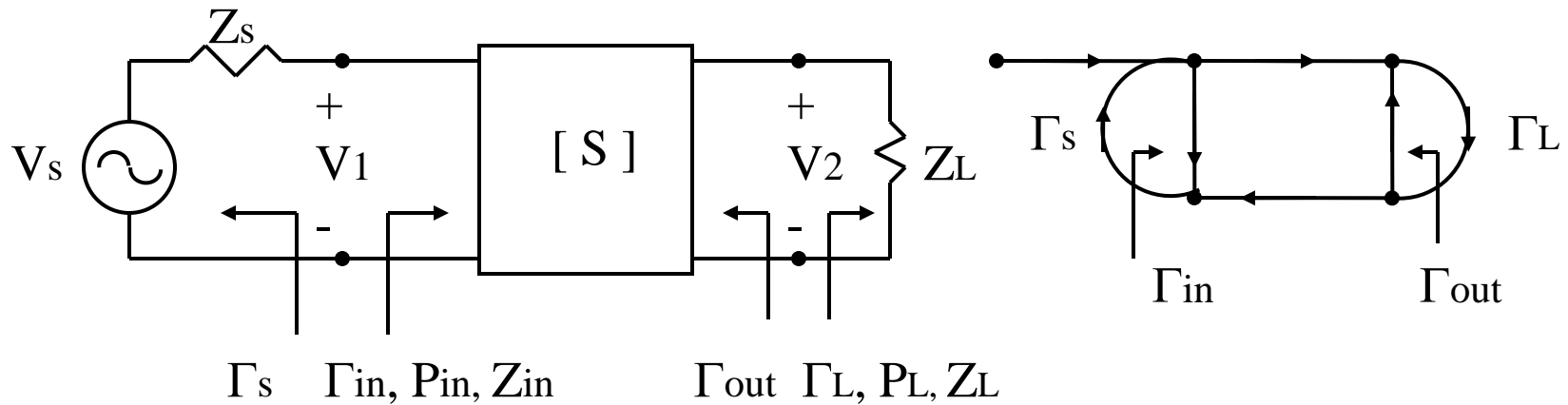


Chapter 11 Microwave Amplifiers Design

- 11.1 Two-port power gains
power gains G , G_T , G_A
- 11.2 Stability
input and output stability circles, stability criterion
- 11.3 Single-stage transistor amplifier design
conjugate match, constant gain circle, noise parameters, constant noise figure circle, LNA (low noise amplifier)
- 11.4 Broadband transistor amplifier design
balanced amplifier, distributed amplifier
- 11.5 Power amplifier
nonlinear operation

11.1 Two-port power gains



$$\text{power gain } G \equiv \frac{P_L}{P_{in}}(S, \Gamma_L)$$

$$\text{available power gain } G_A \equiv \frac{P_{avn}}{P_{avs}}(S, \Gamma_S)$$

$$\text{transducer power gain } G_T \equiv \frac{P_L}{P_{avs}}(S, \Gamma_S, \Gamma_L)$$

$$P_{in}(\Gamma_{in}), P_{avs}(\Gamma_S) = P_{in} \Big|_{\Gamma_{in} = \Gamma_S^*}, P_L(\Gamma_L), P_{avn}(\Gamma_{out}) = P_L \Big|_{\Gamma_L = \Gamma_{out}^*}$$

Discussion

1.

$$V_1 = V_s \frac{Z_{in}}{Z_s + Z_{in}} = V_1^+ + V_1^- = V_1^+ (1 + \Gamma_{in}),$$

$$\Gamma_{in} = \frac{Z_{in} - Z_o}{Z_{in} + Z_o}, \Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o}$$

$$\rightarrow V_1^+ = \frac{V_s}{1 + \Gamma_{in}} \frac{Z_{in}}{Z_s + Z_{in}} = \frac{V_s}{2} \frac{1 - \Gamma_s}{1 - \Gamma_s \Gamma_{in}}$$

$$P_{in} = P_s (1 - |\Gamma_{in}|^2) = \frac{1}{2} \frac{|V_1^+|^2}{Z_o} (1 - |\Gamma_{in}|^2) = \frac{|V_s|^2}{8Z_o} \frac{|1 - \Gamma_s|^2}{|1 - \Gamma_s \Gamma_{in}|^2} (1 - |\Gamma_{in}|^2)$$

2. $V_2^- = S_{21} V_1^+ + S_{22} V_2^+, V_2^+ = \Gamma_L V_2^-, V_1^+ = \frac{V_s}{2} \frac{1 - \Gamma_s}{1 - \Gamma_s \Gamma_{in}}$

$$\rightarrow V_2^- = \frac{V_s}{2} \frac{S_{21} (1 - \Gamma_s)}{(1 - S_{22} \Gamma_L)(1 - \Gamma_s \Gamma_{in})}$$

$$P_L = P_{out} (1 - |\Gamma_L|^2) = \frac{1}{2} \frac{|V_2^-|^2}{Z_o} (1 - |\Gamma_L|^2) = \frac{|V_s|^2}{8Z_o} \frac{|S_{21}|^2 |1 - \Gamma_s|^2}{|1 - S_{22} \Gamma_L|^2 |1 - \Gamma_s \Gamma_{in}|^2} (1 - |\Gamma_L|^2)$$

$$3. P_{avs} = P_{in} \Big|_{\Gamma_{in}=\Gamma_s^*} = \frac{|V_s|^2}{8Z_o} \frac{|1-\Gamma_s|^2}{1-|\Gamma_s|^2}$$

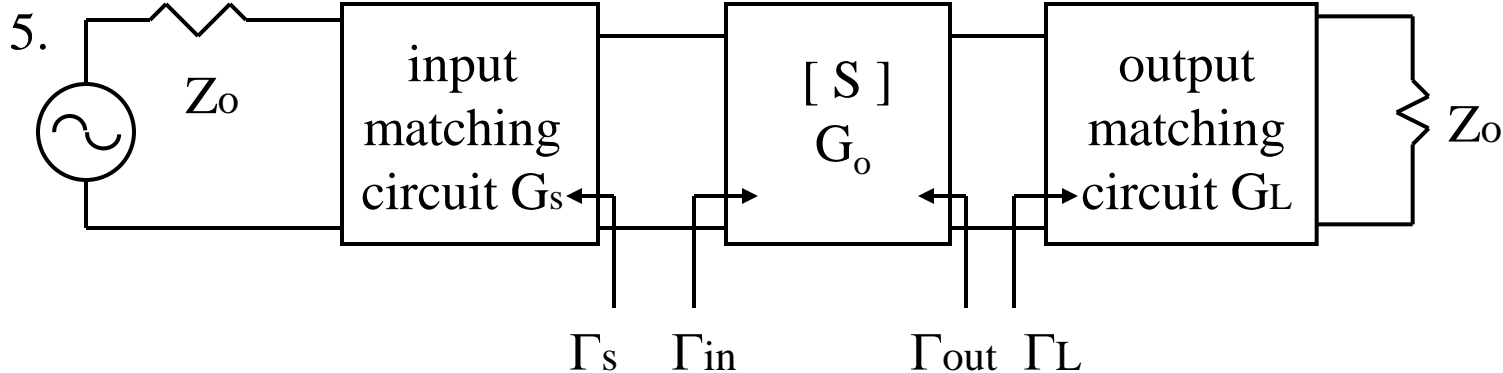
$$P_{avn} = P_L \Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_s|^2}{8Z_o} \frac{|S_{21}|^2 |1-\Gamma_s|^2 (1-|\Gamma_{out}|^2)}{|1-S_{22}\Gamma_{out}^*|^2 |1-\Gamma_s\Gamma_{in}|^2}, \Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

$$\rightarrow P_{avn} = \frac{|V_s|^2}{8Z_o} \frac{|S_{21}|^2 |1-\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2 (1-|\Gamma_{out}|^2)}$$

$$4. G(S, \Gamma_L) \equiv \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2) |1-S_{22}\Gamma_L|^2}$$

$$G_A(S, \Gamma_s) \equiv \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2 (1-|\Gamma_s|^2)}{(1-|\Gamma_{out}|^2) |1-S_{11}\Gamma_s|^2}$$

$$G_T(S, \Gamma_s, \Gamma_L) \equiv \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2 (1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_s\Gamma_{in}|^2 |1-S_{22}\Gamma_L|^2} (= |S_{21}|^2, \text{ if } \Gamma_s = \Gamma_L = 0)$$



$$G_T = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_s \Gamma_{in}|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \Gamma_L|^2} = G_s G_o G_L$$

$$\Gamma_{in} = \Gamma_s^*, \Gamma_{out} = \Gamma_L^* \rightarrow G_{T \max} = \frac{1}{1 - |\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \Gamma_L|^2}$$

$$S_{12} = 0, \text{ unilateral transducer gain } G_{TU} = \frac{1 - |\Gamma_s|^2}{|1 - S_{11} \Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \Gamma_L|^2}$$

$$\Gamma_s = S_{11}^*, \Gamma_L = S_{22}^* \rightarrow G_{TU \max} = \frac{1}{1 - |S_{11}|^2} |S_{21}|^2 \frac{1}{1 - |S_{22}|^2}$$

6. Ex.11.1 $S_{11} = 0.45 \angle 150^\circ$, $S_{12} = 0.01 \angle -10^\circ$, $S_{21} = 2.05 \angle 10^\circ$, $S_{22} = 0.4 \angle -150^\circ$
 $Z_s = 20\Omega$, $Z_L = 30\Omega$

