Chapter 7 Power dividers and directional couplers

7.1 Basic properties of dividers and couplers
three-port network (T-junction), four-port network (directional coupler), directivity measurement
7.2 The T-junction power divider
lossless divider, lossy divider
7.3 The Wilkinson power divider
even-odd mode analysis, unequal power division divider, n-way Wilkinson divider
7.5 The quadrature (90°) hybrid
branch-line coupler
7.6 Coupled line directional couplers
even- and odd-mode Z_o, single-section and multisection coupled line couplers

7.7 The Lange coupler

7.8 The 180° hybrid

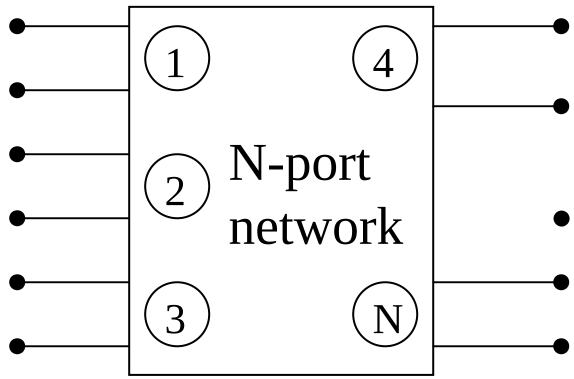
rat-race hybrid, tapered coupled line hybrid

7.9 Other couplers

reflectometer

7.1 Basic properties of dividers and couplers

• N-port network



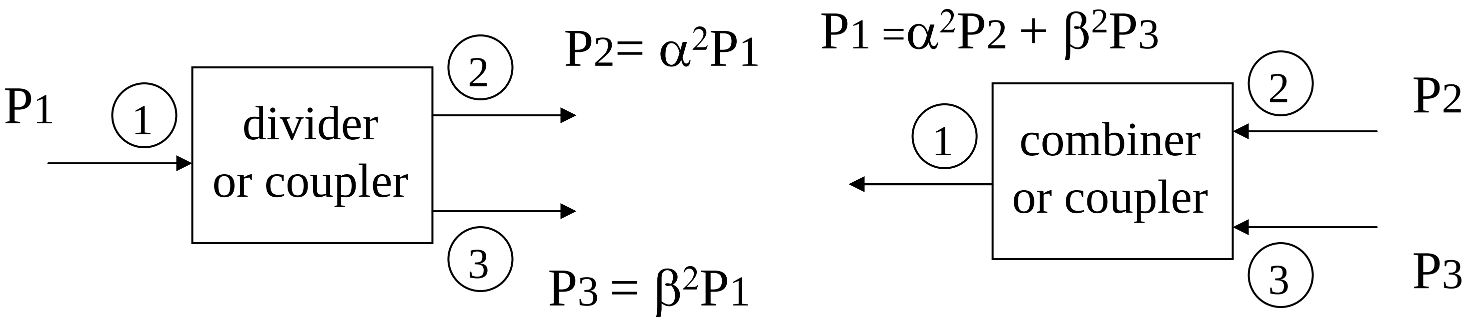
$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & \bullet & S_{1N} \\ S_{21} & S_{22} & S_{23} & \bullet & S_{2N} \\ S_{31} & S_{32} & S_{33} & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ S_{N1} & S_{N2} & \bullet & \bullet & S_{NN} \end{bmatrix} \qquad S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+=0, k\neq j}$$

<u>Discussion</u>

1. matched ports → $S_{ii} = 0$
2. reciprocal network → symmetric property $S_{ij} = S_{ji}$
3. lossless network → unitary property

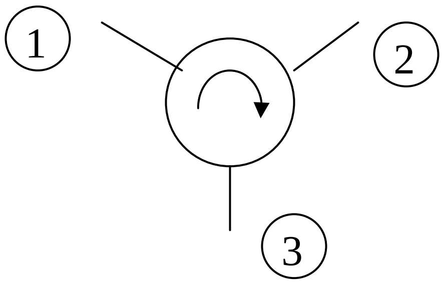
$$\sum_{i=1}^{N} \left| S_{ij} \right|^2 = 1 \ \forall j, \quad \sum_{i=1}^{N} S_{ik} S_{kj}^* = 0 \quad k \neq j$$

• three-port network (T-junction)

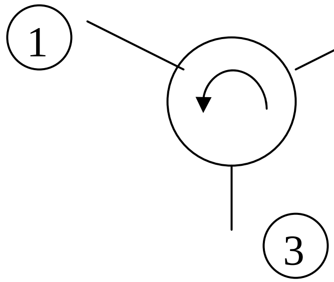


Discussion

1. Three-port network cannot be lossless, reciprocal and matched at all ports.
2. A lossless and matched three-port network is nonreciprocal
   $\rightarrow$ circulator



$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

3. A matched and reciprocal three-port network is lossy $\rightarrow$ resistive divider.
4. A matched and lossy three-port network can have ∞ isolation at two output ports ($S_{23}=S_{32}=0$) $\rightarrow$ Wilkinson power divider .

(derivation of 1)

For a matched, reciprocal three-port network

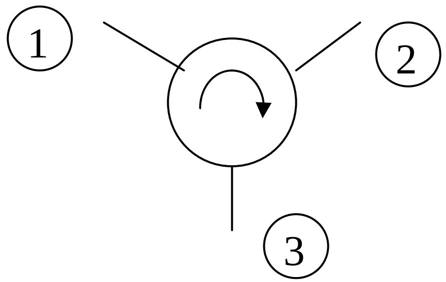
$$\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \xrightarrow{lossless} \begin{matrix} |S_{12}|^2 + |S_{13}|^2 = 1 \\ |S_{12}|^2 + |S_{23}|^2 = 1 \\ |S_{13}|^2 + |S_{23}|^2 = 1 \end{matrix} \; , \begin{matrix} S_{13}^{*}S_{23} = 0 \\ S_{12}^{*}S_{13} = 0 \\ S_{12}^{*}S_{23} = 0 \end{matrix} \rightarrow \text{if } S_{13} = 0, \; \begin{matrix} |S_{12}| = 1 \\ |S_{23}| = 0 \\ |S_{13}| = 1 \neq 0 \end{matrix} \rightarrow \text{lossy}$$

(derivation of 2)

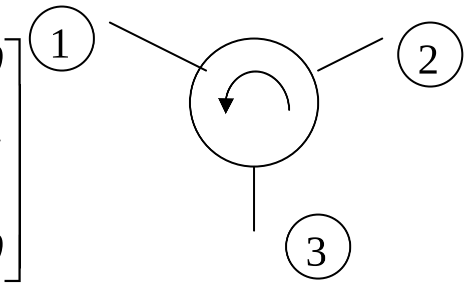
For a matched, lossless, nonreciprocal three-port network

$$\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \xrightarrow{lossless} \begin{matrix} |S_{21}|^2 + |S_{31}|^2 = 1 \\ |S_{12}|^2 + |S_{32}|^2 = 1 \\ |S_{13}|^2 + |S_{23}|^2 = 1 \end{matrix} \; , \begin{matrix} S_{31}^{*}S_{32} = 0 \\ S_{12}^{*}S_{13} = 0 \\ S_{21}^{*}S_{23} = 0 \end{matrix} \rightarrow \begin{matrix} \text{if } S_{21}=1 \rightarrow S_{31}=0, S_{32}=1, S_{23}=0, S_{13}=1, S_{12}=0 \\ \text{if } S_{21}=0 \rightarrow S_{31}=1, S_{32}=0, S_{23}=1, S_{13}=0, S_{12}=1 \end{matrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



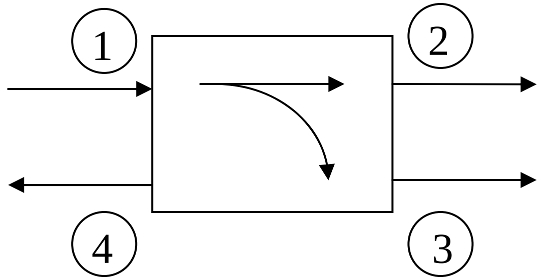
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

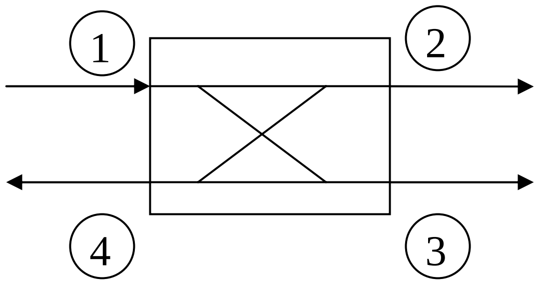


• four-port network (directional coupler)

input port P1 — P2 through port

isolated port P4 — P3 coupled port

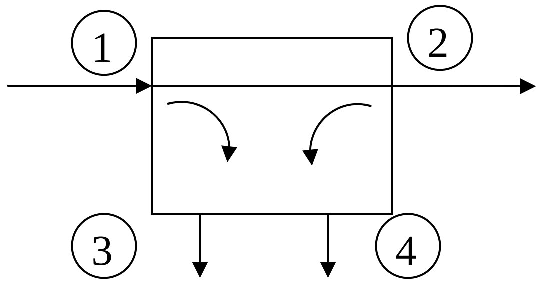




coupling $C(dB) \equiv 10\log\dfrac{P_1}{P_3} = -20\log|S_{31}|$

directivity $D(dB) \equiv 10\log\dfrac{P_3}{P_4} = 10\log\dfrac{P_3}{P_1}\dfrac{P_1}{P_4} = 20\log\dfrac{|S_{31}|}{|S_{41}|}$

isolation $I(dB) \equiv 10\log\dfrac{P_1}{P_4} = C + D = -20\log|S_{41}|$



voltage coupling factor $C = 10^{-C(dB)/20} = \left|\dfrac{V_3^-}{V_1^+}\right| < 1$

directivity $D = 10^{D(dB)/20} = \left|\dfrac{V_3^-}{V_4^-}\right| > 1$

Discussion

1. Matched, reciprocal and lossless four-port network → symmetrical (90°) directional coupler or antisymmetrical (180°) directional coupler

$$\begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

2. C=3dB → 90° hybrid (quadrature hybrid, symmetrical coupler), 180° hybrid (magic-T hybrid, rate-race hybrid)

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

(derivation of 1)

For a matched, reciprocal and lossless four-port network

$$\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix} \rightarrow \begin{array}{l} \text{row } 1^*, 2: S_{13}^* S_{23} + S_{14}^* S_{24} = 0 \ldots (1) \\ \text{row } 3, 4^*: S_{14}^* S_{13} + S_{24}^* S_{23} = 0 \ldots (2) \\ \text{row } 1^*, 3: S_{12}^* S_{23} + S_{14}^* S_{34} = 0 \ldots (3) \\ \text{row } 2, 4^*: S_{14}^* S_{12} + S_{34}^* S_{23} = 0 \ldots (4) \end{array}$$

$$\xrightarrow{(1)S_{24}^* - (2)S_{13}^*} S_{14}^* (|S_{13}|^2 - |S_{24}|^2) = 0$$

$$\xrightarrow{(3)S_{12} - (4)S_{34}} S_{23} (|S_{12}|^2 - |S_{34}|^2) = 0$$

$$\text{case 1}: S_{14} = S_{23} = 0 \rightarrow \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix} \rightarrow \begin{array}{l} |S_{12}|^2 + |S_{13}|^2 = 1 \\ |S_{12}|^2 + |S_{24}|^2 = 1 \\ |S_{13}|^2 + |S_{34}|^2 = 1 \\ |S_{24}|^2 + |S_{34}|^2 = 1 \end{array} \rightarrow \begin{array}{l} |S_{13}| = |S_{24}| \\ |S_{12}| = |S_{34}| \end{array} \xrightarrow{choose} \begin{array}{l} S_{12} = S_{34} = \alpha \\ S_{13} = \beta e^{j\theta} \\ S_{24} = \beta e^{j\phi} \\ \alpha^2 + \beta^2 = 1 \end{array}$$

$$\text{row } 2^*, 3 \rightarrow S_{12}^* S_{13} + S_{24}^* S_{34} = 0 \rightarrow e^{j\theta} + e^{-j\phi} = 0 \rightarrow \theta + \phi = \pi$$

(a) 90^o directional coupler with $\theta = \phi = \dfrac{\pi}{2}$ (b) 180^o directional coupler with $\theta = 0, \phi = \pi$

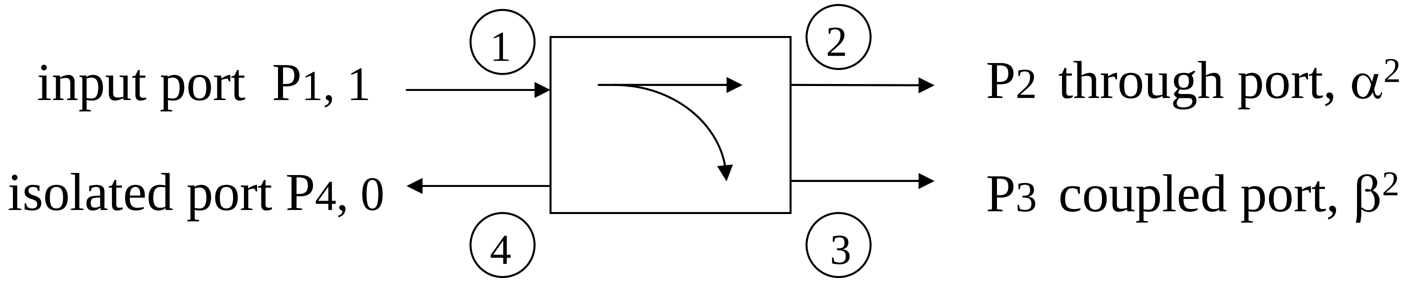
$$\begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix} \rightarrow \begin{array}{l} \text{row } 1^*, 2: S_{13}^* S_{23} + S_{14}^* S_{24} = 0 \ldots (1) \\ \text{row } 3, 4^*: S_{14}^* S_{13} + S_{24}^* S_{23} = 0 \ldots (2) \\ \text{row } 1^*, 3: S_{12}^* S_{23} + S_{14}^* S_{34} = 0 \ldots (3) \\ \text{row } 2, 4^*: S_{14}^* S_{12} + S_{34}^* S_{23} = 0 \ldots (4) \end{array} \begin{array}{l} \xrightarrow{(1)S_{24}^* - (2)S_{13}^*} S_{14}^* (|S_{13}|^2 - |S_{24}|^2) = 0 \\ \xrightarrow{(3)S_{12} - (4)S_{34}} S_{23} (|S_{12}|^2 - |S_{34}|^2) = 0 \end{array}$$

$$\text{case } 2: \text{if } \begin{array}{l} |S_{13}| = |S_{24}| \\ |S_{12}| = |S_{34}| \end{array} \xrightarrow{choose} \begin{array}{l} S_{13} = S_{24} = j\beta \\ S_{12} = S_{34} = \alpha \end{array} \rightarrow \begin{array}{l} (1): j\beta(-S_{23} + S_{14}^*) = 0 \\ (3): \alpha(S_{23} + S_{14}^*) = 0 \end{array}$$

