrow V_1 = V_1^e + V_1^o = -j\sqrt{2}V, \left.\begin{array}{l} V_3^e = V \\ V_3^o = -V \end{array}\right\} \rightarrow V_3 = V_3^e + V_3^o = 0$$

$$\Rightarrow S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{S_{22}=0, \Gamma_{L1}=0} = \frac{V_1}{V_2} = \frac{-j\sqrt{2}V}{2V} = -j\frac{1}{\sqrt{2}} = S_{21} = S_{31} = S_{13}$$

$$\Rightarrow S_{32} = \frac{V_3^-}{V_2^+} = \left.\frac{V_3}{V_2}\right|_{S_{22}=0, \Gamma_{L3}=0} = \frac{0}{2V} = 0 = S_{23}$$

Discussion

1. A 3dB Wilkinson power divider has equal amplitude and phase outputs at port 2 and port 3.
2. Ex. 7.2 A 3dB Wilkinson power divider has Zo=70.7Ω and R=100Ω. frequency response (p.332, Fig. 7.12)
3. 3dB Wilkinson power combiner



$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -j & -j \\ -j & 0 & 0 \\ -j & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -j\frac{1}{\sqrt{2}}(a_2+a_3) \\ 0 \\ 0 \end{bmatrix}$$

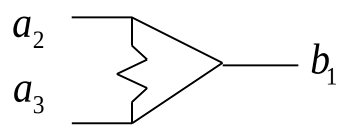
$$\text{if } a_2 = a_3 \rightarrow P_1 = \frac{1}{2}\left(\frac{1}{2}|a_2+a_3|^2\right) = \frac{1}{2}\left(\frac{1}{2}|2a_2|^2\right) = |a_2|^2 = \frac{1}{2}\left(|a_2|^2+|a_3|^2\right) = P_2 + P_3$$

$$\text{if } a_2 \neq a_3,\ P_1 = \frac{1}{2}\left(\frac{1}{2}|a_2+a_3|^2\right) = \frac{1}{4}|a_2+a_3|^2$$

$$\rightarrow i/p \begin{matrix} a_2 \\ a_3 \end{matrix} = even \begin{matrix} \frac{a_2+a_3}{2} \\ \frac{a_2+a_3}{2} \end{matrix}, odd \begin{matrix} \frac{a_2-a_3}{2} \\ \frac{a_3-a_2}{2} \end{matrix} \Rightarrow P_1 = \frac{1}{2}\left(\frac{1}{2}\left|\frac{a_2+a_3}{2}+\frac{a_2+a_3}{2}\right|^2 + \frac{1}{2}\left|\frac{a_2-a_3}{2}+\frac{a_3-a_2}{2}\right|^2\right)$$

$$= 2\times\frac{1}{2}\left|\frac{a_2+a_3}{2}\right|^2 = \frac{1}{4}|a_2+a_3|^2 \leq \frac{1}{2}\left(|a_2|^2+|a_3|^2\right) = P_2+P_3$$

(Proof of $P_1 = \frac{1}{4}|a_2 + a_3|^2 \le \frac{1}{2}(|a_2|^2 + |a_3|^2) = P_2 + P_3$)



$$P_1 = \frac{1}{4}|a_2 + a_3|^2 = \frac{1}{4}(a_2 + a_3)(a_2^* + a_3^*) = \frac{1}{4}\left[|a_2|^2 + |a_3|^2 + a_2 a_3^* + a_2^* a_3\right]$$

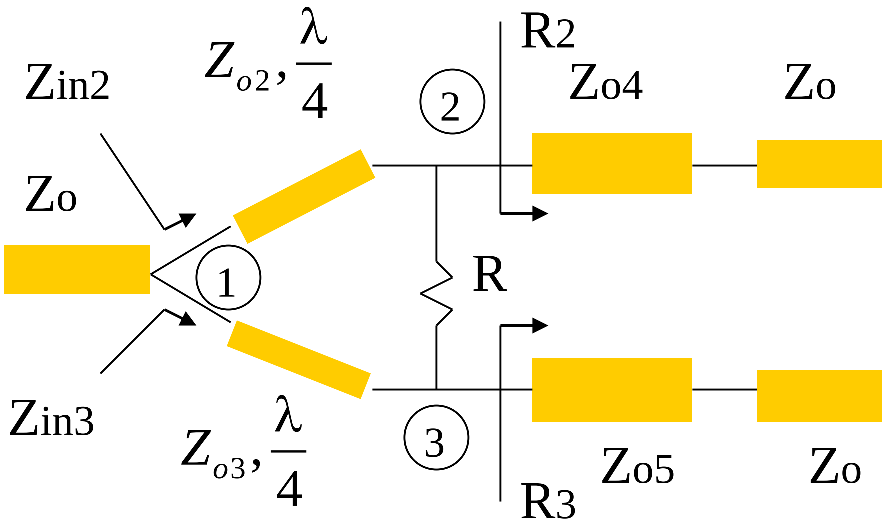
$$= \frac{1}{4}\left[|a_2|^2 + |a_3|^2\right] + \frac{1}{2}\mathrm{Re}[a_2 a_3^*] \le \frac{1}{4}\left[|a_2|^2 + |a_3|^2\right] + \frac{1}{4}\left[|a_2|^2 + |a_3|^2\right] = \frac{1}{2}|a_2|^2 + \frac{1}{2}|a_3|^2 = P_2 + P_3$$

$$\because |a_2 - a_3|^2 = (a_2 - a_3)(a_2^* - a_3^*) = |a_2|^2 + |a_3|^2 - a_2 a_3^* - a_2^* a_3$$

$$= |a_2|^2 + |a_3|^2 - 2\,\mathrm{Re}[a_2 a_3^*] \ge 0$$

$$\Rightarrow 2\,\mathrm{Re}[a_2 a_3^*] \le |a_2|^2 + |a_3|^2 \Rightarrow \frac{1}{2}\mathrm{Re}[a_2 a_3^*] \le \frac{1}{4}\left[|a_2|^2 + |a_3|^2\right]$$

4. unequal power division (Wilkinson power divider)



(1) port1 match $\rightarrow Z_o = Z_{in2} // Z_{in3}$

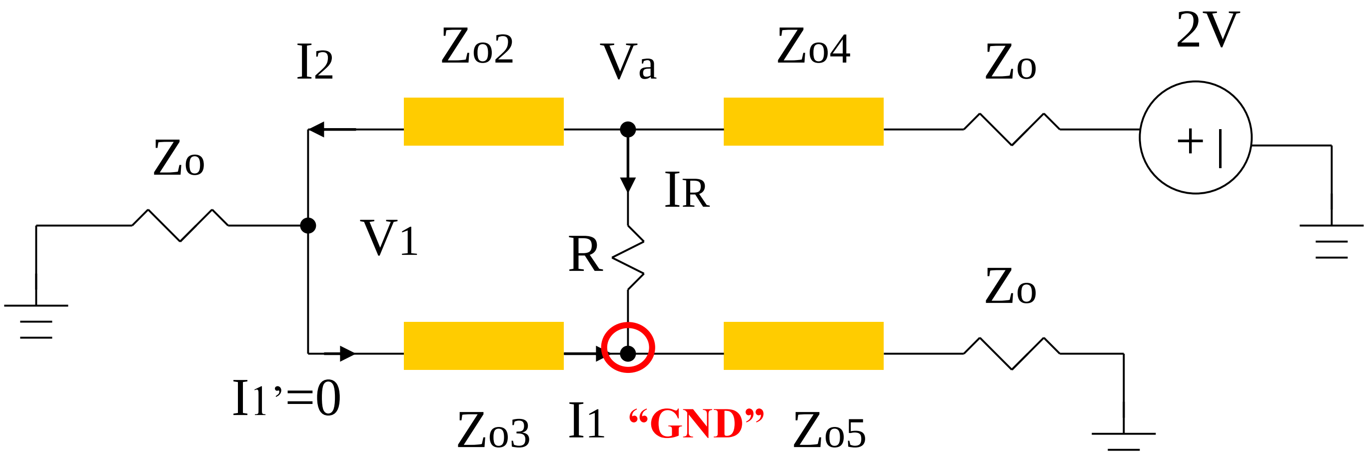
(2) $\frac{P_3}{P_2} = K^2 \rightarrow \frac{V_3^2}{Z_{in3}} = K^2 \frac{V_2^2}{Z_{in2}}$

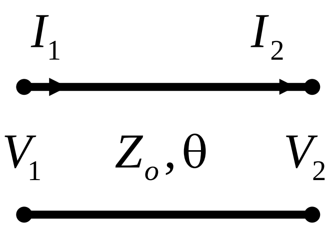
(3) $V_2 = V_3 \rightarrow Z_{in2} = K^2 Z_{in3}$

$$(1),(3) \rightarrow Z_{in2} = (1+K^2)Z_o, Z_{in3} = \frac{1+K^2}{K^2} Z_o$$

$$(2) \rightarrow R_2 = K^2 R_3, \text{ let } R_2 = KZ_o \rightarrow R_3 = \frac{Z_o}{K}, Z_{o4} = \sqrt{K} Z_o, Z_{o5} = \frac{Z_o}{\sqrt{K}}$$

$$Z_{o2} = \sqrt{Z_{in2} R_2} = \sqrt{K(1+K^2)} Z_o, Z_{o3} = \sqrt{Z_{in3} R_3} = \sqrt{\frac{1+K^2}{K^3}} Z_o$$





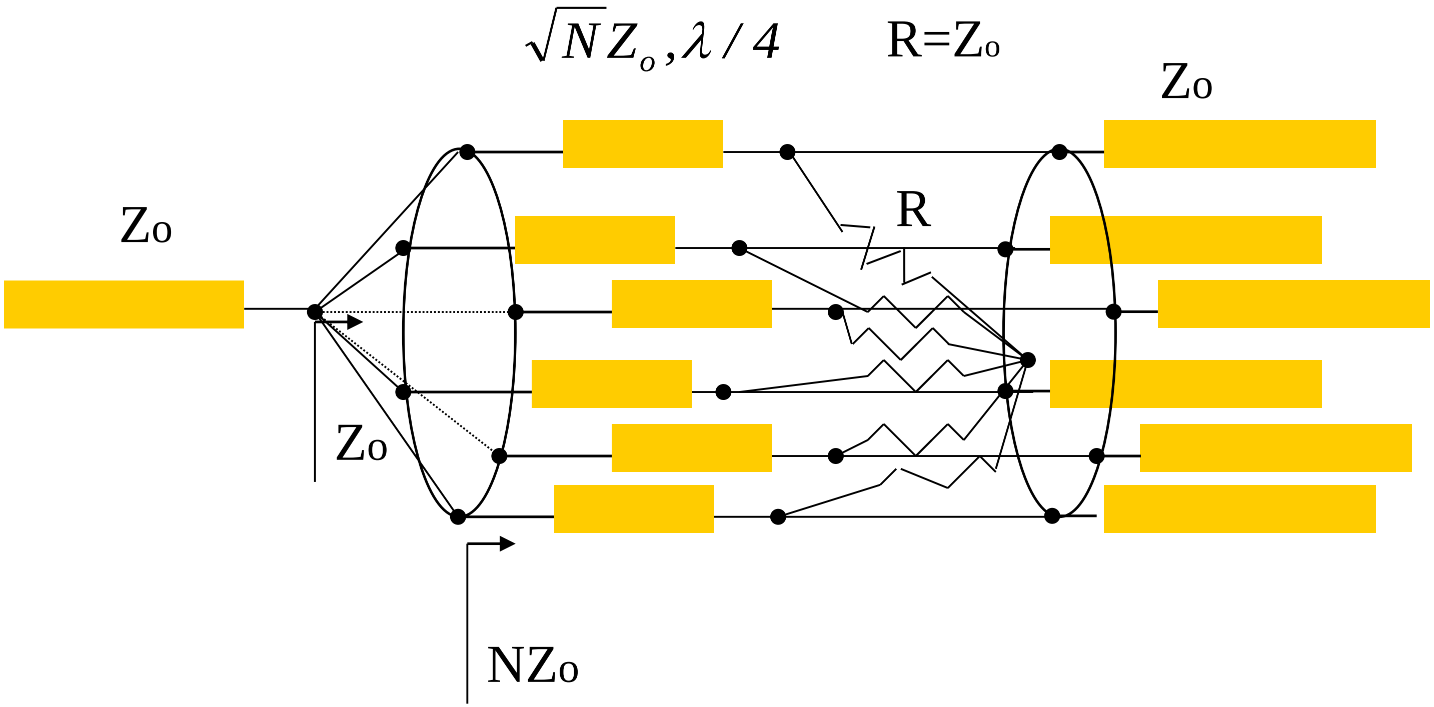
$$\begin{bmatrix}V_1\\ I_1\end{bmatrix} \overset{table\ 4.1}{=} \begin{bmatrix}\cos\theta & jZ_o\sin\theta\\ \dfrac{j\sin\theta}{Z_o} & \cos\theta\end{bmatrix}_{\theta=\frac{\pi}{2}} \begin{bmatrix}V_2\\ I_2\end{bmatrix} = \begin{bmatrix}0 & jZ_o\\ \dfrac{j}{Z_o} & 0\end{bmatrix}\begin{bmatrix}V_2\\ I_2\end{bmatrix} \to \boxed{V_1 = jZ_oI_2},\quad I_1 = \frac{j}{Z_o}V_2$$