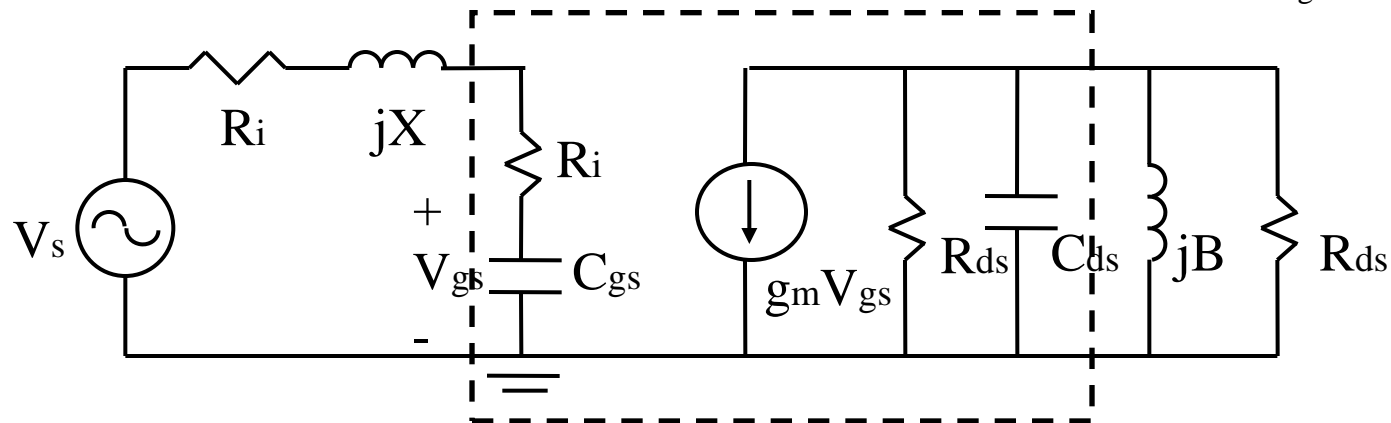
$$\Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o} = -0.429, \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = -0.25$$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.455 \angle 150^\circ$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} = 0.408 \angle -151^\circ$$

$$G = 5.94, G_A = 5.85, G_T = 5.49$$

7. conjugate match using FET equivalent circuit ($S_{12}=0$, or $C_{gd}=0$)



$$Z_{in} = Z_S^* \rightarrow X = \frac{1}{\omega C_{gs}}, Z_{out} = Z_L^* \rightarrow B = -\omega C_{ds}$$

$$V_{gs} = \frac{V_s}{2R_i} \frac{1}{j\omega C_{gs}}$$

$$G_{TU} = \frac{P_L}{P_{avs}} = \frac{\frac{1}{2} \left(\frac{1}{2} |g_m V_{gs}| \right)^2 R_{ds}}{\frac{1}{2} \left(\frac{1}{2} V_s \right)^2 / R_i} = \frac{g_m^2 R_{ds}}{4\omega^2 R_i C_{gs}^2} = \frac{R_{ds}}{4R_i} \left(\frac{f_T}{f} \right)^2, f_T = \frac{g_m}{2\pi C_{gs}}$$

11.2 Stability (S, f)

unconditional stable $\forall Z_s, Z_L \rightarrow |\Gamma_{in}| < 1, |\Gamma_{out}| < 1$

conditional stable $\exists Z_s, Z_L \rightarrow |\Gamma_{in}| < 1, |\Gamma_{out}| < 1$

Discussion

1. $S_{12} = 0, |\Gamma_{in}| < 1, |\Gamma_{out}| < 1 \rightarrow |S_{11}| < 1, |S_{22}| < 1$

2. $|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = 1 \rightarrow$ output stability circle $|\Gamma_L - C_L| = R_L$

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}, R_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$$

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| = 1 \rightarrow$$
 input stability circle $|\Gamma_s - C_s| = R_s$

$$C_s = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}, R_s = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|$$

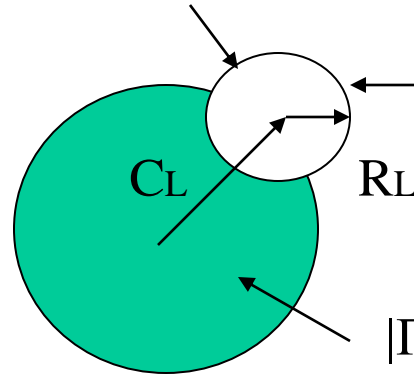
(derivation in p.543)

3. conditional stable

$$|S_{11}| < 1$$

Γ_L -plane

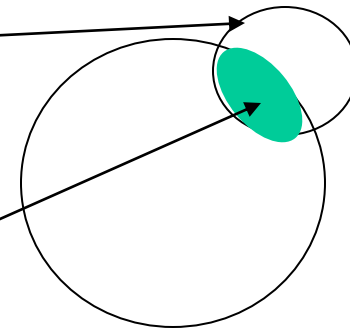
$$|\Gamma_{in}| = 1$$



$$|S_{11}| > 1$$

Γ_L -plane

output
stability
circle

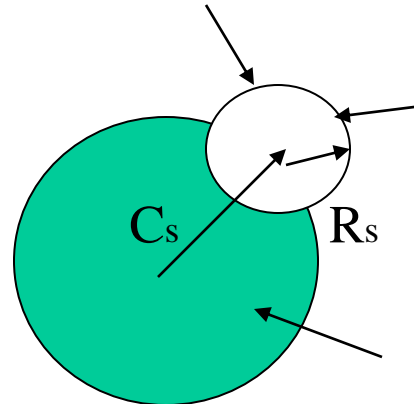


$$|\Gamma_{in}| < 1$$

$$|S_{22}| < 1$$

Γ_s -plane

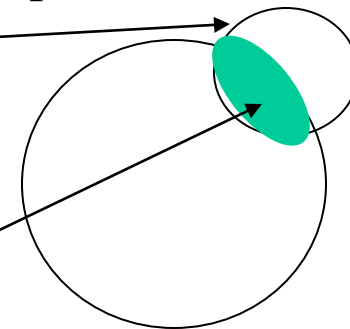
$$|\Gamma_{out}| = 1$$



$$|S_{22}| > 1$$

Γ_s -plane

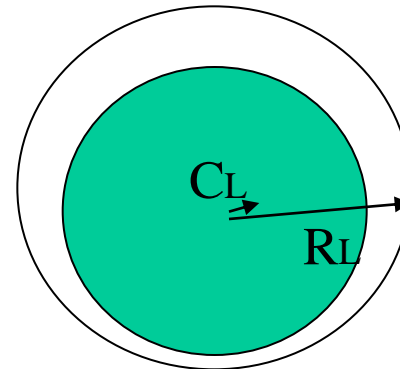
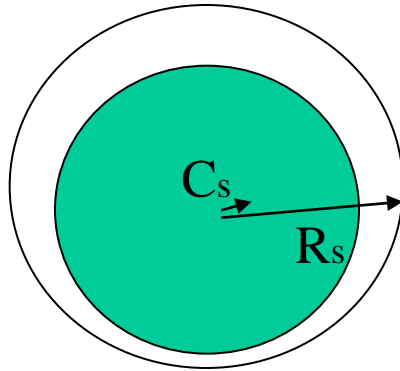
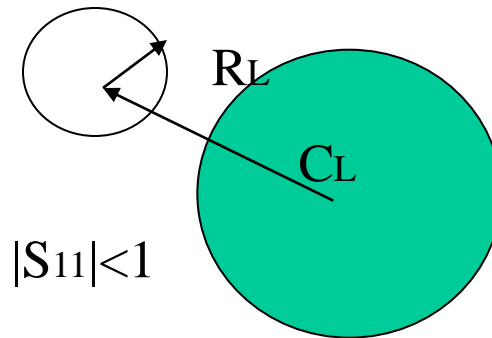
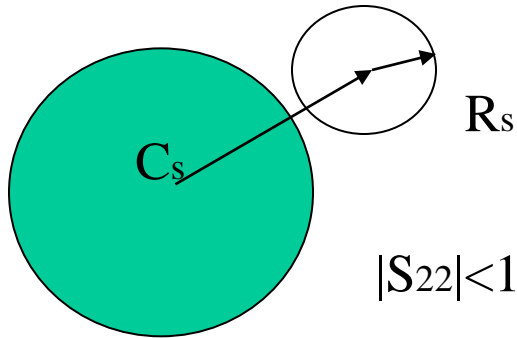
input
stability
circle



$$|\Gamma_{out}| < 1$$

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4. unconditional stable, stability factor K



unconditional stable $|S_{11}| < 1, |S_{22}| < 1, ||C_s| - R_s| > 1, ||C_L| - R_L| > 1$

$$\Leftrightarrow |\Delta| < 1, K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1, \Delta = \det[S] \text{ (derivation in p.546, 547)}$$

5. In practice, one should consider stability over a wide bandwidth for the possible oscillations.

6. Ex.11.2 HP HFET-102 @ 2GHz, $V_{gs}=0V$

$$S_{11} = 0.894 \angle -60.6^\circ, S_{12} = 0.02 \angle 62.4^\circ, S_{21} = 3.122 \angle 123.6^\circ, \\ S_{22} = 0.781 \angle -27.6^\circ$$

$$|\Delta| = 0.696 < 1, K = 0.607 < 1$$

input stability circle $C_s = 1.132 \angle 68^\circ, R_s = 0.199$

output stability circle $C_L = 1.361 \angle 47^\circ, R_L = 0.5$

(p.549, Fig.11.6)

11.3 Single-stage transistor amplifier design

- conjugate match (maximum transducer power gain)

if $|\Delta| < 1$, $K > 1$

→ input and output simultaneously conjugate match $\Gamma_{in} = \Gamma_s^*$, $\Gamma_{out} = \Gamma_L^*$

$$\rightarrow G_T = G_{T_{\max}} = \frac{1}{1 - |\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} = \left| \frac{S_{21}}{S_{12}} \right| (K - \sqrt{K^2 - 1})$$

$$\Gamma_s^* = \Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \rightarrow \Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

$$\Gamma_L^* = \Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \rightarrow \Gamma_s = \frac{B_1 - \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2, B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$C_1 = S_{11} - \Delta S_{22}^*, C_2 = S_{22} - \Delta S_{11}^*$$

(derivation in p. 550)

Discussion

1. linear amplifier design procedure

if $|\Delta| < 1, K > 1$ then uses input and output simultaneously conjugate matches for G_{Tmax}

if $K < 1$ then draws input and output stability circles to see if input and output simultaneously conjugate matches possible, otherwise selects the proper Γ_s and Γ_L for gain or noise figure considerations.

