Chapter 8 Microwave filters

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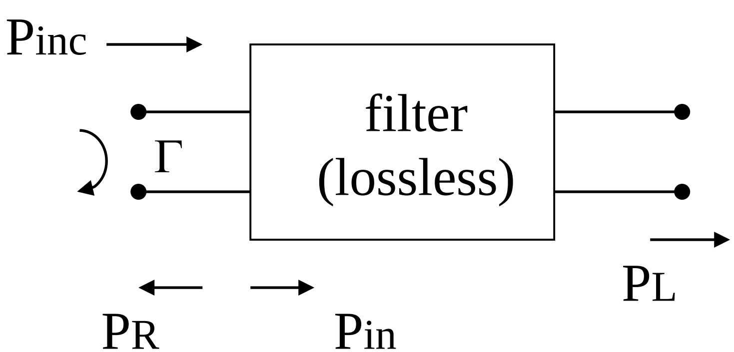
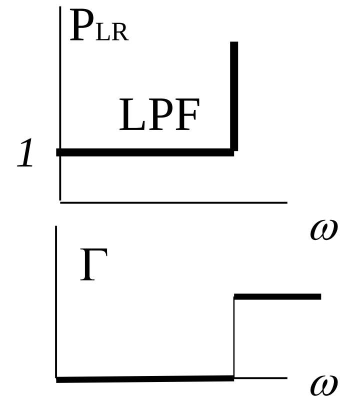
8.3 Filter design by the insertion loss method

- Power loss ratio (insertion loss)





$$P_{LR} \equiv \frac{P_{inc}}{P_L} = \frac{P_{inc}}{P_{inc} - P_R}$$

$$= \frac{1}{1 - |\Gamma(\omega)|^2}$$

$$= \frac{1}{1 - \dfrac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}}$$

$$= 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

<u>Discussion</u>

1. $P_{inc}(= P_{avs}$, available power from source$) = P_{in} + P_R$

   $P_{in} = P_L$ for a lossless filter

   insertion loss $IL \equiv 10\log P_{LR} = 10\log \dfrac{P_{inc}}{P_L}$ $(= -10\log G_T)$ (12.13)

   $(= -20\log|S_{21}|$, if $\Gamma_S = \Gamma_L = 0)$

   return loss $RL \equiv 10\log \dfrac{P_{inc}}{P_R} = -20\log|\Gamma|$

2. $|\Gamma(w)|^2$ is an even function of w $\rightarrow |\Gamma(w)|^2 = \dfrac{M(w^2)}{M(w^2)+N(w^2)}$ (p.173)

$\because v(t): \text{real} \rightarrow \text{Re}V(w): even, \text{Im}V(w): odd \rightarrow V(-w) = V^*(w)$

$I(-w) = I^*(w), Z(-w) = Z^*(w), \Gamma(-w) = \Gamma^*(w)$

$|\Gamma(w)|^2 = \Gamma(w)\Gamma^*(w) = \Gamma(w)\Gamma(-w), |\Gamma(-w)|^2 = \Gamma(-w)\Gamma^*(-w) = \Gamma(-w)\Gamma(w)$

3. Maximally flat (Butterworth, binomial) LPF

$P_{LR}(w) = 1 + \varepsilon(\dfrac{w}{w_c})^{2N}, w_c$:cutoff frequency, $\varepsilon = 1 \rightarrow P_{LR}(w_c) = 2 = 3dB$

stopband attenuation $20NdB/decade$

4. Equal ripple (Chebyshev, optimal) LPF

$P_{LR}(w) = 1 + \varepsilon T_N^2(\dfrac{w}{w_c})^{2N}, P_{LR}(w_c) = 1 + \varepsilon$

ripple $10^{\log(1+\varepsilon)}$ in passband, stopband attenuation $20NdB/decade$

frequency responses (p.400, Fig. 8.21)

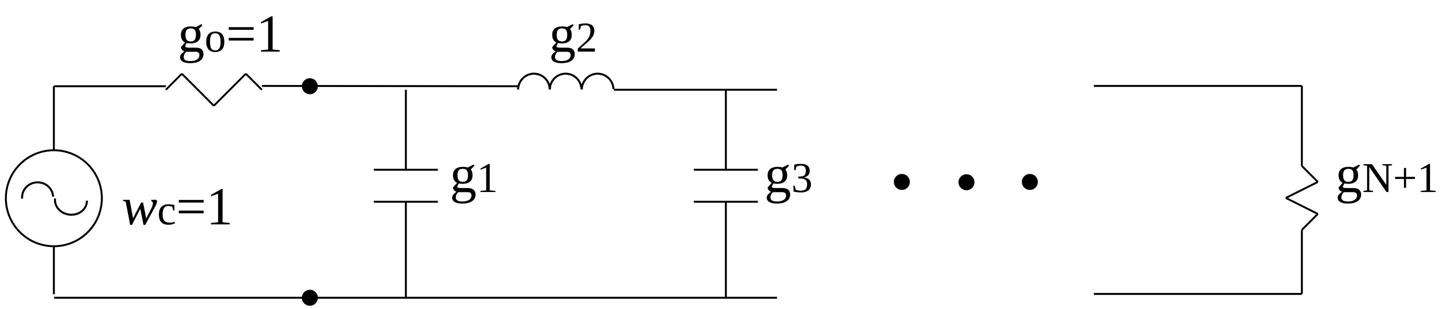
5. Filter design procedure:

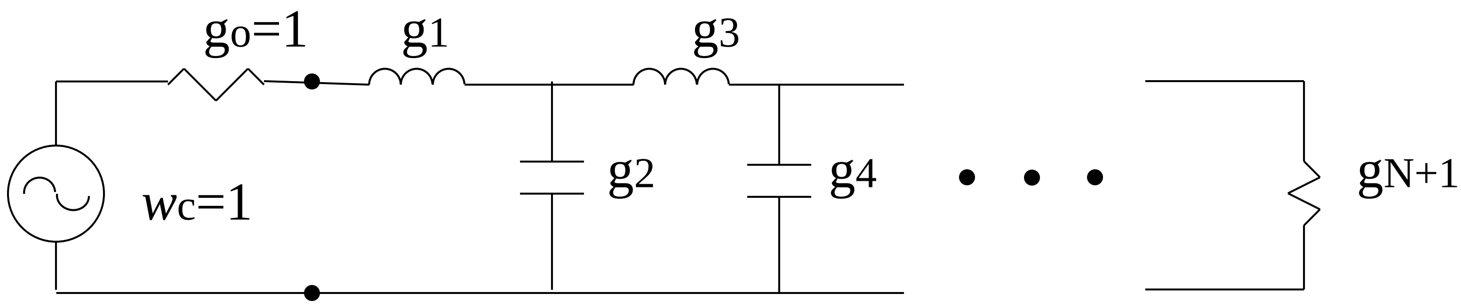
specification → LPF prototype → LPF, HPF, BPF, or BSF transformation → microstrip realization

6. Filter specification:

frequency range, BW, IL, stopband attenuation and frequencies, input and output impedances, VSWR, group delay, phase linearity, temperature range, and transient response.

- LPF prototype





<u>Discussion</u>

1. Maximally flat LPF prototype design equations

$$IL = 10\log[1+(\frac{w}{w_c})^{2N}]$$

$$w_c = 1, g_o = g_{N+1} = 1, g_k = 2\sin\frac{(2k-1)\pi}{2N} \quad k = 1,2,\ldots N$$

$$N = \frac{\log(10^{IL(w)/10}-1)}{2\log\frac{w}{w_c}}$$

2. Element values for maximally flat LPF prototype are given in Table 8.3 of p.404, frequency response given in Fig. 8.26 of p.405.

3. Equal-ripple LPF prototype design equations

$$IL = 10\log[1+\varepsilon T_N^2(\frac{w}{w_c})]$$

$$\varepsilon = 10^{\frac{ripple(dB)}{10}} - 1$$

$$T_N(w) = \begin{cases} \cos(N\cos^{-1}\frac{w}{w_c}) & 0 \le w \le w_c \\ \cosh(N\cosh^{-1}\frac{w}{w_c}) & w > w_c \end{cases}$$

$$N = \frac{\cosh^{-1}[10^{IL(w)/10} - 1]/[10^{\frac{ripple(dB)}{10}} - 1]}{\cos^{-1}(\frac{w}{w_c})}$$

$$w_c = 1,\ g_o = 1,\ g_1 = \frac{2a_1}{\gamma},\ g_{N+1} = \begin{cases} 1, & N\ odd \\ \coth^2 \frac{\beta}{4}, & N\ even \end{cases}$$

$$\gamma = \sinh \frac{\beta}{2N}$$

$$\beta = \ln[\coth \frac{10\log(\varepsilon + 1)}{17.37}]$$

$$g_k = \frac{4a_{k-1}a_k}{b_{k-1}b_k},\ k = 2,3,\ldots N$$

$$a_k = \sin \frac{(2k-1)\pi}{2N},\ b_k = \gamma^2 + \sin^2 \frac{k\pi}{N},\ k = 1,2,\ldots N$$

4. Element values for equal-ripple LPF prototype (p.406, Table 8.4) and frequency response (p.407, Fig. 8.27)

8.4 Filter transformations

• Impedance scaling $g_o = 1 \rightarrow R_o \Rightarrow impedance \times R_o$

scaled values: $R_S' = R_o, R_L' = R_o R_L, L' = R_o L, C' = \dfrac{C}{R_o}$

• Frequency scaling

$$w : 1 \rightarrow w_c \Rightarrow P'_{LR}(w) = P_{LR}(\frac{w}{w_c}) \Rightarrow \text{same impedance}, \quad w \leftarrow \frac{w}{w_c}$$

scaled values:

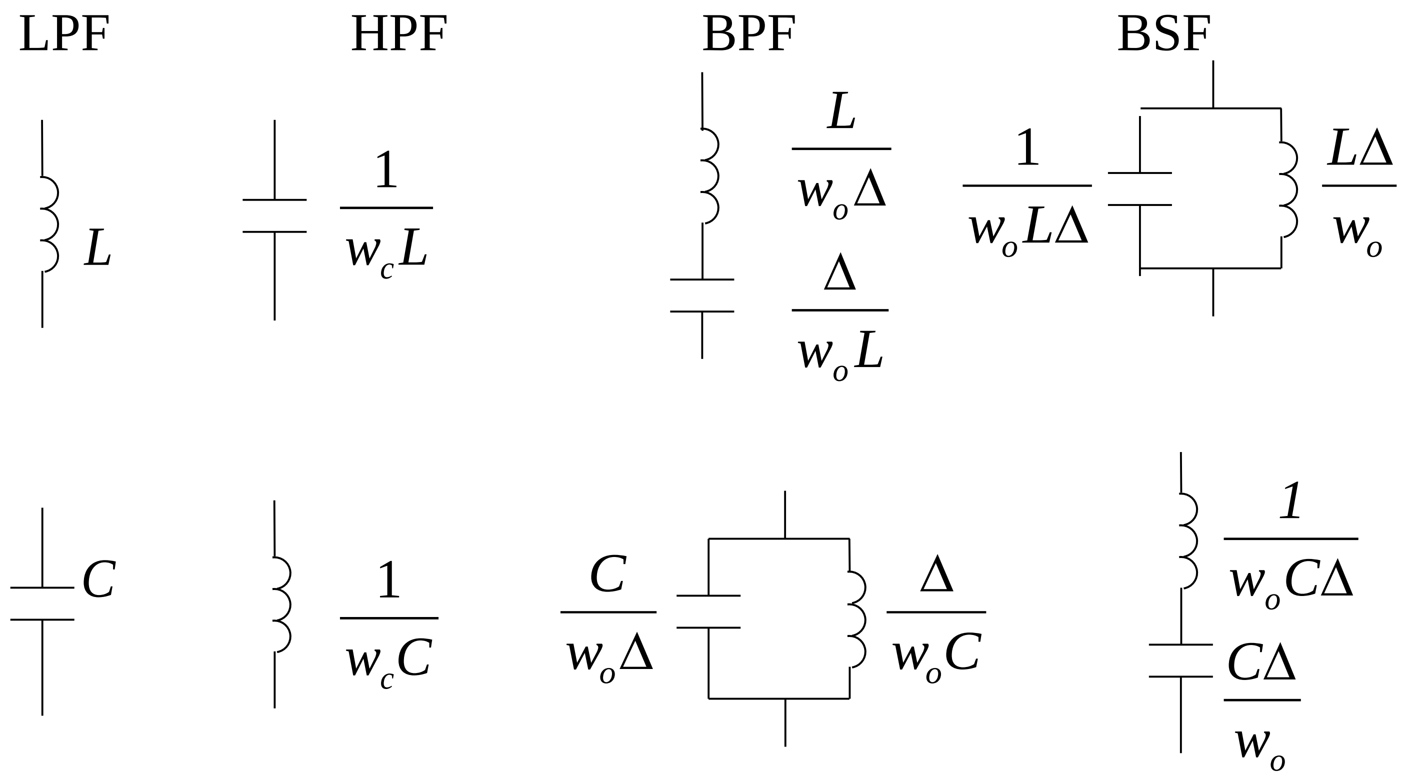
$$jX_L = j\frac{w}{w_c}L = jwL' \rightarrow L' = \frac{L}{w_c}$$

$$jB_C = j\frac{w}{w_c}C = jwC' \rightarrow C' = \frac{C}{w_c}$$

• Impedance and frequency scaling

scaled values: $R_S' = R_o, R_L' = R_o R_L, L' = \dfrac{R_o L}{w_c}, C' = \dfrac{C}{R_o w_c}$

- Frequency scaling for HPF, BPF and BSF



$$\Delta = \frac{w_2 - w_1}{w_o}$$: fractional BW

Discussion

1. Ex. 8.3 design a maximally flat LPF, fc=2GHz, Zo=50Ω, IL(3GHz) =15dB

from Fig.8.26, $IL = 15dB$ for $\left|\frac{w}{w_c}\right| - 1 = 0.5 @ 3GHz \rightarrow N = 5$

from Table 8.3, $C_1 = C_5 = 0.618, L_2 = L_4 = 1.618, C_3 = 2$

$$\rightarrow C_1' = C_5' = \frac{C_1}{50 w_c} = 0.984 pF$$

$$L_2' = L_4' = \frac{50 L_2}{w_c} = 6.438 nH$$

$$C_3' == \frac{C_3}{50 w_c} = 3.183 pF$$

frequency response (p.412, Fig. 8.30) with the comparison of equal-ripple LPF and linear phase LPF

