Chapter 4 Resonant circuits

4.1 Series resonant circuits

steady state response, Q factor, half power frequencies, input impedance,

4.2 Parallel resonant circuits

steady state response, input impedance, loaded Q, external Q

4.3 Transformer coupled circuits

equivalent resonator circuit

4.4 Transmission line resonant circuits

short-circuited $\lambda/2$ and $\lambda/4$ lines, open-circuited $\lambda/2$ and $\lambda/4$ lines

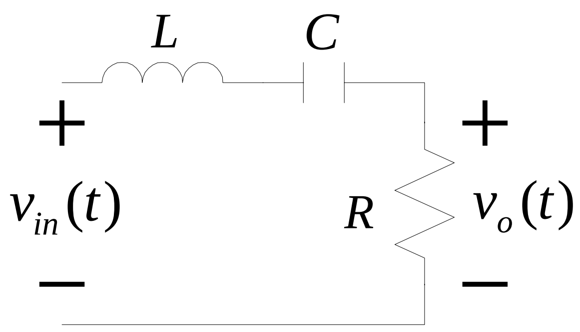
4.5 Microwave resonators

rectangular cavity, circular cylindrical cavity, dielectric resonator

4.1 Series resonant circuits

<u>Basics</u>

1. Resonator applications: filter, frequency selective components in amplifier and oscillator, impedance transformer, matching element.
2. Steady state response



$$A(jw)=\frac{V_o(jw)}{V_{in}(jw)}$$

$$=\frac{R}{jwL+\frac{1}{jwC}+R}=\frac{1}{\frac{jwL}{R}+\frac{1}{jwRC}+1}$$

$$=\frac{1}{\frac{j}{RC}(LCw-\frac{1}{w})+1}=\frac{1}{\frac{j}{RC}(\frac{w}{w_o^2}-\frac{1}{w})+1}=\frac{1}{1+\frac{j}{w_oRC}(\frac{w}{w_o}-\frac{w_o}{w})}=\frac{1}{1+jQ(\frac{w}{w_o}-\frac{w_o}{w})}$$

resonant frequency $w_o\equiv\frac{1}{\sqrt{LC}}$

quality factor $Q\equiv w_o\frac{\text{average stored energy}}{\text{power loss}}=w_o\frac{W_m+W_e}{P_{loss}}=\frac{w_oL}{R}=\frac{1}{w_oRC}$

$$P_{loss}=\frac{1}{2}|I|^2R, W_m=\frac{1}{4}|I|^2L, W_e=\frac{1}{4}|V_c|^2C=\frac{1}{4}|I|^2\frac{1}{\omega^2C}$$

3. Half power frequencies $Q = \dfrac{w_o}{w_2 - w_1}$

4. Input impedance near resonance

$$Z_{in}(jw) = R + jwL + \frac{1}{jwC}$$

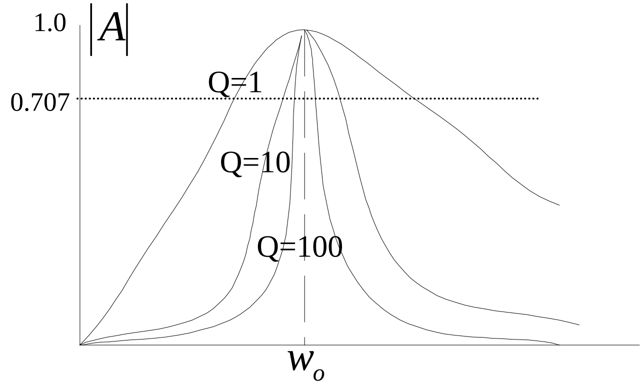
$$= R + jwL\left(1 - \frac{w_o^2}{w^2}\right) = R + jwL\,\frac{(w+w_o)(w-w_o)}{w^2}$$

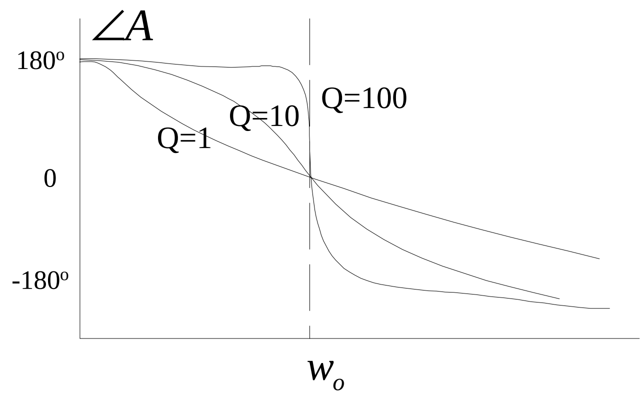
$$\approx R + j2\delta wL$$

$$(1) = R + \frac{j2RQ\,\delta w}{w_o}$$

$$(2) = w_o\frac{L}{Q} + j2(w-w_o)L = j2\left(w - w_o + \frac{w_o}{j2Q}\right)L$$

$$= j2\left[w - w_o\left(1 + j\frac{1}{2Q}\right)\right]L$$





Lossy resonator Z_{in}
= lossless resonator Z_{in} by letting $w_o \rightarrow w_o\left(1 + j\dfrac{1}{2Q}\right)$