$$2. S_{12} = 0 \rightarrow \Gamma_s = S_{11}^*, \Gamma_L = S_{22}^*$$

$$G_{TUmax} = \frac{1}{1 - |S_{11}|^2} |S_{21}|^2 \frac{1}{1 - |S_{22}|^2} = G_{smax} |S_{21}|^2 G_{Lmax}$$

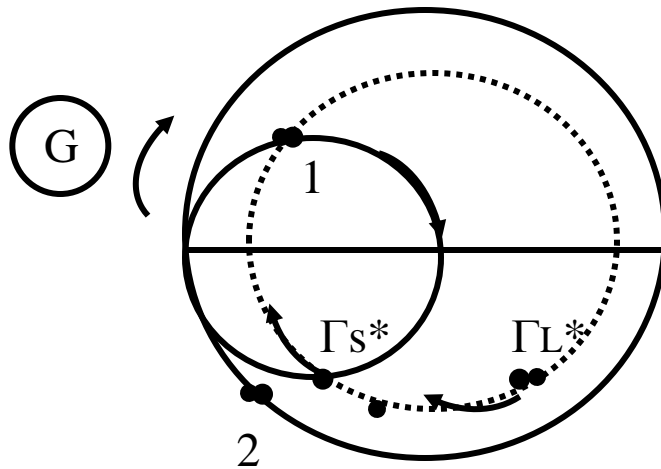
3. Ex.11.3 FET @ 4GHz

$$S_{11} = 0.72 \angle -116^\circ, S_{12} = 0.03 \angle 57^\circ, S_{21} = 2.6 \angle 76^\circ,$$

$$S_{22} = 0.73 \angle -54^\circ$$

$$|\Delta| = 0.488 < 1, K = 1.195 > 1 \rightarrow \Gamma_s = 0.872 \angle 123^\circ, \Gamma_L = 0.876 \angle 61^\circ$$

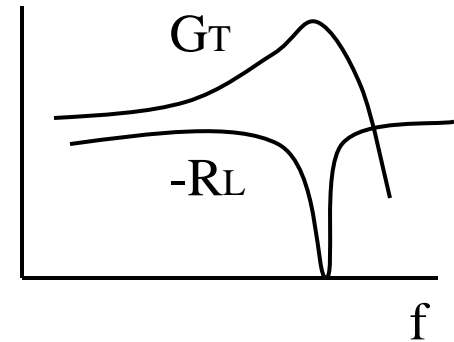
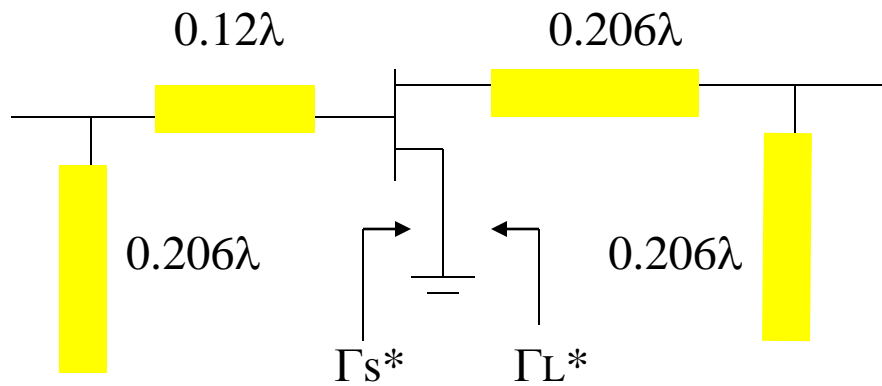
$$G_{Tmax} = 6.2 + 8.3 + 2.22 = 16.7 \text{ dB}$$



$$\Gamma_S^* = 0.872 \angle -123^\circ$$

$$\Gamma_L^* = 0.876 \angle -61^\circ$$

1. $y = 1 - j3.5$
2. $y = j3.5$



frequency response (p.552, Fig.11.7)

- constant gain circle ($S_{12}=0$, unilateral assumption)

$$G_s = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}, G_{s\max} = \frac{1}{1-|S_{11}|^2}, G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}, G_{L\max} = \frac{1}{1-|S_{22}|^2}$$

$$g_s \equiv \frac{G_s}{G_{s\max}} \rightarrow \text{constant gain circle in } \Gamma_s\text{-plane } |\Gamma_s - C_s| = R_s$$

$$g_L \equiv \frac{G_L}{G_{L\max}} \rightarrow \text{constant gain circle in } \Gamma_L\text{-plane } |\Gamma_L - C_L| = R_L$$

$$C_s = \frac{g_s S_{11}^*}{1-(1-g_s)|S_{11}|^2}, R_s = \frac{\sqrt{1-g_s}(1-|S_{11}|^2)}{1-(1-g_s)|S_{11}|^2}$$

$$C_L = \frac{g_L S_{22}^*}{1-(1-g_L)|S_{22}|^2}, R_L = \frac{\sqrt{1-g_L}(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2}$$

(derivation in p. 554)

Discussion

1. $G_s=0\text{dB}$ and $G_L=0\text{dB}$ circles pass through the Smith chart center.

$$G_s = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2} (=0\text{dB}) = 1, G_{s\max} = \frac{1}{1-|S_{11}|^2},$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} (=0\text{dB}) = 1, G_{L\max} = \frac{1}{1-|S_{22}|^2}$$

$$g_s = \frac{G_s}{G_{s\max}} = \frac{1}{G_{s\max}} = 1-|S_{11}|^2 \rightarrow 1-g_s = |S_{11}|^2,$$

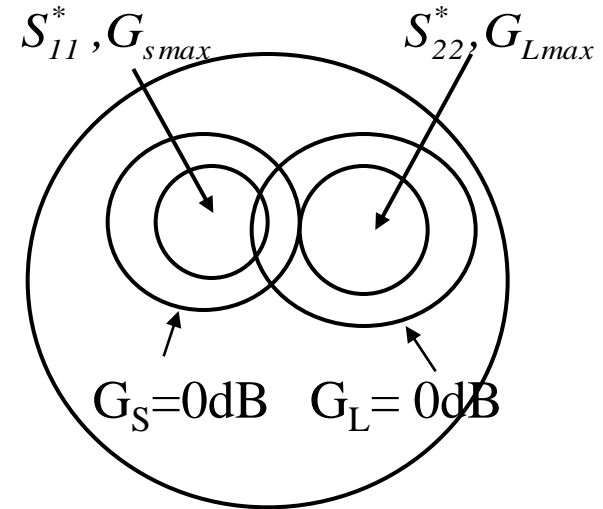
$$g_L = \frac{G_L}{G_{L\max}} = \frac{1}{G_{L\max}} = 1-|S_{22}|^2 \rightarrow 1-g_L = |S_{22}|^2$$

constant gain circles $|\Gamma_s - C_s| = R_s$, $|\Gamma_L - C_L| = R_L$

$$C_s = \frac{g_s S_{11}^*}{1-(1-g_s)|S_{11}|^2} = \frac{(1-|S_{11}|^2)S_{11}^*}{1-|S_{11}|^4} = \frac{S_{11}^*}{1+|S_{11}|^2}, R_s = \frac{\sqrt{1-g_s}(1-|S_{11}|^2)}{1-(1-g_s)|S_{11}|^2} = \frac{|S_{11}|(1-|S_{11}|^2)}{1-|S_{11}|^4} = \frac{|S_{11}|}{1+|S_{11}|^2}$$

$$C_L = \frac{g_L S_{22}^*}{1-(1-g_L)|S_{22}|^2} = \frac{(1-|S_{22}|^2)S_{22}^*}{1-|S_{22}|^4} = \frac{S_{22}^*}{1+|S_{22}|^2}, R_L = \frac{\sqrt{1-g_L}(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2} = \frac{|S_{22}|(1-|S_{22}|^2)}{1-|S_{22}|^4} = \frac{|S_{22}|}{1+|S_{22}|^2}$$

$$\rightarrow |C_s| = R_s, |C_L| = R_L$$



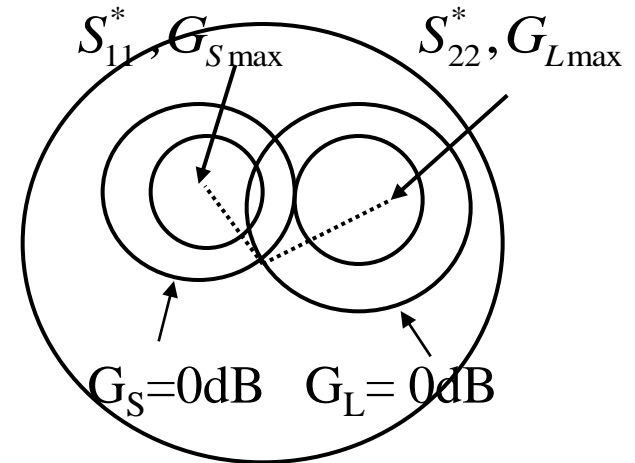
2. Centers of constant gain circles are distributed along the lines from S_{11}^* and S_{22}^* to the Smith chart center, respectively.