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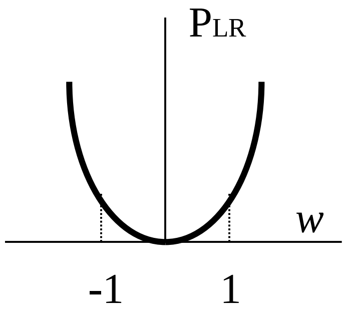
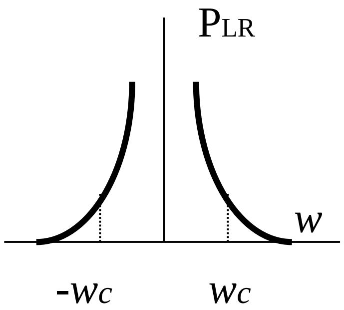
2. LPF → HPF frequency scaling





$$0 \to \pm\infty, 1 \to -w_c, -1 \to w_c \Rightarrow \quad w \leftarrow -\frac{w_c}{w}$$

$$LPF \begin{cases} \dfrac{1}{jwC} \to \dfrac{1}{-j\dfrac{w_c C}{w}} = jwL' \\ jwL \to -j\dfrac{w_c L}{w} = \dfrac{1}{jwC'} \end{cases} \Rightarrow HPF \begin{cases} L' = \dfrac{1}{w_c C} \\ C' = \dfrac{1}{w_c L} \end{cases}$$

3. LPF $\rightarrow$ BPF frequency scaling

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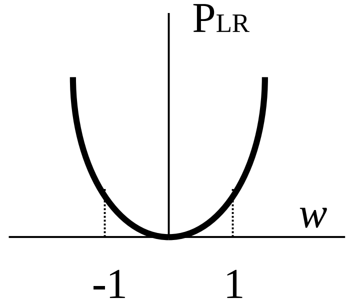
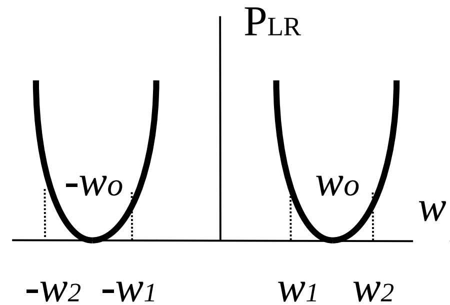
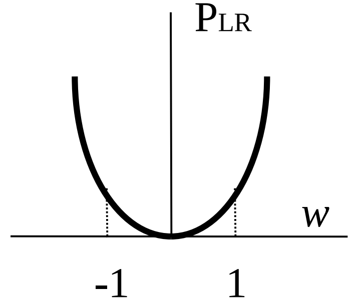


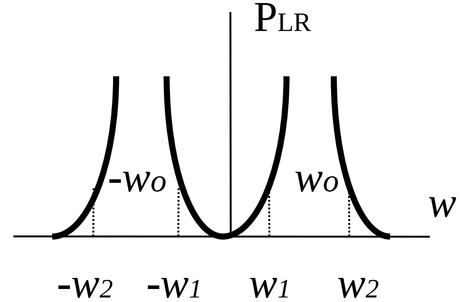


$$0 \rightarrow w_o, 1 \rightarrow w_2, -1 \rightarrow w_1 \Rightarrow \quad w \leftarrow \frac{1}{\Delta}\left(\frac{w}{w_o} - \frac{w_o}{w}\right),\ w_o = \sqrt{w_1 w_2}, \Delta = \frac{w_2 - w_1}{w_o}$$

$$LPF \begin{cases} \dfrac{1}{jwC} \rightarrow \dfrac{1}{j\dfrac{C}{\Delta}\left(\dfrac{w}{w_o} - \dfrac{w_o}{w}\right)} = \dfrac{1}{jw\dfrac{C}{w_o\Delta} + \dfrac{Cw_o}{jw\Delta}} \\ jwL \rightarrow j\dfrac{L}{\Delta}\left(\dfrac{w}{w_o} - \dfrac{w_o}{w}\right) = jw\dfrac{L}{w_o\Delta} + \dfrac{Lw_o}{jw\Delta} \end{cases} \Rightarrow BPF \begin{cases} C' = \dfrac{C}{w_o\Delta}, L' = \dfrac{\Delta}{w_o C} \\ L' = \dfrac{L}{w_o\Delta}, C' = \dfrac{\Delta}{w_o L} \end{cases}$$

4. LPF $\rightarrow$ BSF frequency scaling





$$0 \rightarrow \pm\infty, 1 \rightarrow -w_1, -1 \rightarrow w_2 \Rightarrow \quad w \leftarrow \frac{\Delta}{\frac{w_o}{w} - \frac{w}{w_o}}$$

$$LPF \begin{cases} \dfrac{1}{jwC} \rightarrow \dfrac{1}{jC(\dfrac{\Delta}{\frac{w_o}{w} - \frac{w}{w_o}})} = \dfrac{w_o}{jwC\Delta} + \dfrac{jw}{C\Delta w_o} \\ \\ jwL \rightarrow jL(\dfrac{\Delta}{\frac{w_o}{w} - \frac{w}{w_o}}) = \dfrac{1}{\frac{w_o}{jwL\Delta} + \frac{jw}{Lw_o\Delta}} \end{cases} \Rightarrow BSF \begin{cases} C' = \dfrac{C\Delta}{w_o}, L' = \dfrac{1}{w_o C\Delta} \\ \\ L' = \dfrac{L\Delta}{w_o}, C' = \dfrac{1}{w_o L\Delta} \end{cases}$$

5. Ex. 8.4 design a N=3, 0.5dB equal-ripple BPF, f_o=1GHz, Z_o=50Ω, BW=10%

from Table 8.4, $L_1 = L_3 = 1.5963, C_2 = 1.0967, R_L = 1$

$$\rightarrow L_1' = L_3' = \frac{L_1 50}{w_o \Delta} = 127nH$$

$$C_1' = C_3' = \frac{\Delta}{w_o L_1 50} = 0.199\,pF$$

$$L_2' = \frac{50\Delta}{w_o C_2} = 0.726nH$$

$$C_2' == \frac{C_2}{50 w_o \Delta} = 34.91pF$$

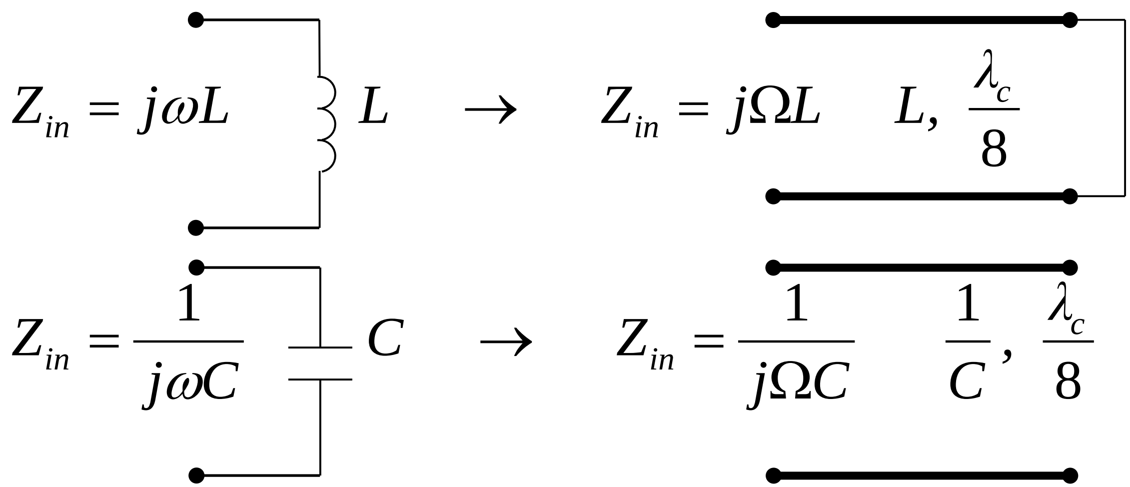
frequency response (p.415, Fig. 8.33)

8.5 Filter implementation

• Richards' transformation

$\omega \to \Omega,\ \Omega \equiv \tan\beta l = \tan(\omega l / v_p)$

lumped elements $\to$ commensurate lines with S.C. or O.C. stub



<u>Discussion</u>

1. $\omega_c = 1 \to \Omega_c = \tan\beta_c l = \tan\frac{2\pi}{\lambda_c}\frac{\lambda_c}{8} = \tan\frac{\pi}{4} = 1$

2. $\beta\frac{\lambda_c}{8} = \pi \to \frac{\omega}{c}\frac{\lambda_c}{8} = \pi \to \omega = 8\pi f_c = 4\omega_c \Rightarrow$ stub response repeats every $4\omega_c$.

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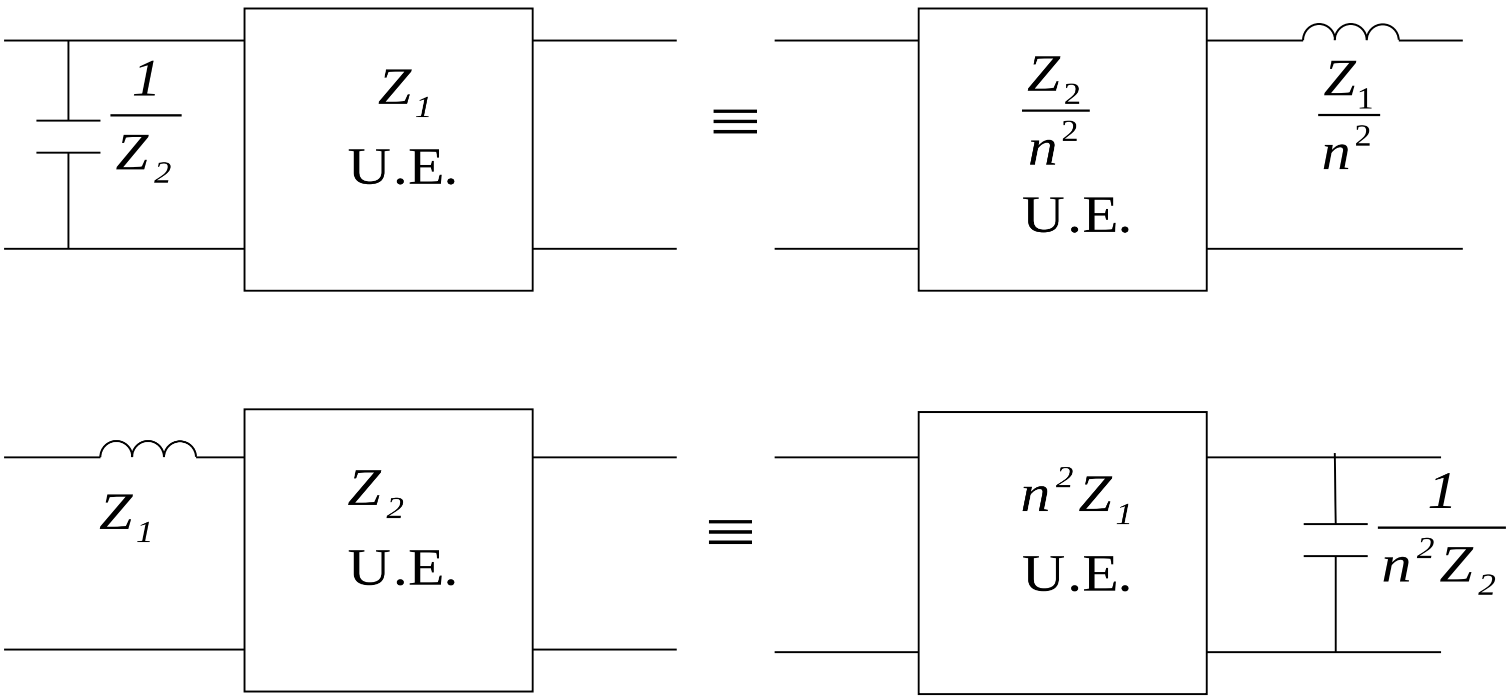
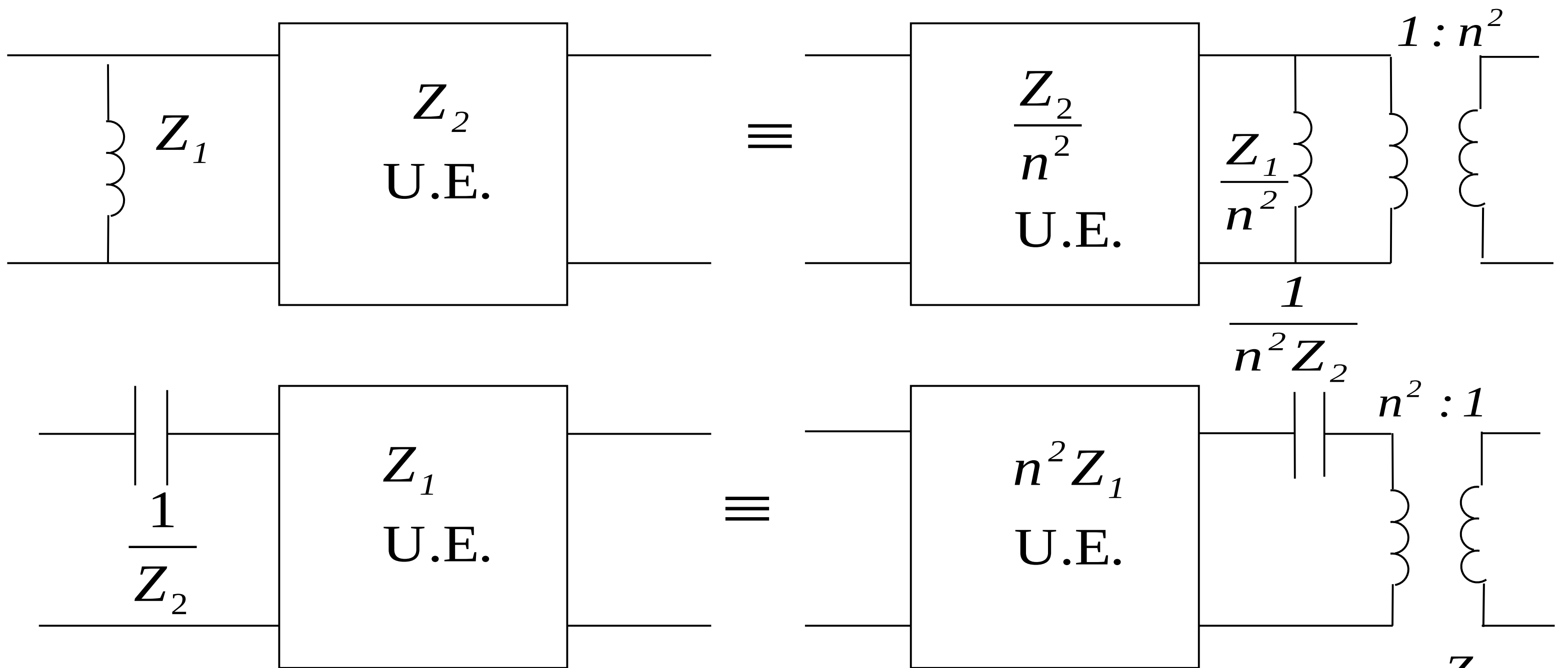
• Kuroda’s identites