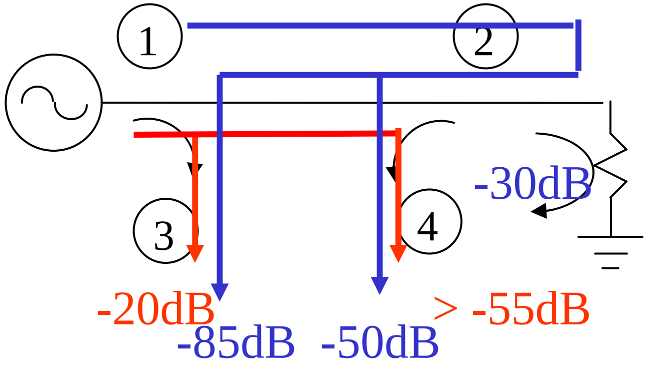
(a) $S_{14} = S_{23} = 0 \rightarrow$ directional coupler as case 1

$$(b)\alpha = \beta = 0 \rightarrow \begin{bmatrix} 0 & 0 & 0 & S_{14} \\ 0 & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & 0 \\ S_{14} & 0 & 0 & 0 \end{bmatrix}, \text{two decoupled two-port networks}$$



3. directivity measurement

If C=20dB, D=35dB, RL=30dB(require <35dB)



For $|\Gamma| < \dfrac{1}{D}$

with the use of a sliding load $|\Gamma| e^{j\angle\Gamma}$

$$P_{4\max} = P_1\left(\frac{C}{D} + C|\Gamma|\right)^2$$

$$P_{4\min} = P_1\left(\frac{C}{D} - C|\Gamma|\right)^2$$

1→3 (C ,-20dB)→4 (C/D,-55dB)

$$V_3^- = C V_1^+ ,\ D = \frac{V_3^-}{V_4^-} \rightarrow V_4^- = \frac{C}{D} V_1^+$$

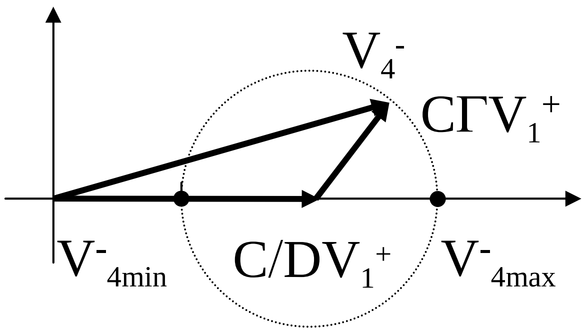
1→2→4 (ΓC,-50dB)→3 (ΓC/D-85dB)

$$V_4^- \approx C\Gamma V_1^+ (= C\Gamma\sqrt{1 - C^2} V_1^+), V_3^- \approx \frac{C\Gamma}{D} V_1^+$$

$$P_3 = C^2 P_1, M = \sqrt{\frac{P_3}{P_{4\max}}} = \frac{D}{1 + |\Gamma| D}$$

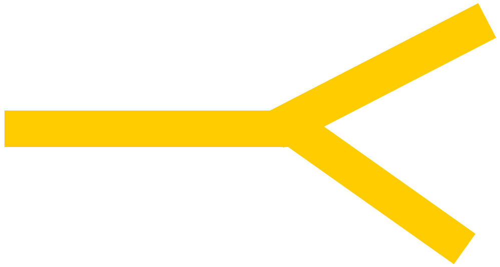
$$m = \sqrt{\frac{P_{4\max}}{P_{4\min}}} = \frac{1 + |\Gamma| D}{1 - |\Gamma| D}$$

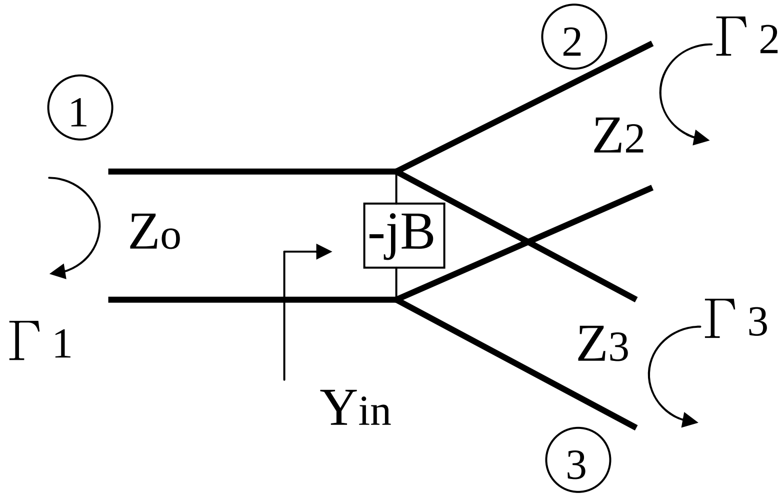
$$\Rightarrow D = M \frac{2m}{m+1}$$



7.2 The T-junction power divider

- lossless divider





$$Y_{in} = jB + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{Z_o} \quad \rightarrow \quad B = 0 \ \ \text{"not practical"}$$

$\Rightarrow$ A lossless divider has mismatched ports.

Discussion

1. Ex. 7.1 Zo=50Ω, P2:P3=1:2, calculate Γ2 and Γ3.

$$P_1 : P_2 : P_3 = 1 : \frac{1}{3} : \frac{2}{3} = \frac{1}{Z_o} : \frac{1}{Z_2} : \frac{1}{Z_3}$$

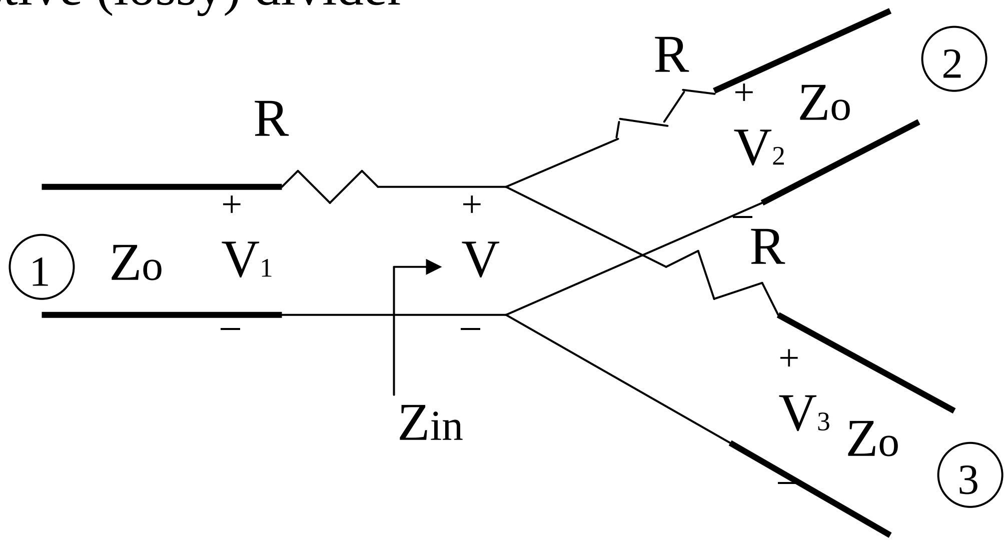
$$\rightarrow Z_2 = 3Z_o = 150\Omega, \quad Z_3 = \frac{3}{2} Z_o = 75\Omega$$

$$Z_{in} = Z_2 // Z_3 = 50\Omega$$

$$\Gamma_2 = \frac{50//75 - Z_2}{50//75 + Z_2} = -0.666, \Gamma_3 = \frac{50//150 - Z_3}{50//150 + Z_3} = -0.333$$

2. It’s a lossless and mismatched three-port divider, but not good in isolation.

• resistive (lossy) divider



matched ports $\Rightarrow (R+Z_o)//(R+Z_o)+R=Z_o \quad \rightarrow \quad R=\frac{Z_o}{3}$

Discussion

1.

$$[S]=\frac{1}{2}\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix} \because Z_{in}=\frac{R+Z_o}{2}=\frac{2}{3}Z_o, V=V_1\frac{\frac{2}{3}Z_o}{R+\frac{2}{3}Z_o}=\frac{2}{3}V_1$$

$$V_2,V_3=V\frac{Z_o}{\frac{Z_o}{3}+Z_o}=\frac{3}{4}V=\frac{1}{2}V_1$$

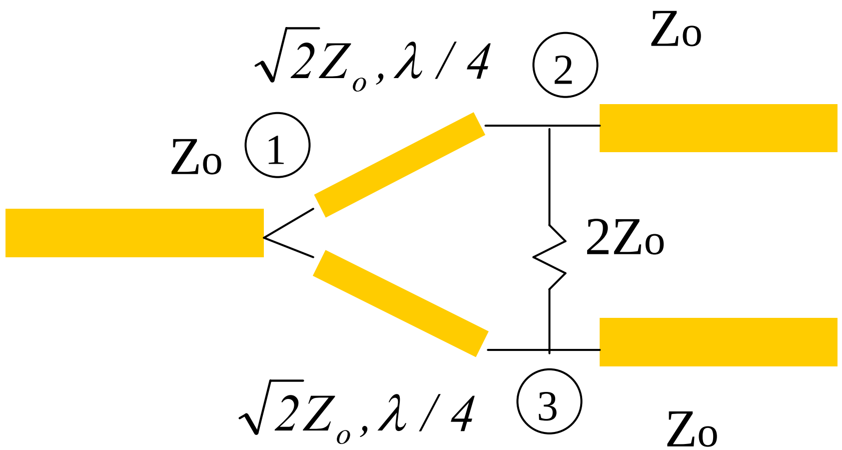
2. $P_{in} = \frac{1}{2}\frac{V_1^2}{Z_o}$, $P_2 = P_3 = \frac{1}{8}\frac{V_1^2}{Z_o} = \frac{P_{in}}{4}$, $P_{loss} = \frac{P_{in}}{4}$ for each R

$\rightarrow$ lossy divider

3. It's a lossy and matched three-port divider, but not good in isolation.

7.3 The Wilkinson power divider

• basic concept



$$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -j & -j \\ -j & 0 & 0 \\ -j & 0 & 0 \end{bmatrix}$$

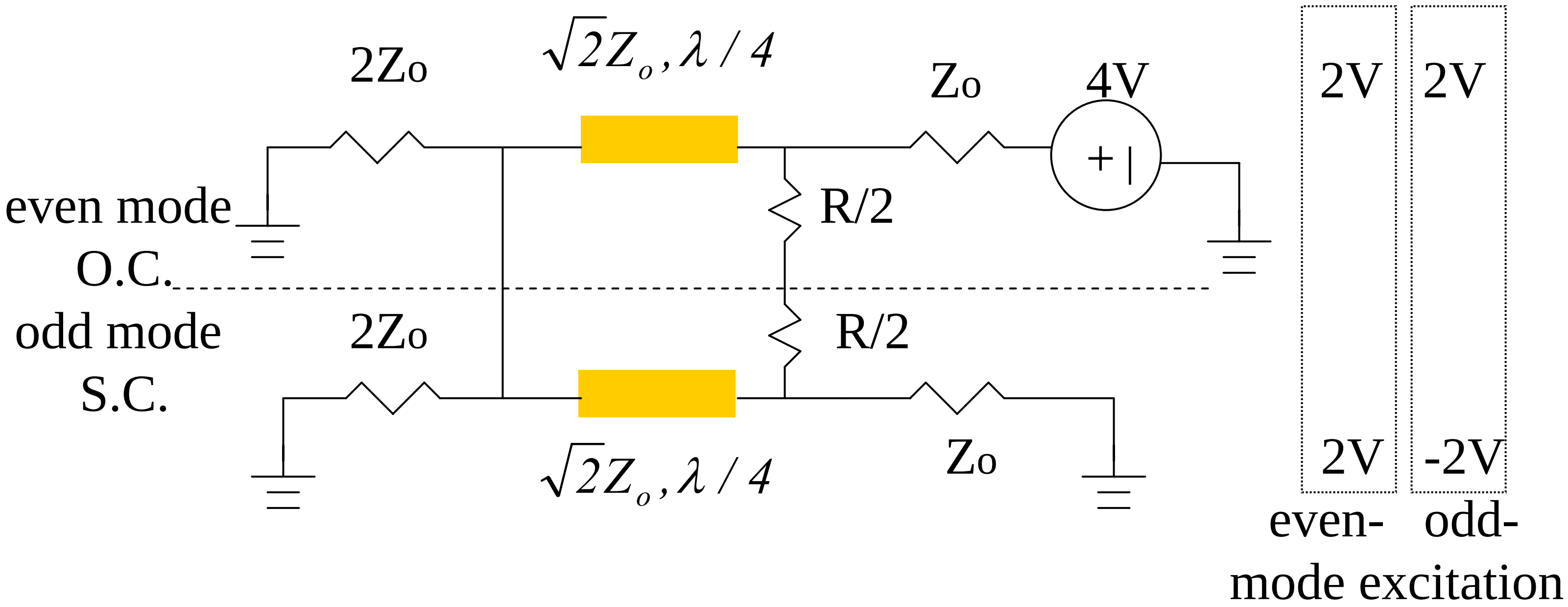
Input port 1 matched, port 2 and port 3 have equal potential

$\rightarrow \quad \sqrt{2}Z_o, \lambda/4$

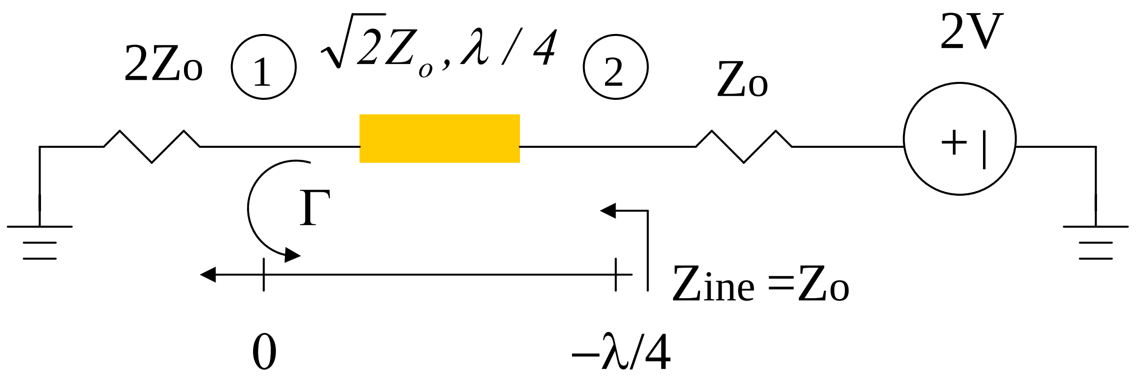
Input port 2, port 2 and port 3 have perfect isolation

=> a lossy, matched and good isolation (equal phase) three-port divider

• even-odd mode analysis
A linear, symmetric network



even-mode



ports 2, 3 matched, $V_{2e} = V$, symmetry of ports 2 and 3 $\rightarrow V_{3e} = V$