$$C_s = \frac{g_s S_{11}^*}{1 - (1 - g_s) |S_{11}|^2}, C_L = \frac{g_L S_{22}^*}{1 - (1 - g_L) |S_{22}|^2}$$

$$g_s = 1, g_L = 1 \rightarrow C_{s,1} = S_{11}^*, C_{L,1} = S_{22}^*$$

$$G_s = 1, G_L = 1 \rightarrow C_{s,2} = \frac{S_{11}^*}{1 + |S_{11}|^2}, C_{L,2} = \frac{S_{22}^*}{1 + |S_{22}|^2}$$

$$\Rightarrow \tan^{-1} \frac{\text{Re}(C_{s,1})}{\text{Im}(C_{s,1})} = \tan^{-1} \frac{\text{Re}(C_{s,2})}{\text{Im}(C_{s,2})}, \tan^{-1} \frac{\text{Re}(C_{L,1})}{\text{Im}(C_{L,1})} = \tan^{-1} \frac{\text{Re}(C_{L,2})}{\text{Im}(C_{L,2})}$$



$$2. \quad \frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}$$

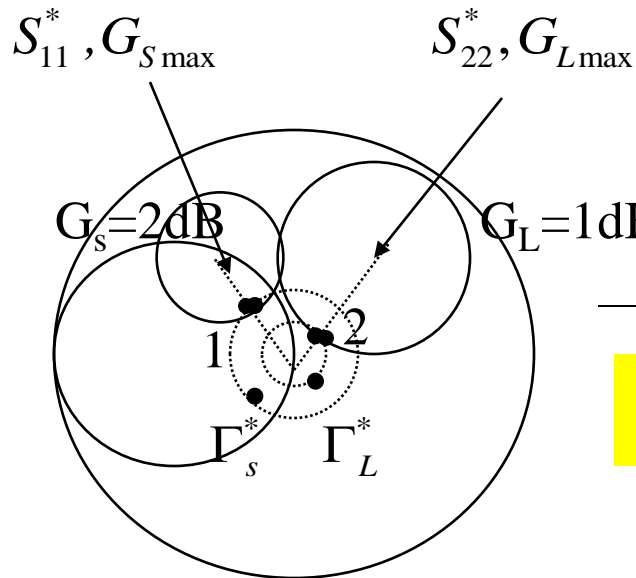
$$U = \frac{|S_{11}| |S_{21}| |S_{12}| |S_{22}|}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)} \quad \text{unilateral figure of merit}$$

3. Ex.11.4 design an amplifier with $G_T=11\text{dB}$ @ 4GHz

$$S_{11} = 0.75 \angle -120^\circ, S_{12} = 0, S_{21} = 2.5 \angle 80^\circ, S_{22} = 0.6 \angle -70^\circ$$

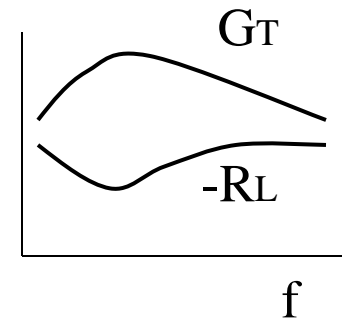
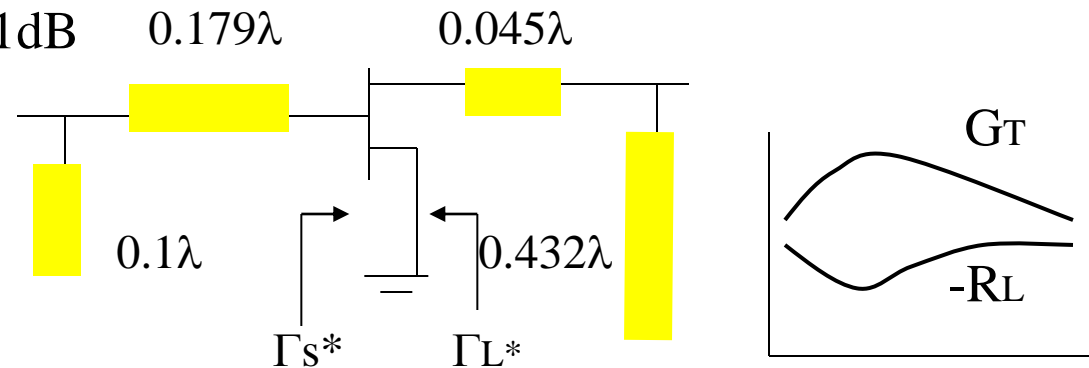
$$G_{TU \max} = 3.6 + 8 + 1.9 = 13.5 \text{ dB}$$

$$\text{choose } G_{TU} = 2 + 8 + 1 = 11 \text{ dB}$$



$$1. \Gamma_s = 0.33 \angle 120^\circ, \Gamma_s^* = 0.33 \angle -120^\circ$$

$$2. \Gamma_L = 0.22 \angle 70^\circ, \Gamma_L^* = 0.22 \angle -70^\circ$$



frequency response (p.556, Fig.11.8)

- constant noise figure circle
for a two-port amplifier

$$F = F_{\min} + \frac{R_N}{G_s} |Y_s - Y_{opt}|^2 = F_{\min} + \frac{4R_N}{Z_o} \frac{|\Gamma_s - \Gamma_{opt}|^2}{(1 - |\Gamma_s|^2) |1 + \Gamma_{opt}|^2}$$

noise parameter: F_{\min}, Y_{opt}, R_N equivalent noise resistance of transistor

$$N \equiv \frac{|\Gamma_s - \Gamma_{opt}|^2}{1 - |\Gamma_s|^2} = \frac{F - F_{\min}}{4R_N / Z_o} |1 + \Gamma_{opt}|^2$$

→ constant noise figure circle $|\Gamma_s - C_F| = R_F$

$$C_F = \frac{\Gamma_{opt}}{N+1}, R_F = \frac{\sqrt{N(N+1 - |\Gamma_{opt}|^2)}}{N+1}$$

(derivation in p. 558, 559)

Discussion

1. Ex.11.5 design a LNA with $F=2\text{dB}$ and max. gain @ 4GHz

$$S_{11} = 0.6\angle -60^\circ, S_{12} = 0.05\angle 26^\circ, S_{21} = 1.9\angle 81^\circ, S_{22} = 0.5\angle -60^\circ$$

$$F_{\min} = 1.6\text{dB}, \Gamma_{opt} = 0.62\angle 100^\circ, R_N = 20\Omega$$

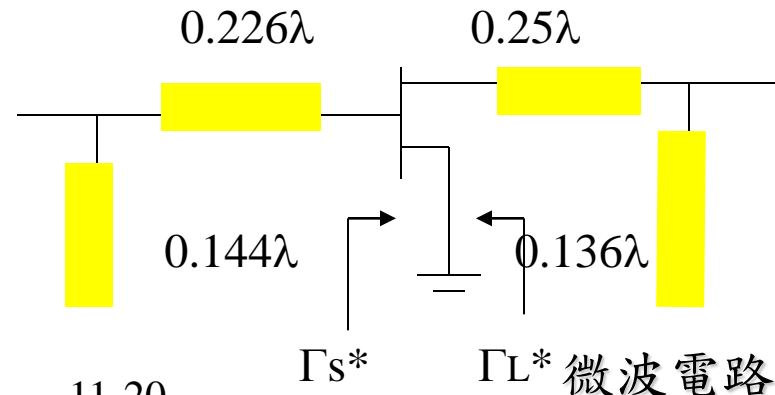
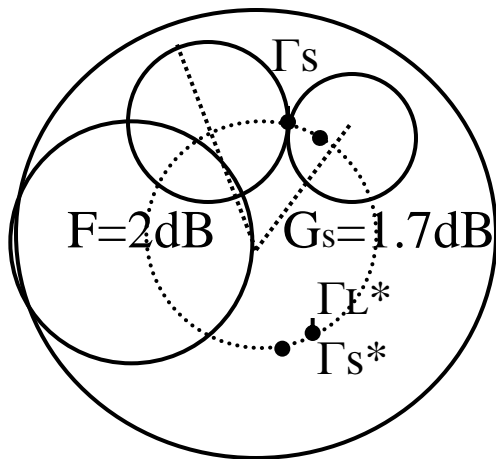
$$U = 0.059$$

$$0.89 = \frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} = 1.13, -0.5\text{dB} < G_T - G_{TU} < 0.53\text{dB}$$

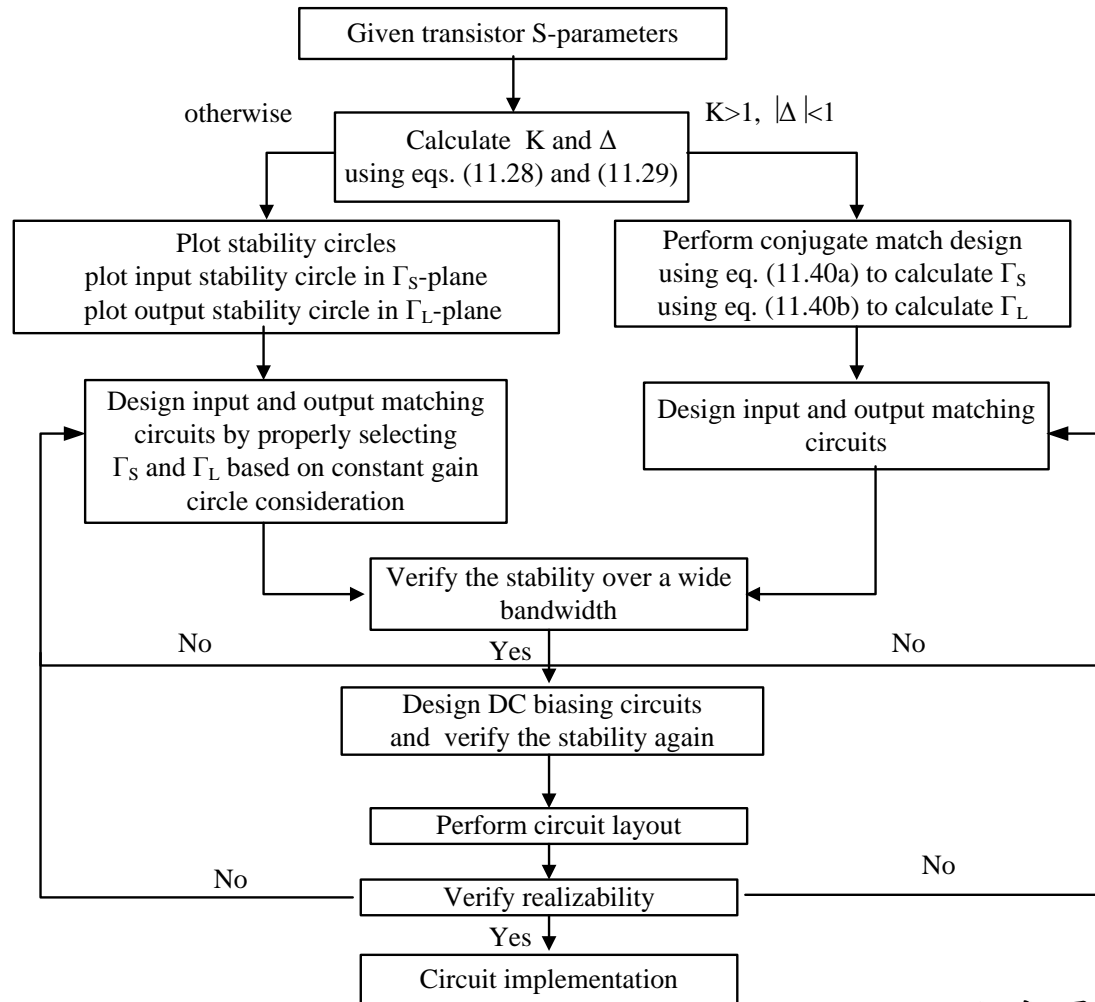
$$F = 2\text{dB} \rightarrow C_F = 0.56\angle 100^\circ, R_F = 0.24$$

$$G_s = 1.7\text{dB} \rightarrow C_s = 0.58\angle 60^\circ, R_s = 0.15 \rightarrow \Gamma_s = 0.53\angle 75^\circ$$

$$\Gamma_L = S_{22}^* = 0.5\angle 60^\circ \rightarrow G_L = \frac{1}{1-|S_{22}|^2} = 1.25\text{dB} \rightarrow G_{TU} = 1.7 + |S_{21}|^2 + 1.25 = 8.53\text{dB}$$