$$n^2 = 1 + \frac{Z_2}{Z_1}$$

U.E. (unit element) : λc/8 line

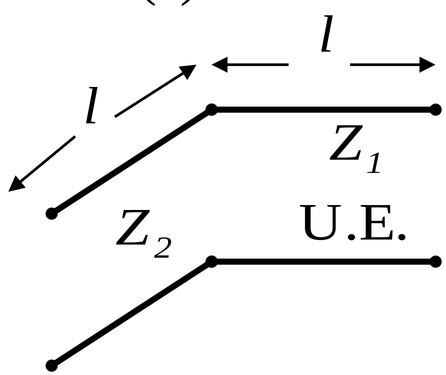


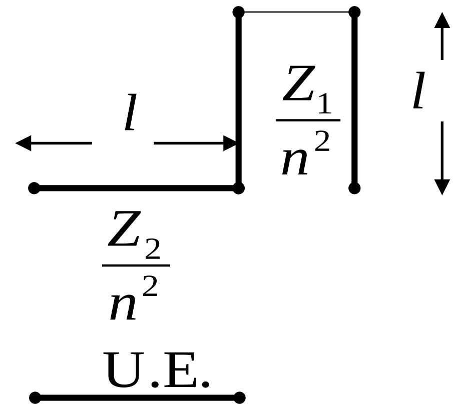
$$n^2 = 1 + \frac{Z_2}{Z_1}$$

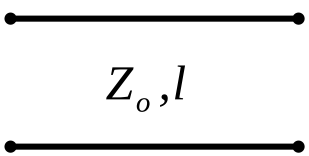
<u>Discussion</u>

1. Use $\lambda_c/8$ redundant lines to separate stubs.
2. series L (short stub) ↔ shunt C (open stub)
   series C and shunt L change positions

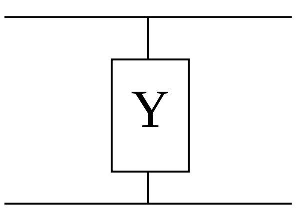
3. derivation of (a)



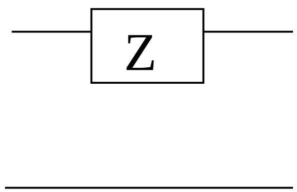




$$\begin{bmatrix}A & B\\ C & D\end{bmatrix}=\begin{bmatrix}\cos\beta l & jZ_o\sin\beta l\\ jY_o\sin\beta l & \cos\beta l\end{bmatrix}=\frac{1}{\sqrt{1+\Omega^2}}\begin{bmatrix}1 & j\Omega Z_o\\ \dfrac{j\Omega}{Z_o} & 1\end{bmatrix}$$



$$\begin{bmatrix}1 & 0\\ Y & 1\end{bmatrix}=\begin{cases}\begin{bmatrix}1 & 0\\ \dfrac{j\Omega}{Z_o} & 1\end{bmatrix} \text{open-circuited shunt stub}\\ \begin{bmatrix}1 & 0\\ \dfrac{1}{j\Omega Z_o} & 1\end{bmatrix} \text{short-circuited shunt stub}\end{cases}$$



$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} = \begin{cases} \begin{bmatrix} 1 & \dfrac{Z_o}{j\Omega} \\ 0 & 1 \end{bmatrix} \text{ open-circuited series stub} \\ \begin{bmatrix} 1 & j\Omega\, Z_o \\ 0 & 1 \end{bmatrix} \text{short-circuited series stub} \end{cases}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_L = \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & 0 \\ \dfrac{j\Omega}{Z_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & j\Omega\, Z_1 \\ \dfrac{j\Omega}{Z_1} & 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & j\Omega\, Z_1 \\ j\Omega\, (\dfrac{1}{Z_1} + \dfrac{1}{Z_2}) & 1-\Omega^2 \dfrac{Z_1}{Z_2} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_R = \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & \dfrac{j\Omega Z_2}{n^2} \\ \dfrac{j\Omega\, n^2}{Z_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & \dfrac{j\Omega Z_1}{n^2} \\ 0 & 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & \dfrac{j\Omega}{n^2}(Z_1 + Z_2) \\ \dfrac{j\Omega n^2}{Z_2} & 1-\Omega^2 \dfrac{Z_1}{Z_2} \end{bmatrix} \rightarrow n^2 = 1 + \frac{Z_2}{Z_1}$$

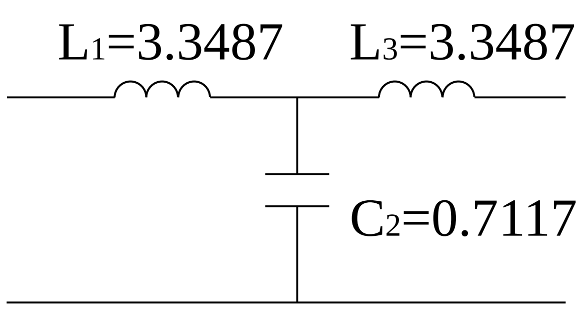
4. Microstrp LPF design procedure

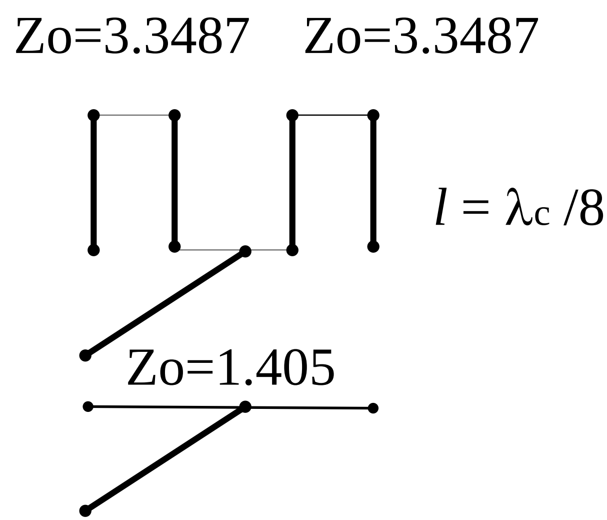
series L, shunt C → $\lambda_c/8$ series short stub, $\lambda_c/8$ shunt open stub

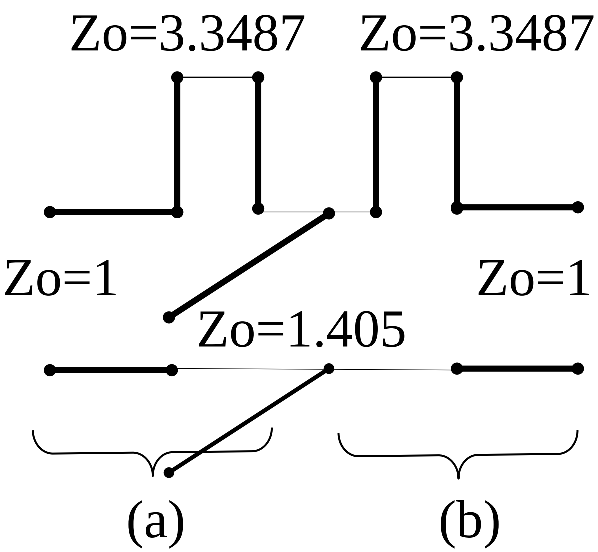
→ add redundant $\lambda_c/8$ Z_o lines → $\lambda_c/8$ shunt open stubs

→ consider discontinuity effects

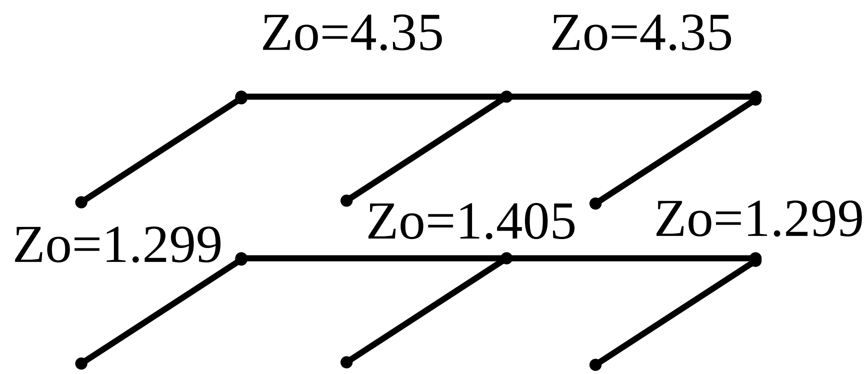
5. Ex.8.5 design a 3dB equal-ripple LPF with f_c=4GHz, N=3, Zin, Zout=50Ω

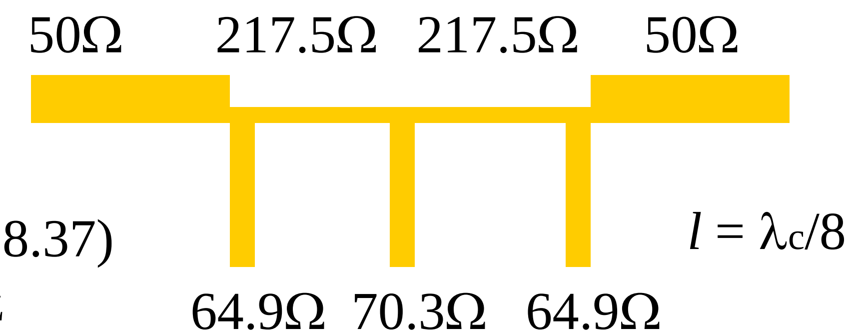






Kuroda’s identity



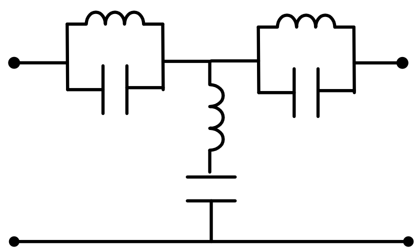


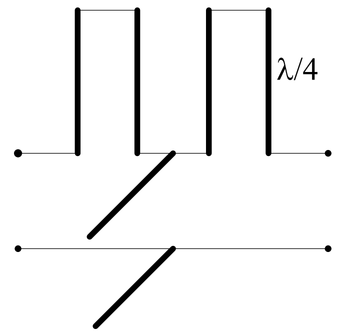
frequency response (p.421, Fig.8.37)

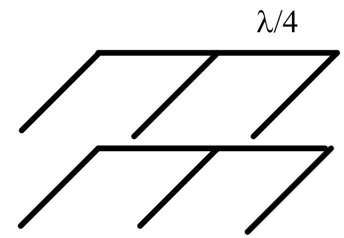
frequency repeats every 16GHz

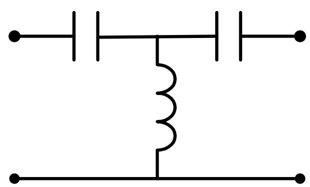
S_{21} @8GHz, 24GHz (l=λ’/4, 3λ’/4,…)

6. Similar procedures can be used for bandstop filters, but Kuroda identities are not useful for highpass or bandpass filters. (p.421)