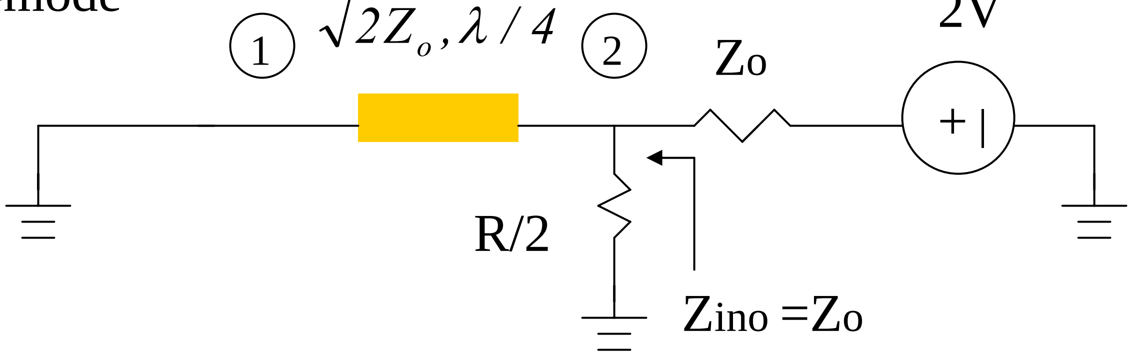
$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}} \rightarrow V_{1e} = jV\frac{\Gamma+1}{\Gamma-1} = -j\sqrt{2}V$$

(derivation of V_{1e}) $V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$

$$V_{2e} = V^+ e^{-j\frac{2\pi}{\lambda}\frac{-\lambda}{4}} + V^- e^{j\frac{2\pi}{\lambda}\frac{-\lambda}{4}} = V^+ e^{j\frac{\pi}{2}} + V^- e^{-j\frac{\pi}{2}} = jV^+(1-\Gamma) \equiv V$$

$$V_{1e} = V^+ + V^- = V^+(1+\Gamma) = -j\frac{1+\Gamma}{1-\Gamma}V = -j\frac{1+\frac{2-\sqrt{2}}{2+\sqrt{2}}}{1-\frac{2-\sqrt{2}}{2+\sqrt{2}}}V = -j\frac{4}{2\sqrt{2}}V = -j\sqrt{2}V$$

odd-mode



$$\frac{R}{2} = Z_o \rightarrow R = 2Z_o \rightarrow \text{ports 2 and 3 matched, } V_{2o} = V, V_{3o} = -V, \text{symmetry of ports 2 and 3}$$

$$\left.\begin{array}{l} V_{2e} = V \\ V_{2o} = V \end{array}\right\} \rightarrow V_2 = V_{2e} + V_{2o} = 2V, \left.\begin{array}{r} V_{1e} = -j\sqrt{2}V \\ V_{1o} = 0 \end{array}\right\} \rightarrow V_1 = V_{1e} + V_{1o} = -j\sqrt{2}V$$

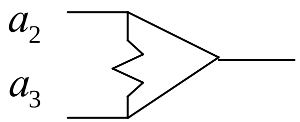
$$\left.\begin{array}{l} V_{3e} = V \\ V_{3o} = -V \end{array}\right\} \rightarrow V_3 = V_{3e} + V_{3o} = 0$$

$$\Rightarrow S_{12} = \frac{V_1^-}{V_2^+} = \frac{V_1}{V_2}\bigg|_{S_{11}=S_{22}=0} = \frac{-j\sqrt{2}V}{2V} = -j\frac{1}{\sqrt{2}} = S_{21} = S_{31} = S_{13}$$

$$\Rightarrow S_{32} = \frac{V_3^-}{V_2^+} = \frac{V_3}{V_2}\bigg|_{S_{11}=S_{33}=0} = \frac{0}{2V} = 0 = S_{23}$$

Discussion

1. 3dB Wilkinson power divider has equal amplitude and phase outputs at port 2 and port 3.
2. Ex. 7.2 3dB Wilkinson power divider Zo=70.7Ω, R=100Ω. frequency response (p.322, Fig. 7.12)
3. 3dB Wilkinson power combiner



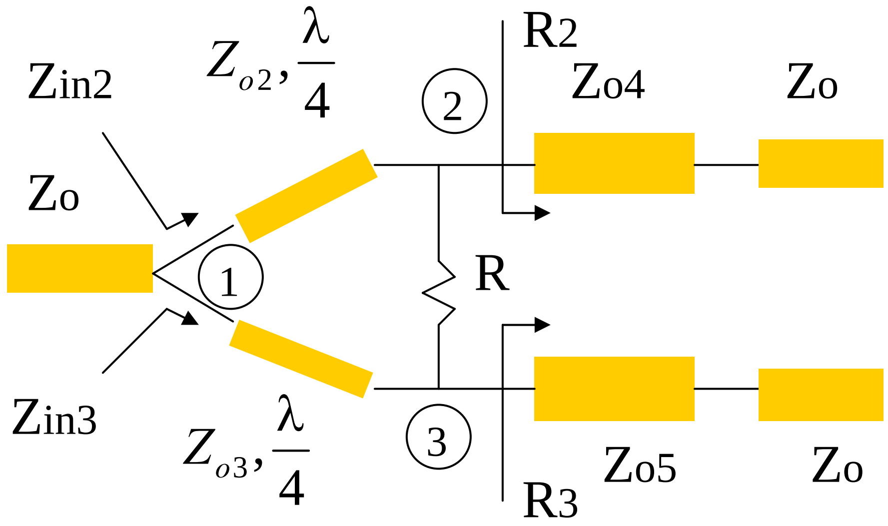
$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -j & -j \\ -j & 0 & 0 \\ -j & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -j\frac{1}{\sqrt{2}}(a_2 + a_3) \\ 0 \\ 0 \end{bmatrix}$$

$$\text{if } a_2 = a_3 \rightarrow P_1 = \frac{1}{2}\left(\frac{1}{2}|a_2 + a_3|^2\right) = \frac{1}{2}\left(|a_2|^2 + |a_3|^2\right) = P_2 + P_3$$

$$\text{if } a_2 \neq a_3,\ P_1 = \frac{1}{2}\left(\frac{1}{2}|a_2 + a_3|^2\right) \neq \frac{1}{2}\left(|a_2|^2 + |a_3|^2\right)$$

$$\rightarrow even \begin{array}{c} \frac{a_2 + a_3}{2} \\ \frac{a_2 + a_3}{2} \end{array}, odd \begin{array}{c} \frac{a_2 - a_3}{2} \\ \frac{a_3 - a_2}{2} \end{array} \Rightarrow 2 \times \frac{1}{2}\left|\frac{a_2 + a_3}{2}\right|^2$$

4. unequal power division (Wilkinson power divider)



(1) port1 match $\rightarrow Z_o = Z_{in2} // Z_{in3}$

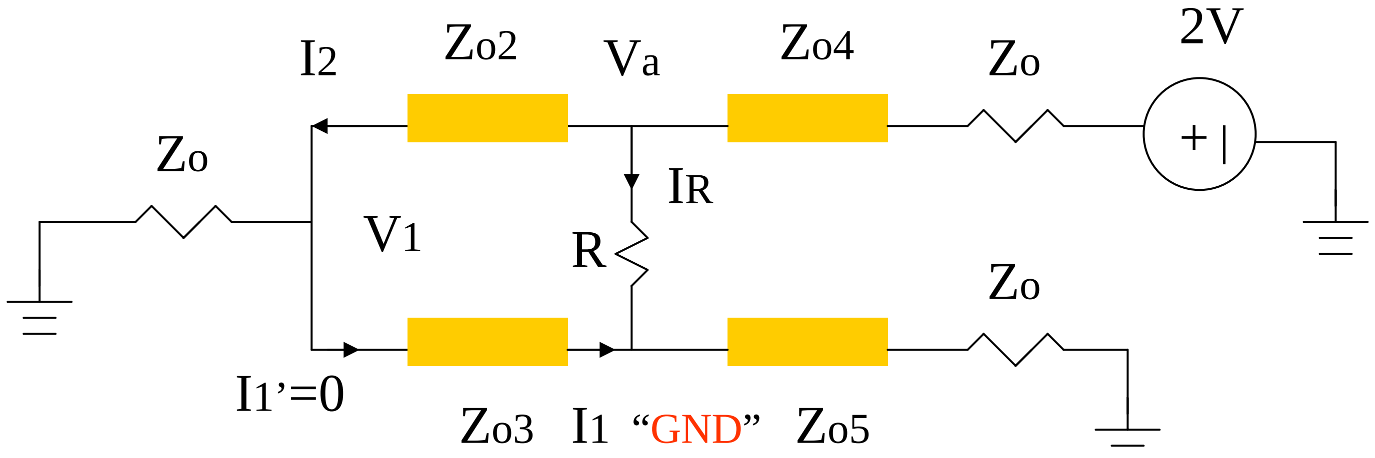
(2) $\frac{P_3}{P_2} = K^2 \rightarrow \frac{V_3^2}{Z_{in3}} = K^2 \frac{V_2^2}{Z_{in2}}$

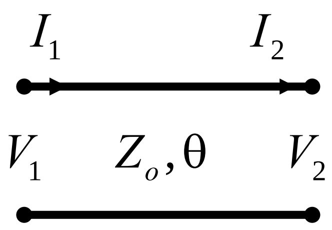
(3) $V_2 = V_3 \rightarrow Z_{in2} = K^2 Z_{in3}$

$$(1),(3) \rightarrow Z_{in2} = (1+K^2)Z_o, Z_{in3} = \frac{1+K^2}{K^2} Z_o$$

$$R_2 = K^2 R_3, R_2 = KZ_o \rightarrow R_3 = \frac{Z_o}{K}, Z_{o4} = \sqrt{K} Z_o, Z_{o5} = \frac{Z_o}{\sqrt{K}}$$

$$Z_{o2} = \sqrt{Z_{in2} R_2} = \sqrt{K(1+K^2)} Z_o, Z_{o3} = \sqrt{Z_{in3} R_3} = \sqrt{\frac{1+K^2}{K^3}} Z_o$$





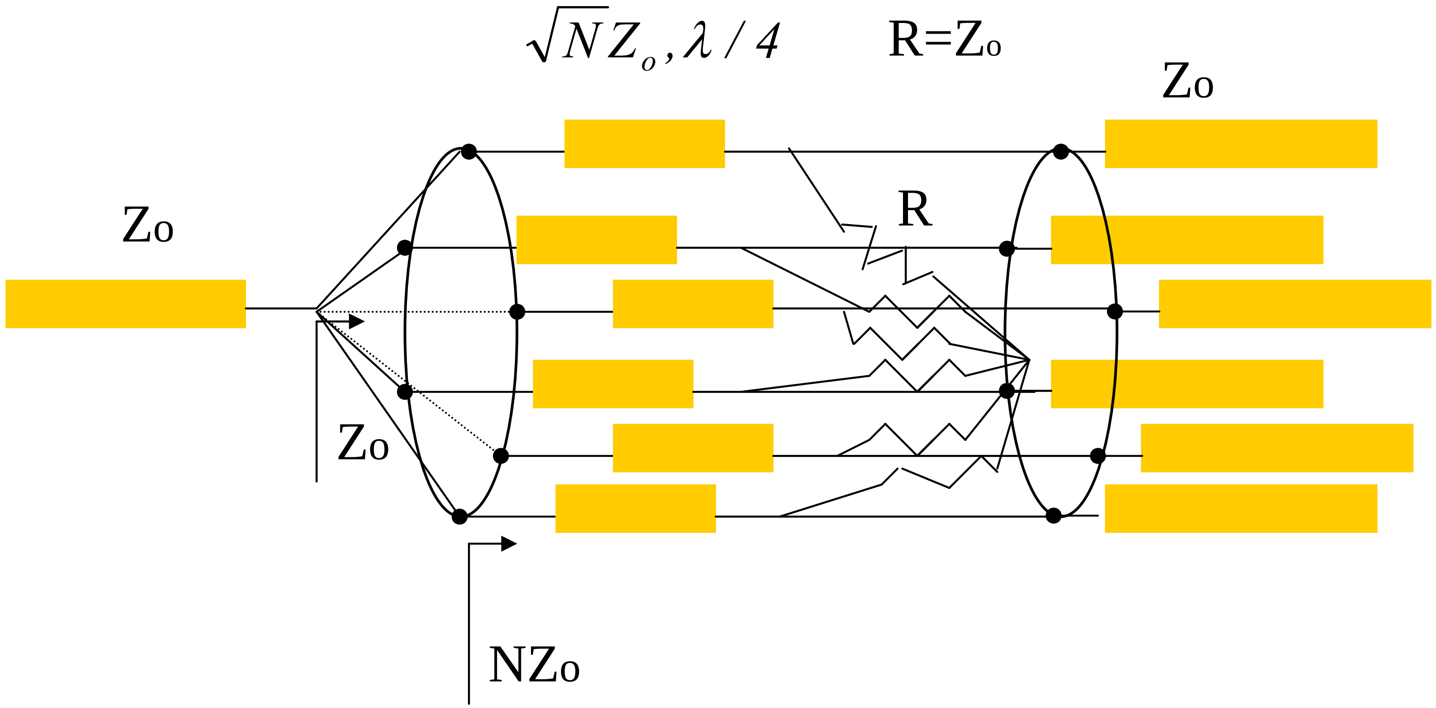
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_o \sin\theta \\ \dfrac{j\sin\theta}{Z_o} & \cos\theta \end{bmatrix}_{\theta=\frac{\pi}{2}} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & jZ_o \\ \dfrac{j}{Z_o} & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \rightarrow \begin{matrix} V_1 = jZ_o I_2 \\ I_1 = \dfrac{j}{Z_o} V_2 \end{matrix}$$

$$I_1 = \frac{V_1}{jZ_{o3}}, I_2 = \frac{V_a}{jZ_{o2}}, \frac{V_1}{I_2} = Z_o$$

$$I_R + I_1 = 0 = \frac{V_a}{R} + \frac{V_1}{jZ_{o3}} = \frac{jZ_{o2} I_2}{R} + \frac{Z_o I_2}{jZ_{o3}} = jI_2 \left(\frac{Z_{o2}}{R} - \frac{Z_o}{Z_{o3}}\right)$$

$$\rightarrow R = \frac{Z_{o2} Z_{o3}}{Z_o} = \frac{1+K^2}{K} Z_o, K = 1 \text{ for a 3dB Wilkinson divider}$$

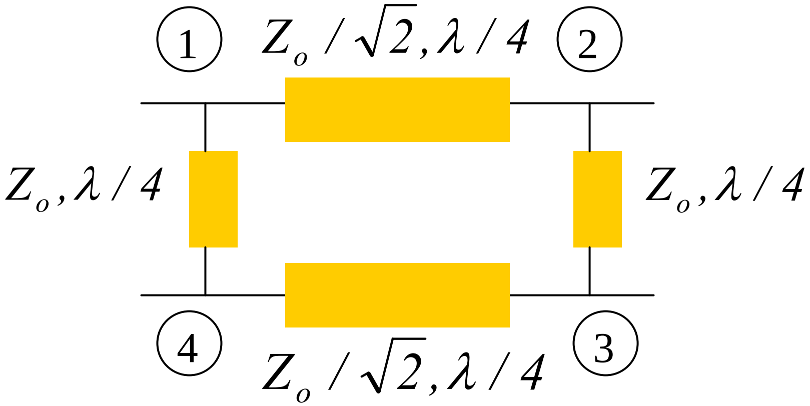
5. N-way Wilkinson power divider (not in planar shape)