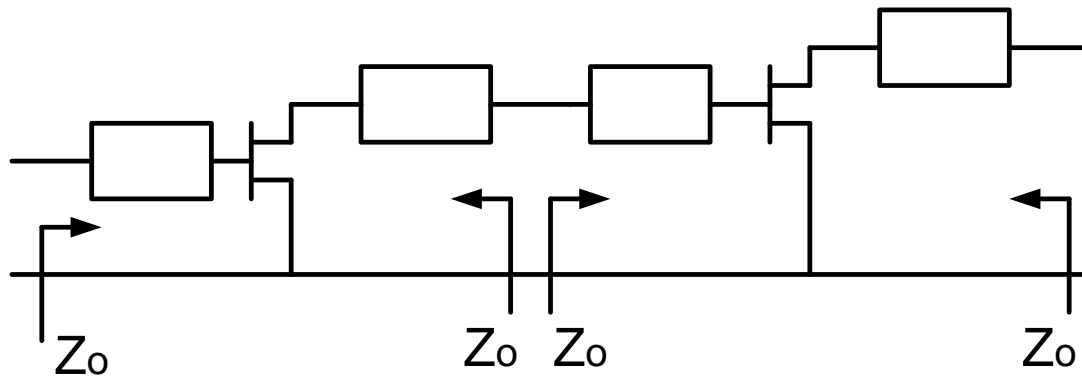


2. Approach for single-stage linear amplifier design

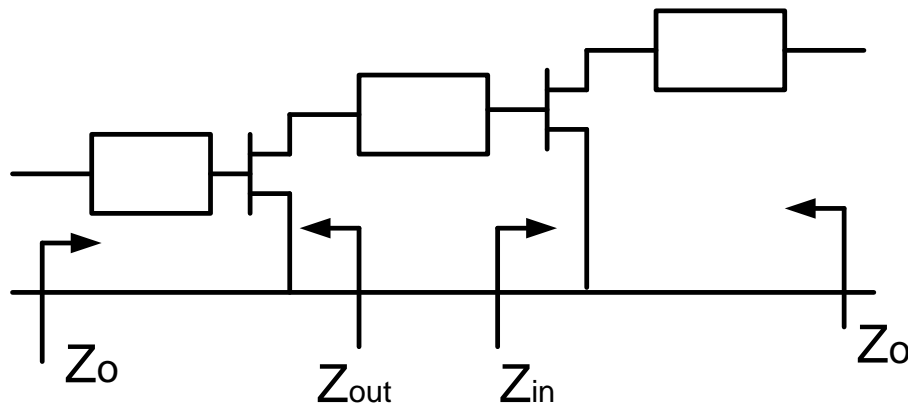


3. two approaches for multi-stage amplifier design

(1)

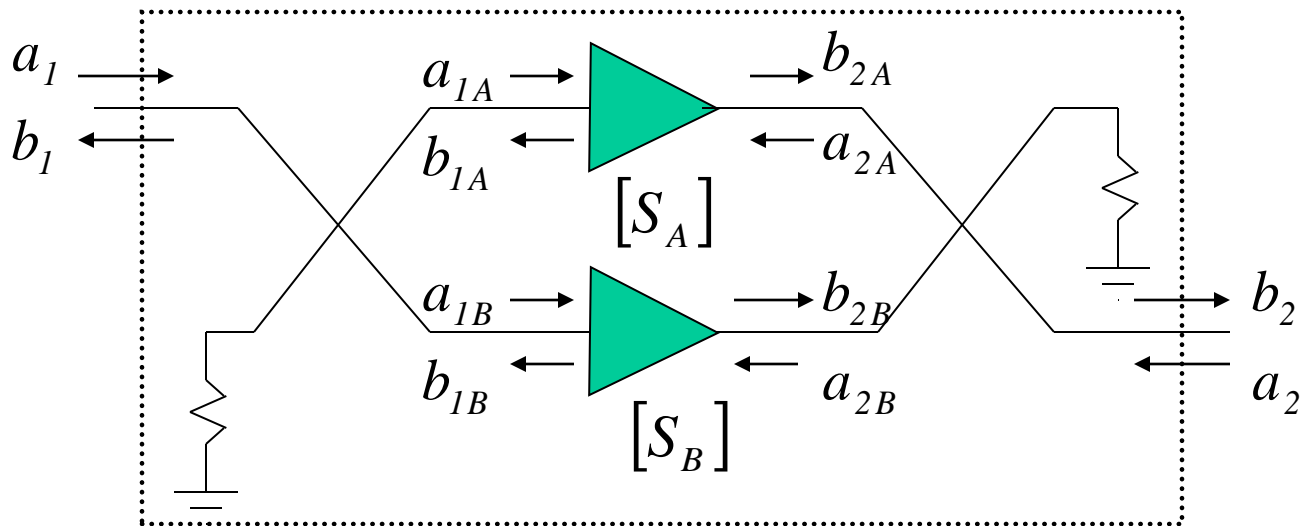


(2)



11.4 Broadband transistor amplifier design

- balanced amplifier



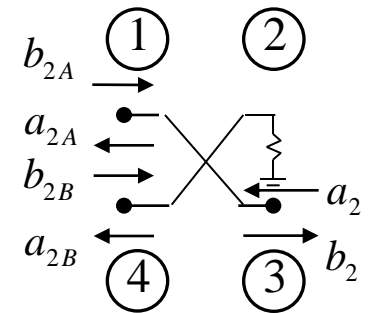
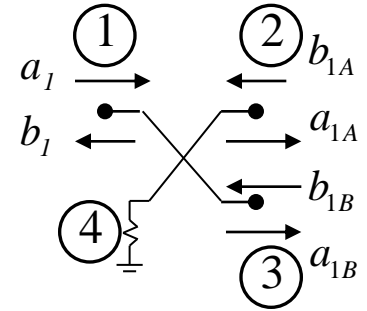
$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} S_{11A} - S_{11B} & -j(S_{12A} + S_{12B}) \\ -j(S_{21A} + S_{21B}) & -(S_{22A} - S_{22B}) \end{bmatrix}$$

Discussion

1. derivation of S-parameters

$$90^\circ \text{ hybrid} \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{-j}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{-j}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{-j}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}, \begin{bmatrix} b_1 \\ a_{1A} \\ a_{1B} \\ - \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{-j}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{-j}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{-j}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ b_{1A} \\ b_{1B} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} b_{1A,B} \\ b_{2A,B} \end{bmatrix} = \begin{bmatrix} S_{11A,B} & S_{12A,B} \\ S_{21A,B} & S_{22A,B} \end{bmatrix} \begin{bmatrix} a_{1A,B} \\ a_{2A,B} \end{bmatrix}, \begin{bmatrix} a_{2A} \\ - \\ b_2 \\ a_{2B} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{-j}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{-j}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{-j}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} b_{2A} \\ 0 \\ a_2 \\ b_{2B} \end{bmatrix}$$



$$\begin{aligned}
b_1 &= \frac{1}{\sqrt{2}} b_{1A} + \frac{-j}{\sqrt{2}} b_{1B} \\
&= \frac{1}{\sqrt{2}} (S_{11A} a_{1A} + S_{12A} a_{2A}) + \frac{-j}{\sqrt{2}} (S_{11B} a_{1B} + S_{12B} a_{2B}) \\
&= \frac{1}{\sqrt{2}} (S_{11A} \frac{1}{\sqrt{2}} a_1 + S_{12A} \frac{-j}{\sqrt{2}} a_2) + \frac{-j}{\sqrt{2}} (S_{11B} \frac{-j}{\sqrt{2}} a_1 + S_{12B} \frac{1}{\sqrt{2}} a_2) \\
&= \frac{1}{2} (S_{11A} - S_{11B}) a_1 + \frac{-j}{2} (S_{12A} + S_{12B}) a_2
\end{aligned}$$