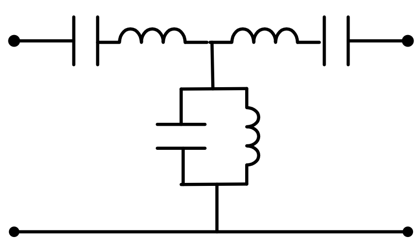


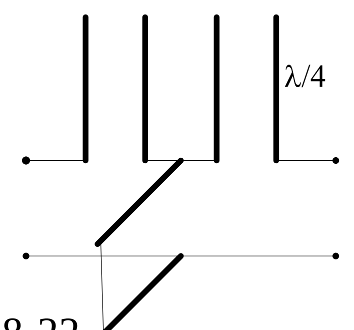






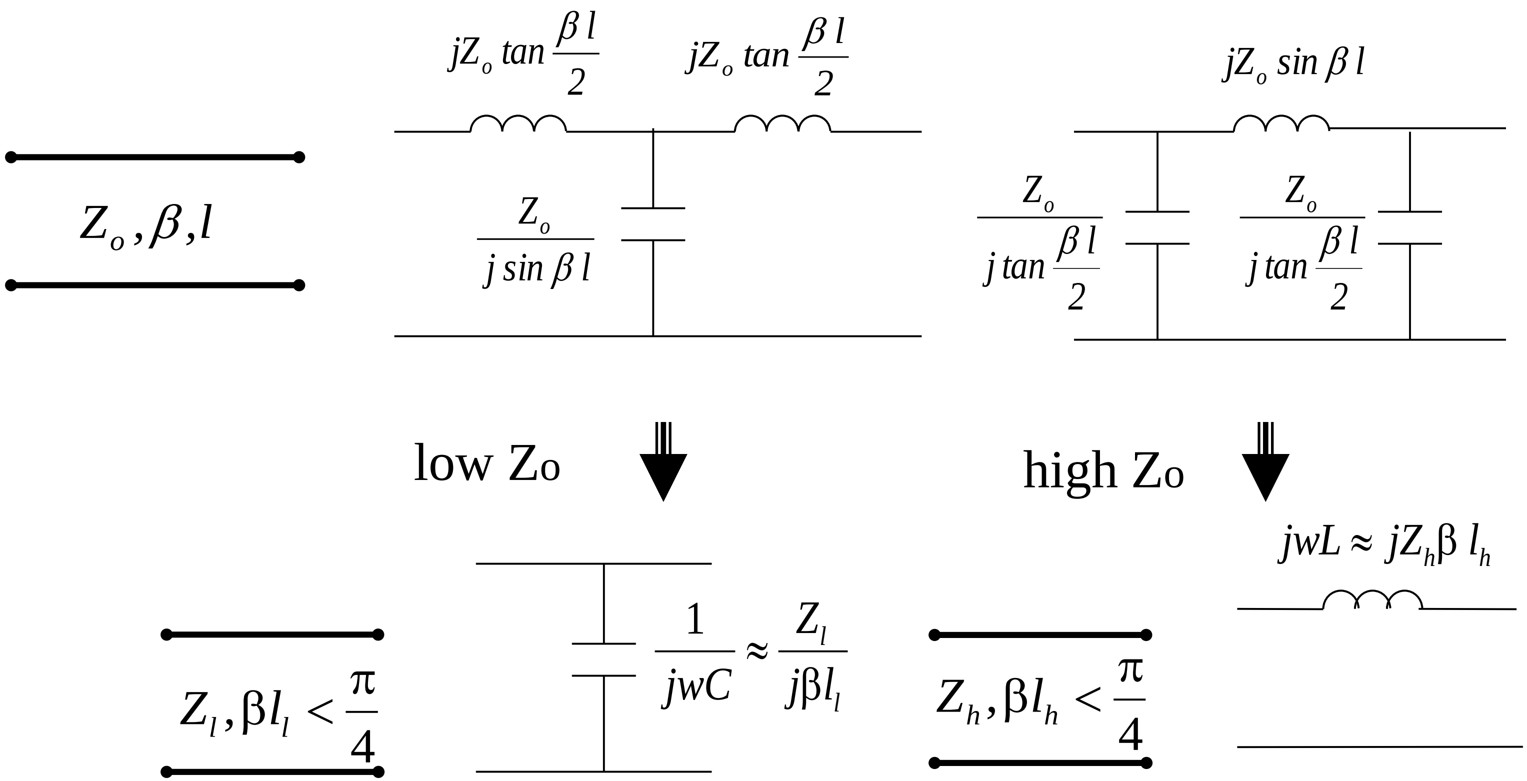
Series capacitor transformation is not available in Kuroda identities.





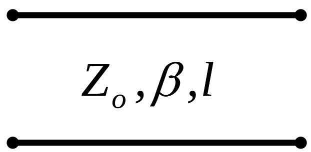
8.6 Stepped-impedance LPF

• Short transmission line section



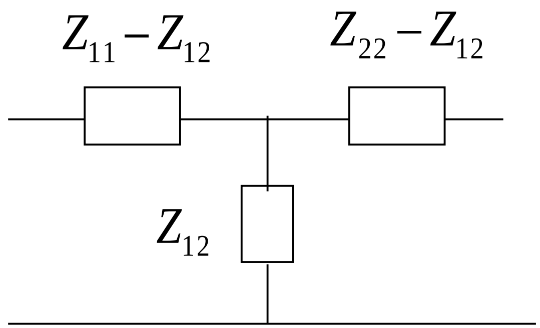
Discussion

1. (derivation from notes 4-12, 13)



$$\theta = \beta l$$

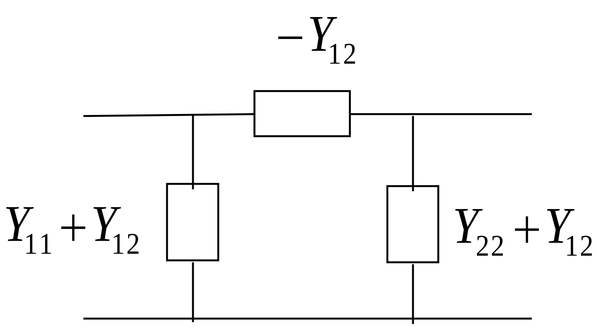
$$[Z] = -jZ_o \begin{bmatrix} \cot\theta & \csc\theta \\ \csc\theta & \cot\theta \end{bmatrix}, [Y] = -jY_o \begin{bmatrix} \cot\theta & -\csc\theta \\ -\csc\theta & \cot\theta \end{bmatrix}$$



$$Z_{12} = -jZ_o \csc\theta$$

$$Z_{11} - Z_{12} = -jZ_o(\cot\theta - \csc\theta) = -jZ_o\left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= -jZ_o \frac{-2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = jZ_o \tan\frac{\theta}{2}$$



$$Y_{12} = jY_o \csc\theta$$

$$Y_{11} + Y_{12} = \frac{1}{jZ_o}\left[\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right] = \frac{j\tan\frac{\theta}{2}}{Z_o}$$

2. Microstrp LPF design procedure

L, C $\rightarrow$ select proper Z_h, Z_l $\rightarrow$ at cutoff frequency $R_o L = Z_h \beta l_h$, $R_o/C = Z_l/\beta l_l$ $\rightarrow$ l_h, l_l $\rightarrow$ consider parasitics of L,C and discontinuity effects

3. Considerations of Z_l

$$\frac{Z_l}{j\sin\beta l_l} = \frac{1}{j\omega C} \rightarrow \sin\beta l_l = \omega C Z_l \overset{\beta l \le \frac{\pi}{4}}{\le} \frac{1}{\sqrt{2}} \rightarrow$$

$$Z_l \le \frac{1}{\omega C\sqrt{2}}$$

$$l_l = \frac{1}{\beta}\sin^{-1}\omega C Z_l \rightarrow \text{open stubs?}$$

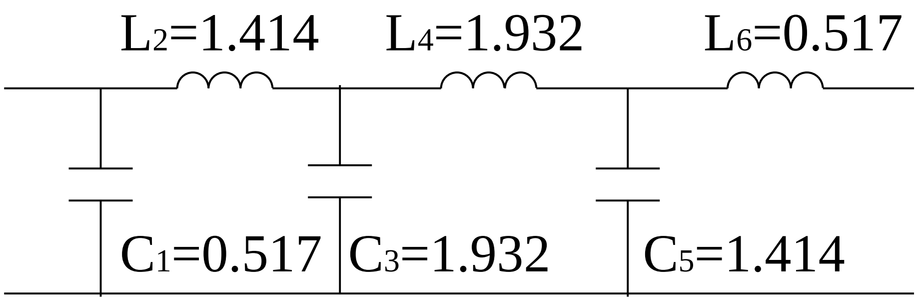
4. Considerations of Z_h

$$jZ_h \sin\beta l_h = j\omega L \rightarrow \sin\beta l_h = \frac{\omega L}{Z_h} \overset{\beta l \le \frac{\pi}{4}}{\le} \frac{1}{\sqrt{2}} \rightarrow$$

$$Z_h \ge \sqrt{2}\omega L \rightarrow \text{fabrication?}$$

$$l_h = \frac{1}{\beta}\sin^{-1}\frac{\omega L}{Z_h} \rightarrow \text{capacitor coupling?}$$

5. Ex.8.6 design a maximally flat LPF with f_c=2.5GHz, IL(4GHz)=20 dB, Z_{in}, Z_{out} =50Ω, Z_h=120Ω, Z_l =20Ω→N=6

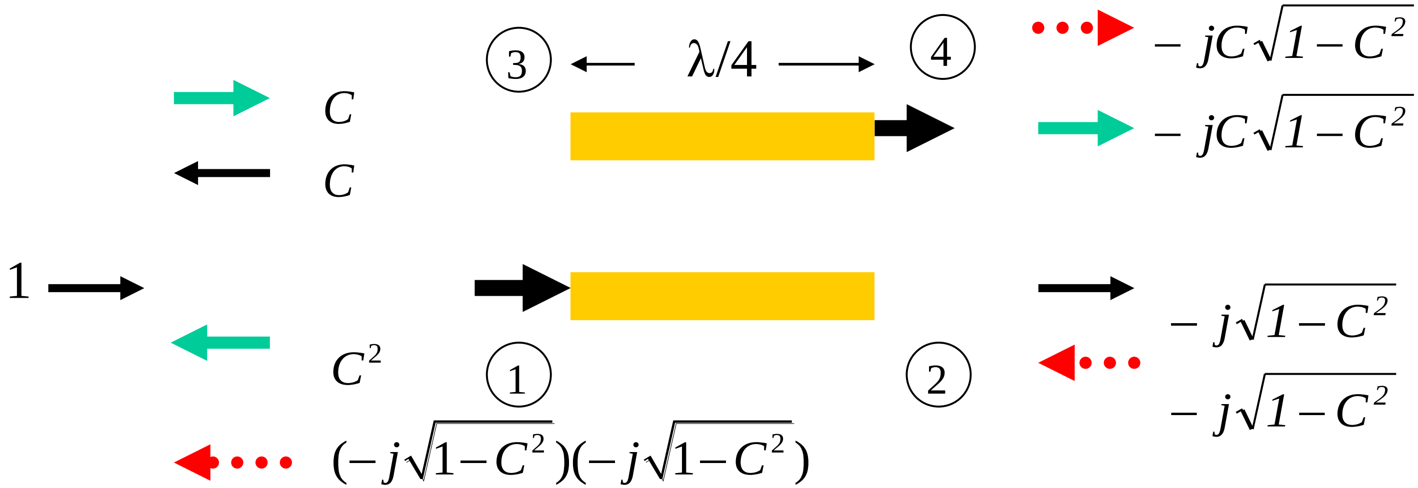


$l_h = R_o L/\beta Z_h$,
$l_l = C Z_l/\beta R_o$

frequency response (p.425, Fig.8.41)
no repetition of frequency response

8.7 Coupled line filters

- Coupled line BPF element



① $[C^2-(1-C^2)]^2 = 4C^4 - 4C^2 + 1$

④ $[-j2C\sqrt{1-C^2}]^2 = 4C^2(1-C^2) = -4C^4 + 4C^2$

① + ④ =1

if $C = \dfrac{1}{\sqrt{2}}, \rightarrow$ ① =0, ④ =1

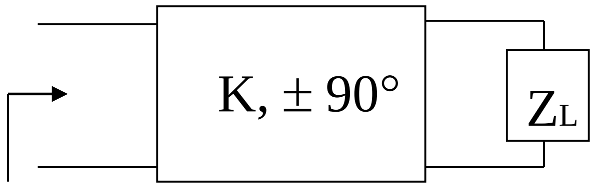
$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & -j\sqrt{1-C^2} & C & 0 \\ -j\sqrt{1-C^2} & 0 & 0 & C \\ C & 0 & 0 & -j\sqrt{1-C^2} \\ 0 & C & -j\sqrt{1-C^2} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -j\sqrt{1-C^2}a_1 \\ Ca_1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & -j\sqrt{1-C^2} & C & 0 \\ -j\sqrt{1-C^2} & 0 & 0 & C \\ C & 0 & 0 & -j\sqrt{1-C^2} \\ 0 & C & -j\sqrt{1-C^2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -j\sqrt{1-C^2}a_1 \\ Ca_1 \\ 0 \end{bmatrix}$$

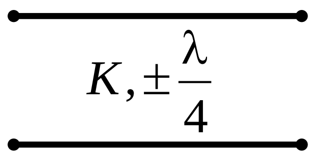
$$= \begin{bmatrix} [-(1-C^2)+C^2]a_1 \\ 0 \\ 0 \\ [-j\sqrt{1-C^2}C - j\sqrt{1-C^2}C]a_1 \end{bmatrix} \overset{C=\frac{1}{\sqrt{2}}}{=} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -ja_1 \end{bmatrix}$$

• Impedance and admittance inverters (p.422, Fig. 8.38)

impedance inverter

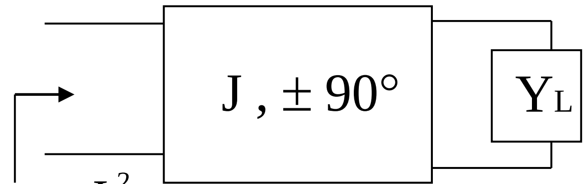


$$Z_{in} = \frac{K^2}{Z_L}$$

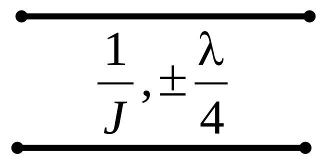


$$K, \pm\frac{\lambda}{4}$$

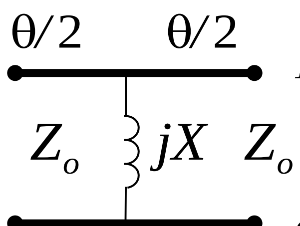
admittance inverter



$$Y_{in} = \frac{J^2}{Y_L}$$

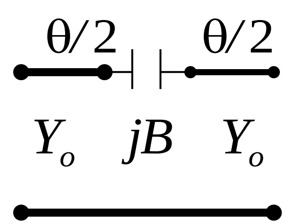


$$\frac{1}{J}, \pm\frac{\lambda}{4}$$



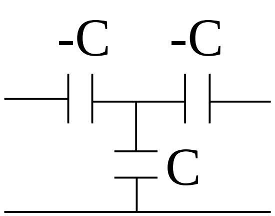
$$K = Z_o \tan\left|\frac{\theta}{2}\right|, X = \frac{K}{1-(\frac{K}{Z_o})^2}$$

$$\theta = -\tan^{-1}\frac{2X}{Z_o}$$

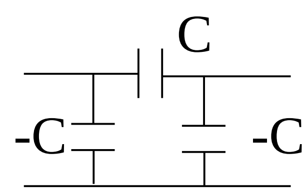


$$J = Y_o \tan\left|\frac{\theta}{2}\right|, B = \frac{J}{1-(\frac{J}{Y_o})^2}$$

$$\theta = -\tan^{-1}\frac{2B}{Y_o}$$

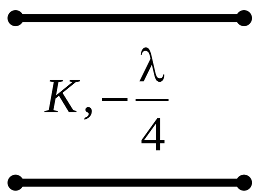


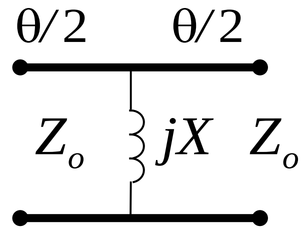
$$K = \frac{1}{\omega C}$$

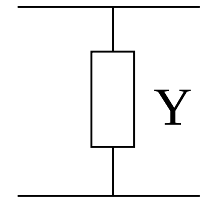


$$J = \omega C$$

(derivation of impedance inverter)