$$I_1 = \frac{V_1}{jZ_{o3}},\ I_2 = \frac{V_a}{jZ_{o2}},\ \frac{V_1}{I_2} = Z_o$$

$$I_R + I_1 = 0 = \frac{V_a}{R} + \frac{V_1}{jZ_{o3}} = \frac{jZ_{o2}I_2}{R} + \frac{Z_oI_2}{jZ_{o3}} = jI_2\left(\frac{Z_{o2}}{R} - \frac{Z_o}{Z_{o3}}\right)$$

$$\to R = \frac{Z_{o2}Z_{o3}}{Z_o} = \frac{1+K^2}{K}Z_o,\ K = 1 \text{ for a 3dB Wilkinson divider}$$

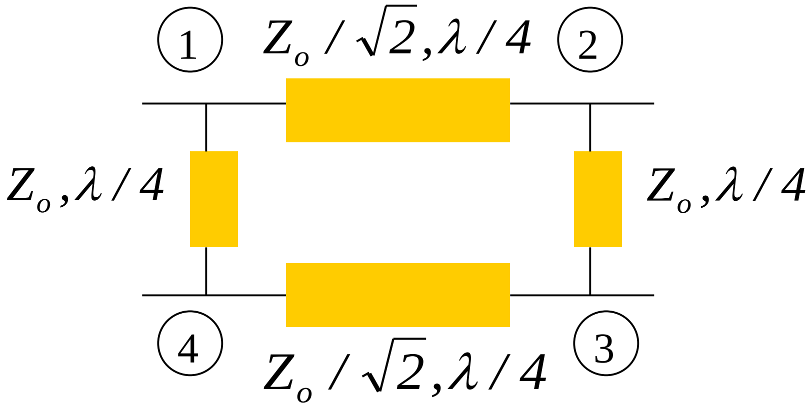
5. N-way Wilkinson power divider (not in a planar shape)



7.5 The quadrature (90°) hybrid

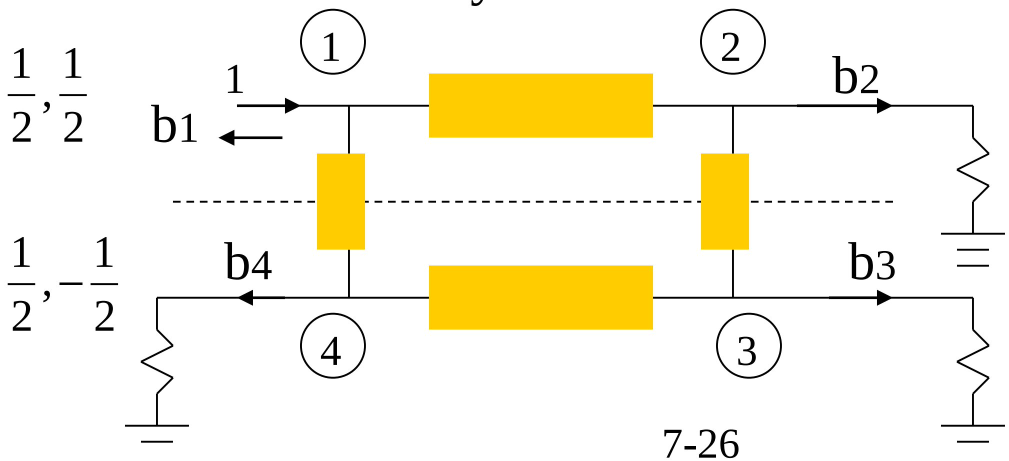
- branch-line hybrid

  Port 2 and port 3 have equal amplitude and 90° phase difference.

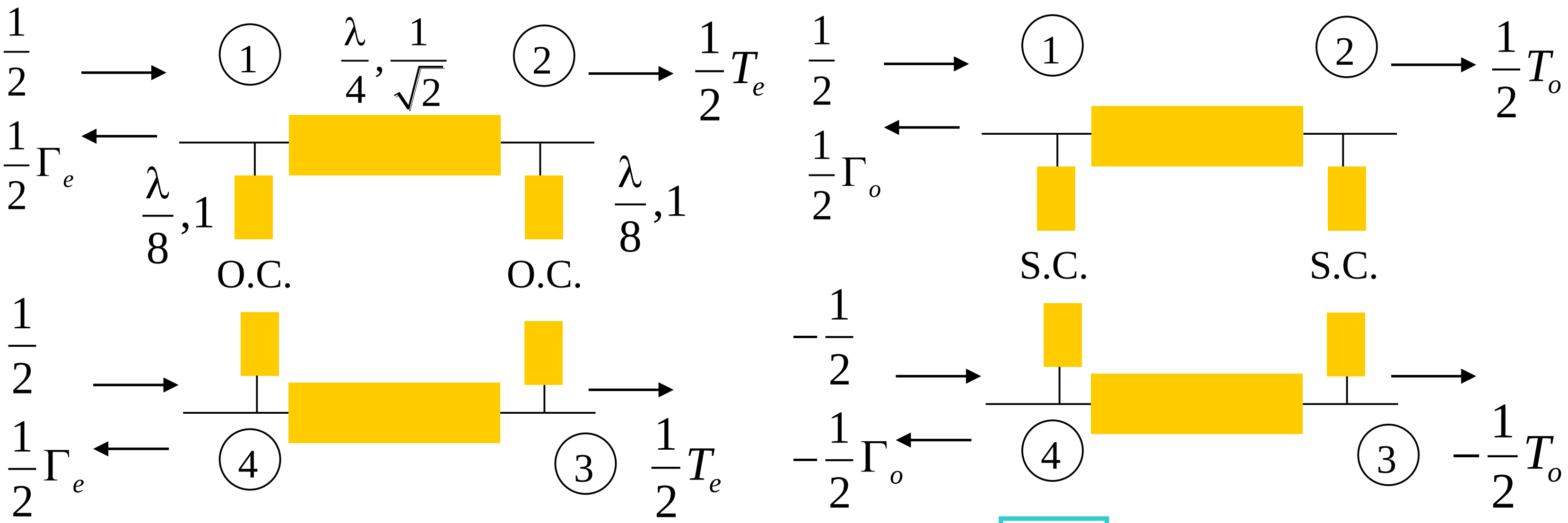


$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

- even-odd mode analysis



even-mode odd-mode



$$b_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o = S_{11}, b_2 = \frac{1}{2}T_e + \frac{1}{2}T_o = S_{21}$$

$$b_3 = \frac{1}{2}T_e - \frac{1}{2}T_o = S_{31}, b_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o = S_{41}$$

$$\begin{bmatrix} b_1 & b_2 & b_3 & b_4 \\ b_2 & b_1 & b_4 & b_3 \\ b_3 & b_4 & b_1 & b_2 \\ b_4 & b_3 & b_2 & b_1 \end{bmatrix}$$

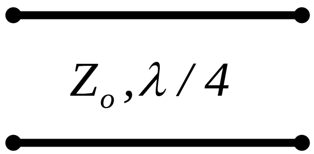
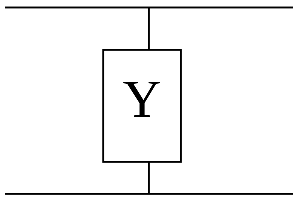


table 4.1 $\rightarrow$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_o \sin\theta \\ jY_o \sin\theta & \cos\theta \end{bmatrix}_{\theta=\frac{\pi}{2}} = \begin{bmatrix} 0 & jZ_o \\ jY_o & 0 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$\frac{\lambda}{8}$ open-circuit stub $\rightarrow Z_{in} = -j \dfrac{Z_o}{\tan \frac{\lambda}{4}} = -jZ_o \rightarrow Y_{in} = jY_o$

$\frac{\lambda}{8}$ short-circuit stub $\rightarrow Z_{in} = jZ_o \tan \frac{\lambda}{4} = jZ_o \rightarrow Y_{in} = -jY_o$

$\Rightarrow$ even-mode: $\frac{\lambda}{8}$ open-circuit stub $\begin{bmatrix} 1 & 0 \\ jY_o & 1 \end{bmatrix}$

odd-mode: $\frac{\lambda}{8}$ short-circuit stub $\begin{bmatrix} 1 & 0 \\ -jY_o & 1 \end{bmatrix}$

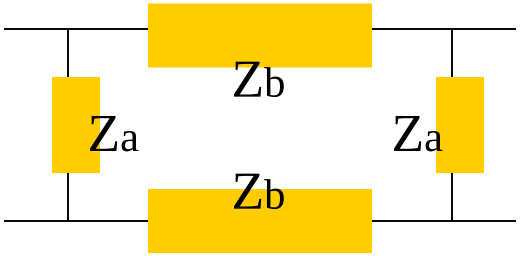
$$\Gamma = S_{11} = \frac{AZ_o + B - CZ_o^2 - DZ_o}{AZ_o + B + CZ_o^2 + DZ_o}, T = S_{21} = \frac{2Z_o}{AZ_o + B + CZ_o^2 + DZ_o}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{j}{\sqrt{2}} \\ j\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{j}{\sqrt{2}} \\ j\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

$$\rightarrow \Gamma_e = 0, \Gamma_o = 0, T_e = \frac{-1}{\sqrt{2}}(1+j), T_o = \frac{1}{\sqrt{2}}(1-j)$$