7.5 The quadrature (90°) hybrid

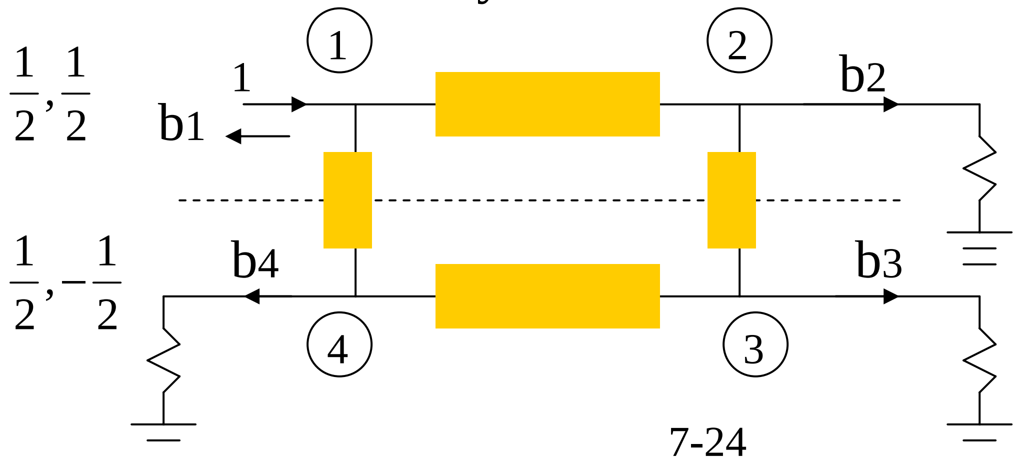
- branch-line coupler

  Port 2 and port 3 have equal amplitude and 90° phase difference

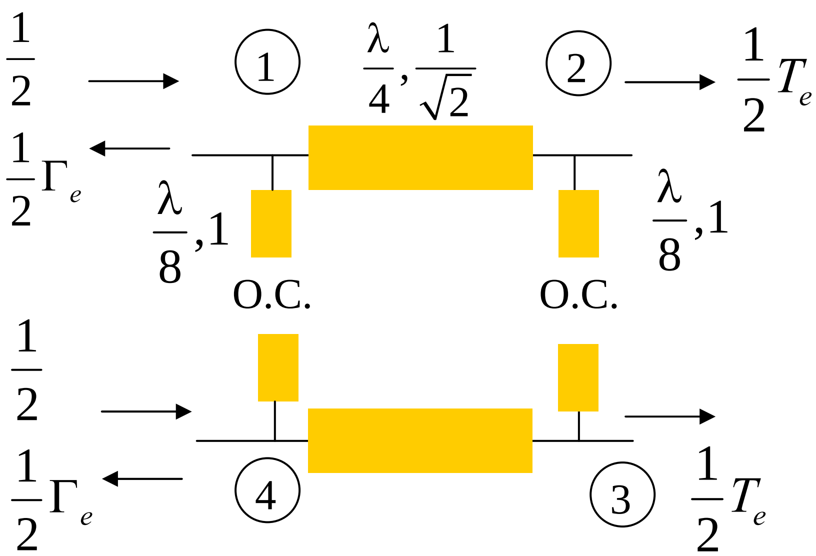


$$\frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

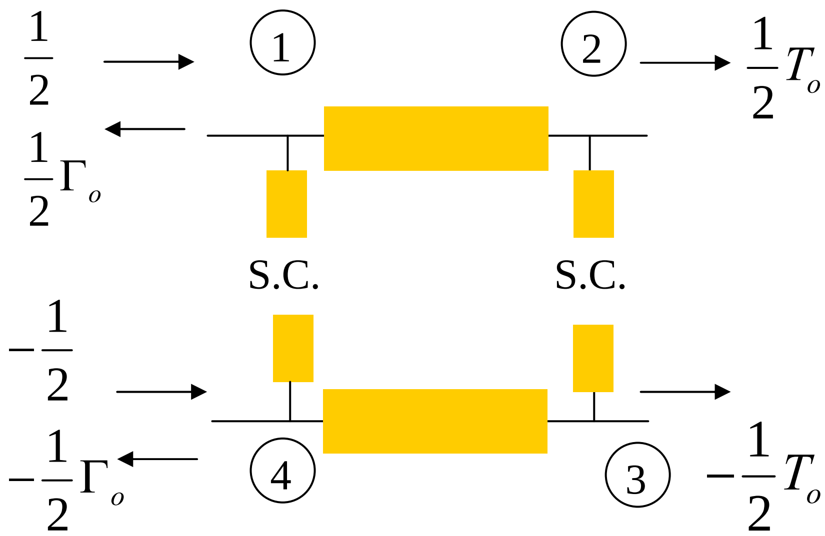
- even-odd mode analysis



even-mode



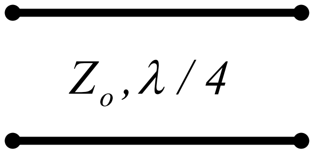
odd-mode



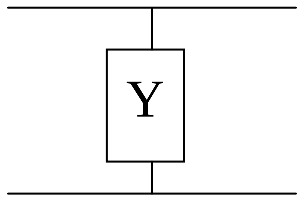
$$b_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o = S_{11}, b_2 = \frac{1}{2}T_e + \frac{1}{2}T_o = S_{21}$$

$$b_3 = \frac{1}{2}T_e - \frac{1}{2}T_o = S_{31}, b_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o = S_{41}$$

$$\begin{bmatrix} b_1 & b_2 & b_3 & b_4 \\ b_2 & b_1 & b_4 & b_3 \\ b_3 & b_4 & b_1 & b_2 \\ b_4 & b_3 & b_2 & b_1 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_o\sin\theta \\ jY_o\sin\theta & \cos\theta \end{bmatrix}_{\theta=\frac{\pi}{2}} = \begin{bmatrix} 0 & jZ_o \\ jY_o & 0 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$\Rightarrow$ even-mode: $\dfrac{\lambda}{8}$ open-circuit stub $\begin{bmatrix} 1 & 0 \\ jY_o & 1 \end{bmatrix}$

odd-mode: $\dfrac{\lambda}{8}$ short-circuit stub $\begin{bmatrix} 1 & 0 \\ -jY_o & 1 \end{bmatrix}$

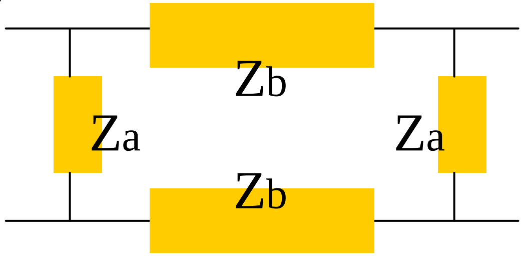
$$\Gamma = S_{11} = \frac{AZ_o + B - CZ_o^2 - DZ_o}{AZ_o + B + CZ_o^2 + DZ_o}, T = S_{21} = \frac{2Z_o}{AZ_o + B + CZ_o^2 + DZ_o}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{j}{\sqrt{2}} \\ j\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{j}{\sqrt{2}} \\ j\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

$$\rightarrow \Gamma_e = 0, \Gamma_o = 0, T_e = \frac{-1}{\sqrt{2}}(1+j), T_o = \frac{1}{\sqrt{2}}(1-j)$$

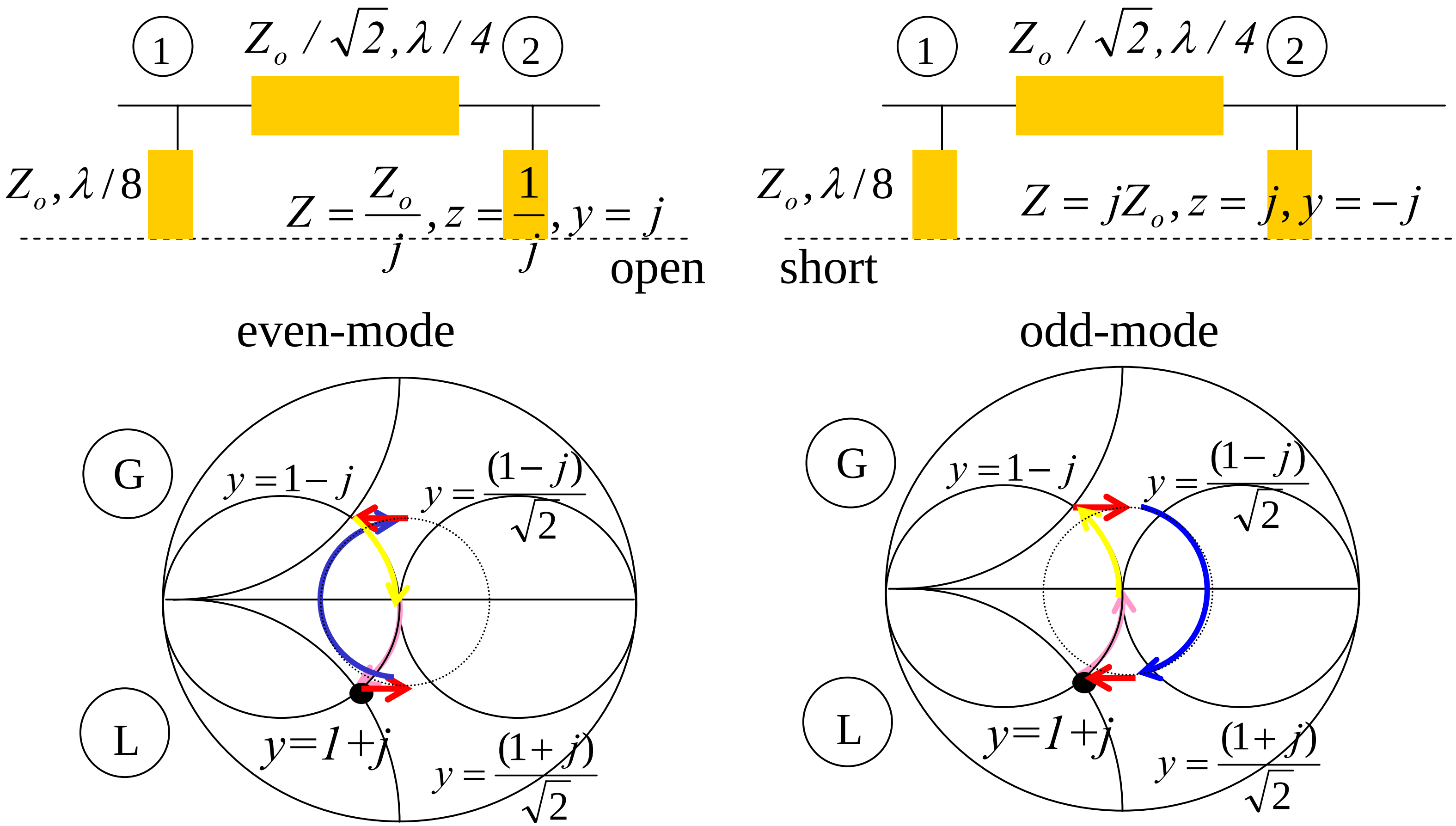
$$\Rightarrow b_1 = 0, b_2 = \frac{-j}{\sqrt{2}}, b_3 = \frac{-1}{\sqrt{2}}, b_4 = 0$$



Discussion

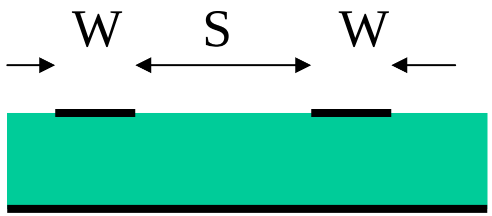
1. Unequal power division branch-line coupler uses Za, Zb λ/4 lines (prob. 7.17).
2. Ex. 7.5, frequency response (p.336, Fig. 7.25), BW: 10% ~ 20%
3. Multisection branch-line couplers can increase the operation BW.

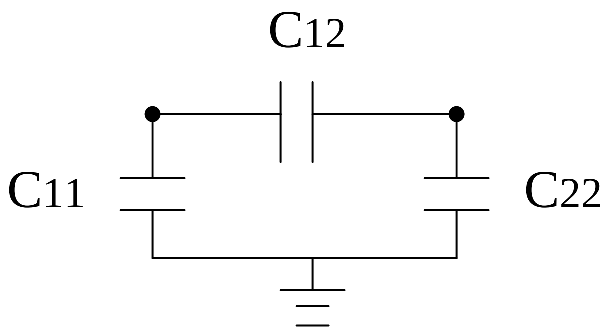
4. Smith chart consideration



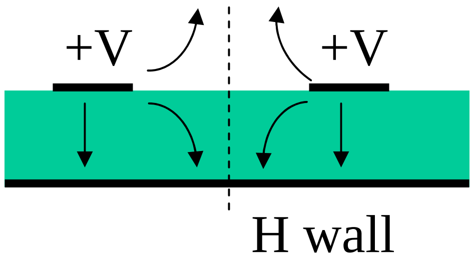
7.6 Coupled line directional couplers

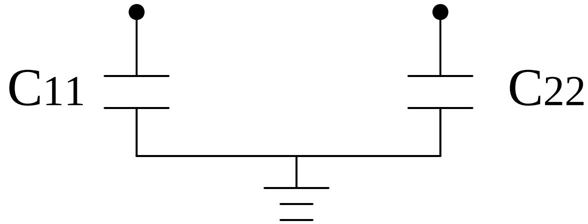
- coupled line theory





even-mode excitation





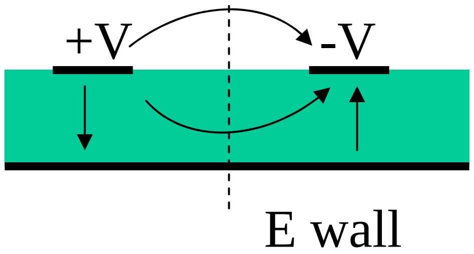
$$C_e = C_{11} = C_{22}$$

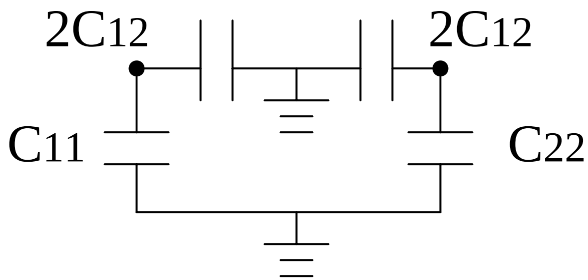
$$Z_{oe} = \frac{1}{v_e C_e}$$

$$C_o = C_{11} + 2C_{12}$$

$$Z_{oo} = \frac{1}{v_o C_o}$$

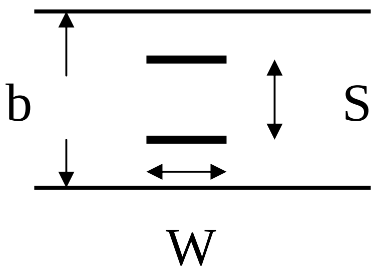
odd-mode excitation





Discussion

1. In general $v_e \neq v_o$ ($\because \varepsilon_{effe} > \varepsilon_{effo}$), for TEM mode $v_e = v_o = v$.
2. Z_{oe}, Z_{oo} (W/b, S/b) (p.339, Fig. 7.29 and p.340, Fig. 7.30) for coupled striplines and microstrip lines.
3. $Z_{oe} > Z_{oo}$, W/b ↑→ Z_{oe} ↓ Z_{oo} ↓, S/b ↑→ Z_{oe} ↓ Z_{oo} ↑and Z_{oe}, Z_{oo} → Z_o.
4. Ex.7.6 derive Z_{oe}, Z_{oo} of coupled striplines (p.337, Fig.7.26(b))