$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_o \sin\beta l \\ jY_o \sin\beta l & \cos\beta l \end{bmatrix}, \quad \begin{matrix} Z_o = K \\ l = -\frac{\lambda}{4} \end{matrix} \xrightarrow{\beta l = -\frac{\pi}{2}} \begin{bmatrix} 0 & -jK \\ -\frac{j}{K} & 0 \end{bmatrix}$$

$$\begin{bmatrix} \cos\frac{\theta}{2} & jZ_o \sin\frac{\theta}{2} \\ \frac{j}{Z_o}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{jX} & 1 \end{bmatrix} \begin{bmatrix} \cos\frac{\theta}{2} & jZ_o \sin\frac{\theta}{2} \\ \frac{j}{Z_o}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} = \begin{bmatrix} \cos\theta + \frac{Z_o}{2X}\sin\theta & jZ_o(\sin\theta + \frac{Z_o}{X}\sin^2\frac{\theta}{2}) \\ j(\frac{1}{Z_o}\sin\theta - \frac{1}{X}\cos^2\frac{\theta}{2}) & \cos\theta + \frac{Z_o}{2X}\sin\theta \end{bmatrix}$$

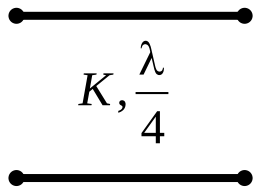
$$\cos\theta + \frac{Z_o}{2X}\sin\theta = 0 \qquad \theta = -\tan^{-1}\frac{2X}{Z_o} \qquad (1)-(2) \rightarrow X = \frac{K}{1-(\frac{K}{Z_o})^2}$$

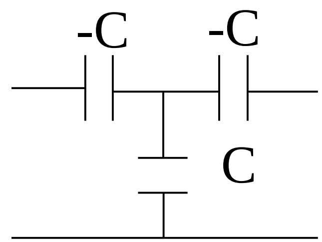
$$\rightarrow j(\frac{1}{Z_o}\sin\theta - \frac{1}{X}\cos^2\frac{\theta}{2}) = -\frac{j}{K} \rightarrow \sin\theta - \frac{Z_o}{X}\cos^2\frac{\theta}{2} = -\frac{Z_o}{K} \ldots(1) \rightarrow$$

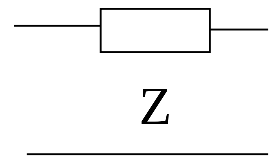
$$jZ_o(\sin\theta + \frac{Z_o}{X}\sin^2\frac{\theta}{2}) = -jK \qquad \sin\theta + \frac{Z_o}{X}\sin^2\frac{\theta}{2} = -\frac{K}{Z_o} \ldots(2)$$

$$\tan|\theta| = \frac{2X}{Z_o} = \frac{2K/Z_o}{1-(\frac{K}{Z_o})^2} = \frac{2\tan\left|\frac{\theta}{2}\right|}{1-\tan^2\left|\frac{\theta}{2}\right|}, K = Z_o \tan\left|\frac{\theta}{2}\right|$$

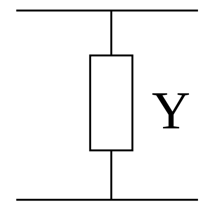
(derivation of impedance inverter)







$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

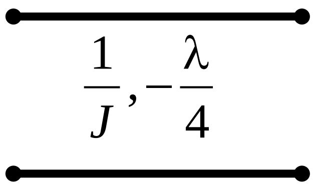


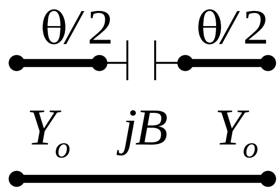
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

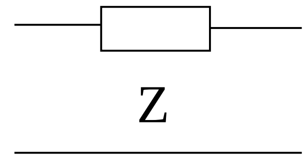
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_o \sin\beta l \\ jY_o \sin\beta l & \cos\beta l \end{bmatrix}, \quad \begin{matrix} Z_o = K & \beta l = \frac{\pi}{2} \\ l = \frac{\lambda}{4} & \end{matrix} \rightarrow \begin{bmatrix} 0 & jK \\ \frac{j}{K} & 0 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{j\omega C} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{j\omega C} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & j\frac{1}{\omega C} \\ j\omega C & 0 \end{bmatrix} \rightarrow K = \frac{1}{\omega C}$$

(derivation of admittance inverter)







$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_o \sin\beta l \\ jY_o \sin\beta l & \cos\beta l \end{bmatrix}, \begin{matrix} Y_o = J \\ l = -\frac{\lambda}{4} \end{matrix} \xrightarrow{\beta l = -\frac{\pi}{2}} \begin{bmatrix} 0 & -\frac{j}{J} \\ -jJ & 0 \end{bmatrix}$$

$$\begin{bmatrix} \cos\frac{\theta}{2} & \frac{j}{Y_o}\sin\frac{\theta}{2} \\ jY_o \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{jB} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\frac{\theta}{2} & \frac{j}{Y_o}\sin\frac{\theta}{2} \\ jY_o \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} = \begin{bmatrix} \cos\theta + \frac{Y_o}{2B}\sin\theta & j(\frac{1}{Y_o}\sin\theta - \frac{1}{B}\cos^2\frac{\theta}{2}) \\ jY_o(\sin\theta + \frac{Y_o}{B}\sin^2\frac{\theta}{2}) & \cos\theta + \frac{Y_o}{2B}\sin\theta \end{bmatrix}$$

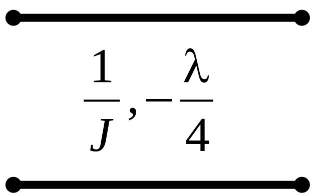
$$\cos\theta + \frac{Y_o}{2B}\sin\theta = 0 \qquad \theta = -\tan^{-1}\frac{2B}{Y_o} \qquad (1)-(2) \rightarrow B = \frac{J}{1-(\frac{J}{Y_o})^2}$$

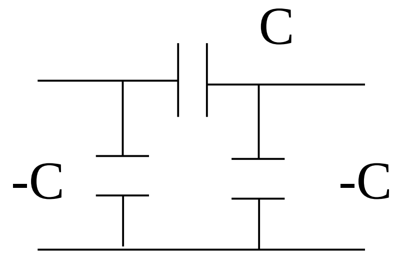
$$\rightarrow j(\frac{1}{Y_o}\sin\theta - \frac{1}{B}\cos^2\frac{\theta}{2}) = -\frac{j}{J} \rightarrow \sin\theta - \frac{Y_o}{B}\cos^2\frac{\theta}{2} = -\frac{Y_o}{J} \ldots(1) \rightarrow$$

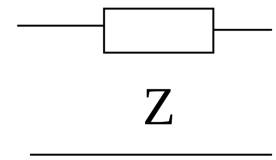
$$jY_o(\sin\theta + \frac{Y_o}{B}\sin^2\frac{\theta}{2}) = -jJ \qquad \sin\theta + \frac{Y_o}{B}\sin^2\frac{\theta}{2} = -\frac{J}{Y_o} \ldots(2)$$

$$\tan|\theta| = \frac{2B}{Y_o} = \frac{2J/Y_o}{1-(\frac{J}{Y_o})^2} = \frac{2\tan\left|\frac{\theta}{2}\right|}{1-\tan^2\left|\frac{\theta}{2}\right|}, J = Y_o \tan\left|\frac{\theta}{2}\right|$$

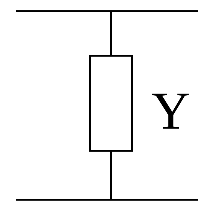
(derivation of admittance inverter)







$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

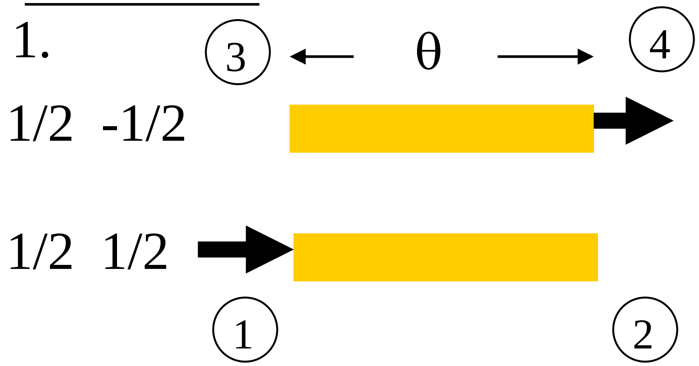
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_o \sin\beta l \\ jY_o \sin\beta l & \cos\beta l \end{bmatrix}, \begin{matrix} Y_o = J \\ l = -\frac{\lambda}{4} \end{matrix} \xrightarrow{\beta l = -\frac{\pi}{2}} \begin{bmatrix} 0 & \frac{1}{jJ} \\ -jJ & 0 \end{bmatrix}$$

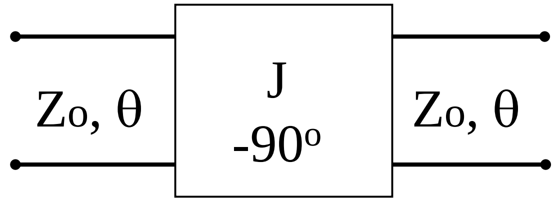
$$\rightarrow J = \omega C$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -j\omega C & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{j\omega C} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j\omega C & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{j\omega C} \\ -j\omega C & 0 \end{bmatrix}$$

Discussion

1.





design equation #1

$$\theta = \frac{\pi}{2}, \begin{cases} Z_{oe} = Z_o[1 + JZ_o + (JZ_o)^2] \\ Z_{oo} = Z_o[1 - JZ_o + (JZ_o)^2] \end{cases}$$

For the left circuit: $V_1 = V_{1e}^+ + V_{1e}^- + V_{1o}^+ + V_{1o}^-, I_1 = \frac{V_{1e}^+}{Z_{oe}} - \frac{V_{1e}^-}{Z_{oe}} + \frac{V_{1o}^+}{Z_{oo}} - \frac{V_{1o}^-}{Z_{oo}}$

$$V_2 = V_{1e}^+ e^{-j\theta} + V_{1e}^- e^{j\theta} + V_{1o}^+ e^{-j\theta} + V_{1o}^- e^{j\theta}, I_2 = \frac{V_{1e}^+}{Z_{oe}} e^{-j\theta} - \frac{V_{1e}^-}{Z_{oe}} e^{j\theta} + \frac{V_{1o}^+}{Z_{oo}} e^{-j\theta} - \frac{V_{1o}^-}{Z_{oo}} e^{j\theta}$$

$$V_3 = V_{1e}^+ + V_{1e}^- - V_{1o}^+ - V_{1o}^-, I_3 = \frac{V_{1e}^+}{Z_{oe}} - \frac{V_{1e}^-}{Z_{oe}} - \frac{V_{1o}^+}{Z_{oo}} + \frac{V_{1o}^-}{Z_{oo}}$$

$$V_4 = V_{1e}^+ e^{-j\theta} + V_{1e}^- e^{j\theta} - V_{1o}^+ e^{-j\theta} - V_{1o}^- e^{j\theta}, I_4 = \frac{V_{1e}^+}{Z_{oe}} e^{-j\theta} - \frac{V_{1e}^-}{Z_{oe}} e^{j\theta} - \frac{V_{1o}^+}{Z_{oo}} e^{-j\theta} + \frac{V_{1o}^-}{Z_{oo}} e^{j\theta}$$

$$\begin{bmatrix} V_1 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{14} \\ Z_{14} & Z_{11} \end{bmatrix} \begin{bmatrix} I_1 \\ I_4 \end{bmatrix}, Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_4=0}, Z_{14} = \left.\frac{V_4}{I_1}\right|_{I_4=0}$$

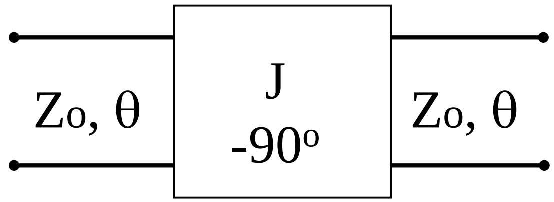
$$V_{1o}^- = V_{1o}^+ e^{-j2\theta}$$

ports 2, 3 open, $I_2 = I_3 = 0$ $\Rightarrow$ $\frac{V_{1e}^+}{Z_{oe}} = \frac{V_{1o}^+}{Z_{oo}}$ $\Rightarrow$ $Z_{11} = -\frac{j}{2}(Z_{oe} + Z_{oo})\cot\theta$

$I_4 = 0$ $\qquad Z_{14} = -\frac{j}{2}(Z_{oe} - Z_{oo})\csc\theta$

$$\frac{V_{1e}^-}{Z_{oe}} = \frac{V_{1o}^+}{Z_{oo}}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{41}} & \frac{Z_{11}Z_{44}}{Z_{41}} - Z_{14} \\ \frac{1}{Z_{41}} & \frac{Z_{44}}{Z_{41}} \end{bmatrix} = \begin{bmatrix} \frac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}\cos\theta & 2j\frac{(Z_{oe}-Z_{oo})^2-(Z_{oe}+Z_{oo})^2\cos\theta}{(Z_{oe}-Z_{oo})\sin\theta} \\ 2j\frac{\sin\theta}{Z_{oe}-Z_{oo}} & \frac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}\cos\theta \end{bmatrix}$$



$$\begin{bmatrix} \cos\theta & jZ_o\sin\theta \\ jY_o\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 0 & -\frac{j}{J} \\ -jJ & 0 \end{bmatrix}\begin{bmatrix} \cos\theta & jZ_o\sin\theta \\ jY_o\sin\theta & \cos\theta \end{bmatrix}$$