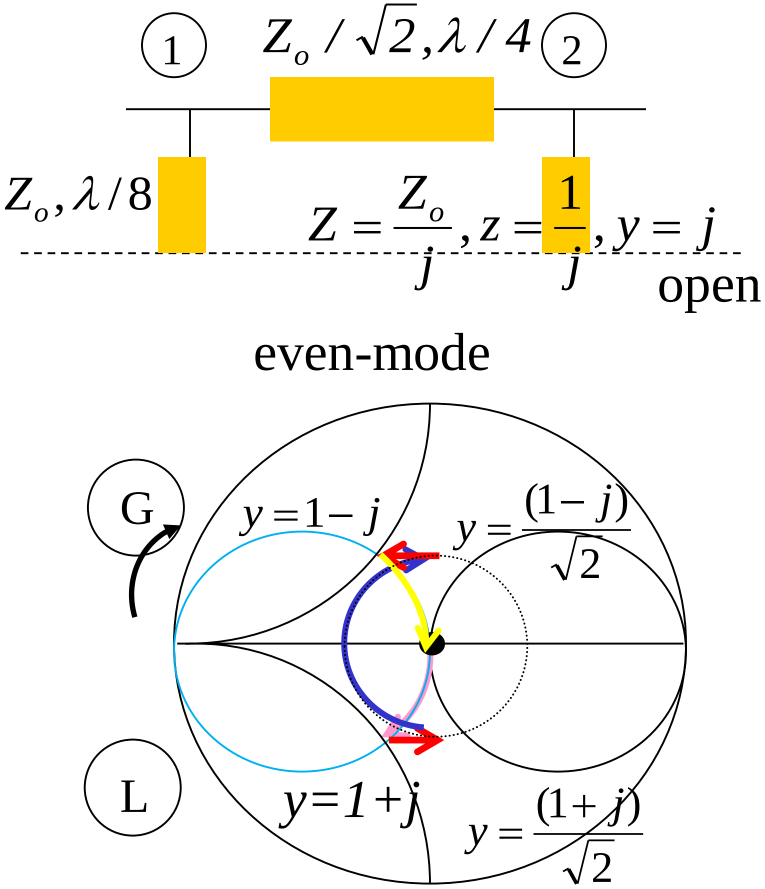
$$\Rightarrow b_1 = 0, b_2 = \frac{-j}{\sqrt{2}}, b_3 = \frac{-1}{\sqrt{2}}, b_4 = 0$$

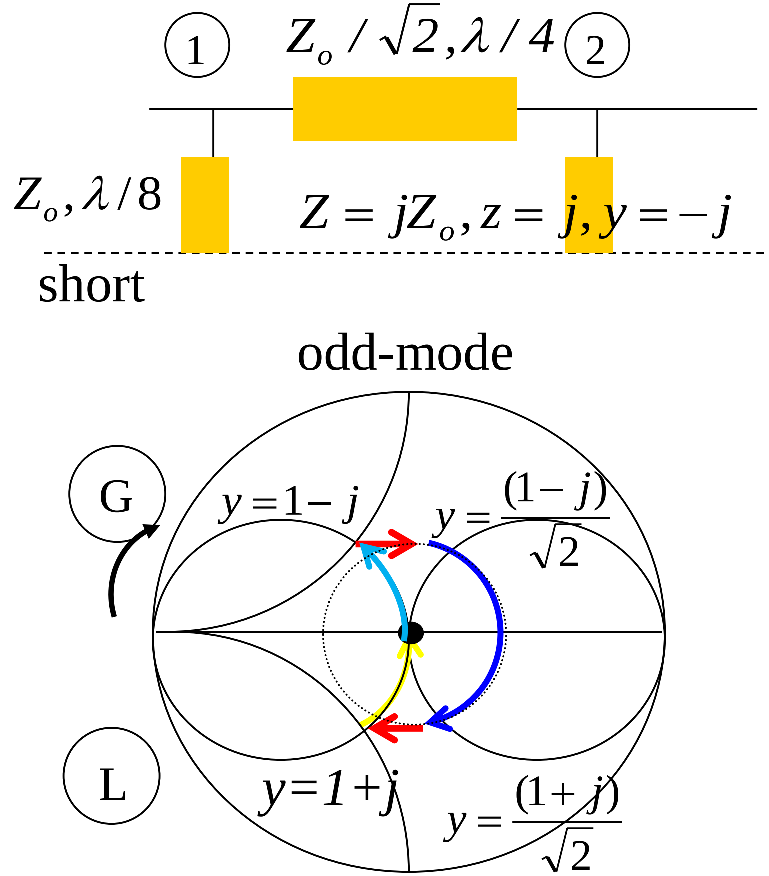


Discussion

1. Unequal power division branch-line coupler uses Za, Zb λ/4 lines (prob. 7.18).
2. Ex. 7.5, frequency response (p.346, Fig. 7.25), BW: 10% ~ 20%
3. Multisection branch-line couplers can increase the operation BW.

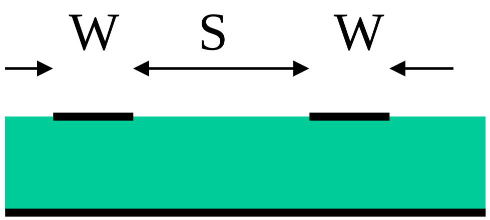
4. Smith chart consideration

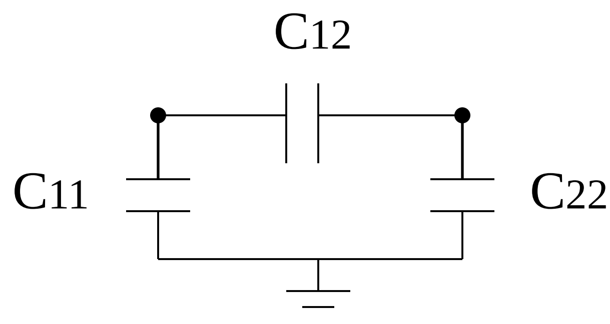




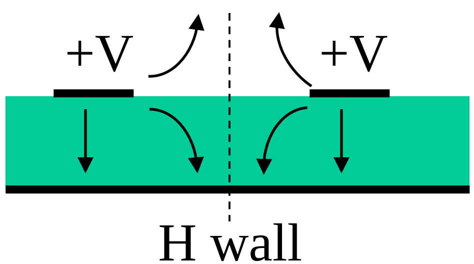
7.6 Coupled line directional couplers

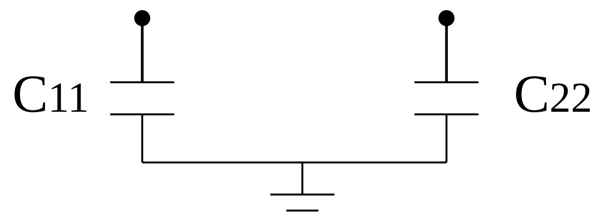
- coupled line theory





even-mode excitation

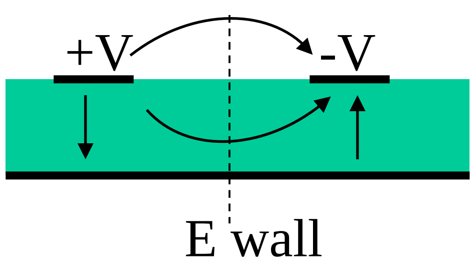


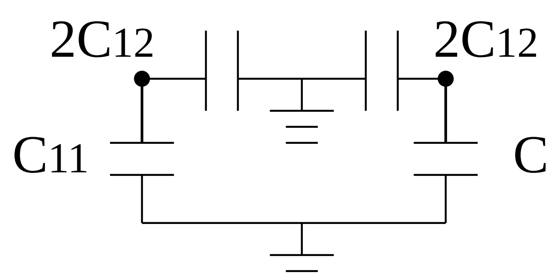


$$C_e = C_{11} = C_{22}$$

$$Z_{oe} = \frac{1}{v_e C_e}$$

odd-mode excitation



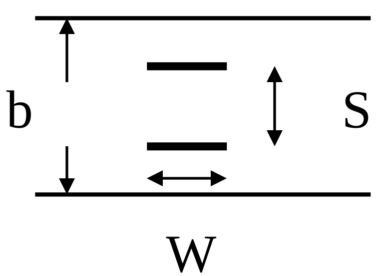


$$C_o = C_{11} + 2C_{12}$$

$$Z_{oo} = \frac{1}{v_o C_o}$$

Discussion

1. In general $v_e \neq v_o$ ($\because \varepsilon_{effe} > \varepsilon_{effo}$), only for TEM mode $v_e = v_o = v$.
2. Z_{oe}, Z_{oo} (W/b, S/b) (p.349, Fig. 7.29 and p.350, Fig. 7.30) are given for coupled striplines and microstrip lines.
3. $Z_{oe} > Z_{oo}$, W/b $\uparrow \rightarrow Z_{oe} \downarrow Z_{oo} \downarrow$, S/b $\uparrow \rightarrow Z_{oe} \downarrow Z_{oo} \uparrow$ and $Z_{oe}, Z_{oo} \rightarrow Z_o$.
4. Ex.7.6 derive Z_{oe}, Z_{oo} of coupled striplines (p.347, Fig.7.26(b))



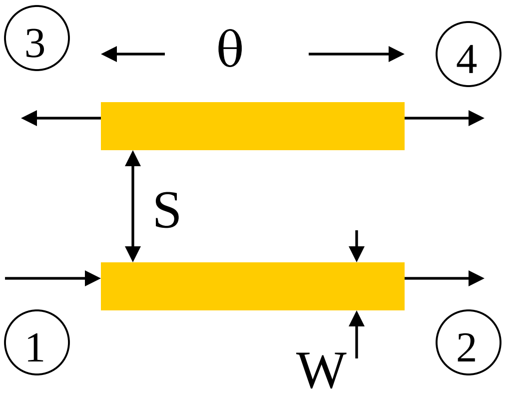
$$C_{11} = \frac{\varepsilon W}{(b-S)/2} + \frac{\varepsilon W}{(b+S)/2} = \frac{4b\varepsilon W}{b^2 - S^2}, C_{12} = \frac{\varepsilon W}{S}$$

$$C_e = C_{11}, C_o = C_{11} + 2C_{12}$$

$$Z_{oe} = \frac{1}{vC_e} = Z_o \frac{b^2 - S^2}{4bW\sqrt{\varepsilon_r}}$$

$$Z_{oo} = \frac{1}{vC_o} = Z_o \frac{1}{2W\sqrt{\varepsilon_r}[2b/(b^2 - S^2) + 1/S]}$$

- coupled line coupler



matched port $\leftarrow Z_o = \sqrt{Z_{oe} Z_{oo}}$

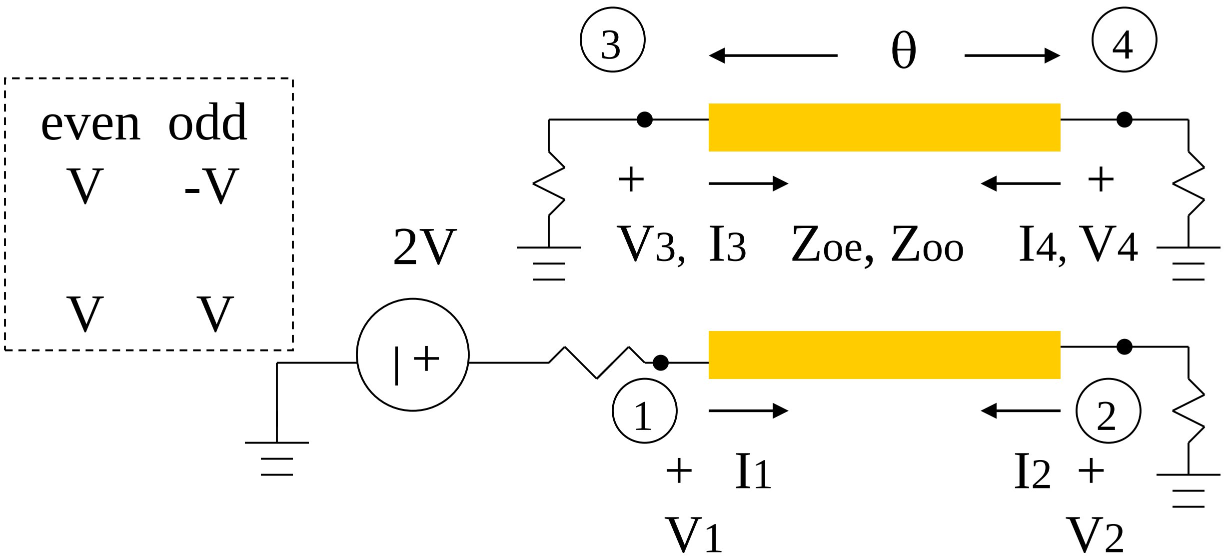
$$C \equiv \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}, \theta = \frac{\pi}{2} \Rightarrow$$

$$[S] = \begin{bmatrix} 0 & -j\sqrt{1-C^2} & C & 0 \\ -j\sqrt{1-C^2} & 0 & 0 & C \\ C & 0 & 0 & -j\sqrt{1-C^2} \\ 0 & C & -j\sqrt{1-C^2} & 0 \end{bmatrix}$$

design equations: $Z_{oe} = Z_o \sqrt{\frac{1+C}{1-C}},\ Z_{oo} = Z_o \sqrt{\frac{1-C}{1+C}} \Rightarrow S,\ W$