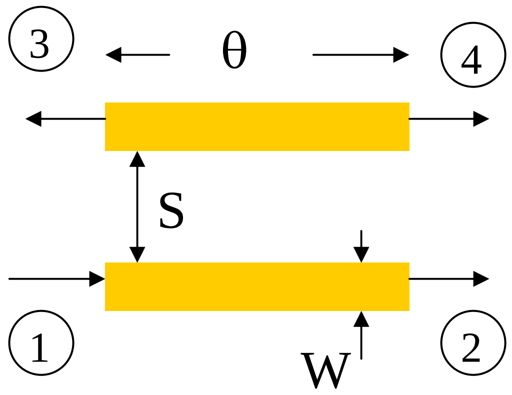
$$C_{11} = \frac{\varepsilon W}{(b-S)/2} + \frac{\varepsilon W}{(b+S)/2} = \frac{4b\varepsilon W}{b^2 - S^2}, C_{12} = \frac{\varepsilon W}{S}$$

$$C_e = C_{11}, C_o = C_{11} + 2C_{12}$$

$$Z_{oe} = \frac{1}{vC_e} = Z_o \frac{b^2 - S^2}{4bW\sqrt{\varepsilon_r}}$$

$$Z_{oo} = \frac{1}{vC_o} = Z_o \frac{1}{2W\sqrt{\varepsilon_r}[2b/(b^2 - S^2) + 1/S]}$$

• coupled line coupler



matched port $\rightarrow Z_o = \sqrt{Z_{oe} Z_{oo}}$

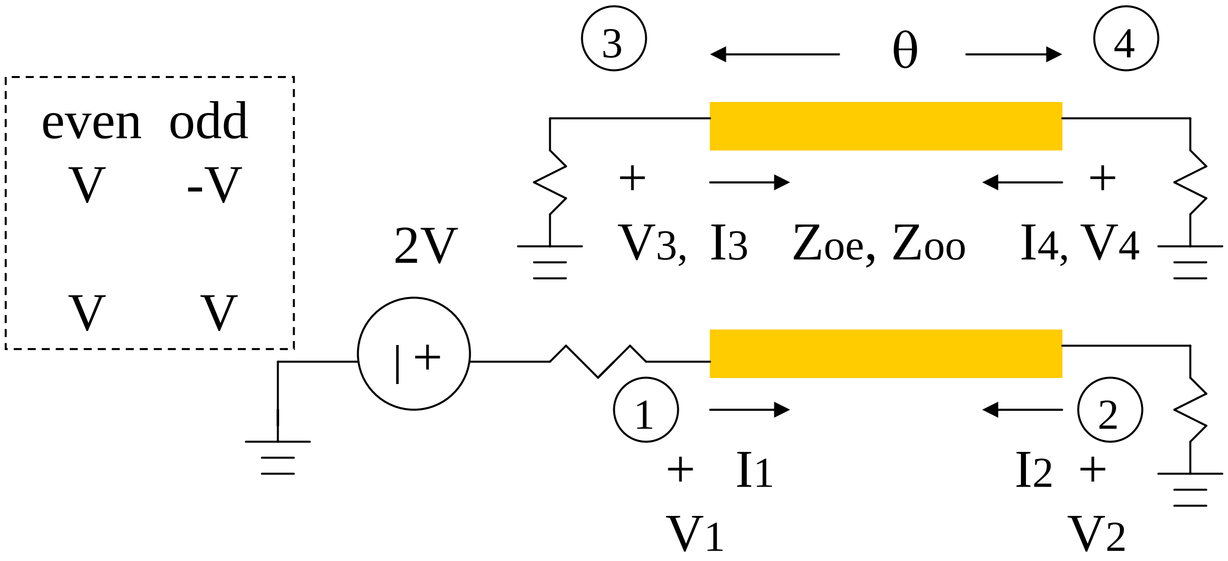
$$C \equiv \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}, \theta = \frac{\pi}{2} \Rightarrow$$

$$\begin{bmatrix} 0 & -j\sqrt{1-C^2} & C & 0 \\ -j\sqrt{1-C^2} & 0 & 0 & C \\ C & 0 & 0 & -j\sqrt{1-C^2} \\ 0 & C & -j\sqrt{1-C^2} & 0 \end{bmatrix}$$

design equations: $Z_{oe} = Z_o \sqrt{\frac{1+C}{1-C}}, Z_{oo} = Z_o \sqrt{\frac{1-C}{1+C}} \Rightarrow S, W$

Discussion

1. Design procedure: given C and Z_o, calculate Z_{oe} and Z_{oo}, then use Fig. 7.29 or 7.30 to find W/b and S/b for stripline or microstrip.
2. even-odd mode analysis



even mode: $I_1^e = I_3^e, I_2^e = I_4^e, V_1^e = V_3^e, V_2^e = V_4^e$

odd mode: $I_1^o = -I_3^o, I_2^o = -I_4^o, V_1^o = -V_3^o, V_2^o = -V_4^o$

$$I_1 = I_1^e + I_1^o, I_2 = I_2^e + I_2^o, I_3 = I_1^e - I_1^o, I_4 = I_2^e - I_2^o$$

$$V_1 = V_1^e + V_1^o, V_2 = V_2^e + I_2^o, V_3 = V_1^e - V_1^o, V_4 = V_2^e - V_2^o$$

$$Z_{in}^{e,o} = Z_{oe,o} \frac{Z_o + jZ_{oe,o}\tan\theta}{Z_{oe,o} + jZ_o \tan\theta} \quad V_1^{e,o} = V \frac{Z_{in}^{e,o}}{Z_{in}^{e,o} + Z_o}, I_1^{e,o} = \frac{V}{Z_{in}^{e,o} + Z_o}$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o} = Z_o + \frac{2(Z_{in}^e Z_{in}^o - Z_o^2)}{Z_{in}^e + Z_{in}^o + 2Z_o} \ldots(1)$$

$$= Z_o \Leftarrow \ if \ Z_o^2 (= Z_{in}^e Z_{in}^o) = Z_{oe} Z_{oo} \ldots(2)$$

$$\rightarrow V_1^{e,o} = V \frac{Z_o + jZ_{oe,o}\tan\theta}{2Z_o + j(Z_{oe} + Z_{oo})\tan\theta} \ldots(3)$$

$$V_3 = V_1^e - V_1^o = V \frac{j(Z_{oe} - Z_{oo})\tan\theta}{2Z_o + j(Z_{oe} + Z_{oo})\tan\theta}$$

$$C \equiv \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} \rightarrow V_3 = V \frac{jC\tan\theta}{\sqrt{1 - C^2} + j\tan\theta} \ldots(4)$$

derivation of (1): $Z_{in} = \dfrac{V_1}{I_1} = \dfrac{V_1^e + V_1^o}{I_1^e + I_1^o} = \dfrac{V\dfrac{Z_{in}^e}{Z_{in}^e + Z_o} + V\dfrac{Z_{in}^o}{Z_{in}^o + Z_o}}{\dfrac{V}{Z_{in}^e + Z_o} + \dfrac{V}{Z_{in}^o + Z_o}}$

$$= \frac{Z_{in}^e(Z_{in}^o + Z_o) + Z_{in}^o(Z_{in}^e + Z_o)}{Z_{in}^e + Z_{in}^o + 2Z_o} = \frac{Z_o(Z_{in}^e + Z_{in}^o + 2Z_o) + 2Z_{in}^o Z_{in}^e - 2Z_o^2}{Z_{in}^e + Z_{in}^o + 2Z_o}$$

$$= Z_o + \frac{2(Z_{in}^e Z_{in}^o - Z_o^2)}{Z_{in}^e + Z_{in}^o + 2Z_o}$$

derivation of (2): $Z_{in}^e Z_{in}^o = Z_{oe}\dfrac{Z_o + jZ_{oe}\tan\theta}{Z_{oe} + jZ_o\tan\theta} Z_{oo}\dfrac{Z_o + jZ_{oo}\tan\theta}{Z_{oo} + jZ_o\tan\theta}$

$$\overset{Z_{oe}Z_{oo}=Z_o^2}{=} Z_{oe}Z_{oo}\frac{\sqrt{Z_{oe}Z_{oo}} + jZ_{oe}\tan\theta}{Z_{oe} + j\sqrt{Z_{oe}Z_{oo}}\tan\theta} \times \frac{1/\sqrt{Z_{oe}}}{1/\sqrt{Z_{oe}}} \frac{\sqrt{Z_{oe}Z_{oo}} + jZ_{oo}\tan\theta}{Z_{oo} + j\sqrt{Z_{oe}Z_{oo}}\tan\theta} \times \frac{1/\sqrt{Z_{oo}}}{1/\sqrt{Z_{oo}}}$$

$= Z_{oe}Z_{oo}\dfrac{\sqrt{Z_{oo}} + j\sqrt{Z_{oe}}\tan\theta}{\sqrt{Z_{oe}} + j\sqrt{Z_{oo}}\tan\theta} \dfrac{\sqrt{Z_{oe}} + j\sqrt{Z_{oo}}\tan\theta}{\sqrt{Z_{oo}} + j\sqrt{Z_{oe}}\tan\theta} = Z_{oe}Z_{oo} = Z_o^2 \rightarrow Z_{in} = Z_o$: i/p match derivation of (3): $V_1^e = V\frac{Z_{in}^e}{Z_{in}^e + Z_o} = V\frac{Z_{oe}\frac{Z_o + jZ_{oe}\tan\theta}{Z_{oe} + jZ_o\tan\theta}}{Z_{oe}\frac{Z_o + jZ_{oe}\tan\theta}{Z_{oe} + jZ_o\tan\theta} + Z_o}$

$$= V\frac{Z_{oe}Z_o + jZ_{oe}^2\tan\theta}{2Z_{oe}Z_o + jZ_{oe}^2\tan\theta + jZ_o^2\tan\theta} \times \frac{1/Z_{oe}}{1/Z_{oe}} = V\frac{Z_o + jZ_{oe}\tan\theta}{2Z_o + jZ_{oe}\tan\theta + jZ_{oo}\tan\theta}$$

$$V_3 = V_1^e - V_1^o = V\frac{j(Z_{oe} - Z_{oo})\tan\theta}{2Z_o + j(Z_{oe} + Z_{oo})\tan\theta}$$

derivation of (4): $C \equiv \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} \rightarrow V_3 = V\frac{j\frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}\tan\theta}{\frac{2Z_o}{Z_{oe} + Z_{oo}} + j\tan\theta} = V\frac{jC\tan\theta}{\frac{2Z_o}{Z_{oe} + Z_{oo}} + j\tan\theta}$

$$\sqrt{1 - C^2} = \sqrt{1 - (\frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}})^2} = \frac{\sqrt{(Z_{oe} + Z_{oo})^2 - (Z_{oe} - Z_{oo})^2}}{Z_{oe} + Z_{oo}} = \frac{2\sqrt{Z_{oe}Z_{oo}}}{Z_{oe} + Z_{oo}} = \frac{2Z_o}{Z_{oe} + Z_{oo}}$$

$$\rightarrow V_3 = V\frac{jC\tan\theta}{\sqrt{1 - C^2} + j\tan\theta}$$

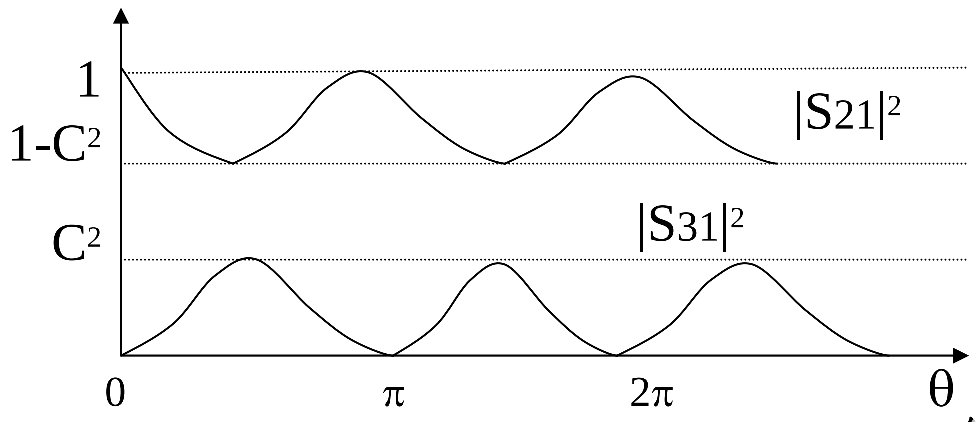
$$\begin{bmatrix} V_1^{e,o} \\ I_1^{e,o} \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_{oe,o}\sin\theta \\ jY_{oe,o}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_2^{e,o} \\ I_2^{e,o} \end{bmatrix}$$

$$\rightarrow V_2^{e,o} = V \frac{Z_o}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta} \text{.......(5)}$$

$$V_2 = V_2^e + V_2^o = V \frac{2Z_o}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta} = V \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}$$

$$V_4 = V_2^e - V_2^e = 0$$

3. frequency response at port 2 and port 3



derivation of (5)

$$\begin{bmatrix} V_1^{e,o} \\ I_1^{e,o} \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_{oe,o}\sin\theta \\ jY_{oe,o}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_2^{e,o} \\ I_2^{e,o} \end{bmatrix} \rightarrow \begin{bmatrix} V_2^{e,o} \\ I_2^{e,o} \end{bmatrix} = \begin{bmatrix} \cos\theta & -jZ_{oe,o}\sin\theta \\ -jY_{oe,o}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_1^{e,o} \\ I_1^{e,o} \end{bmatrix}$$

$$V_1^{e,o} = V\frac{Z_o + jZ_{oe,o}\tan\theta}{2Z_o + j(Z_{oe}+Z_{oo})\tan\theta} = V\frac{Z_o\cos\theta + jZ_{oe,o}\sin\theta}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta}$$

$$I_1^e = \frac{V}{Z_{in}^e + Z_o},\ Z_{in}^e = Z_{oe}\frac{Z_o + jZ_{oe}\tan\theta}{Z_{oe}+jZ_o\tan\theta}$$

$$I_1^e = \frac{V}{Z_{in}^e + Z_o} = \frac{V}{Z_{oe}\dfrac{Z_o + jZ_{oe}\tan\theta}{Z_{oe}+jZ_o\tan\theta} + Z_o} = V\frac{Z_{oe}+jZ_o\tan\theta}{Z_{oe}(Z_o + jZ_{oe}\tan\theta) + Z_o(Z_{oe}+jZ_o\tan\theta)}$$

$$= V\frac{Z_{oe}+jZ_o\tan\theta}{2Z_{oe}Z_o + j(Z_{oe}^2+Z_o^2)\tan\theta}\times\frac{1/Z_{oe}}{1/Z_{oe}} = V\frac{1+j\sqrt{Z_{oo}/Z_{oe}}\tan\theta}{2Z_o + j(Z_{oe}+Z_{oo})\tan\theta} = V\frac{\cos\theta + j\sqrt{Z_{oo}/Z_{oe}}\sin\theta}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta}$$

$$\rightarrow I_1^{e,o} = V\frac{\cos\theta + j\sqrt{Z_{oo,e}/Z_{oe,o}}\sin\theta}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta}$$

$$\Rightarrow V_2^{e,o} = \cos\theta V_1^{e,o} - jZ_{oe,o}\sin\theta I_1^{e,o}$$

$$= V\frac{(Z_o\cos\theta + jZ_{oe,o}\sin\theta)\cos\theta}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta} - V\frac{jZ_{oe,o}\sin\theta(\cos\theta + j\sqrt{Z_{oo,e}/Z_{oe,o}}\sin\theta)}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta}$$

$$= V\frac{Z_o\cos^2\theta + jZ_{oe,o}\sin\theta - jZ_{oe,o}\sin\theta + \sqrt{Z_{oe}Z_{oo}}\sin^2\theta}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta}$$

$$= V\frac{Z_o}{2Z_o\cos\theta + j(Z_{oe}+Z_{oo})\sin\theta}$$

4. selection of line length

$$\theta = \frac{\pi}{2} \rightarrow l = \frac{\lambda}{4}$$

$$V_1 = 1$$

$$V_2 = -j\sqrt{1-C^2}, P_2 = (1-C^2)P_1$$

$$V_3 = C, P_2 = C^2 P_1$$

$$V_4 = 0$$

$$\theta = \pi \rightarrow l = \frac{\lambda}{2}$$

$$V_1 = 1$$

$$V_2 = 1$$

$$V_3 = 0$$

$$V_4 = 0$$

$\theta = \frac{\pi}{2}$, V_2 and V_3 have 90^o phase difference $\rightarrow$ quadrature coupler