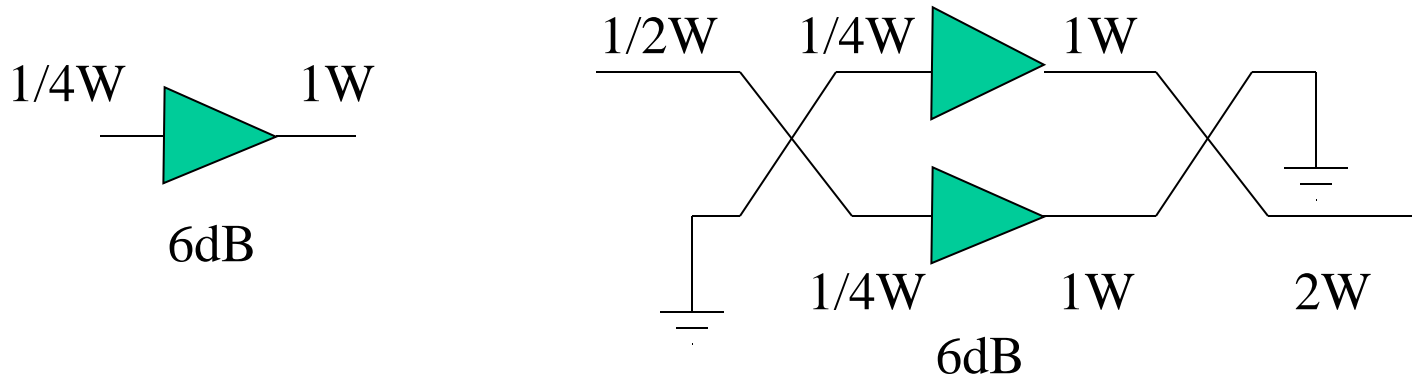
$$\begin{aligned}
b_2 &= \frac{-j}{\sqrt{2}} b_{2A} + \frac{1}{\sqrt{2}} b_{2B} \\
&= \frac{-j}{\sqrt{2}} (S_{21A} a_{1A} + S_{22A} a_{2A}) + \frac{1}{\sqrt{2}} (S_{21B} a_{1B} + S_{22B} a_{2B}) \\
&= \frac{-j}{\sqrt{2}} (S_{21A} \frac{1}{\sqrt{2}} a_1 + S_{22A} \frac{-j}{\sqrt{2}} a_2) + \frac{1}{\sqrt{2}} (S_{21B} \frac{-j}{\sqrt{2}} a_1 + S_{22B} \frac{1}{\sqrt{2}} a_2) \\
&= \frac{-j}{2} (S_{21A} + S_{21B}) a_1 + \frac{-1}{2} (S_{22A} - S_{22B}) a_2
\end{aligned}$$

2. amplifier A=amplifier B, good i/p and o/p match
→ good stability

$$\begin{bmatrix} 0 & -jS_{12A} \\ -jS_{21A} & 0 \end{bmatrix}$$

3. high reliability and less tuning work
4. I/p and o/p matching are improved by the two 90° hybrids, and mismatch reflections are absorbed by two resistors.
5. If one transistors fails, gain losses 6dB.
→ graceful degradation
6. disadvantages: larger size and lower efficiency
7. Bandwidth is limited by the hybrids.

8. power amplifier application



9. Balanced amplifiers can be implemented in a tree structure with a very high power in radar and communication applications.

10. Ex 11.6, two amplifiers of ex.11.4 are implemented as a balanced amplifier to improve its i/p and o/p return loss at 4 GHz. Then, the stub lengths are optimize to give better matching and gain flatness from 3 to 5 GHz bandwidth.

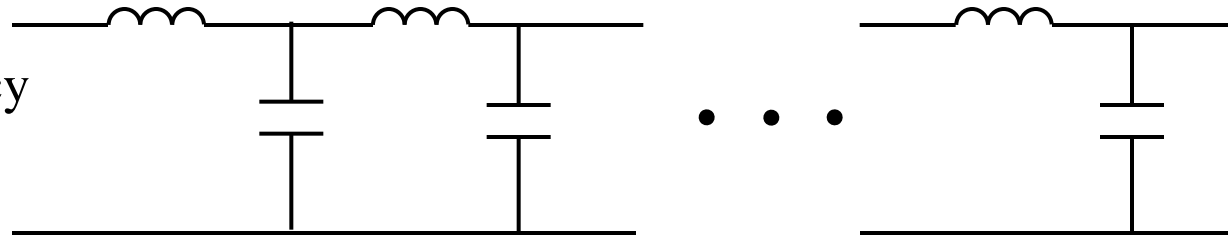
frequency response (p.564, Fig.11.11)

- distributed (traveling wave) amplifier

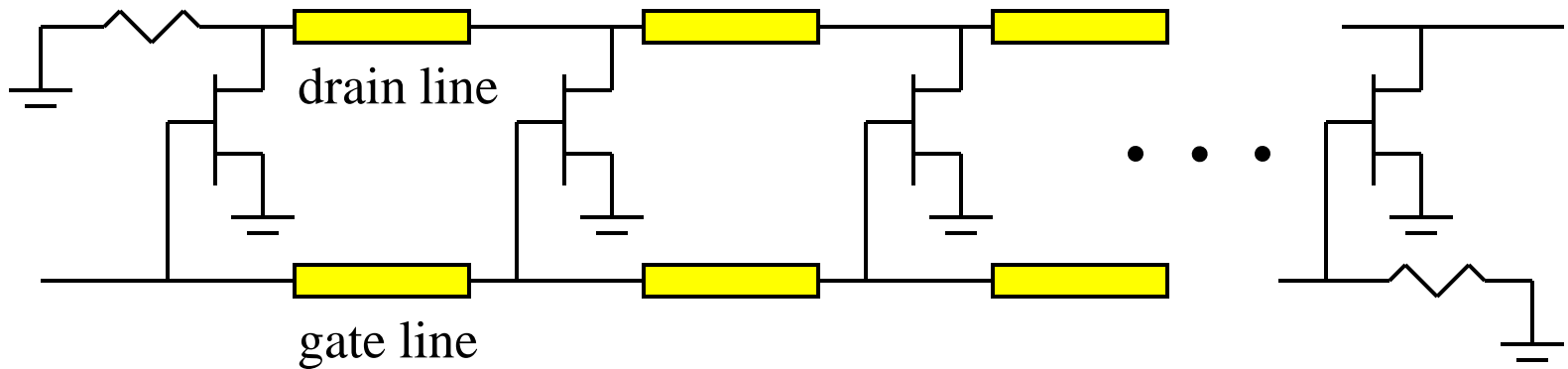
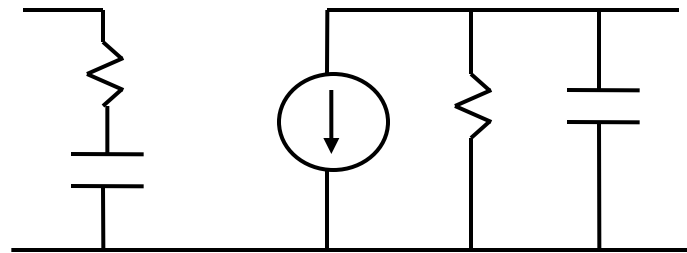
TEM line “extreme wide operation bandwidth”

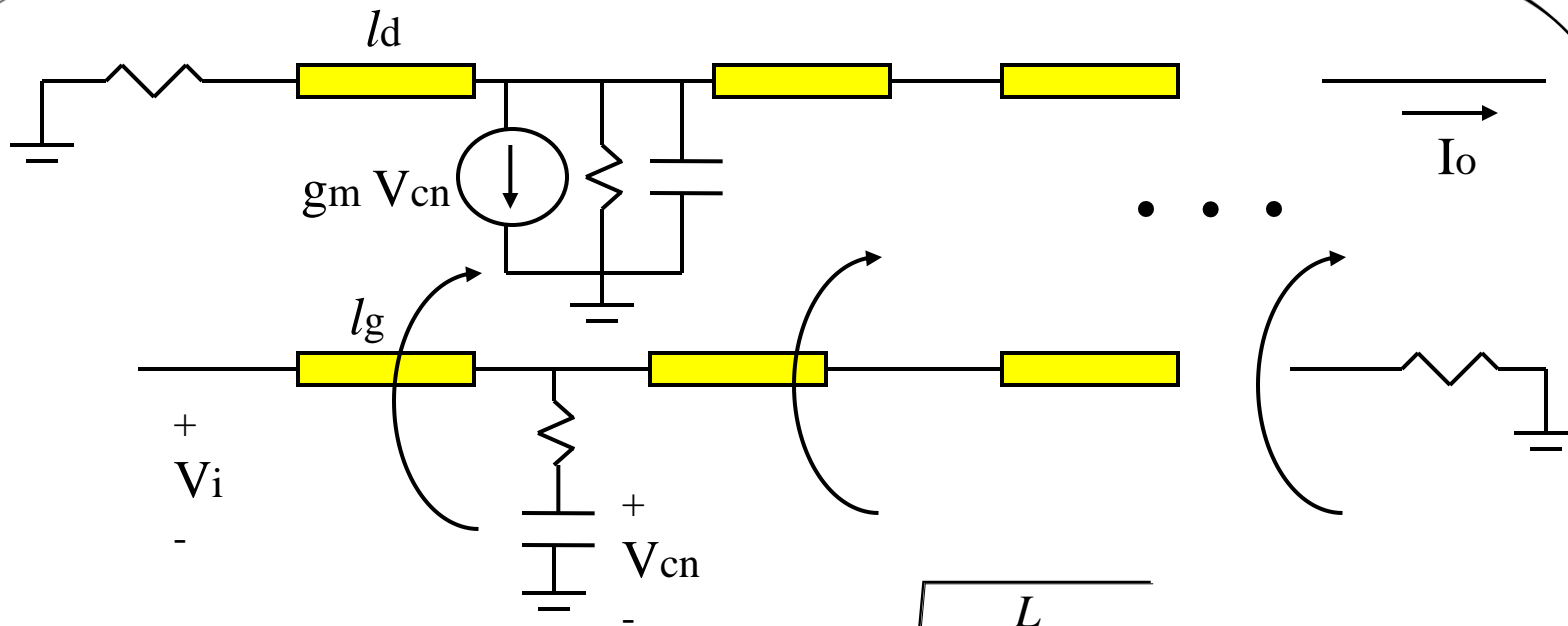
cut off frequency

$$f_c = \frac{1}{\pi\sqrt{LC}}$$



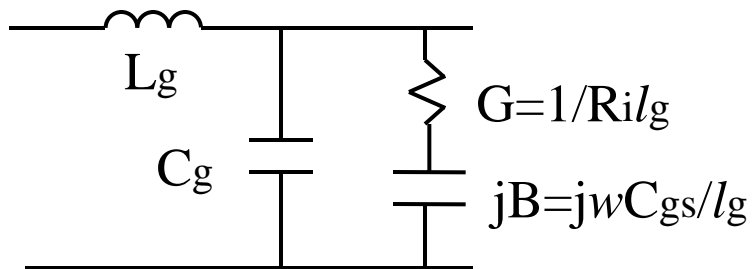
FET equivalent circuit





Discussion

1. unit cell of gate line



$$Z_g \approx \sqrt{\frac{L_g}{C_g + C_{gs}/l_g}}$$

$$\gamma_g = \sqrt{j\omega L_g \left(j\omega C_g + \frac{j\omega C_{gs}/l_g}{1 + j\omega R_i C_{gs}} \right)}$$

$$\underset{\substack{\text{small loss} \\ \omega R_i C_{gs} \ll 1}}{\approx} \frac{\omega^2 R_i Z_g C_{gs}^2}{2l_g} + j\omega \sqrt{L_g \left(C_g + \frac{C_{gs}}{l_g} \right)}$$

$$= \alpha_g + j\beta_g$$

(derivation of 1)

$$Z = j\omega L_g, Y = j\omega C_g + \frac{j\omega C_{gs} / l_g}{1 + j\omega R_i C_{gs}}$$