Discussion

1. Design procedure: given C and Z_o, calculate Z_{oe} and Z_{oo}, then use Fig. 7.29 or 7.30 to find W/b and S/b for stripline or microstrip.
2. even-odd mode analysis



even mode: $I_1^e = I_3^e, I_2^e = I_4^e, V_1^e = V_3^e, V_2^e = V_4^e$

odd mode: $I_1^o = -I_3^o, I_2^o = -I_4^o, V_1^o = -V_3^o, V_2^o = -V_4^o$

$$I_1 = I_1^e + I_1^o, I_2 = I_2^e + I_2^o, I_3 = I_1^e - I_1^o, I_4 = I_2^e - I_2^o$$

$$V_1 = V_1^e + V_1^o, V_2 = V_2^e + I_2^o, V_3 = V_1^e - V_1^o, V_4 = V_2^e - V_2^o$$

$$Z_{in}^{e,o} \overset{\theta_e=\theta_o=\theta}{=} Z_{oe,o}\frac{Z_o + jZ_{oe,o}\tan\theta}{Z_{oe,o} + jZ_o\tan\theta}, \quad V_1^{e,o} = V\frac{Z_{in}^{e,o}}{Z_{in}^{e,o} + Z_o}, I_1^{e,o} = \frac{V}{Z_{in}^{e,o} + Z_o}$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o} = Z_o + \frac{2(Z_{in}^e Z_{in}^o - Z_o^2)}{Z_{in}^e + Z_{in}^o + 2Z_o} ...(1)$$

$$= Z_o \Leftarrow \ if \ \ Z_o^2 (= Z_{in}^e Z_{in}^o) = Z_{oe} Z_{oo} ...(2)$$

$$\rightarrow V_1^{e,o} = V\frac{Z_o + jZ_{oe,o}\tan\theta}{2Z_o + j(Z_{oe} + Z_{oo})\tan\theta} ...(3)$$

$$V_3 = V_1^e - V_1^o = V\frac{j(Z_{oe} - Z_{oo})\tan\theta}{2Z_o + j(Z_{oe} + Z_{oo})\tan\theta}$$

$$C \equiv \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} \rightarrow V_3 = V\frac{jC\tan\theta}{\sqrt{1 - C^2} + j\tan\theta} ...(4)$$

$$\text{derivation of (1)}: Z_{in}=\frac{V_1}{I_1}=\frac{V_1^e+V_1^o}{I_1^e+I_1^o}=\frac{V\dfrac{Z_{in}^e}{Z_{in}^e+Z_o}+V\dfrac{Z_{in}^o}{Z_{in}^o+Z_o}}{\dfrac{V}{Z_{in}^e+Z_o}+\dfrac{V}{Z_{in}^o+Z_o}}$$

$$=\frac{Z_{in}^e(Z_{in}^o+Z_o)+Z_{in}^o(Z_{in}^e+Z_o)}{Z_{in}^e+Z_{in}^o+2Z_o}=\frac{Z_o(Z_{in}^e+Z_{in}^o+2Z_o)+2Z_{in}^oZ_{in}^e-2Z_o^2}{Z_{in}^e+Z_{in}^o+2Z_o}$$

$$=Z_o+\frac{2(Z_{in}^eZ_{in}^o-Z_o^2)}{Z_{in}^e+Z_{in}^o+2Z_o}$$

$$\text{derivation of (2)}: Z_{in}^eZ_{in}^o=Z_{oe}\frac{Z_o+jZ_{oe}\tan\theta}{Z_{oe}+jZ_o\tan\theta}Z_{oo}\frac{Z_o+jZ_{oo}\tan\theta}{Z_{oo}+jZ_o\tan\theta}$$

$$\overset{let\ Z_{oe}Z_{oo}=Z_o^2}{=}Z_{oe}Z_{oo}\frac{\sqrt{Z_{oe}Z_{oo}}+jZ_{oe}\tan\theta}{Z_{oe}+j\sqrt{Z_{oe}Z_{oo}}\tan\theta}\times\frac{1/\sqrt{Z_{oe}}}{1/\sqrt{Z_{oe}}}\frac{\sqrt{Z_{oe}Z_{oo}}+jZ_{oo}\tan\theta}{Z_{oo}+j\sqrt{Z_{oe}Z_{oo}}\tan\theta}\times\frac{1/\sqrt{Z_{oo}}}{1/\sqrt{Z_{oo}}}$$

$$=Z_{oe}Z_{oo}\frac{\sqrt{Z_{oo}}+j\sqrt{Z_{oe}}\tan\theta}{\sqrt{Z_{oe}}+j\sqrt{Z_{oo}}\tan\theta}\frac{\sqrt{Z_{oe}}+j\sqrt{Z_{oo}}\tan\theta}{\sqrt{Z_{oo}}+j\sqrt{Z_{oe}}\tan\theta}=Z_{oe}Z_{oo}=Z_o^2\rightarrow Z_{in}=Z_o:\text{i/p match}$$

derivation of (3): $V_1^e = V \dfrac{Z_{in}^e}{Z_{in}^e + Z_o} = V \dfrac{Z_{oe} \dfrac{Z_o + jZ_{oe}\tan\theta}{Z_{oe} + jZ_o\tan\theta}}{Z_{oe} \dfrac{Z_o + jZ_{oe}\tan\theta}{Z_{oe} + jZ_o\tan\theta} + Z_o}$

$$= V \frac{Z_{oe}Z_o + jZ_{oe}^2\tan\theta}{2Z_{oe}Z_o + jZ_{oe}^2\tan\theta + jZ_o^2\tan\theta} \times \frac{1/Z_{oe}}{1/Z_{oe}} = V \frac{Z_o + jZ_{oe}\tan\theta}{2Z_o + jZ_{oe}\tan\theta + jZ_{oo}\tan\theta}$$

$$V_3 = V_1^e - V_1^o = V \frac{j(Z_{oe} - Z_{oo})\tan\theta}{2Z_o + j(Z_{oe} + Z_{oo})\tan\theta}$$

derivation of (4): $C \equiv \dfrac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} \rightarrow V_3 = V \dfrac{j\dfrac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}\tan\theta}{\dfrac{2Z_o}{Z_{oe} + Z_{oo}} + j\tan\theta} = V \dfrac{jC\tan\theta}{\dfrac{2Z_o}{Z_{oe} + Z_{oo}} + j\tan\theta}$

$$\because \sqrt{1 - C^2} = \sqrt{1 - \left(\frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}\right)^2} = \frac{\sqrt{(Z_{oe} + Z_{oo})^2 - (Z_{oe} - Z_{oo})^2}}{Z_{oe} + Z_{oo}} = \frac{2\sqrt{Z_{oe}Z_{oo}}}{Z_{oe} + Z_{oo}} = \frac{2Z_o}{Z_{oe} + Z_{oo}}$$

$$\rightarrow V_3 = V \frac{jC\tan\theta}{\sqrt{1 - C^2} + j\tan\theta}$$

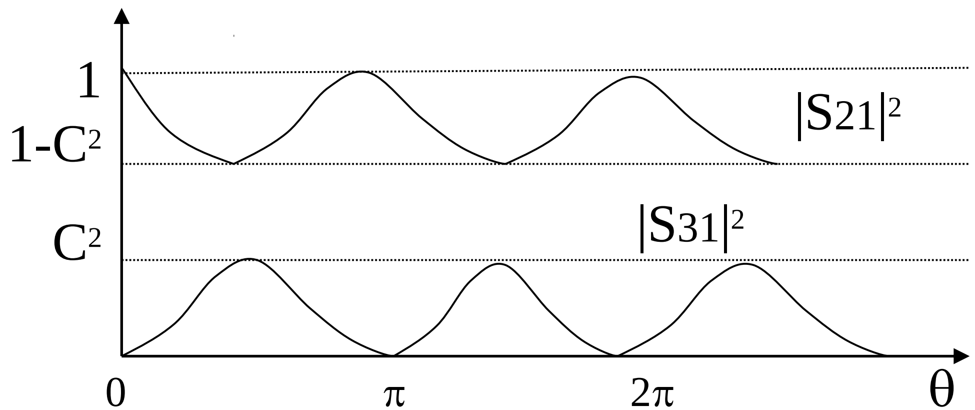
$$\begin{bmatrix} V_1^{e,o} \\ I_1^{e,o} \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_{oe,o}\sin\theta \\ jY_{oe,o}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_2^{e,o} \\ I_2^{e,o} \end{bmatrix}$$

$$\rightarrow V_2^{e,o} = V\frac{Z_o}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta} \quad ...(5)$$

$$V_2 = V_2^e + V_2^o = V\frac{2Z_o}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta} = V\frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}$$

$$V_4 = V_2^e - V_2^e = 0$$

3. frequency response at port 2 and port 3



derivation of (5)

$$\begin{bmatrix} V_1^{e,o} \\ I_1^{e,o} \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_{oe,o}\sin\theta \\ jY_{oe,o}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_2^{e,o} \\ I_2^{e,o} \end{bmatrix} \rightarrow \begin{bmatrix} V_2^{e,o} \\ I_2^{e,o} \end{bmatrix} = \begin{bmatrix} \cos\theta & -jZ_{oe,o}\sin\theta \\ -jY_{oe,o}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_1^{e,o} \\ I_1^{e,o} \end{bmatrix}$$

$$(3) \rightarrow V_1^{e,o} = V\frac{Z_o + jZ_{oe,o}\tan\theta}{2Z_o + j(Z_{oe}+Z_{oo})\tan\theta} = V\frac{Z_o\cos\theta + jZ_{oe,o}\sin\theta}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta}$$

$$I_1^e = \frac{V}{Z_{in}^e + Z_o}, Z_{in}^e = Z_{oe}\frac{Z_o + jZ_{oe}\tan\theta}{Z_{oe} + jZ_o\tan\theta}$$

$$I_1^e = \frac{V}{Z_{in}^e + Z_o} = \frac{V}{Z_{oe}\dfrac{Z_o + jZ_{oe}\tan\theta}{Z_{oe} + jZ_o\tan\theta} + Z_o} = V\frac{Z_{oe} + jZ_o\tan\theta}{Z_{oe}(Z_o + jZ_{oe}\tan\theta) + Z_o(Z_{oe} + jZ_o\tan\theta)}$$

$$= V\frac{Z_{oe} + jZ_o\tan\theta}{2Z_{oe}Z_o + j(Z_{oe}^2 + Z_o^2)\tan\theta} \times \frac{1/Z_{oe}}{1/Z_{oe}} = V\frac{1 + j\sqrt{Z_{oo}/Z_{oe}}\tan\theta}{2Z_o + j(Z_{oe}+Z_{oo})\tan\theta} = V\frac{\cos\theta + j\sqrt{Z_{oo}/Z_{oe}}\sin\theta}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta}$$

$$\rightarrow I_1^{e,o} = V\frac{\cos\theta + j\sqrt{Z_{oo,e}/Z_{oe,o}}\sin\theta}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta}$$

$$\Rightarrow V_2^{e,o} = \cos\theta V_1^{e,o} - jZ_{oe,o}\sin\theta I_1^{e,o}$$

$$= V\frac{(Z_o\cos\theta + jZ_{oe,o}\sin\theta)\cos\theta}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta} - V\frac{jZ_{oe,o}\sin\theta(\cos\theta + j\sqrt{Z_{oo,e}/Z_{oe,o}}\sin\theta)}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta}$$

$$= V\frac{Z_o\cos^2\theta + jZ_{oe,o}\sin\theta\cos\theta - jZ_{oe,o}\sin\theta\cos\theta + \sqrt{Z_{oe}Z_{oo}}\sin^2\theta}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta}$$

$$= V\frac{Z_o}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta}$$

4. selection of line length

$$\theta = \frac{\pi}{2} \rightarrow l = \frac{\lambda}{4}$$

$$V_1 = 1$$

$$V_2 = -j\sqrt{1-C^2}, P_2 = (1-C^2)P_1$$

$$V_3 = C, P_2 = C^2 P_1$$

$$V_4 = 0$$

$$\theta = \pi \rightarrow l = \frac{\lambda}{2}$$

$$V_1 = 1$$

$$V_2 = 1$$

$$V_3 = 0$$

$$V_4 = 0$$

$\theta = \frac{\pi}{2}$, V_2 and V_3 have 90^o phase difference $\rightarrow$ quadrature coupler