5. In general $v_e \neq v_o \rightarrow \theta_e \neq \theta_o$, coupled line coupler is suited for a weakly coupled coupler.
6. C=3dB $\rightarrow$ Zoe=120.7Ω, Zoo=20.7Ω $\rightarrow$ S becomes not practical and coupled line theory is not applicable $\rightarrow$ Lange coupler

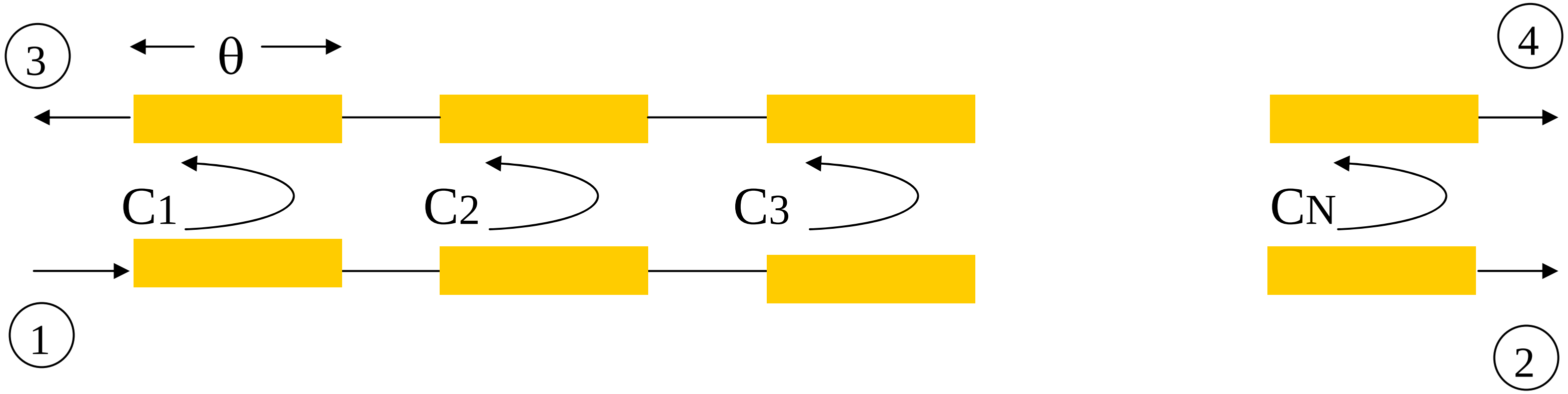
7. Ex.7.7 design a 20dB coupled stripline coupler, b=0.32cm, ε_r=2.2, Zo=50Ω, f=3GHz.

$$C = 10^{-20/20} = 0.1 \rightarrow Z_{oe} = 55.28\Omega, Z_{oo} = 45.23\Omega$$

from p.339, Fig. 7.29, $W/b = 0.809, S/b = 0.306 \rightarrow S = 0.098cm$

frequency response (p.346, Fig. 7.34)

- multisection coupled line coupler



$C << 1, N : odd$

single section $\frac{V_3}{V_1} = \frac{jC\tan\theta}{\sqrt{1-C^2} + j\tan\theta} \approx \frac{jC\tan\theta}{1+j\tan\theta} = jC\sin\theta e^{-j\theta}$

$$\frac{V_2}{V_1} = \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta} \approx \frac{1}{\cos\theta + j\sin\theta} = e^{-j\theta}$$

multisection $\frac{V_3}{V_1} = jC_1\sin\theta e^{-j\theta} + jC_2\sin\theta e^{-j\theta}e^{-j2\theta} + ...jC_N\sin\theta e^{-j\theta}e^{-j2(N-1)\theta}$

if $C_1 = C_N, C_2 = C_{N-1}, ...$

$$\frac{V_3}{V_1} = j2\sin\theta e^{-jN\theta}[C_1\cos(N-1)\theta + C_2\cos(N-3)\theta + ... + \frac{1}{2}C_M], M = \frac{N+1}{2}$$

$$C = \frac{V_3}{V_1}(\theta = \frac{\pi}{2}), \quad \frac{V_3}{V_1}(\theta) \rightarrow C_n \Rightarrow S_n, W_n$$

Discussion

1. Ex. 7.8 design a 20dB coupler with binomial response, N=3,
   Z_0=50Ω, f_0=3GHz.

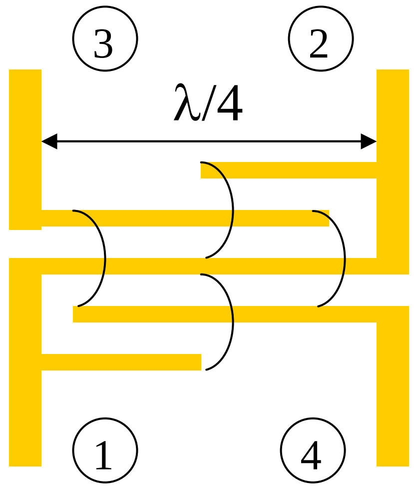
$$C=\left|\frac{V_3}{V_1}\right|=2\sin\theta[C_1\cos 2\theta+\frac{1}{2}C_2]=C_1\sin 3\theta+(C_2-C_1)\sin\theta$$

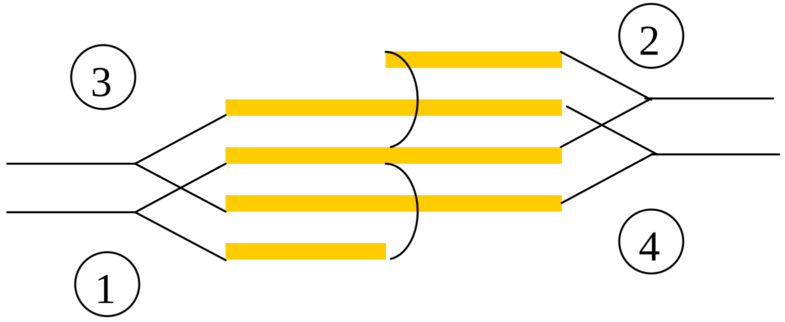
$$\left.\frac{dC}{d\theta}\right|_{\theta=\frac{\pi}{2}}=0,\ \left.\frac{d^2C}{d\theta^2}\right|_{\theta=\frac{\pi}{2}}=10C_1-C_2=0,\ C(\frac{\pi}{2})=\mathrm{C}_2\text{-}2\mathrm{C}_1=10^{-20/20}=0.1$$

$$\rightarrow \begin{matrix} C_1=C_3=0.0125\rightarrow Z_{oe}^1=Z_{oe}^3=50.63\Omega, Z_{oo}^1=Z_{oo}^3=49.38\Omega \\ C_2=0.125\rightarrow Z_{oe}^2=56.69\Omega,\ Z_{oo}^2=44.1\Omega \end{matrix}$$

frequency response (p.348, Fig. 7.37)

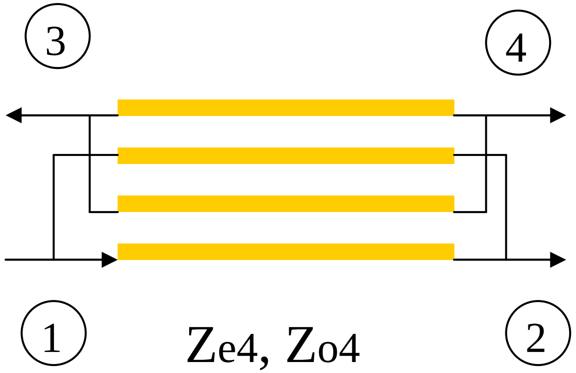
7.7 The Lange coupler (interdigitated coupler)

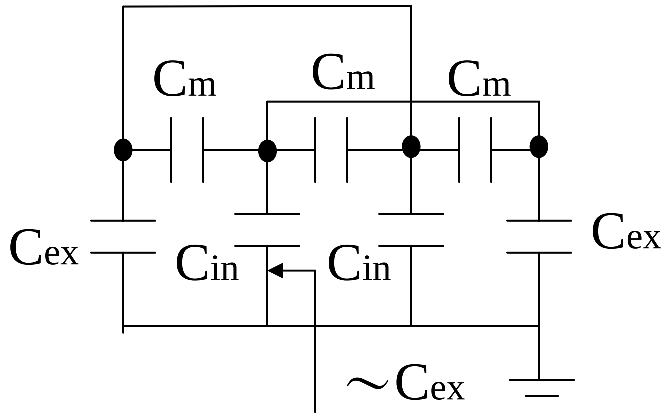




Unfolded Lange coupler

four-wire coupled line model

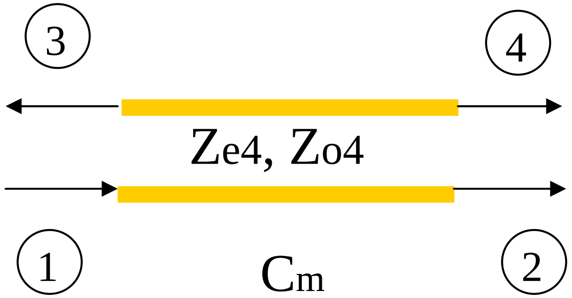




approximate two-wire coupled line model

$$C_{ex} \approx C_{in} + C_m \text{ in series with } C_{ex}$$

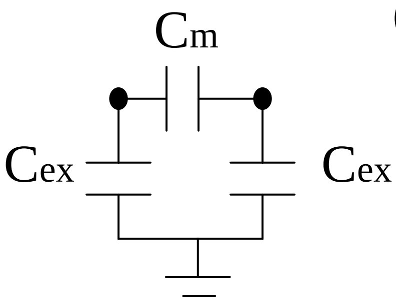
$$\rightarrow C_{in} \approx C_{ex} - \frac{C_{ex}C_m}{C_{ex} + C_m}$$



nearest neighbor coupling

$$C_{e4} = C_{ex} + C_{in}$$

$$C_{o4} = C_{ex} + C_{in} + 3 \times 2C_m$$



$$C_e = C_{ex}$$

$$C_o = C_{ex} + 2C_m$$

$$\text{4-line model}: C_{e4} = C_{ex} + C_{in}, C_{o4} = C_{ex} + C_{in} + 6C_m$$

$$\text{2-line model}: C_e = C_{ex}, C_o = C_{ex} + 2C_m$$

$$\rightarrow C_{e4} = \frac{C_e(3C_e + C_o)}{C_e + C_o}, C_{o4} = \frac{C_o(3C_o + C_e)}{C_e + C_o} \ldots(1)$$

$$Z_o = \frac{1}{vC} \Rightarrow Z_{e4} = \frac{Z_{oo} + Z_{oe}}{3Z_{oo} + Z_{oe}} Z_{oe}, Z_{o4} = \frac{Z_{oo} + Z_{oe}}{3Z_{oe} + Z_{oo}} Z_{oo} \ldots(2)$$

$$Z_o = \sqrt{Z_{e4} Z_{o4}}, C \equiv \frac{Z_{e4} - Z_{o4}}{Z_{e4} + Z_{o4}} \rightarrow \begin{cases} Z_{oe} = \dfrac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{(1-C)/(1+C)}} Z_o \\ Z_{oo} = \dfrac{4C - 3 - \sqrt{9 - 8C^2}}{2C\sqrt{(1+C)/(1-C)}} Z_o \end{cases} \Rightarrow W, S$$

<u>Discussion</u>

1. Lange coupler is suitable for wideband 3dB 90° hybrid, and MMIC design uses air bridges instead of bond wires.

derivation of (1) : 4-line : $C_{e4} = C_{ex} + C_{in} \overset{C_{in} \approx C_{ex} - \frac{C_{ex}C_m}{C_{ex}+C_m}}{=} 2C_{ex} - \dfrac{C_{ex}C_m}{C_{ex}+C_m}$

2-line : $C_e = C_{ex}, C_o = C_{ex} + 2C_m \rightarrow C_m = \dfrac{C_o - C_e}{2}$

$$C_{e4} = 2C_e - \frac{C_e \frac{C_o - C_e}{2}}{C_e + \frac{C_o - C_e}{2}} = 2C_e - \frac{C_e C_o - C_e^2}{C_e + C_o} = \frac{2C_e^2 + 2C_e C_o - C_e C_o + C_e^2}{C_e + C_o} = \frac{C_e(3C_e + C_o)}{C_e + C_o}$$

$$C_{o4} = C_{ex} + C_{in} + 6C_m = \frac{C_e(3C_e + C_o)}{C_e + C_o} + 3(C_o - C_e) = \frac{3C_e^2 + C_e C_o + 3C_o^2 - 3C_e^2}{C_e + C_o} = \frac{C_o(3C_o + C_e)}{C_e + C_o}$$