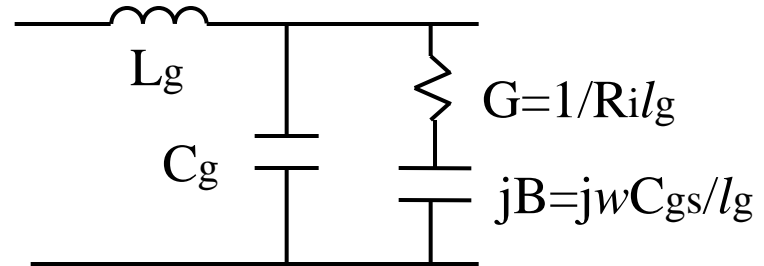
$$Z_g = \sqrt{\frac{Z}{Y}} \stackrel{\text{small loss}}{\approx} \sqrt{\frac{L_g}{C_g + C_{gs} / l_g}} \quad wR_i C_{gs} \ll 1$$

$$\gamma_g = \sqrt{ZY} = \sqrt{j\omega L_g \left(j\omega C_g + \frac{j\omega C_{gs} / l_g}{1 + j\omega R_i C_{gs}} \right)} \stackrel{wR_i C_{gs} \ll 1}{\approx} \sqrt{j\omega L_g \left[j\omega C_g + \frac{j\omega C_{gs} (1 - j\omega R_i C_{gs})}{l_g} \right]}$$

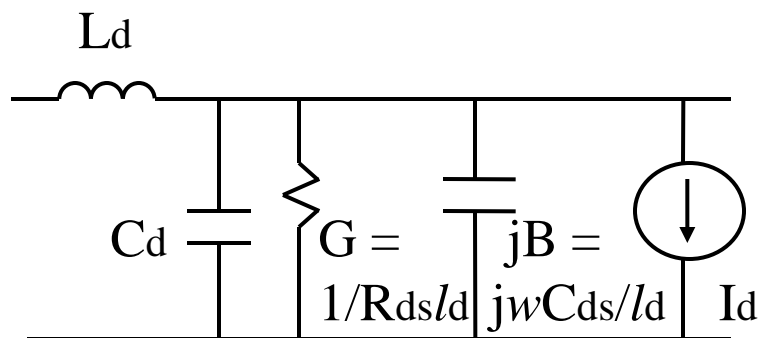
$$= \sqrt{(j\omega)^2 L_g \left(C_g + \frac{C_{gs}}{l_g} \right) - \frac{(j\omega)^3 L_g R_i C_{gs}^2}{l_g}}$$

$$\stackrel{(a-b)^2 \approx a^2 - 2ab}{\approx} \sqrt{(j\omega)^2 L_g \left(C_g + \frac{C_{gs}}{l_g} \right) - \frac{1}{2} \frac{(j\omega)^3 L_g R_i C_{gs}^2 / l_g}{j\omega \sqrt{L_g (C_g + C_{gs} / l_g)}}$$

$$= j\omega \sqrt{L_g \left(C_g + \frac{C_{gs}}{l_g} \right)} + \frac{\omega^2}{2} \sqrt{\frac{L_g}{C_g + \frac{C_{gs}}{l_g}}} \frac{R_i C_{gs}^2}{l_g} = j\omega \sqrt{L_g \left(C_g + \frac{C_{gs}}{l_g} \right)} + \frac{\omega^2 R_i C_{gs}^2 Z_g}{2l_g} = \alpha_g + j\beta_g$$



2. unit cell of drain line



$$Z_d \approx \sqrt{\frac{L_d}{C_d + C_{ds}/l_d}}$$

$$\gamma_d = \sqrt{j\omega L_d \left[\frac{1}{R_{ds}l_d} + j\omega \left(C_d + \frac{C_{ds}}{l_d} \right) \right]}$$

$$\text{small loss} \approx \frac{Z_d}{2R_{ds}l_d} + j\omega \sqrt{L_d \left(C_d + \frac{C_{ds}}{l_d} \right)}$$

$$= \alpha_d + j\beta_d$$

3. o/p current

$$I_o = -\frac{1}{2} \sum_{n=1}^N I_{dn} e^{-(N-n)\gamma_d l_d}, I_{dn} = g_m V_{cn}, V_{cn} = V_i e^{-(n-1)\gamma_g l_g} \left(\frac{1}{1 + j\omega R_i C_{gs}} \right)$$

$$\rightarrow I_o = -\frac{g_m V_i}{2} e^{-N\gamma_d l_d} e^{\gamma_g l_g} \sum_{n=1}^N e^{-n(\gamma_g l_g - \gamma_d l_d)} = -\frac{g_m V_i}{2} \frac{e^{-N\gamma_g l_g} - e^{-N\gamma_d l_d}}{e^{-\gamma_g l_g} - e^{-\gamma_d l_d}}$$

(derivation of 2)

$$Z = j\omega L_d, Y = \frac{1}{R_{ds}l_d} + j\omega(C_d + \frac{C_{ds}}{l_d})$$

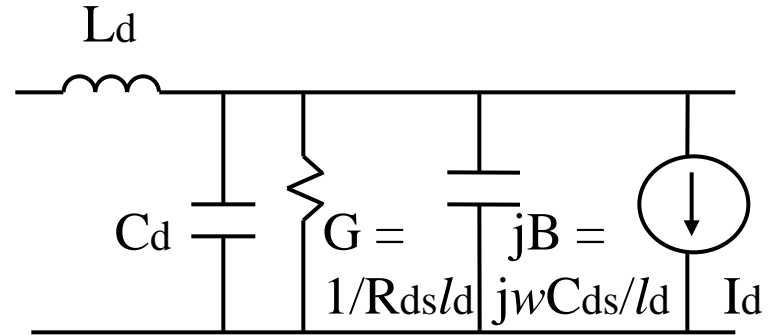
$$Z_d = \sqrt{\frac{Z}{Y}} \stackrel{\text{small loss}}{\approx} \sqrt{\frac{L_d}{C_d + C_{ds}/l_d}} \quad R_{ds}l_d \ll 1$$

$$\gamma_d = \sqrt{ZY} = \sqrt{j\omega L_d \left[\frac{1}{R_{ds}l_d} + j\omega(C_d + \frac{C_{ds}}{l_d}) \right]} = \sqrt{(j\omega)^2 L_d (C_d + \frac{C_{ds}}{l_d}) + \frac{j\omega L_d}{R_{ds}l_d}}$$

$$\stackrel{(a+b)^{\frac{1}{2}} \approx a^{\frac{1}{2}} + \frac{1}{2}a^{-\frac{1}{2}}b}{\approx} j\omega \sqrt{L_d (C_d + \frac{C_{ds}}{l_d})} + \frac{1}{2} \frac{1}{\sqrt{(j\omega)^2 L_d (C_d + C_{ds}/l_d)}} \frac{j\omega L_d}{R_{ds}l_d}$$

$$= j\omega \sqrt{L_d (C_d + \frac{C_{ds}}{l_d})} + \frac{1}{2} \frac{L_d}{R_{ds}l_d} \frac{1}{\sqrt{L_d (C_d + C_{ds}/l_d)}} = j\omega \sqrt{L_d (C_d + \frac{C_{ds}}{l_d})} + \frac{1}{2} \frac{1}{R_{ds}l_d} \sqrt{\frac{L_d}{C_d + C_{ds}/l_d}}$$

$$= j\omega \sqrt{L_d (C_d + \frac{C_{ds}}{l_d})} + \frac{1}{2} \frac{Z_d}{R_{ds}l_d} = \alpha_d + j\beta_d$$



(derivation of 3)

$$I_o = -\frac{1}{2} \sum_{n=1}^N I_{dn} e^{-(N-n)\gamma_d l_d}, I_{dn} = g_m V_{cn}, V_{cn} = V_i e^{-(n-1)\gamma_g l_g} \left(\frac{1}{1 + j\omega R_i C_{gs}} \right)$$

$$\rightarrow I_o = -\frac{g_m}{2} \sum_{n=1}^N V_{cn} e^{-(N-n)\gamma_d l_d} = -\frac{g_m V_i}{2} e^{-N\gamma_d l_d} e^{\gamma_g l_g} \sum_{n=1}^N e^{-n(\gamma_g l_g - \gamma_d l_d)}, \sum r^n = \frac{r(1-r^N)}{1-r}$$

$$= -\frac{g_m V_i}{2} e^{-N\gamma_d l_d} e^{\gamma_g l_g} \frac{e^{-(N+1)(\gamma_g l_g - \gamma_d l_d)} - e^{-(\gamma_g l_g - \gamma_d l_d)}}{e^{-(\gamma_g l_g - \gamma_d l_d)} - 1} \times \frac{e^{-\gamma_d l_d}}{e^{-\gamma_d l_d}}$$

$$= -\frac{g_m V_i}{2} e^{-(N+1)\gamma_d l_d} e^{\gamma_g l_g} \frac{e^{-(N+1)(\gamma_g l_g - \gamma_d l_d)} - e^{-(\gamma_g l_g - \gamma_d l_d)}}{e^{-\gamma_g l_g} - e^{-\gamma_d l_d}}$$

$$= -\frac{g_m V_i}{2} \frac{e^{-N\gamma_g l_g} - e^{-N\gamma_d l_d}}{e^{-\gamma_g l_g} - e^{-\gamma_d l_d}}$$