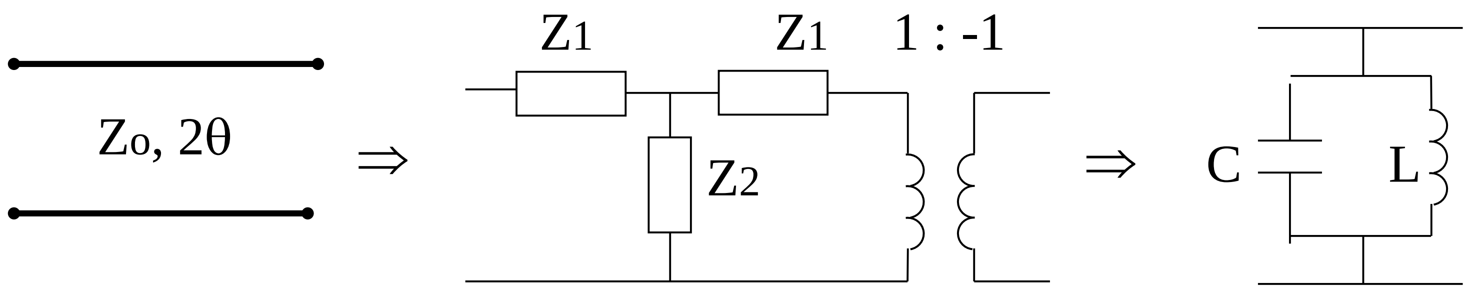
$$=\begin{bmatrix} (Z_oJ+\frac{Y_o}{J})\sin\theta\cos\theta & jZ_o^2J\sin^2\theta-\frac{j}{J}\cos^2\theta \\ -jJ\cos^2\theta+j\frac{Y_o^2}{J}\sin^2\theta & (Z_oJ+\frac{Y_o}{J})\sin\theta\cos\theta \end{bmatrix}$$

$$"=" \Rightarrow (Z_oJ+\frac{Y_o}{J})\sin\theta\cos\theta=\frac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}\cos\theta, -jJ\cos^2\theta+j\frac{Y_o^2}{J}\sin^2\theta=\frac{j2\sin\theta}{Z_{oe}-Z_{oo}}$$

$$\theta=\frac{\pi}{2}\Rightarrow\frac{Z_{oe}+Z_{oo}}{Z_{oe}-Z_{oo}}=Z_oJ+\frac{Y_o}{J}, Z_{oe}-Z_{oo}=2\frac{J}{Y_o^2}=2JZ_o^2$$

$$\Rightarrow \begin{matrix} Z_{oe}=Z_o(1+JZ_o+J^2Z_o^2) \\ Z_{oo}=Z_o(1-JZ_o-J^2Z_o^2) \end{matrix}$$

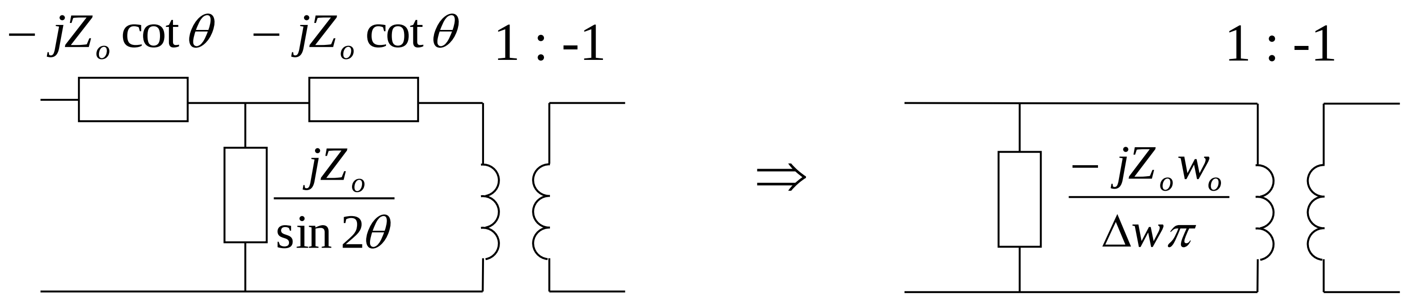
2.



design equation #2 $L = \dfrac{2Z_o}{\omega_o \pi}, C = \dfrac{\pi}{2\omega_o Z_o}$

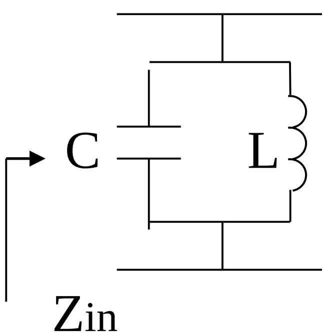
$$\begin{bmatrix} \cos 2\theta & jZ_o \sin 2\theta \\ jY_o \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} 1+\dfrac{Z_1}{Z_2} & 2Z_1 + \dfrac{Z_1^2}{Z_2} \\ \dfrac{1}{Z_2} & 1+\dfrac{Z_1}{Z_2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -\begin{bmatrix} 1+\dfrac{Z_1}{Z_2} & 2Z_1 + \dfrac{Z_1^2}{Z_2} \\ \dfrac{1}{Z_2} & 1+\dfrac{Z_1}{Z_2} \end{bmatrix}$$

$$\Rightarrow \begin{array}{c} Z_2 = \dfrac{jZ_o}{\sin 2\theta} \\ -(1+\dfrac{Z_1}{Z_2}) = \cos 2\theta \end{array} \Rightarrow \begin{array}{c} Z_2 = \dfrac{jZ_o}{\sin 2\theta} \\ Z_1 = -jZ_o \cot\theta \end{array}$$



for $\theta \approx \frac{\pi}{2}$, $\cot\theta \approx 0$

$$2\theta = \beta l = \frac{w}{c} l = \frac{w_o + \Delta w}{w_o}\pi \rightarrow \sin 2\theta = \sin(\pi + \frac{\Delta w}{w_o}\pi) = -\sin\frac{\Delta w}{w_o}\pi \approx -\frac{\Delta w}{w_o}\pi$$



$$Z_{in} = \frac{1}{jwC + \frac{1}{jwL}} = \frac{jwL}{1 - w^2 LC} = \frac{j(w_o + \Delta w)L}{1 - (w_o + \Delta w)^2 LC}$$

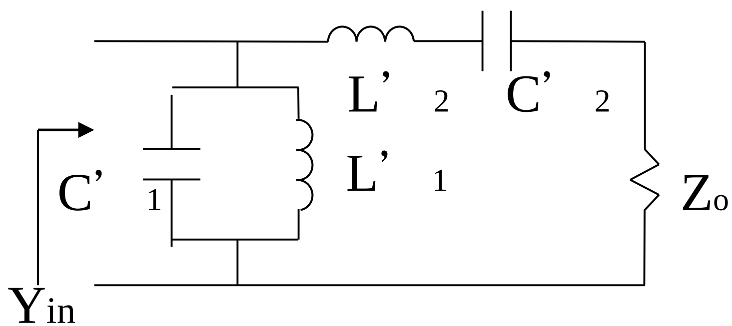
$$= \frac{-j(w_o + \Delta w)L}{(2w_o\Delta w + \Delta w^2)\frac{1}{w_o^2}} \approx \frac{-jw_o(w_o + \Delta w)L}{2\Delta w} \approx \frac{-jw_o^2}{2\Delta w}L$$

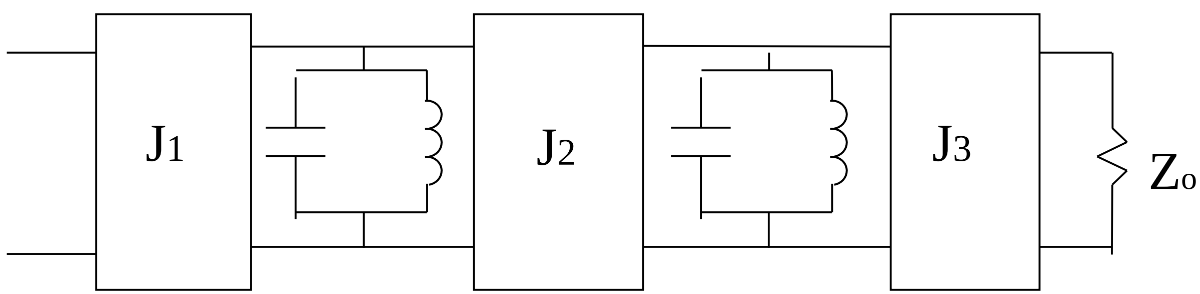
$$"=" \Rightarrow \frac{-jw_o^2}{2\Delta w}L = \frac{-jw_o Z_o}{\Delta w\pi} \Rightarrow L = \frac{2Z_o}{w_o\pi}, C = \frac{1}{Lw_o^2} = \frac{\pi}{2w_o Z_o}$$

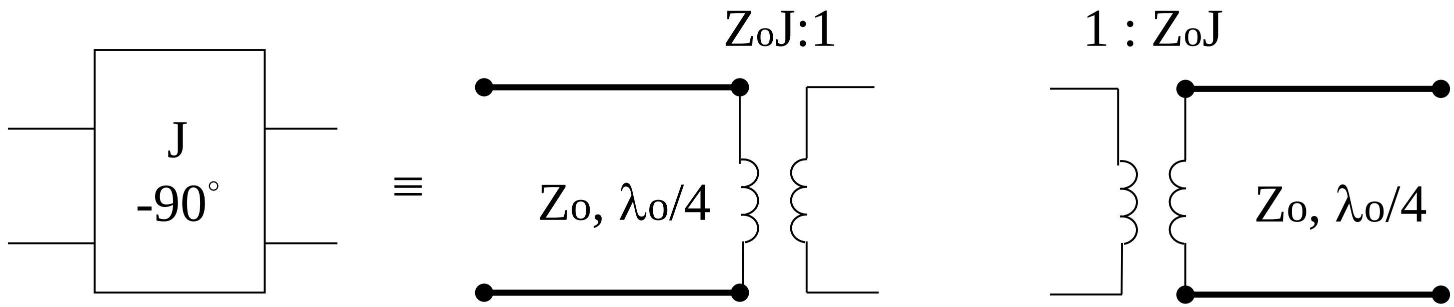
3. derivation of design equation #3 (8.121)

$$J_1 = \frac{1}{Z_o}\sqrt{\frac{\pi\Delta}{2g_1}}, J_i = \frac{\pi\Delta}{2Z_o\sqrt{g_{i-1}g_i}}, J_{N+1} = \frac{1}{Z_o}\sqrt{\frac{\pi\Delta}{2g_N g_{N+1}}}$$

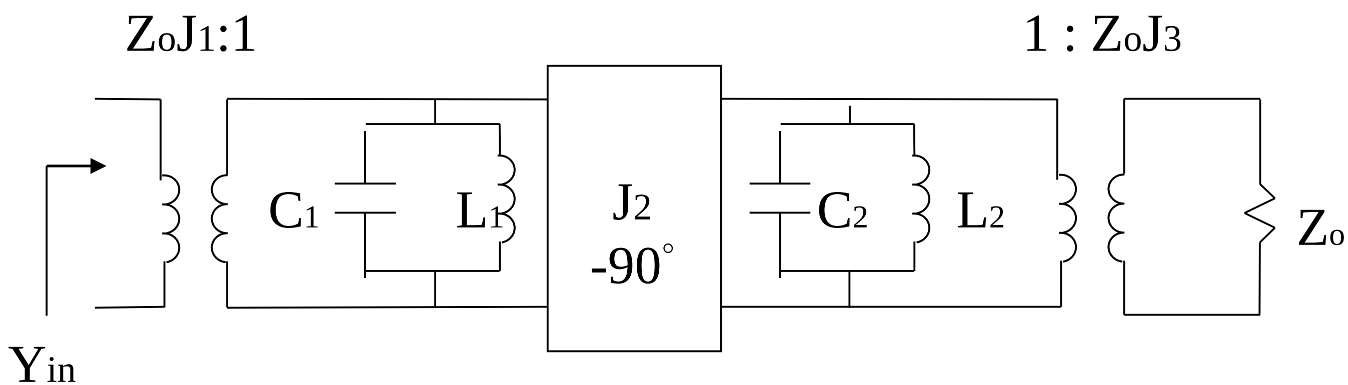
"N=2" (p.434-435, Fig. 8.45 (e), (f))



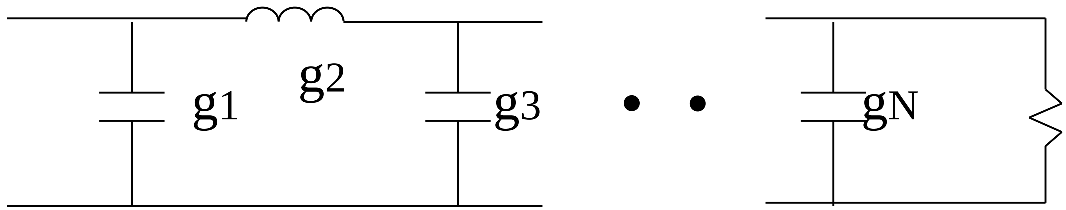


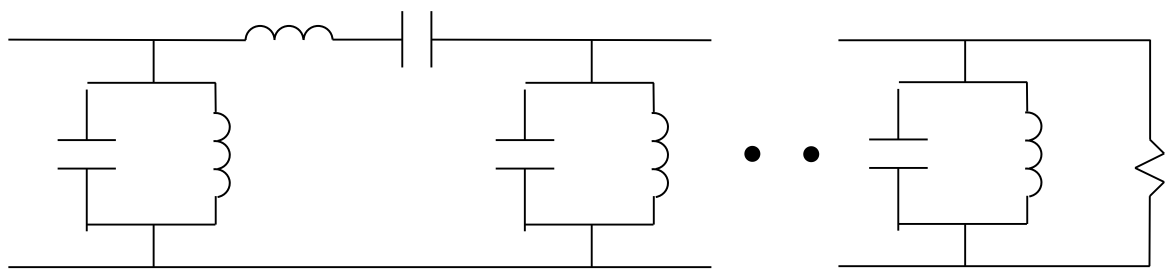


$$\text{left}:\begin{bmatrix} 0 & \dfrac{-j}{J} \\ -jJ & 0 \end{bmatrix} = \text{right}:\begin{bmatrix} 0 & -jZ_o \\ -jY_o & 0 \end{bmatrix}\begin{bmatrix} JZ_o & 0 \\ 0 & \dfrac{1}{JZ_o} \end{bmatrix},\begin{bmatrix} \dfrac{1}{JZ_o} & 0 \\ 0 & JZ_o \end{bmatrix}\begin{bmatrix} 0 & -jZ_o \\ -jY_o & 0 \end{bmatrix}$$