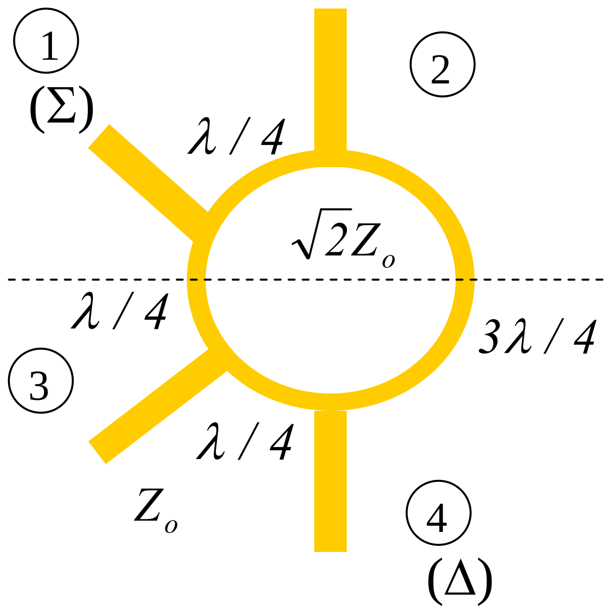
derivation of (2) : $Z_o = \dfrac{1}{vC}$

$$\Rightarrow Z_{e4} = \frac{1}{vC_{e4}} = \frac{1}{v\frac{C_e(3C_e + C_o)}{C_e + C_o}} = \frac{1}{vC_e}\frac{C_e + C_o}{3C_e + C_o} \times \frac{1/vC_eC_o}{1/vC_eC_o} = \frac{1}{vC_e}\frac{1/vC_o + 1/vC_e}{3/vC_o + 1/vC_e} = Z_{oe}\frac{Z_{oo} + Z_{oe}}{3Z_{oo} + Z_{oe}}$$

$$Z_{o4} = \frac{1}{vC_{o4}} = \frac{1}{v\frac{C_o(3C_o + C_e)}{C_e + C_o}} = \frac{1}{vC_o}\frac{C_e + C_o}{3C_o + C_e} \times \frac{1/vC_eC_o}{1/vC_eC_o} = \frac{1}{vC_o}\frac{1/vC_o + 1/vC_e}{3/vC_e + 1/vC_o} = Z_{oo}\frac{Z_{oo} + Z_{oe}}{3Z_{oe} + Z_{oo}}$$

7.8 The 180° hybrid

- rat-race coupler



(1) input port 1

port 1 match $\rightarrow \sqrt{2Z_o Z_o} = \sqrt{2}Z_o$

$\frac{3}{4}\lambda - \frac{1}{4}\lambda = \frac{\lambda}{2} \rightarrow$ port 4 "GND"

or isolated port

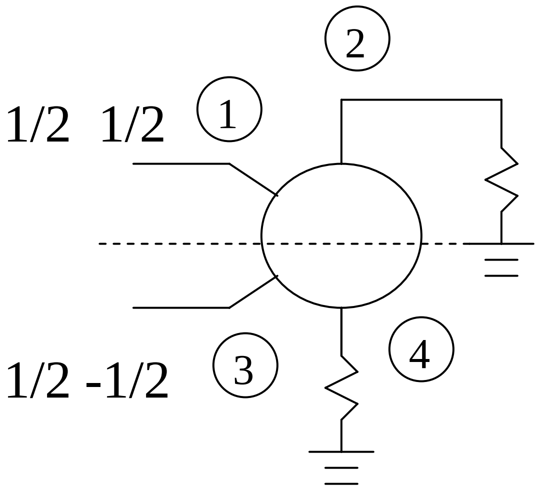
(2) input ports 2 and 3

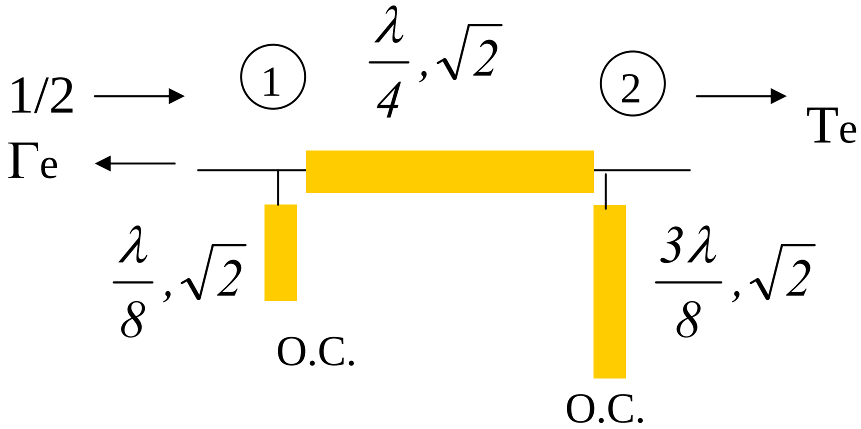
$\rightarrow$ port 1 : Σ port, port 4 : Δ port

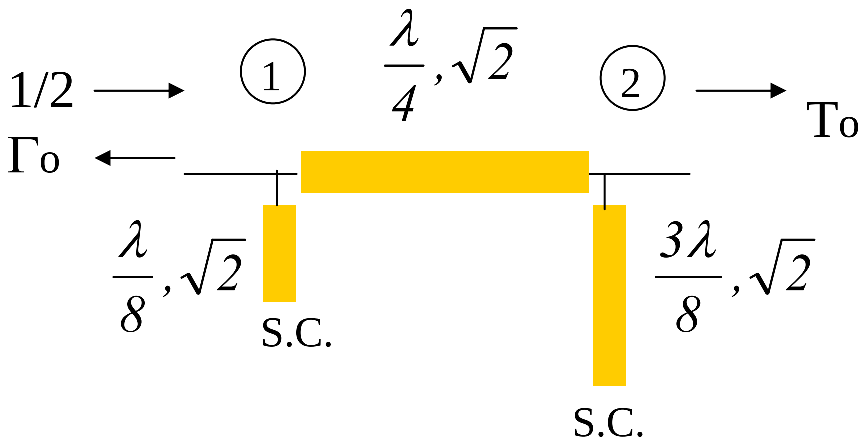
$$\frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

Discussion

1. even-odd mode analysis





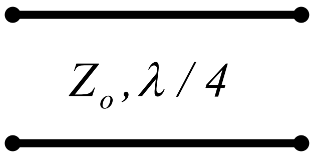


$$b_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o,$$

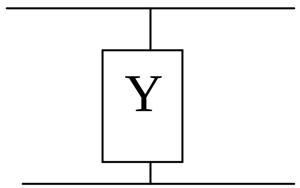
$$b_2 = \frac{1}{2}T_e + \frac{1}{2}T_o$$

$$b_3 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o,$$

$$b_4 = \frac{1}{2}T_e - \frac{1}{2}T_o$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & jZ_o \\ jY_o & 0 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{cases} \begin{bmatrix} 1 & 0 \\ jY_o & 1 \end{bmatrix} \text{even - mode open - circuit } \lambda/8 \text{ stub} \\ \begin{bmatrix} 1 & 0 \\ -jY_o & 1 \end{bmatrix} \text{even - mode open - circuit } 3\lambda/8 \text{ stub} \\ \begin{bmatrix} 1 & 0 \\ -jY_o & 1 \end{bmatrix} \text{odd - mode short - circuit } \lambda/8 \text{ stub} \\ \begin{bmatrix} 1 & 0 \\ jY_o & 1 \end{bmatrix} \text{odd - mode short - circuit } 3\lambda/8 \text{ stub} \end{cases}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ \frac{j}{\sqrt{2}} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ \frac{j}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{j}{\sqrt{2}} & 1 \end{bmatrix} = \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -\frac{j}{\sqrt{2}} & 1 \end{bmatrix} \begin{bmatrix} 0 & j\sqrt{2} \\ \frac{j}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{j}{\sqrt{2}} & 1 \end{bmatrix} = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix}$$

$$\rightarrow \Gamma_e = -\frac{j}{\sqrt{2}}, \Gamma_o = \frac{j}{\sqrt{2}}, T_e = -\frac{j}{\sqrt{2}}, T_o = -\frac{j}{\sqrt{2}}$$

$$\Rightarrow b_1 = 0, b_2 = -\frac{j}{\sqrt{2}}, b_3 = -\frac{j}{\sqrt{2}}, b_4 = 0$$

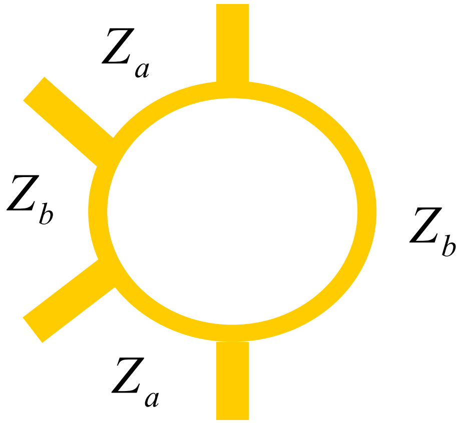
2. input port 2 $\rightarrow$ ports 1, 4 180° phase difference, port 3 isolated port
   input port 4 $\rightarrow$ ports 2, 3 180° phase difference, port 1 isolated port

3. Ex. 7.9 3dB rat-race hybrid Zo=70.7Ω, BW 20~30% (p.357, Fig.7.46)

4.

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = -\frac{j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ a_2 \\ a_3 \\ 0 \end{bmatrix} = -\frac{j}{\sqrt{2}} \begin{bmatrix} a_2 + a_3 \\ 0 \\ 0 \\ -a_2 + a_3 \end{bmatrix}$$

5. Unequal power division rat-race coupler uses Za, Zb lines



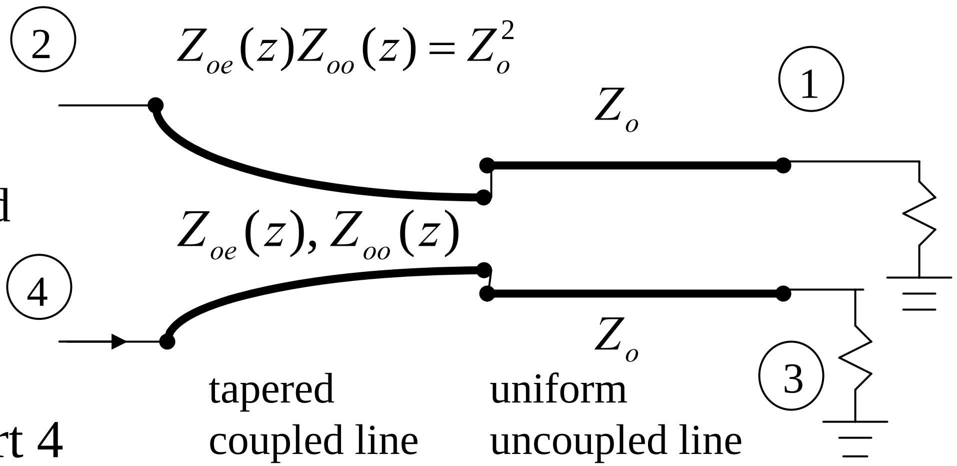
• tapered coupled line hybrid

1/2 -1/2

even odd

1/2 1/2

input at port 4

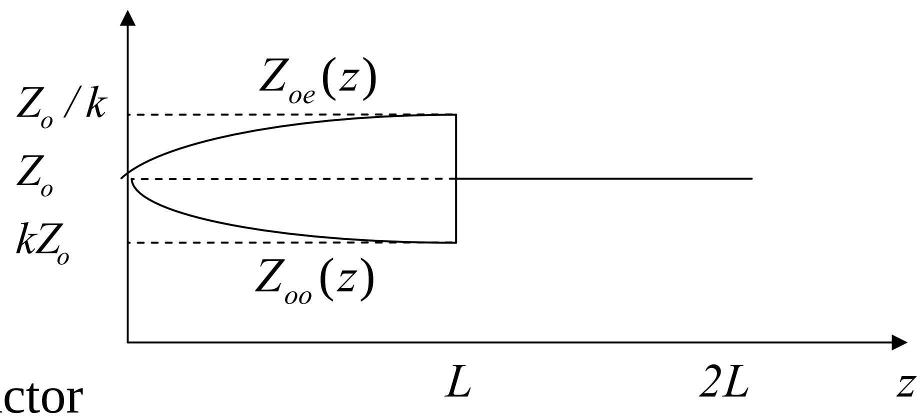


$$b_1 = \frac{1}{2}T_e - \frac{1}{2}T_o$$

$$b_2 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

$$b_3 = \frac{1}{2}T_e + \frac{1}{2}T_o$$

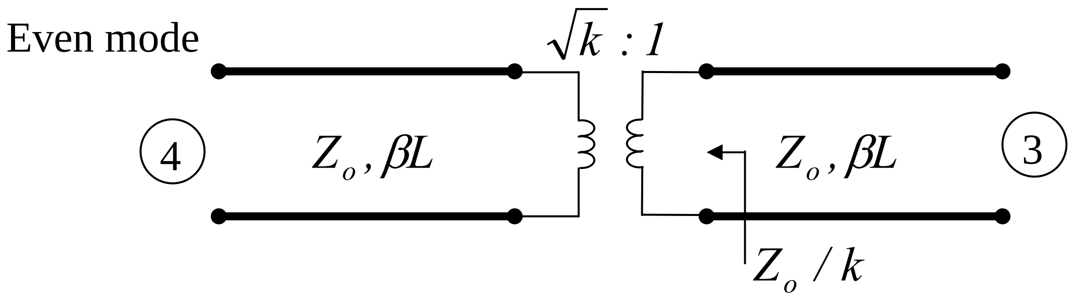
$$b_4 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o$$

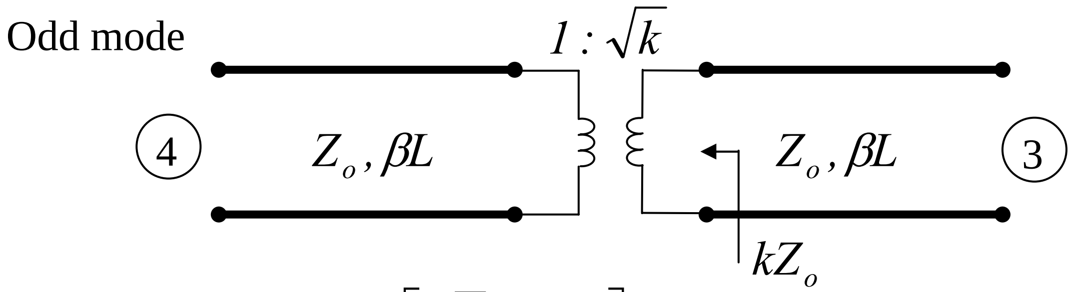


$0 \le k \le 1$

coupling factor

$$\begin{bmatrix} b_1 & b_2 & b_3 & b_4 \\ b_2 & b_1 & b_4 & b_3 \\ b_3 & b_4 & b_1 & b_2 \\ b_4 & b_3 & b_2 & b_1 \end{bmatrix}$$