5. In general $v_e \neq v_o \rightarrow \theta_e \neq \theta_o$, coupled line coupler is suited for a weakly coupled coupler.
6. C=3dB $\rightarrow$ Zoe=120.7Ω, Zoo=20.7Ω $\rightarrow$ S becomes not practical and coupled line theory is not applicable $\rightarrow$ Lange coupler

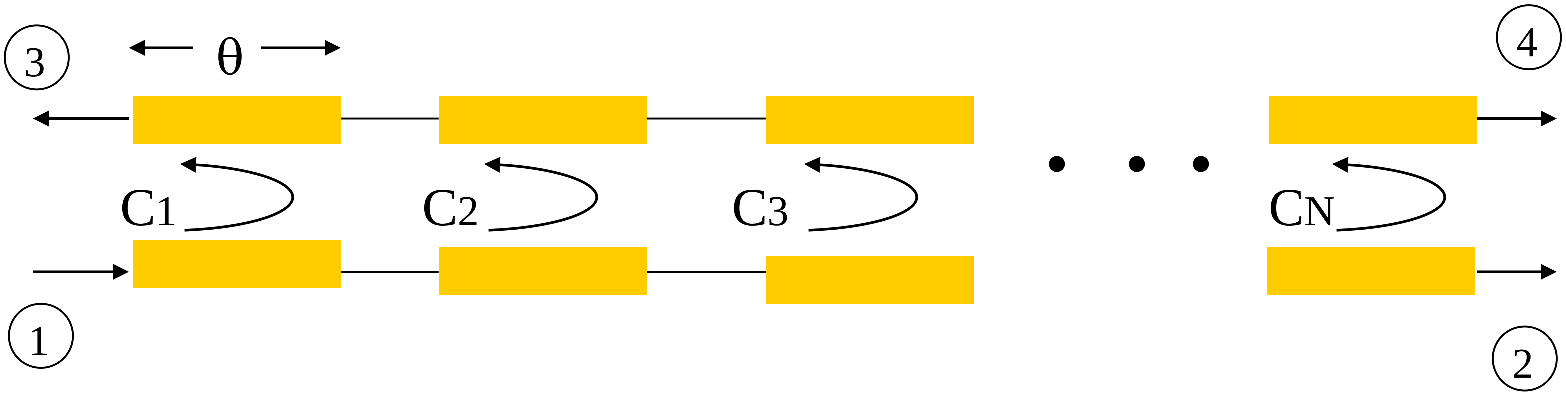
7. Ex.7.7 design a 20dB coupled stripline coupler, b=0.32cm, ε_r=2.2, Zo=50Ω, f=3GHz.

$$C = 10^{-20/20} = 0.1 \rightarrow Z_{oe} = 55.28\Omega, Z_{oo} = 45.23\Omega$$

from p.349, Fig. 7.29, $W/b = 0.809, S/b = 0.306 \rightarrow S = 0.098cm$

frequency response (p.356, Fig. 7.34)

- multisection coupled line coupler



$C << 1, N: odd$

single section $\dfrac{V_3}{V_1} = \dfrac{jC\tan\theta}{\sqrt{1-C^2} + j\tan\theta} \approx \dfrac{jC\tan\theta}{1 + j\tan\theta} = jC\sin\theta e^{-j\theta}$

$$\frac{V_2}{V_1} = \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta} \approx \frac{1}{\cos\theta + j\sin\theta} = e^{-j\theta}$$

multisection $\dfrac{V_3}{V_1} = jC_1\sin\theta e^{-j\theta} + jC_2\sin\theta e^{-j\theta}e^{-j2\theta} + \ldots . jC_N\sin\theta e^{-j\theta}e^{-j2(N-1)\theta}$

if $C_1 = C_N, C_2 = C_{N-1}, \ldots .$

$$\frac{V_3}{V_1} = j\sin\theta e^{-j\theta}[C_1(1+e^{-j2(N-1)\theta}) + C_2(e^{-j2\theta}+e^{-j2(N-2)\theta}) + \ldots . + C_M e^{-j(N-1)\theta}]$$

$$= j2\sin\theta e^{-jN\theta}[C_1\cos(N-1)\theta + C_2\cos(N-3)\theta + \ldots . + \frac{1}{2}C_M], M = \frac{N+1}{2}$$

$C = \dfrac{V_3}{V_1}(\theta = \dfrac{\pi}{2}),\ \ \dfrac{V_3}{V_1}(\theta) \rightarrow C_n \Rightarrow S_n, W_n$

Discussion

1. Ex. 7.8 design a 20dB coupler with binomial response, N=3, $Z_o=50\Omega$, $f_o=3\text{GHz}$.

$$C=\left|\frac{V_3}{V_1}\right|=2\sin\theta[C_1\cos 2\theta+\frac{1}{2}C_2]=2C_1\frac{1}{2}(\sin 3\theta-\sin\theta)+\frac{1}{2}C_2\sin\theta$$

$$=C_1\sin 3\theta+(C_2-C_1)\sin\theta$$

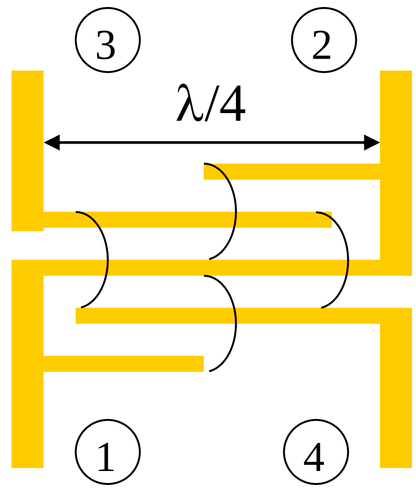
$$\left.\frac{dC}{d\theta}\right|_{\theta=\frac{\pi}{2}}=0,\ \left.\frac{d^2C}{d\theta^2}\right|_{\theta=\frac{\pi}{2}}=10C_1-C_2=0,\ C(\frac{\pi}{2})=\mathrm{C_2\text{-}2C_1}=10^{-20/20}=0.1$$

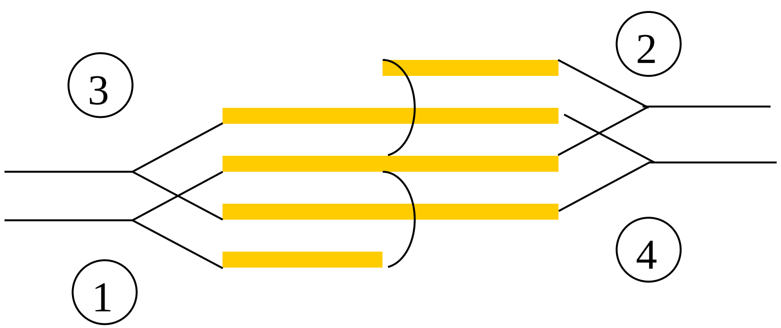
$$\rightarrow \begin{matrix} C_1=C_3=0.0125 \\ C_2=0.125 \end{matrix} \rightarrow Z_{oe}=Z_o\sqrt{\frac{1+C}{1-C}}, Z_{oo}=Z_o\sqrt{\frac{1-C}{1+C}}$$

$$\rightarrow \begin{matrix} Z_{oe}^1=Z_{oe}^3=50.63\Omega, Z_{oo}^1=Z_{oo}^3=49.38\Omega \\ Z_{oe}^2=56.69\Omega,\ Z_{oo}^2=44.1\Omega \end{matrix}$$

frequency response (p.359, Fig. 7.37)

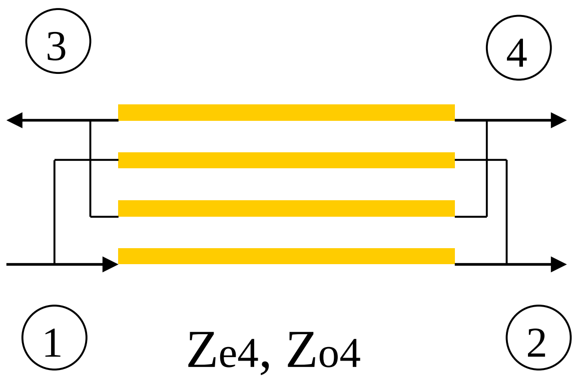
7.7 The Lange coupler (interdigitated coupler)

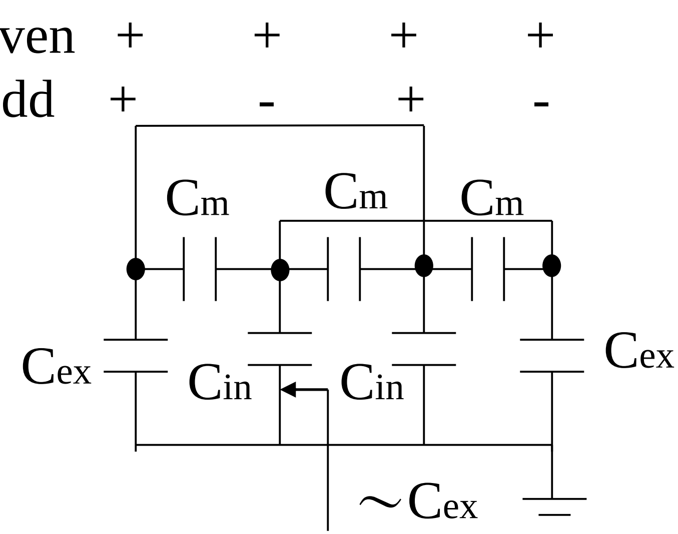




Unfolded Lange coupler

four-wire coupled line model





approximate two-wire coupled line model

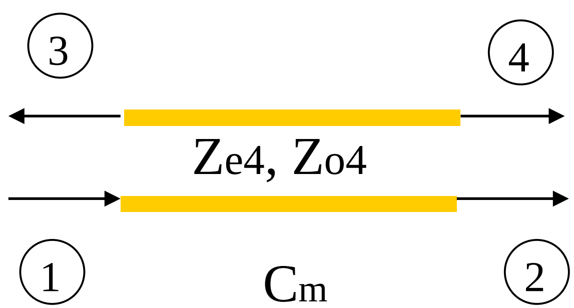
$$C_{ex} \approx C_{in} \,||\, (C_m \text{ in series with } C_{ex})$$

