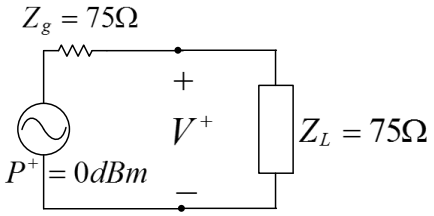
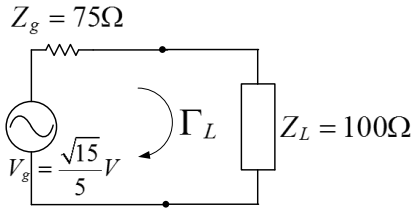
Microwave Review Quiz #1 Solution 2016.10.11

1. A lossless transmission line with a characteristic impedance of $Z_o = 75\Omega$ supports a forward-traveling wave with an average power of 0dBm. The line is terminated in a resistance of 100Ω.
   (1) Plot the lumped-element equivalent circuit at the interface between the transmission line and the load.
   (2) Calculate the reflection coefficient Γ_L, V_L^+, V_L^-, V_L and average power P_L at the load.
   (3) If the lossless transmission line has a length of $\lambda/4$, calculate the reflection coefficient Γ_{in}, V_{in}^+, V_{in}^-, V_{in} and average power P_{in} at the input of transmission line.

(1)





$$P_{inc} = \frac{1}{2}\frac{|V^+|^2}{Z_g} = 0dBm = 1mW = \frac{1}{2}\frac{|V^+|^2}{75} \rightarrow |V^+| = V^+ = \sqrt{150\times10^{-3}} = \frac{\sqrt{15}}{10}V \rightarrow V_g = 2V^+ = \frac{\sqrt{15}}{5}V$$

(2)

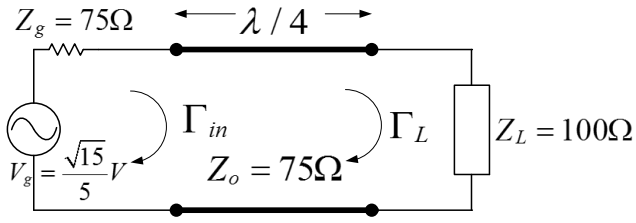
$$\Gamma_L = \frac{Z_L - Z_g}{Z_L + Z_g} = \frac{100-75}{100+75} = \frac{1}{7}$$

$$V_L = V_g\frac{Z_L}{Z_L+Z_g} = \frac{\sqrt{15}}{5}\frac{100}{100+75} = \frac{4\sqrt{15}}{35}V = V_L^+ + V_L^- = V_L^+(1+\Gamma_L) = V_L^+\frac{8}{7} \rightarrow V_L^+ = \frac{\sqrt{15}}{10}V$$

$$V_L^- = V_L^+\Gamma_L = \frac{\sqrt{15}}{10}\times\frac{1}{7} = \frac{\sqrt{15}}{70}V, V_L^+ + V_L^- = \frac{\sqrt{15}}{10} + \frac{\sqrt{15}}{70} = \frac{4\sqrt{15}}{35}V = V_L$$

$$P_{refl} = P_{inc}|\Gamma_L|^2 = 1\times(\frac{1}{7})^2 = \frac{1}{49}mW, P_L = P_{inc} - P_{refl} = 1 - \frac{1}{49} = \frac{48}{49}mW = \frac{1}{2}\frac{V_L^2}{Z_L} = \frac{1}{2}\frac{(\frac{4\sqrt{15}}{35})^2}{100}\times1000 = \frac{48}{49}$$

(3)



$$\Gamma_{in} = \Gamma_L e^{-2j\beta l} = \Gamma_L e^{-2j\frac{2\pi}{\lambda}\frac{\lambda}{4}} = \Gamma_L e^{-j\pi} = -\Gamma_L = -\frac{1}{7}$$

$$Z_{in} = \frac{Z_o^2}{Z_L} = \frac{75^2}{100} = \frac{225}{4}, V_{in} = V_g \frac{Z_{in}}{Z_g + Z_{in}} = \frac{\frac{225}{4}}{75 + \frac{225}{4}} = \frac{\sqrt{15}}{5}\frac{225}{525} = \frac{3\sqrt{15}}{35} V = V_{in}^+ + V_{in}^- = V_{in}^+(1 + \Gamma_{in}) = V_{in}^+ \frac{6}{7}$$

$$\rightarrow V_{in}^+ = \frac{\sqrt{15}}{10} V$$

$$V_{in}^- = V_{in}^+ \Gamma_{in} = \frac{\sqrt{15}}{10} \times (-\frac{1}{7}) = -\frac{\sqrt{15}}{70} V, V_{in}^+ + V_{in}^- = \frac{\sqrt{15}}{10} - \frac{\sqrt{15}}{70} = \frac{3\sqrt{15}}{35} = V_{in}$$

$$P_{refl} = P_{inc} \left|\Gamma_{in}\right|^2 = 1 \times (\frac{1}{7})^2 = \frac{1}{49} mW, P_{in} = P_{inc} - P_{refl} = 1 - \frac{1}{49} = \frac{48}{49} mW = \frac{1}{2}\frac{V_{in}^2}{Z_{in}} = \frac{1}{2}\frac{(\frac{3\sqrt{15}}{35})^2}{\frac{225}{4}} \times 1000 = \frac{48}{49} mW$$