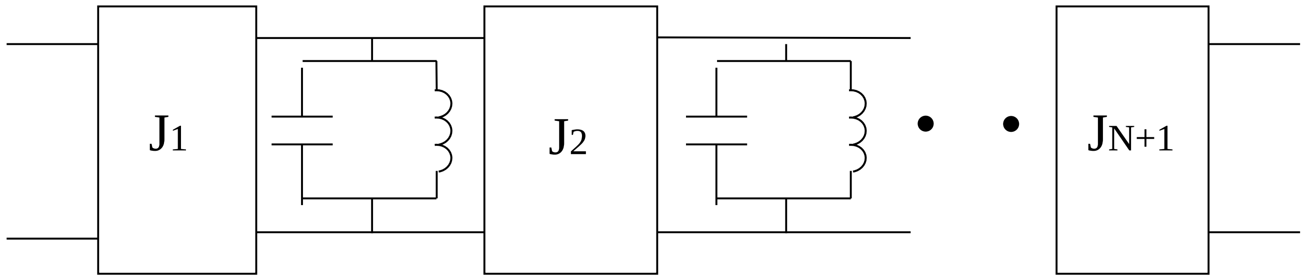


4. Coupled line BPF design

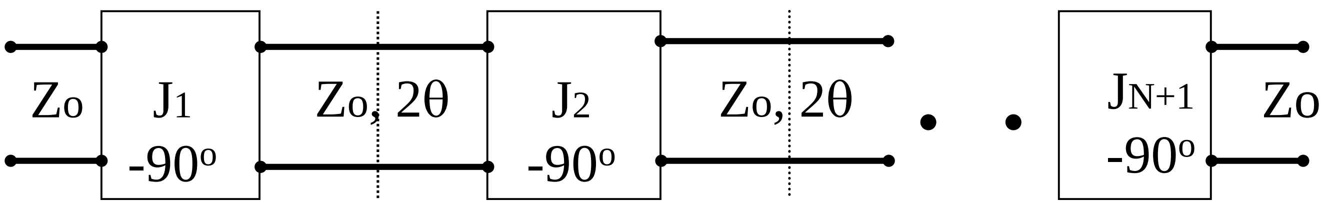




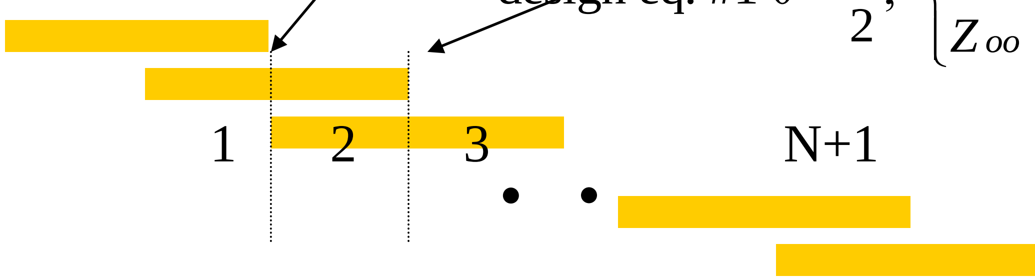
$$L_i = \begin{cases} \dfrac{\Delta Z_o}{\omega_o g_i} & \text{shunt element} \\ \dfrac{g_i Z_o}{\omega_o \Delta} & \text{series element} \end{cases}, \quad C_i = \begin{cases} \dfrac{g_i}{\omega_o \Delta Z_o} & \text{shunt element} \\ \dfrac{\Delta}{\omega_o g_i Z_o} & \text{series element} \end{cases}, \quad \Delta = \frac{\omega_2 - \omega_1}{\omega_o}$$



design equation #3 $J_1 = \frac{1}{Z_o}\sqrt{\frac{\pi\Delta}{2g_1}},\quad J_i = \frac{\pi\Delta}{2Z_o\sqrt{g_{i-1}g_i}},\quad J_{N+1} = \frac{1}{Z_o}\sqrt{\frac{\pi\Delta}{2g_N g_{N+1}}}$



design eq. #1 $\theta = \frac{\pi}{2}$, $\begin{cases} Z_{oe} = Z_o[1 + JZ_o + (JZ_o)^2] \\ Z_{oo} = Z_o[1 - JZ_o + (JZ_o)^2] \end{cases}$



4. Microstrp coupled line BPF design procedure

LPF prototype gi $\rightarrow$ Ji $\rightarrow$ Zoei, Zooi $\rightarrow$ microstrip line Wi, Si

$\rightarrow$ consider discontunuity effects

$$J_1 = \frac{1}{Z_o}\sqrt{\frac{\pi\Delta}{2g_1}}, J_i = \frac{\pi\Delta}{2Z_o\sqrt{g_{i-1}g_i}}, J_{N+1} = \frac{1}{Z_o}\sqrt{\frac{\pi\Delta}{2g_N g_{N+1}}}$$

$$Z_{oe} = Z_o[1 + JZ_o + (JZ_o)^2], Z_{oo} = Z_o[1 - JZ_o + (JZ_o)^2]$$

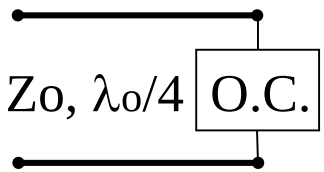
5. Ex.8.7 design a 0.5dB ripple equal-ripple coupled line BPF, N=3, f_o=2GHz, BW=10%, Z_o=50Ω

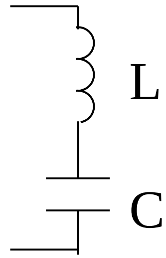
i	g_i	Z_oJ_i	Z_{oe}	Z_{oo}
1	1.5963	0.3137	70.61	39.24
2	1.0967	0.1187	56.64	44.77
3	1.5963	0.1187	56.64	44.77
4	1.0000	0.3137	70.61	39.24

frequency response
(p.436, Fig.8.46)
IL(1.8GHz) ≅ 20dB

8.8 Filters using coupled resonators

• λ/4 stub





design equation

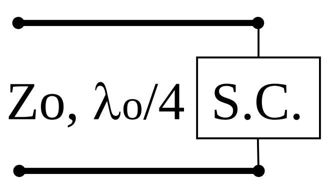
$$L = \frac{\pi Z_o}{4\omega_o}, C = \frac{4}{\omega_o \pi Z_o}$$

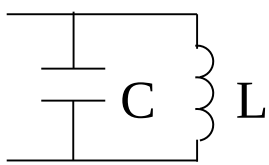
O.C. line: $Z_{in} = \frac{Z_o}{j\tan\theta}, \theta = \beta l = \frac{w}{c}\frac{\lambda_o}{4} = \frac{w}{c}\frac{c}{4f_o} = \frac{\pi}{2}\frac{w}{w_o} = \frac{\pi}{2}\frac{w_o + \Delta w}{w_o}$

$$Z_{in} = \frac{Z_o}{j\tan(\frac{\pi}{2} + \frac{\pi}{2}\frac{\Delta w}{w_o})} = jZ_o \tan\frac{\pi \Delta w}{2w_o} \approx jZ_o \frac{\pi(w - w_o)}{2w_o}$$

series LC: $Z_{in} = jwL + \frac{1}{jwC} = j\sqrt{\frac{L}{C}}(w\sqrt{LC} - \frac{1}{w\sqrt{LC}}) = j\sqrt{\frac{L}{C}}(\frac{w}{w_o} - \frac{w_o}{w})$

$$= j\sqrt{\frac{L}{C}}\frac{(w + w_o)(w - w_o)}{w_o w} \approx j\sqrt{\frac{L}{C}}\frac{2(w - w_o)}{w_o} = j2L(w - w_o) \Rightarrow Z_o = \frac{4w_o L}{\pi}$$



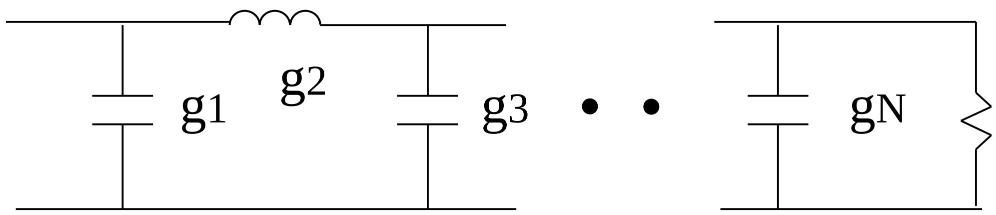


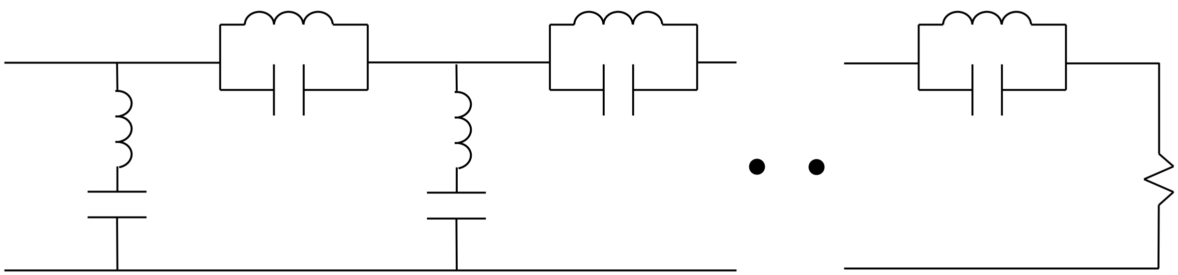
design equation

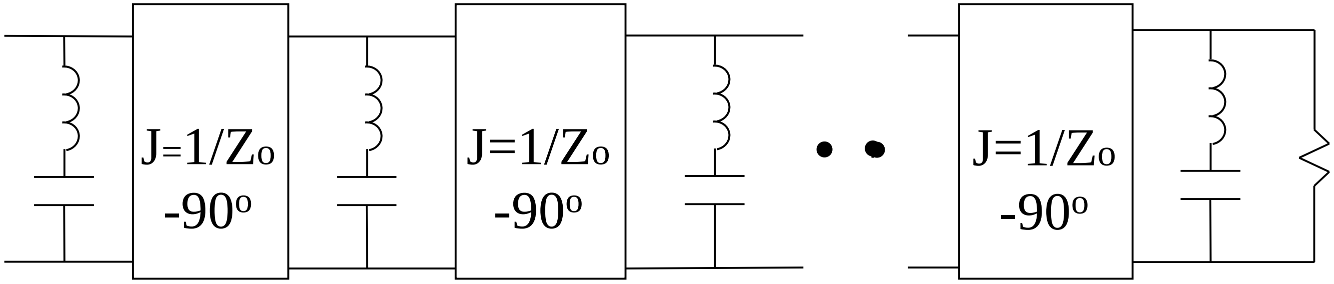
$$C = \frac{\pi Z_o}{4\omega_o}, \ L = \frac{4}{\omega_o \pi Z_o}$$

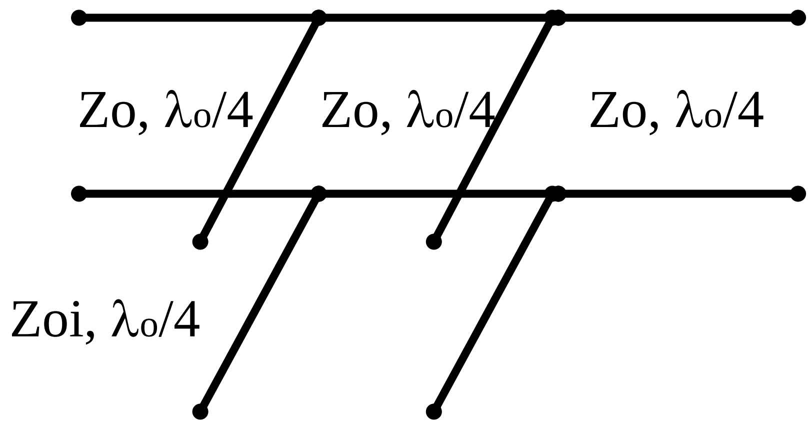
Discussion

1. BSF using $\lambda/4$ open-circuit stub









design equation BSF: $\frac{\lambda}{4}$ open-circuit shunt stub $Z_{oi} = \frac{4Z_o}{\pi g_i \Delta}$

(BPF: $\frac{\lambda}{4}$ short-circuit shunt stub $Z_{oi} = \frac{\pi Z_o \Delta}{4g_i}$)

Derivation for N=2 is given in p.439-440.