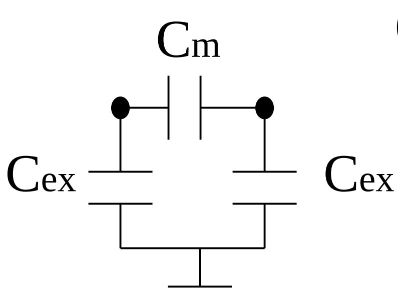
nearest neighbor coupling

$$\rightarrow C_{in} \approx C_{ex} - \frac{C_{ex}C_m}{C_{ex}+C_m}$$

$$C_{e4} = C_{ex} + C_{in}$$

$$C_{o4} = C_{ex} + C_{in} + 3\times 2C_m$$





$$C_e = C_{ex}$$

$$C_o = C_{ex} + 2C_m$$

$$\text{4-line model}: C_{e4} = C_{ex} + C_{in}, C_{o4} = C_{ex} + C_{in} + 6C_m$$

$$\text{2-line model}: C_e = C_{ex}, C_o = C_{ex} + 2C_m$$

$$\rightarrow C_{e4} = \frac{C_e(3C_e + C_o)}{C_e + C_o}, C_{o4} = \frac{C_o(3C_o + C_e)}{C_e + C_o} \ldots(1)$$

$$Z_o = \frac{1}{vC} \Rightarrow Z_{e4} = \frac{Z_{oo} + Z_{oe}}{3Z_{oo} + Z_{oe}} Z_{oe}, Z_{o4} = \frac{Z_{oo} + Z_{oe}}{3Z_{oe} + Z_{oo}} Z_{oo} \ldots(2)$$

$$Z_o = \sqrt{Z_{e4} Z_{o4}}, C \equiv \frac{Z_{e4} - Z_{o4}}{Z_{e4} + Z_{o4}} \xrightarrow{prob.7.27} \begin{cases} Z_{oe} = \dfrac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{(1-C)/(1+C)}} Z_o \\ Z_{oo} = \dfrac{4C - 3 - \sqrt{9 - 8C^2}}{2C\sqrt{(1+C)/(1-C)}} Z_o \end{cases} \Rightarrow W, S$$

<u>Discussion</u>

1. Lange coupler is suitable for wideband 3dB 90° hybrid, and MMIC design uses air bridges instead of bond wires.

$$\text{derivation of (1)}: \text{4-line}: C_{e4} = C_{ex} + C_{in} \overset{C_{in} \approx C_{ex} - \frac{C_{ex}C_m}{C_{ex}+C_m}}{=} 2C_{ex} - \frac{C_{ex}C_m}{C_{ex}+C_m}$$

$$\text{2-line}: C_e = C_{ex}, C_o = C_{ex} + 2C_m \to C_m = \frac{C_o - C_e}{2}$$

$$C_{e4} = 2C_e - \frac{C_e \frac{C_o - C_e}{2}}{C_e + \frac{C_o - C_e}{2}} = 2C_e - \frac{C_eC_o - C_e^2}{C_e + C_o} = \frac{2C_e^2 + 2C_eC_o - C_eC_o + C_e^2}{C_e + C_o} = \frac{C_e(3C_e + C_o)}{C_e + C_o}$$

$$C_{o4} = C_{ex} + C_{in} + 6C_m = \frac{C_e(3C_e + C_o)}{C_e + C_o} + 3(C_o - C_e) = \frac{3C_e^2 + C_eC_o + 3C_o^2 - 3C_e^2}{C_e + C_o} = \frac{C_o(3C_o + C_e)}{C_e + C_o}$$

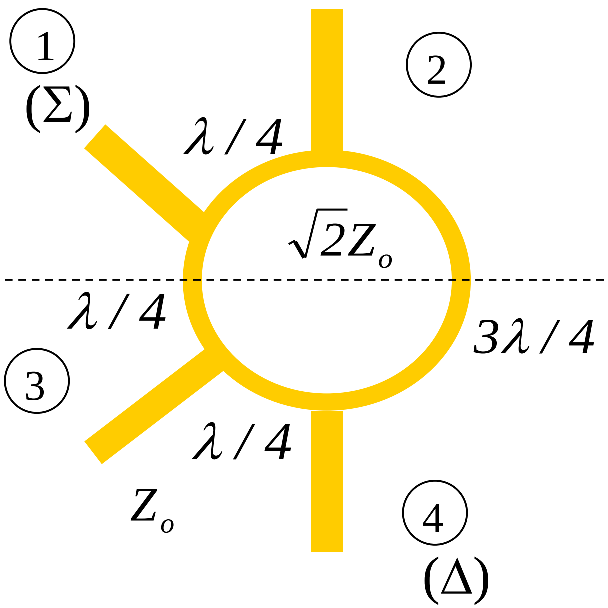
$$\text{derivation of (2)}: Z_o = \frac{1}{vC}$$

$$\Rightarrow Z_{e4} = \frac{1}{vC_{e4}} = \frac{1}{v\frac{C_e(3C_e + C_o)}{C_e + C_o}} = \frac{1}{vC_e}\frac{C_e + C_o}{3C_e + C_o} \times \frac{1/vC_eC_o}{1/vC_eC_o} = \frac{1}{vC_e}\frac{1/vC_o + 1/vC_e}{3/vC_o + 1/vC_e} = Z_{oe}\frac{Z_{oo} + Z_{oe}}{3Z_{oo} + Z_{oe}}$$

$$Z_{o4} = \frac{1}{vC_{o4}} = \frac{1}{v\frac{C_o(3C_o + C_e)}{C_e + C_o}} = \frac{1}{vC_o}\frac{C_e + C_o}{3C_o + C_e} \times \frac{1/vC_eC_o}{1/vC_eC_o} = \frac{1}{vC_o}\frac{1/vC_o + 1/vC_e}{3/vC_e + 1/vC_o} = Z_{oo}\frac{Z_{oo} + Z_{oe}}{3Z_{oe} + Z_{oo}}$$

7.8 The 180° hybrid

• rat-race (ring) hybrid



(1) input port 1

port 1 match $\rightarrow \sqrt{2Z_o \times Z_o} = \sqrt{2}Z_o$

$$\frac{3}{4}\lambda - \frac{1}{4}\lambda = \frac{\lambda}{2} \quad \rightarrow \text{port 4 "GND"}$$

or isolated port

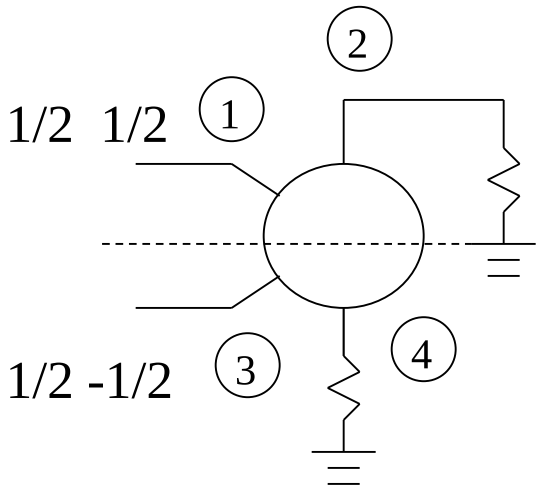
(2) input ports 2 and 3

$\rightarrow$ port 1: Σ port, port 4 :Δ port

$$\frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

Discussion

1. even-odd mode analysis

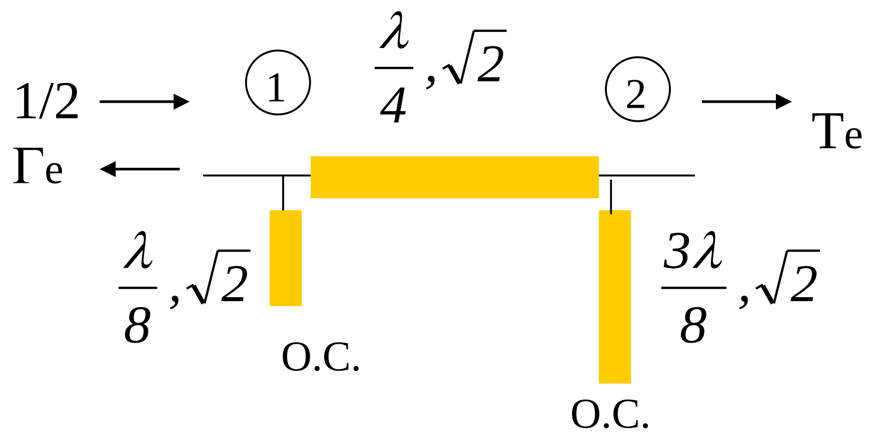


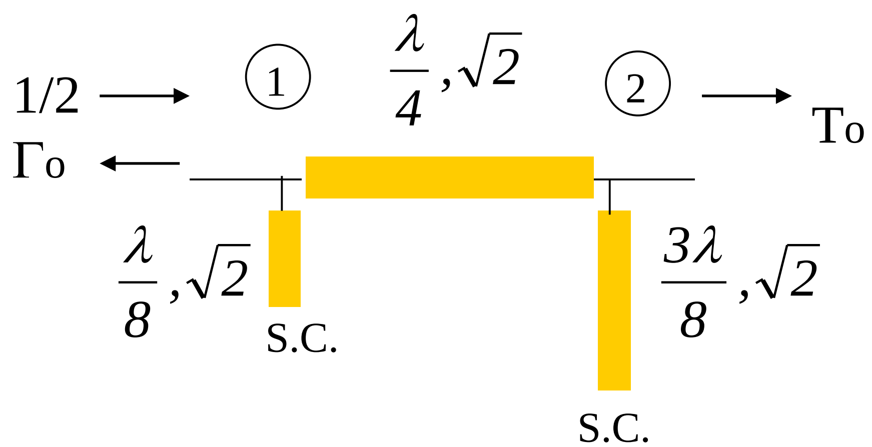
$$b_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o,$$

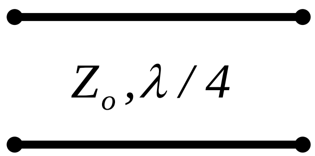
$$b_2 = \frac{1}{2}T_e + \frac{1}{2}T_o$$

$$b_3 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o,$$

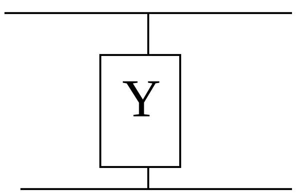
$$b_4 = \frac{1}{2}T_e - \frac{1}{2}T_o$$







$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & jZ_o \\ jY_o & 0 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{cases} \begin{bmatrix} 1 & 0 \\ jY_o & 1 \end{bmatrix} \text{even-mode open-circuit } \lambda/8 \text{ stub} \\ \begin{bmatrix} 1 & 0 \\ -jY_o & 1 \end{bmatrix} \text{even-mode open-circuit } 3\lambda/8 \text{ stub} \\ \begin{bmatrix} 1 & 0 \\ -jY_o & 1 \end{bmatrix} \text{odd-mode short-circuit } \lambda/8 \text{ stub} \\ \begin{bmatrix} 1 & 0 \\ jY_o & 1 \end{bmatrix} \text{odd-mode short-circuit } 3\lambda/8 \text{ stub} \end{cases}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ \frac{j}{\sqrt{2}} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ \frac{j}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{j}{\sqrt{2}} & 1 \end{bmatrix} = \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -\frac{j}{\sqrt{2}} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ \frac{j}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{j}{\sqrt{2}} & 1 \end{bmatrix} = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix}$$

$$\rightarrow \Gamma_e = -\frac{j}{\sqrt{2}}, \Gamma_o = \frac{j}{\sqrt{2}}, T_e = -\frac{j}{\sqrt{2}}, T_o = -\frac{j}{\sqrt{2}}$$

$$\Rightarrow b_1 = 0, b_2 = -\frac{j}{\sqrt{2}}, b_3 = -\frac{j}{\sqrt{2}}, b_4 = 0$$