$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} \cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sqrt{k} & 0 \\ 0 & \frac{1}{\sqrt{k}} \end{bmatrix} \begin{bmatrix} \cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta \end{bmatrix} \quad \text{(p.185, Table 4.1)}$$

$$= \begin{bmatrix} \sqrt{k}\cos^2\theta - \frac{1}{\sqrt{k}}\sin^2\theta & j(\sqrt{k}+\frac{1}{\sqrt{k}})\sin\theta\cos\theta \\ j(\sqrt{k}+\frac{1}{\sqrt{k}})\sin\theta\cos\theta & -\sqrt{k}\sin^2\theta + \frac{1}{\sqrt{k}}\cos^2\theta \end{bmatrix} \rightarrow \begin{aligned} \Gamma_e &= \frac{A+B-C-D}{A+B+C+D} \\ T_e &= \frac{2}{A+B+C+D} \end{aligned}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} \cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{k}} & 0 \\ 0 & \sqrt{k} \end{bmatrix} \begin{bmatrix} \cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{k}}\cos^2\theta - \sqrt{k}\sin^2\theta & j(\sqrt{k}+\frac{1}{\sqrt{k}})\sin\theta\cos\theta \\ j(\sqrt{k}+\frac{1}{\sqrt{k}})\sin\theta\cos\theta & -\frac{1}{\sqrt{k}}\sin^2\theta + \sqrt{k}\cos^2\theta \end{bmatrix} \rightarrow \begin{matrix} \Gamma_o = \frac{A+B-C-D}{A+B+C+D} \\ T_o = \frac{2}{A+B+C+D} \end{matrix}$$

$$\rightarrow \Gamma_e = \frac{k-1}{k+1}e^{-j2\theta}, \Gamma_o = \frac{1-k}{k+1}e^{-j2\theta}, T_e = \frac{2\sqrt{k}}{k+1}e^{-j2\theta}, T_o = \frac{2\sqrt{k}}{k+1}e^{-j2\theta}$$

$$\Rightarrow b_1 = 0 = S_{14}, b_2 = \frac{k-1}{k+1}e^{-j2\theta} \equiv -\alpha e^{-j2\theta} = S_{24},$$

$$b_3 = \frac{2\sqrt{k}}{k+1}e^{-j2\theta} \equiv \beta e^{-j2\theta} = S_{34}, b_4 = 0 = S_{44}$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$b_1 = \frac{1}{2}T_e - \frac{1}{2}T_o$$

$$b_2 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

$$b_3 = \frac{1}{2}T_e + \frac{1}{2}T_o$$

$$b_4 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o$$

$$\text{input at port 2} = \text{input at port 4} \rightarrow S_{12} = S_{34}, S_{22} = S_{44}, S_{32} = S_{14}, S_{42} = S_{24}$$

$$\text{reciprocal} \rightarrow S_{23} = S_{32}, S_{41} = S_{14}$$

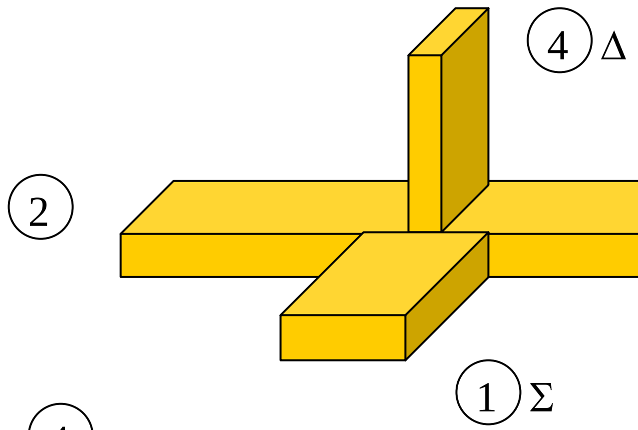
$$\text{input at port 1} \rightarrow \Gamma_e = \frac{1-k}{k+1} e^{-j2\theta}, \Gamma_o = \frac{k-1}{k+1} e^{-j2\theta}$$

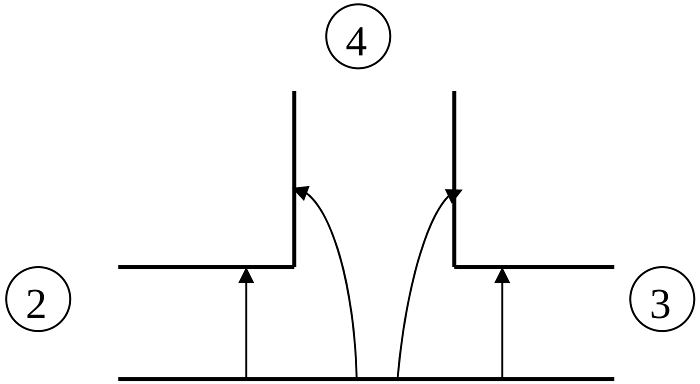
$$\Rightarrow b_1 = \frac{1}{2}(\Gamma_e + \Gamma_o) = 0 = S_{11}, b_3 = \frac{1}{2}(\Gamma_e - \Gamma_o) = \frac{1-k}{k+1} e^{-j2\theta} \equiv \alpha e^{-j2\theta} = S_{31}$$

$$\text{input at port 3} = \text{input at port 1} \rightarrow S_{13} = S_{31}$$

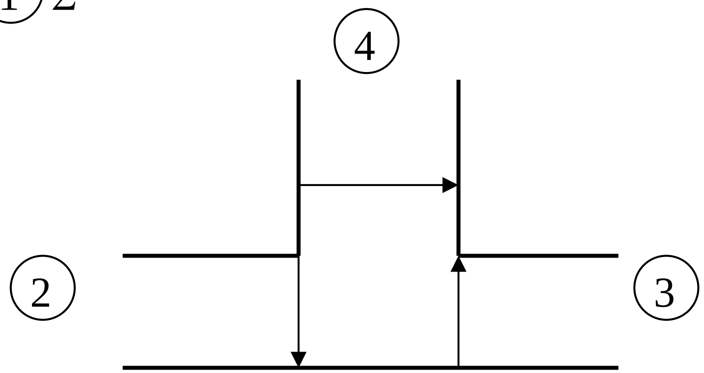
$$\Rightarrow [S] = e^{-j2\theta} \begin{bmatrix} 0 & \beta & \alpha & 0 \\ \beta & 0 & 0 & -\alpha \\ \alpha & 0 & 0 & \beta \\ 0 & -\alpha & \beta & 0 \end{bmatrix}$$

- waveguide magic-T





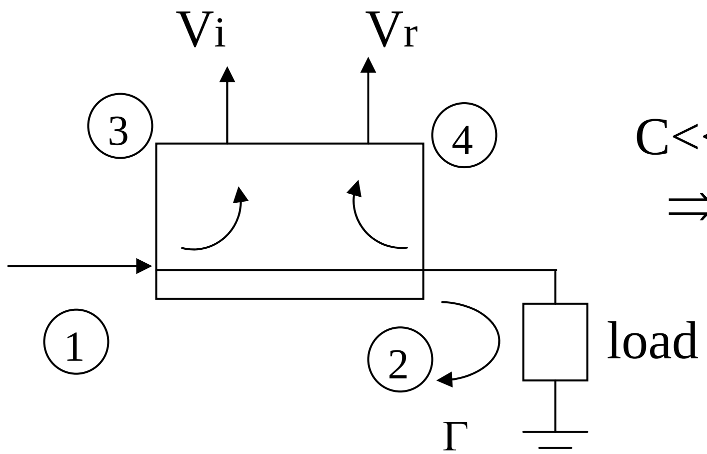
input at port 1
port 4: 0
port 2 and port 3: equal amplitude and phase



input at port 4
port 1: 0
port 2 and port 3: 180° phase difference

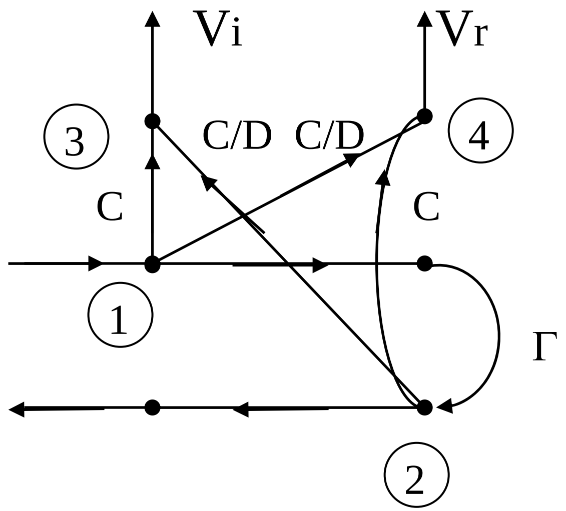
7.9 Other couplers

- reflectometer



C<<1

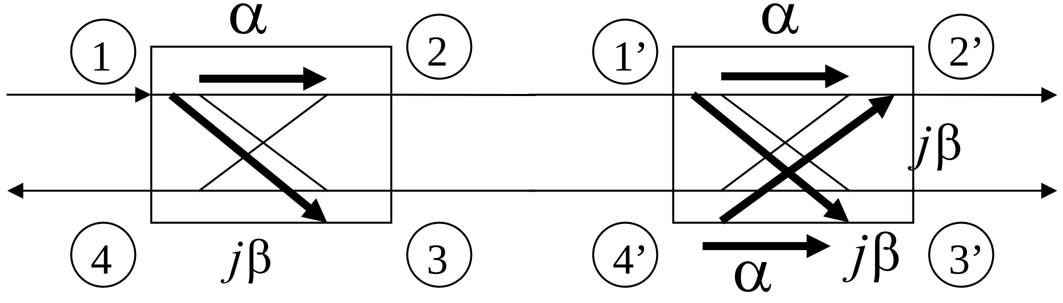
⇒



$$V_i = C + \frac{C}{D}\Gamma\ e^{j\theta}, V_r = \frac{C}{D} + C\Gamma\ e^{j\phi}$$

$$\left|\frac{V_r}{V_i}\right|_{\max,\min} = \frac{|\Gamma| \pm \frac{1}{D}}{1 \mp \frac{|\Gamma|}{D}},\ \text{as } D \to \infty\ \left|\frac{V_r}{V_i}\right| \to \Gamma$$

Prob. 7.3 Two 90° 8.34dB couplers are connected in cascade, find $S_{2'1}, S_{3'1}$



$$C = 8.34dB = -20\log\beta \rightarrow \beta = 0.383, \alpha^2 + \beta^2 = 1 \rightarrow \alpha = 0.924$$

$$V_{3'}^- = \alpha V_{4'}^+ + j\beta V_{1'}^+ = \alpha V_3^- + j\beta V_2^- = \alpha(j\beta V_1^+) + j\beta(\alpha V_1^+)$$

$$= 2j\alpha\beta V_1^+ = 0.707\angle 90^o V_1^+$$

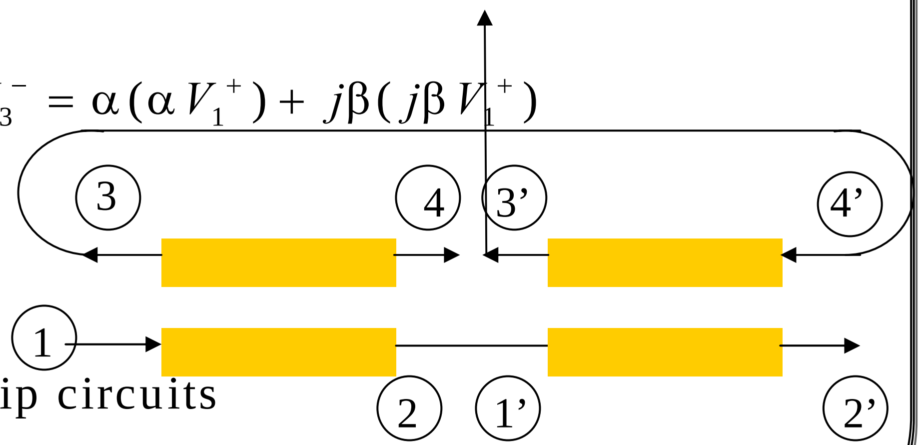
$$V_{2'}^- = \alpha V_{1'}^+ + j\beta V_{4'}^+ = \alpha V_2^- + j\beta V_3^- = \alpha(\alpha V_1^+) + j\beta(j\beta V_1^+)$$

$$= (\alpha^2 - \beta^2)V_1^+ = 0.707 V_1^+$$

$$V_{4'}^- = 0$$

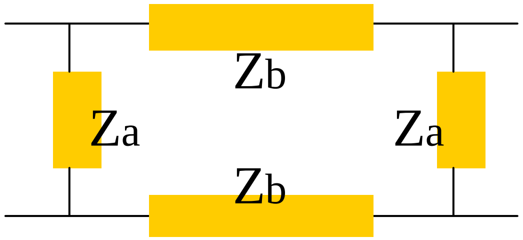
$\Rightarrow$ 3dB quadrature coupler

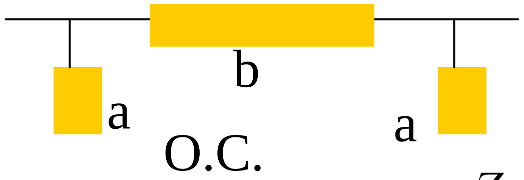
connection problem in microstrip circuits

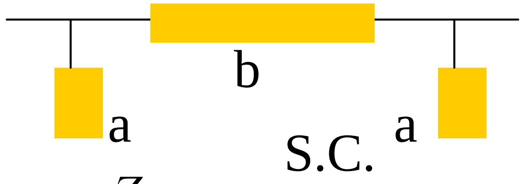


微波電路講義

Prob. 7.17 Design an unequal power branch-line coupler







(even mode) $a=\dfrac{Z_a}{Z_o},\ b=\dfrac{Z_b}{Z_o}$ (odd mode)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ j/a & 1 \end{bmatrix}\begin{bmatrix} 0 & jb \\ j/b & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ j/a & 1 \end{bmatrix} = \begin{bmatrix} -b/a & jb \\ j/b - jb/a^2 & -b/a \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j/a & 1 \end{bmatrix}\begin{bmatrix} 0 & jb \\ j/b & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -j/a & 1 \end{bmatrix} = \begin{bmatrix} b/a & jb \\ j/b - jb/a^2 & b/a \end{bmatrix}$$

$$\rightarrow \Gamma_e = \frac{j(b-1/b+b/a^2)}{-2b/a + j(b+1/b-b/a^2)}, \Gamma_o = \frac{j(b-1/b+b/a^2)}{2b/a + j(b+1/b-b/a^2)}, T_e = \frac{1}{-b/a+jb}, T_o = \frac{1}{b/a+jb}$$

$$\text{input match } b_1 = \frac{1}{2}(\Gamma_e + \Gamma_o) = 0 \rightarrow 1 - \frac{1}{b^2} + \frac{1}{a^2} = 0 \ldots(1)$$

$$b_2 = \frac{1}{2}(T_e + T_o) = \frac{-j}{b(1+1/a^2)}, b_3 = \frac{1}{2}(T_e - T_o) = \frac{-1/a}{b(1+1/a^2)}, P_2 = \alpha P_3 \rightarrow |b_2|^2 = \alpha |b_3|^2 \Rightarrow 1 = \frac{\alpha}{a^2} \rightarrow a = \sqrt{\alpha}$$

$$(1) \rightarrow b = \frac{a}{\sqrt{1+a^2}} = \sqrt{\frac{\alpha}{1+\alpha}} \Rightarrow Z_a = \sqrt{\alpha} Z_o, Z_b = \sqrt{\frac{\alpha}{1+\alpha}} Z_o$$

<u>Suggested homework (due 2 weeks)</u>: 4, 6, 8, 28, 29, 33

<u>ADS examples</u>: Ch7_prj