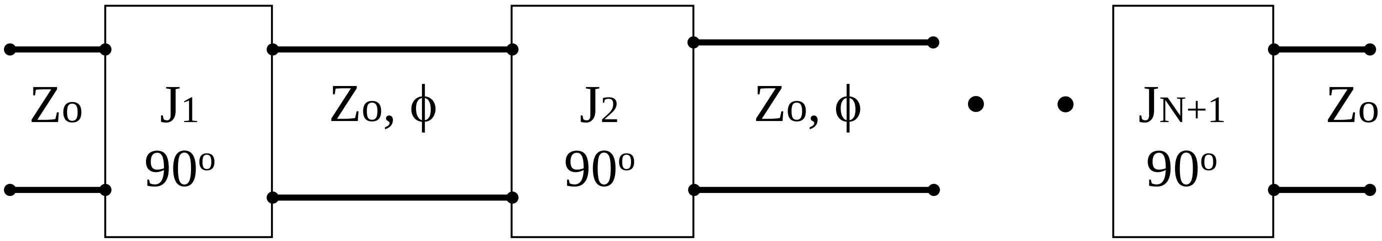
2. Ex.8.8 design a 0.5dB ripple equal-ripple $\lambda/4$ stub BSF, $N=3$, $f_o=2GHz$, $BW=15\%$, $Z_o=50\Omega$

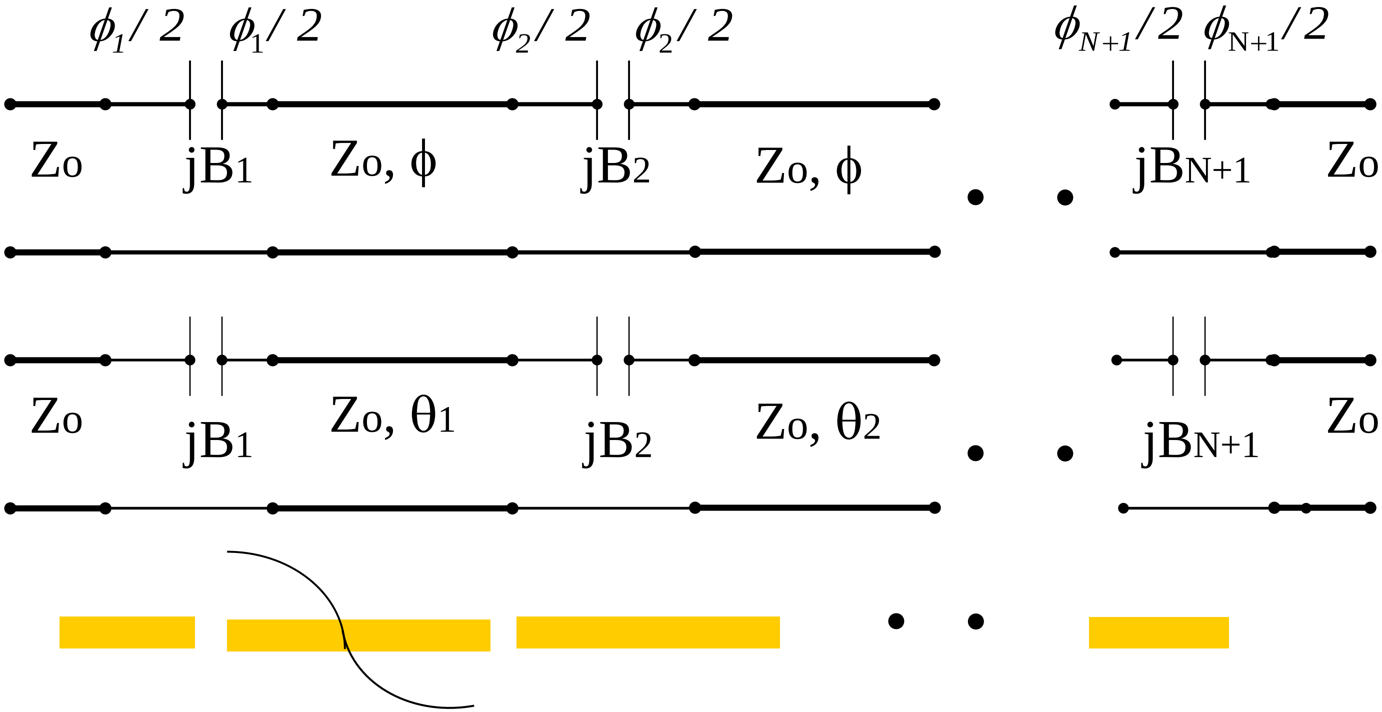
i	g_i	Z_{oi}
1	1.5963	265.9
2	1.0967	387
3	1.5963	265.9

frequency response (p.440, Fig.8.49)

3. Design procedure for microstrip BPF or BSF using $\lambda/4$ stub
LPF prototype g_i $\rightarrow$ $\lambda/4$ open-circuit or short-circui stubs Z_{oi}
$\rightarrow$ microstrip line $\rightarrow$ consider discontunuity effect

4. BPF using capacitively coupled series resonator





design equation

$$\phi_o = \pi\ (l = \frac{\lambda_o}{2}), \theta_i = \pi + \frac{\phi_i}{2} + \frac{\phi_{i+1}}{2} = \pi - (\tan^{-1}\frac{2B_i}{Y_o} + \tan^{-1}\frac{2B_{i+1}}{Y_o})$$

$$B_i = \frac{J_i}{1-(Z_o J_i)^2}$$

5. Design procedure for microstrip BPF using capacitively coupled resonator

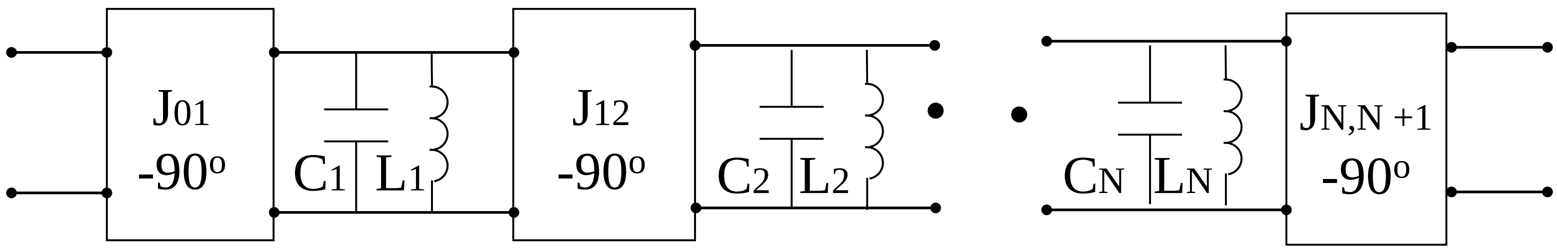
LPF prototype $g_i \rightarrow J_i \rightarrow B_i, \phi_i \rightarrow C_i, \theta_i \rightarrow$ microstrip gap width and line length $\rightarrow$ consider discontunuity effect

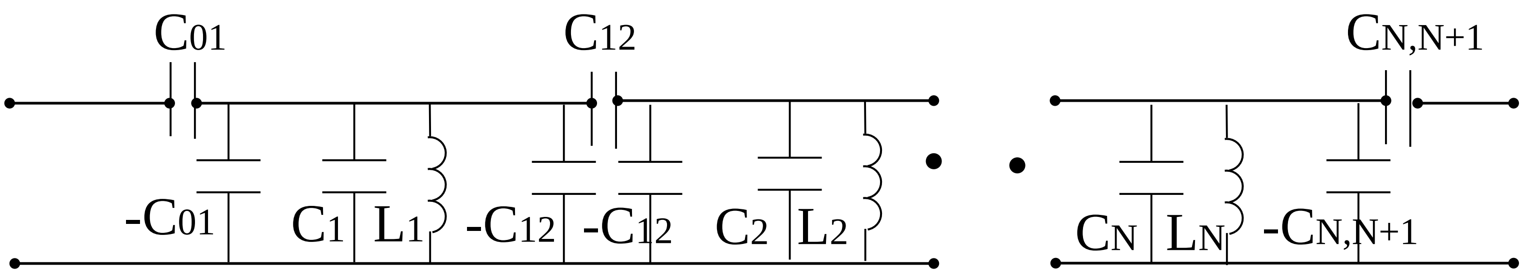
6. Ex.8.9 design a 0.5dB ripple equal-ripple BPF using capacitively coupled resonators, f_o=2GHz, BW=10%, Z_o=50Ω, IL(2.2GHz) > 20dB

i	g_i	Z_oJ_i	$B_i(x10^{-3})$	C_i(pF)	θ_i(deg)
1	1.5963	0.3137	6.96	0.554	155.8
2	1.0967	0.1187	2.41	0.192	166.5
3	1.5963	0.1187	2.41	0.192	166.5
4	1.0000	0.3137	6.96	0.554	

frequency response (p.443, Fig.8.51)

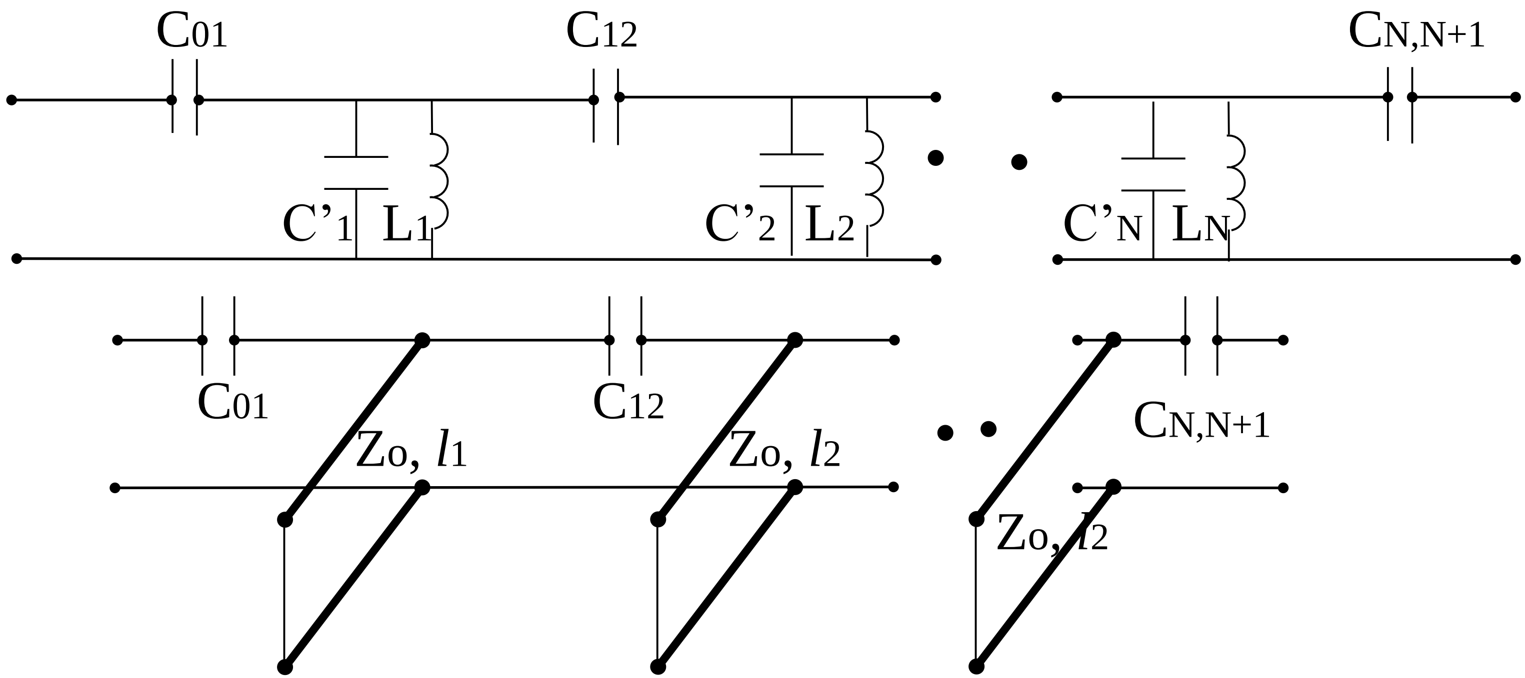
7. BPF using capacitively coupled shunt stub resonator





design equation $$Z_oJ_{01}=\sqrt{\frac{\pi\Delta}{4g_1}},Z_oJ_{n,n+1}=\frac{\pi\Delta}{4\sqrt{g_ng_{n+1}}},Z_oJ_{N,N+1}=\sqrt{\frac{\pi\Delta}{4g_Ng_{N+1}}}$$

$$C_{01}=\frac{J_{01}}{w_0\sqrt{1-(Z_0J_{01})^2}},C_{n,n+1}=\frac{J_{n,n+1}}{w_0},C_{N,N+1}=\frac{J_{N,N+1}}{w_0\sqrt{1-(Z_0J_{N,N+1})^2}}$$

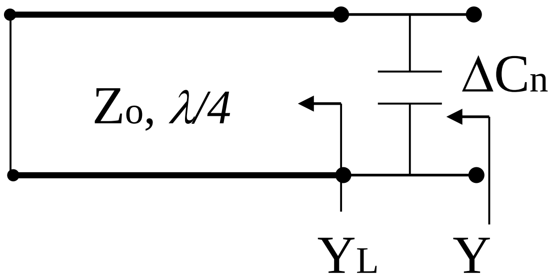


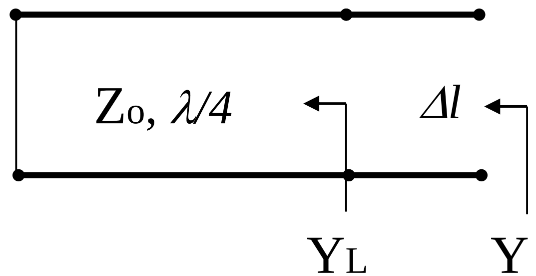
design equation

$$C_n^{'} = C_n - C_{n-1,n} - C_{n,n+1} = C_n + \Delta C_n,\ \Delta C_n = -C_{n-1,n} - C_{n,n+1}$$

$$l_n = \frac{\lambda}{4} + \frac{Z_o w_o \Delta C_n}{2\pi}\lambda$$

(derivation of the change of stub length due to a shunt capacitor)





$$Y = Y_L + jw_o \triangle C_n$$

$$Y = \frac{1}{Z} = \frac{1}{Z_o \dfrac{Z_L + jZ_o \tan \beta\Delta l}{Z_o + jZ_L \tan \beta\Delta l}} = \frac{1}{Z_o} \frac{Z_o + jZ_L \tan \beta\Delta l}{Z_L + jZ_o \tan \beta\Delta l}$$

$$= \frac{1}{Z_o} \frac{Y_L + j\dfrac{1}{Z_o}\tan \beta\Delta l}{\dfrac{1}{Z_o} + jY_L \tan \beta\Delta l} \overset{\beta\Delta l<<1}{\approx} Y_L + j\frac{\beta\Delta l}{Z_o}$$

$$jw_o\Delta C_n = j\frac{\beta_o\Delta l}{Z_o} = j\frac{2\pi\Delta l}{Z_o\lambda} \rightarrow \Delta l = \frac{Z_o w_o \Delta C_n}{2\pi}\lambda$$

8. Design procedure for BPF using capacitively coupled shunt stub resonator

LPF prototype g_i → $J_{i-1,i}$ → C_i → ΔC_i → Δl_i → resonator length → consider discontunuity effect

9. Ex.8.10 design a 0.5dB ripple 3rd order equal-ripple BPF using capacitively coupled shunt resonators, f_o=2GHz, BW=10%, Z_o=50Ω,

i	g_i	$Z_oJ_{i-1,i}$	$C_{i-1,i}$(pF)	ΔC_i(pF)	$\Delta l_i(\lambda)$	l
1	1.5963	0.2218	0.2896	-0.3652	-0.04565	73.6°
2	1.0967	0.0594	0.0756	-0.1512	-0.0189	83.2°
3	1.5963	0.0594	0.0756	-0.3652	-0.04565	73.6°
4	1.0000	0.2218	0.2896			

frequency response (p.447, Fig.8.54)

ADS examples: Ch8_prj