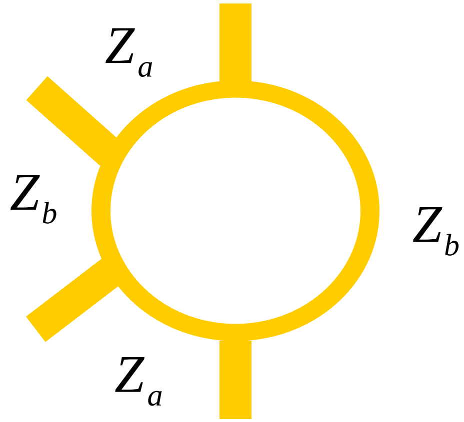
2. input port 2 $\rightarrow$ ports 1, 4 :180° phase difference, port 3 isolated port
input port 4 $\rightarrow$ ports 2, 3 :180° phase difference, port 1 isolated port

3. Ex. 7.9 3dB rat-race hybrid Zo=70.7Ω, BW 20~30% (p.367, Fig.7.46)

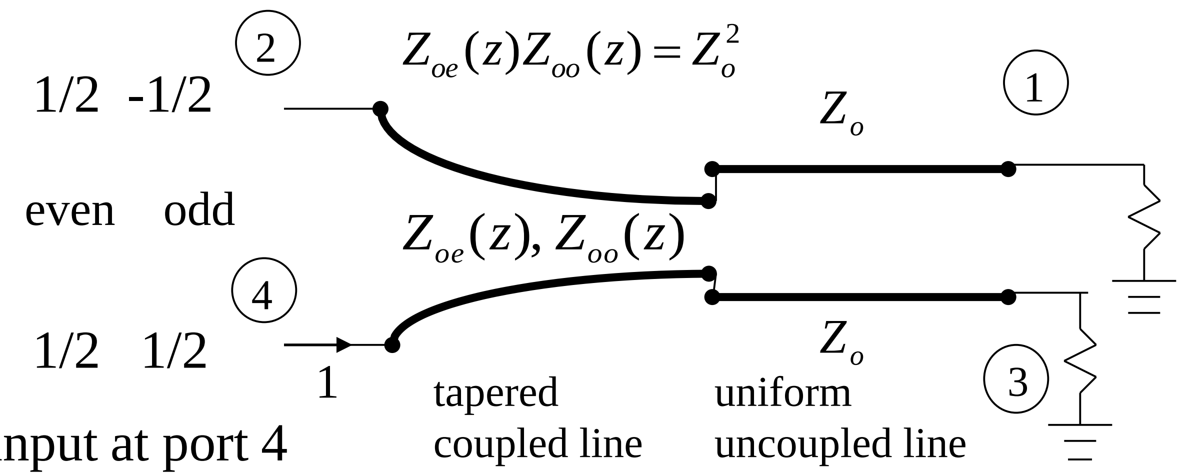
4.

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = -\frac{j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ a_2 \\ a_3 \\ 0 \end{bmatrix} = -\frac{j}{\sqrt{2}} \begin{bmatrix} a_2 + a_3 \\ 0 \\ 0 \\ -a_2 + a_3 \end{bmatrix}$$

5. Unequal power division rat-race coupler uses Za, Zb lines



- tapered coupled line hybrid

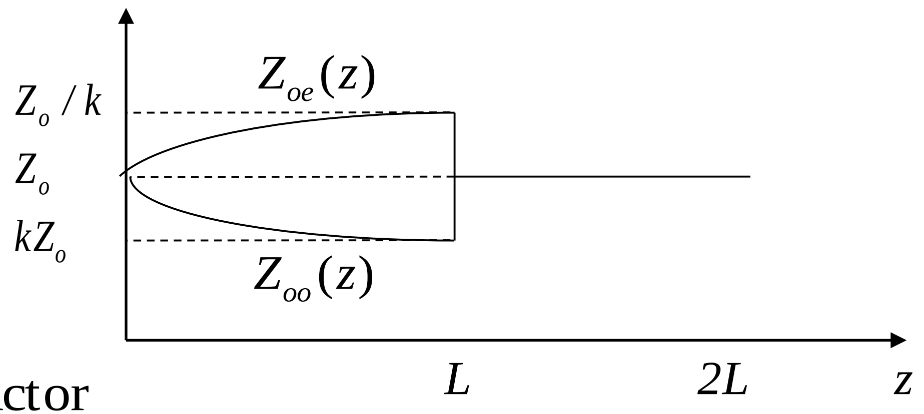


$$b_1=\frac{1}{2}T_e-\frac{1}{2}T_o$$

$$b_2=\frac{1}{2}\Gamma_e-\frac{1}{2}\Gamma_o$$

$$b_3=\frac{1}{2}T_e+\frac{1}{2}T_o$$

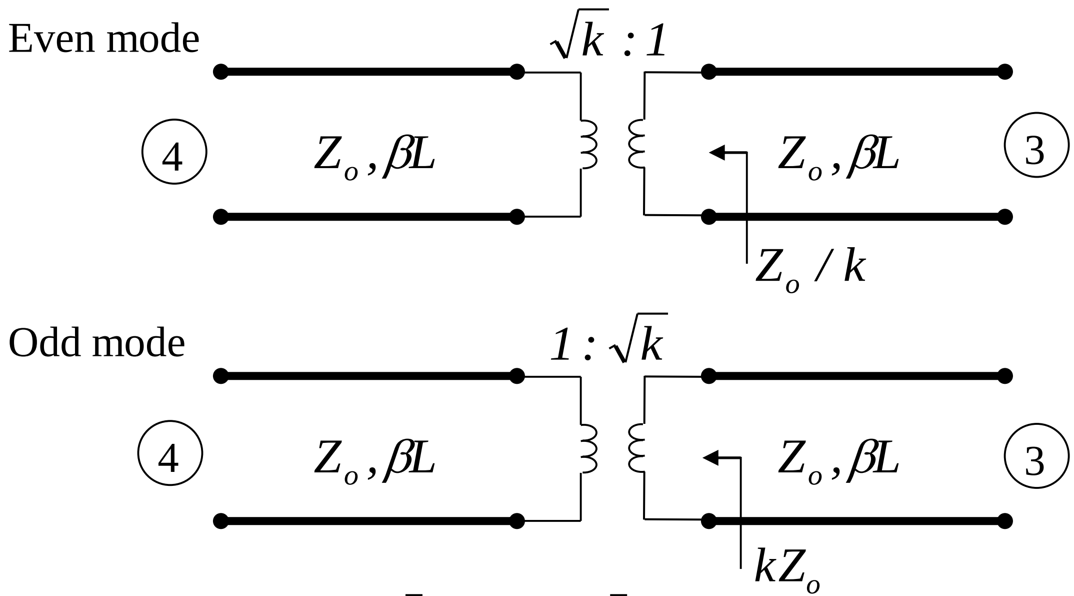
$$b_4=\frac{1}{2}\Gamma_e+\frac{1}{2}\Gamma_o$$



$0\le k\le 1$

coupling factor

$$\begin{bmatrix} b_1 & b_2 & b_3 & b_4 \\ b_2 & b_1 & b_4 & b_3 \\ b_3 & b_4 & b_1 & b_2 \\ b_4 & b_3 & b_2 & b_1 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} \cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sqrt{k} & 0 \\ 0 & \frac{1}{\sqrt{k}} \end{bmatrix} \begin{bmatrix} \cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta \end{bmatrix} \quad \text{(p.190, Table 4.1)}$$

$$= \begin{bmatrix} \sqrt{k}\cos^2\theta - \frac{1}{\sqrt{k}}\sin^2\theta & j(\sqrt{k} + \frac{1}{\sqrt{k}})\sin\theta\cos\theta \\ j(\sqrt{k} + \frac{1}{\sqrt{k}})\sin\theta\cos\theta & -\sqrt{k}\sin^2\theta + \frac{1}{\sqrt{k}}\cos^2\theta \end{bmatrix} \rightarrow \begin{aligned} \Gamma_e &= \frac{A+B-C-D}{A+B+C+D} \\ T_e &= \frac{2}{A+B+C+D} \end{aligned}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} \cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{k}} & 0 \\ 0 & \sqrt{k} \end{bmatrix} \begin{bmatrix} \cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{k}}\cos^2\theta - \sqrt{k}\sin^2\theta & j(\sqrt{k} + \frac{1}{\sqrt{k}})\sin\theta\cos\theta \\ j(\sqrt{k} + \frac{1}{\sqrt{k}})\sin\theta\cos\theta & -\frac{1}{\sqrt{k}}\sin^2\theta + \sqrt{k}\cos^2\theta \end{bmatrix} \rightarrow \begin{matrix} \Gamma_o = \dfrac{A+B-C-D}{A+B+C+D} \\ T_o = \dfrac{2}{A+B+C+D} \end{matrix}$$

$$\rightarrow \Gamma_e = \frac{k-1}{k+1}e^{-j2\theta}, \Gamma_o = \frac{1-k}{k+1}e^{-j2\theta}, T_e = \frac{2\sqrt{k}}{k+1}e^{-j2\theta}, T_o = \frac{2\sqrt{k}}{k+1}e^{-j2\theta}$$

$$\Rightarrow b_1 = 0 = S_{14}, b_2 = \frac{k-1}{k+1}e^{-j2\theta} \equiv -\alpha e^{-j2\theta} = S_{24},$$

$$b_3 = \frac{2\sqrt{k}}{k+1}e^{-j2\theta} \equiv \beta e^{-j2\theta} = S_{34}, b_4 = 0 = S_{44}$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$b_1 = \frac{1}{2}T_e - \frac{1}{2}T_o$$

$$b_2 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

$$b_3 = \frac{1}{2}T_e + \frac{1}{2}T_o$$

$$b_4 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o$$

input at port 2 = input at port 4 $\rightarrow S_{12} = S_{34}, S_{22} = S_{44}, S_{32} = S_{14}, S_{42} = S_{24}$

reciprocal $\rightarrow S_{23} = S_{32}, S_{41} = S_{14}$

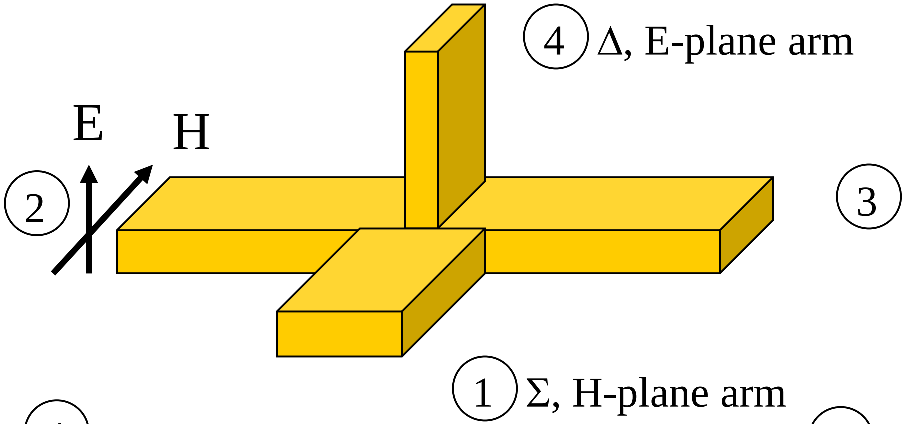
input at port 1 $\rightarrow \Gamma_e = \dfrac{1-k}{k+1} e^{-j2\theta}, \Gamma_o = \dfrac{k-1}{k+1} e^{-j2\theta}$