4. For matched i/p and o/p ports

$$G = \frac{P_{out}}{P_{in}} = \frac{\frac{|I_o|^2 Z_d}{2}}{\frac{|V_i|^2}{2Z_g}} = \frac{|I_o|^2 Z_d Z_g}{|V_i|^2} = \frac{g_m^2 Z_d Z_g}{4} \left| \frac{e^{-N\gamma_g l_g} - e^{-N\gamma_d l_d}}{e^{-\gamma_g l_g} - e^{-\gamma_d l_d}} \right|^2$$

$$= \frac{g_m^2 Z_d Z_g}{4} \left| \frac{e^{-N(\alpha_g l_g + j\beta_g l_g)} - e^{-N(\alpha_d l_d + j\beta_d l_d)}}{e^{-(\alpha_g l_g + j\beta_g l_g)} - e^{-(\alpha_d l_d + j\beta_d l_d)}} \right|^2$$

under synchronization condition $\beta_g l_g = \beta_d l_d$ ($\theta_g = \theta_d$)

$$\rightarrow G = \frac{g_m^2 Z_d Z_g}{4} \frac{(e^{-N\alpha_g l_g} - e^{-N\alpha_d l_d})^2}{(e^{-\alpha_g l_g} - e^{-\alpha_d l_d})^2}, N \nearrow \infty \Rightarrow G \searrow 0$$

$$\frac{dG}{dN} = 0 \rightarrow N_{opt} = \frac{\ln(\alpha_g l_g / \alpha_d l_d)}{\alpha_g l_g - \alpha_d l_d}$$

For lossless amplifier ($R_i=0$, $R_{ds} = \infty$) and if $Z_d = Z_g = Z_o$

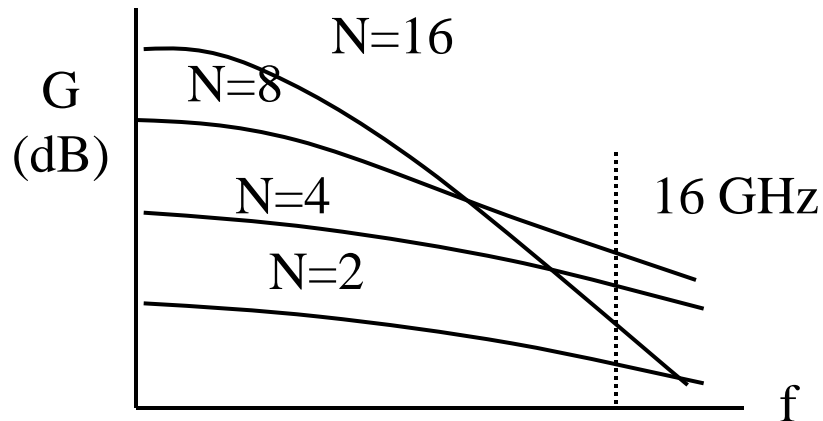
$$\rightarrow G = \frac{g_m^2 Z_d Z_g N^2}{4} = \left(\frac{g_m Z_o N}{2} \right)^2, G \propto N^2$$

5. Ex.11.7 $Z_d = Z_g = Z_o = 50\Omega$, $R_i = 10\Omega$, $R_{ds} = 300\Omega$, $C_{gs} = 0.27\text{pF}$,
 $g_m = 35\text{mS}$

$$\alpha_g l_g = \frac{\omega^2 R_i C_{gs}^2 Z_o}{2} = 0.184 @ 16\text{GHz}$$

$$\alpha_d l_d = \frac{Z_o}{2R_{ds}} = 0.083 @ 16\text{GHz}$$

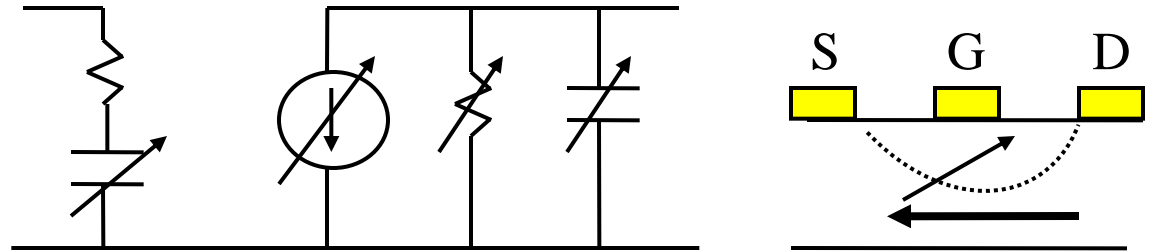
$\rightarrow N_{opt} = 7.9$, frequency response (p.569, Fig.11.15)



11.5 Power amplifiers

- nonlinear operation \rightarrow S(input power, f, DC, T, Z_L)

FET nonlinear
equivalent
circuit (large-signal
S-parameter)



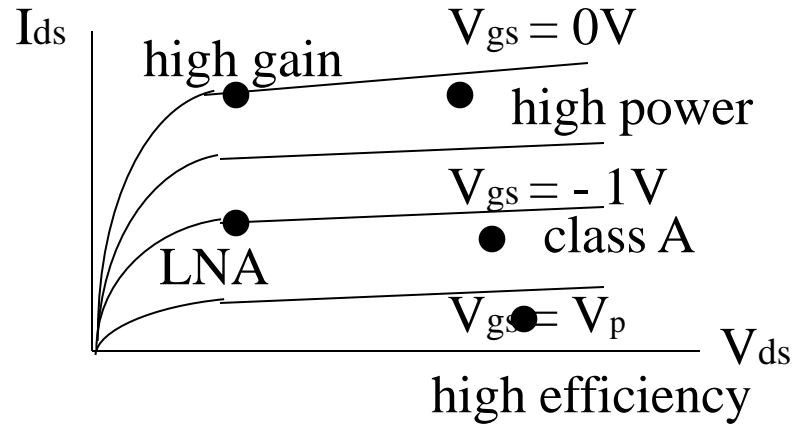
Discussion

1. power amplifier characteristics: efficiency, gain, intermodulation product, thermal conduction

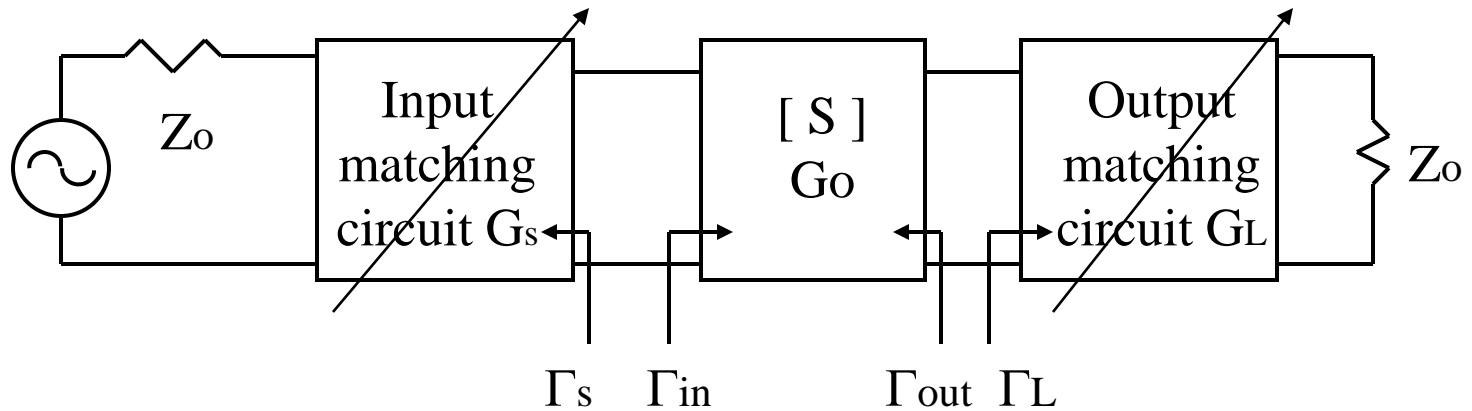
power added efficiency

$$\eta_{\text{PAE}} \equiv \frac{P_{\text{out}} - P_{\text{in}}}{P_{\text{DC}}}$$

2. DC bias consideration



3. design consideration: large-signal source impedance Γ_s (source-pull contour) and load impedance Γ_L (load-pull contour)



4. Ex.11.8 a transistor has small-signal S-parameters at 900MHz as

$$S_{11} = 0.94 \angle 164^\circ, S_{12} = 0.031 \angle 59^\circ, S_{21} = 1.222 \angle 43^\circ, S_{22} = 0.57 \angle -165^\circ$$

For class A operation at $V_{CE} = 24V$ and $I_C = 0.5A$, $P_{1dB,o} = 3.6W$, $G = 12dB$,
 $Z_{in} = 1.2 + j3.5\Omega$, $Z_{out} = 9.0 + j14.5\Omega$, design the input and output matching
circuits to give 3W output power.

From small-signal S-parameter, $|\Delta| = 0.546 < 1$, $K = 1.177 > 1 \rightarrow$ unconditional stable

$$\text{From } Z_{in} \text{ and } Z_{out} \rightarrow \Gamma_{in} = 0.953 \angle 172^\circ, \Gamma_{out} = 0.716 \angle -147^\circ$$

From small-signal S-parameter for $G_{T_{max}} \rightarrow \Gamma_S = 0.935 \angle -164^\circ, \Gamma_L = 0.507 \angle 174^\circ$

$$\Rightarrow \Gamma_S = \Gamma_{in}^* = 0.953 \angle -172^\circ, \Gamma_L = \Gamma_{out}^* = 0.716 \angle 147^\circ$$

For $P_{out} = 3W$, $P_{in} = P_{out} - G = 22.8dBm = 189mW$

$$\eta_{PAE} = \frac{P_{out} - P_{in}}{VI} = \frac{3 - 0.189}{24 \times 0.5} = 23.4\%$$

Suggested homework: 3, 11, 13

ADS examples: Ch11_prj