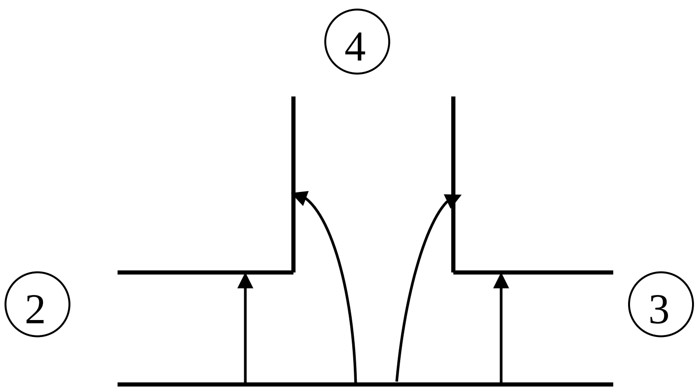
$$\Rightarrow b_1 = \frac{1}{2}(\Gamma_e + \Gamma_o) = 0 = S_{11}, b_3 = \frac{1}{2}(\Gamma_e - \Gamma_o) = \frac{1-k}{k+1} e^{-j2\theta} \equiv \alpha e^{-j2\theta} = S_{31}$$

input at port 3 = input at port 1 $\rightarrow S_{13} = S_{31}$

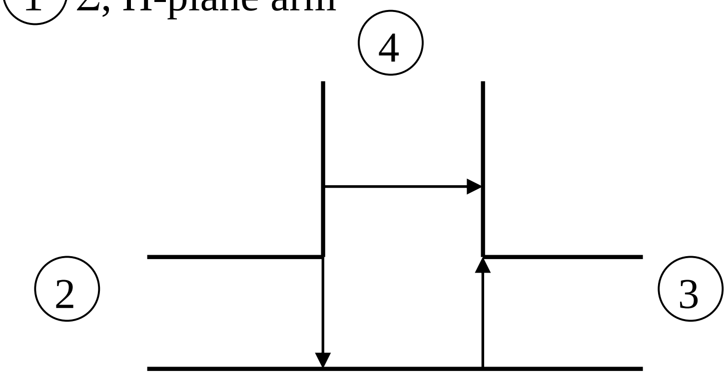
$$\Rightarrow [S] = e^{-j2\theta} \begin{bmatrix} 0 & \beta & \alpha & 0 \\ \beta & 0 & 0 & -\alpha \\ \alpha & 0 & 0 & \beta \\ 0 & -\alpha & \beta & 0 \end{bmatrix}$$

- waveguide magic-T





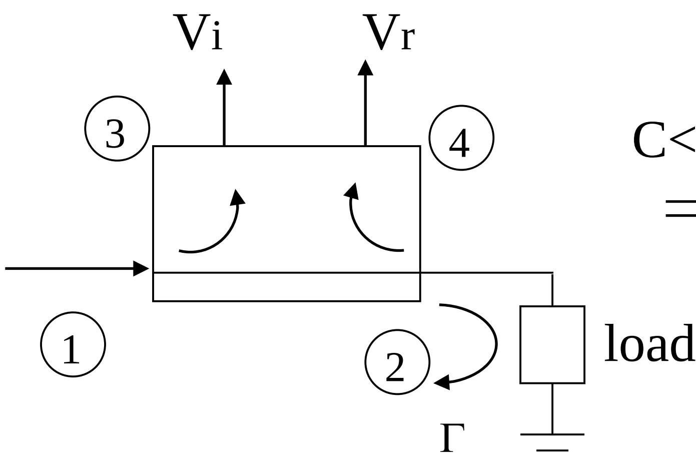
input at port 1
port 4: 0
port 2 and port 3: equal amplitude and phase



input at port 4
port 1: 0
port 2 and port 3: 180° phase difference

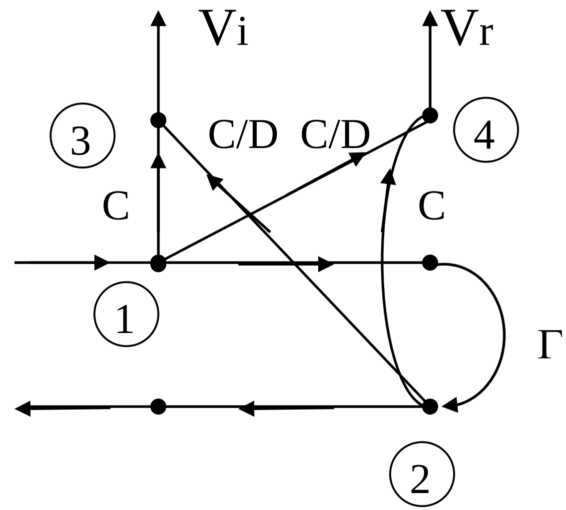
7.9 Other couplers

- reflectometer



C<<1

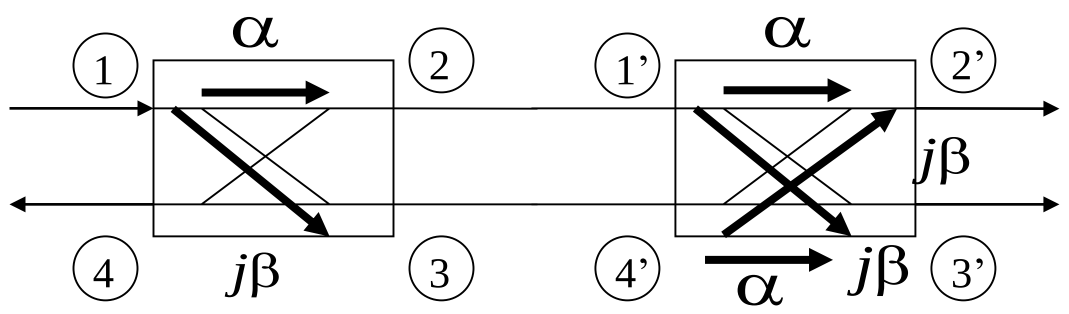
$\Rightarrow$



$$V_i = C + \frac{C}{D}\Gamma\ e^{j\theta}, V_r = \frac{C}{D} + C\Gamma\ e^{j\phi}$$

$$\left|\frac{V_r}{V_i}\right|_{\max,\min} = \frac{|\Gamma| \pm \frac{1}{D}}{1 \mp \frac{|\Gamma|}{D}},\ \text{as } D \to \infty\ \left|\frac{V_r}{V_i}\right| \to \Gamma$$

Prob. 7.4 Two 90° 8.34dB couplers are connected in cascade, find $S_{2'1}, S_{3'1}$



$$C = 8.34dB = -20\log\beta \rightarrow \beta = 0.383, \alpha^2 + \beta^2 = 1 \rightarrow \alpha = 0.924$$

$$V_{3'}^- = \alpha V_{4'}^+ + j\beta V_{1'}^+ = \alpha V_3^- + j\beta V_2^- = \alpha(j\beta V_1^+) + j\beta(\alpha V_1^+)$$

$$= 2j\alpha\beta V_1^+ = 0.707\angle 90^o V_1^+$$

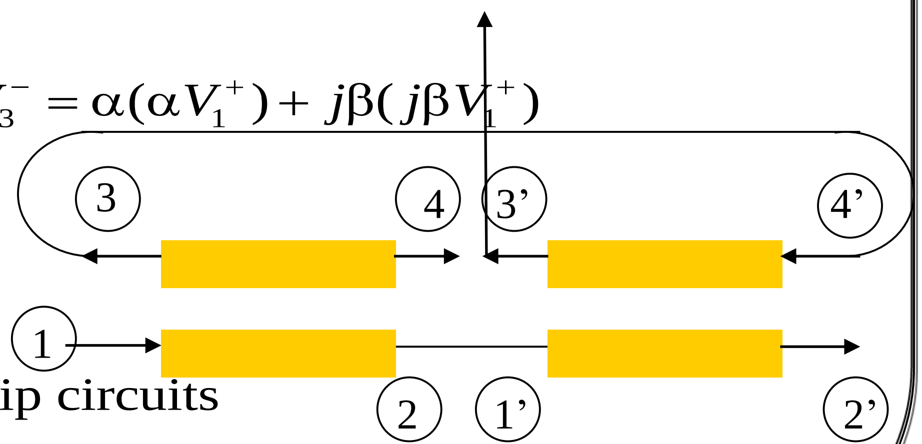
$$V_{2'}^- = \alpha V_{1'}^+ + j\beta V_{4'}^+ = \alpha V_2^- + j\beta V_3^- = \alpha(\alpha V_1^+) + j\beta(j\beta V_1^+)$$

$$= (\alpha^2 - \beta^2)V_1^+ = 0.707 V_1^+$$

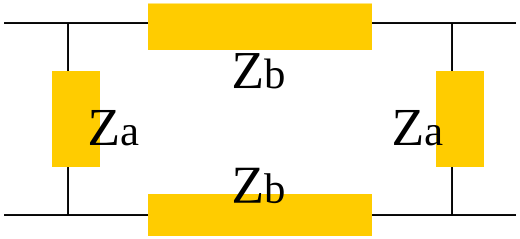
$$V_4^- = 0$$

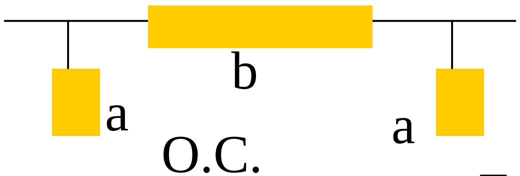
$\Rightarrow 3dB$ quadrature coupler

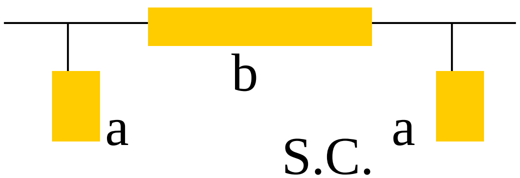
connection problem in microstrip circuits



Prob. 7.18 Design an unequal power branch-line coupler







(even mode) $a=\dfrac{Z_a}{Z_o},\ b=\dfrac{Z_b}{Z_o}$ (odd mode)

$$\begin{bmatrix}A&B\\C&D\end{bmatrix}_e=\begin{bmatrix}1&0\\j/a&1\end{bmatrix}\begin{bmatrix}0&jb\\j/b&0\end{bmatrix}\begin{bmatrix}1&0\\j/a&1\end{bmatrix}=\begin{bmatrix}-b/a&jb\\j/b-jb/a^2&-b/a\end{bmatrix}$$

$$\begin{bmatrix}A&B\\C&D\end{bmatrix}_o=\begin{bmatrix}1&0\\-j/a&1\end{bmatrix}\begin{bmatrix}0&jb\\j/b&0\end{bmatrix}\begin{bmatrix}1&0\\-j/a&1\end{bmatrix}=\begin{bmatrix}b/a&jb\\j/b-jb/a^2&b/a\end{bmatrix}$$

$$\rightarrow \Gamma_e=\frac{j(b-1/b+b/a^2)}{-2b/a+j(b+1/b-b/a^2)},\ \Gamma_o=\frac{j(b-1/b+b/a^2)}{2b/a+j(b+1/b-b/a^2)},\ T_e=\frac{1}{-b/a+jb},\ T_o=\frac{1}{b/a+jb}$$

input match $b_1=\dfrac{1}{2}(\Gamma_e+\Gamma_o)=0\rightarrow 1-\dfrac{1}{b^2}+\dfrac{1}{a^2}=0\ldots(1)$

$$b_2=\frac{1}{2}(T_e+T_o)=\frac{-j}{b(1+1/a^2)},\ b_3=\frac{1}{2}(T_e-T_o)=\frac{-1/a}{b(1+1/a^2)},\ P_2=\alpha P_3\rightarrow |b_2|^2=\alpha|b_3|^2\Rightarrow 1=\frac{\alpha}{a^2}\rightarrow a=\sqrt{\alpha}$$

$$(1)\rightarrow b=\frac{a}{\sqrt{1+a^2}}=\sqrt{\frac{\alpha}{1+\alpha}}\Rightarrow Z_a=\sqrt{\alpha}Z_o,\ Z_b=\sqrt{\frac{\alpha}{1+\alpha}}Z_o$$

ADS examples: Ch7_prj