Discussion

1. Derivation of half power frequencies equation $Q = \dfrac{w_o}{w_2 - w_1}$

$$\because A(jw) = \frac{V_o(jw)}{V_{in}(jw)} = \frac{1}{1 + jQ(\frac{w}{w_o} - \frac{w_o}{w})}$$

$$\frac{1}{2} = \frac{1}{1 + Q^2(\frac{w}{w_o} - \frac{w_o}{w})^2} \rightarrow 2 = 1 + Q^2(\frac{w}{w_o} - \frac{w_o}{w})^2 \rightarrow Q(\frac{w}{w_o} - \frac{w_o}{w}) = \pm 1 \ldots(1)$$

$$\rightarrow \frac{w_2}{w_o} - \frac{w_o}{w_2} = -(\frac{w_1}{w_o} - \frac{w_o}{w_1}) \rightarrow w_2 - \frac{w_o^2}{w_2} = -w_1 + \frac{w_o^2}{w_1} \rightarrow w_1 + w_2 = w_o^2(\frac{1}{w_1} + \frac{1}{w_2})$$

$$\rightarrow w_1 w_2 = w_o^2 \ldots(2)$$

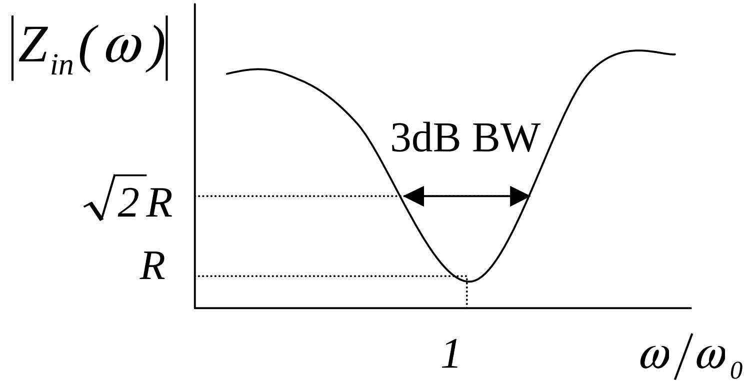
$$(1) \rightarrow \frac{w_1}{w_o} - \frac{w_o}{w_1} = -\frac{1}{Q} \rightarrow w_1 - \frac{w_o^2}{w_1} = -\frac{w_o}{Q} \overset{(2)}{\rightarrow} w_1 - w_2 = -\frac{w_o}{Q} \rightarrow Q = \frac{w_o}{w_2 - w_1}$$

2. $Z_{in}(jw)=R+jX$, and R=X at half power frequencies

$$Z_{in}(jw_1) = R + jw_1 L \frac{w_1^2 - w_o^2}{w_1^2}$$

$$\xrightarrow{\frac{1}{Q}=\frac{w_1^2-w_o^2}{w_o w_1}} Z_{in}(jw_1) = R + j\frac{w_o L}{Q}$$

$$\xrightarrow{Q=w_o\frac{L}{R}} R + jR$$



3. Ex.4.1 A series LC circuit passes signals from 9MHz to 11MHz, and is connected to a communication system with Z_{in}=50Ω. →L, C

$$f_o = \sqrt{f_1 f_2} = \sqrt{9\times 11} = 9.5MHz$$

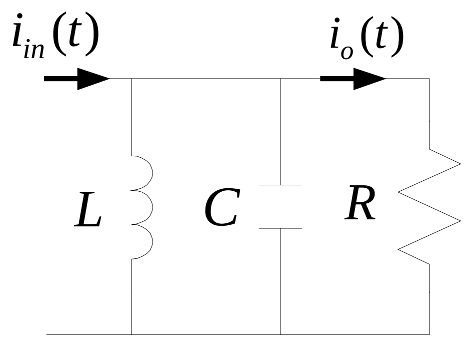
$$Q = \frac{w_o}{w_2 - w_1} = \frac{w_o L}{R} \rightarrow L = \frac{R}{w_2 - w_1} = \frac{50}{2\pi\times(11-9)\times 10^6} = 4uH$$

$$C = \frac{1}{Lw_o^2} = 64.3pF$$

4.2 Parallel resonant circuits

<u>Basics</u>

1. Duality with series resonator: V→I, R→G, L→C, C→L
2. Steady state response



$$A(jw) = \frac{I_o(jw)}{I_{in}(jw)}$$

$$= \frac{1/R}{jwC + \frac{1}{jwL} + \frac{1}{R}} = \frac{1}{jwRC + \frac{R}{jwL} + 1}$$

$$= \frac{1}{\frac{jR}{L}(LCw - \frac{1}{w}) + 1} = \frac{1}{\frac{jR}{L}(\frac{w}{w_o^2} - \frac{1}{w}) + 1} = \frac{1}{1 + \frac{jR}{w_o L}(\frac{w}{w_o} - \frac{w_o}{w})} = \frac{1}{1 + jQ(\frac{w}{w_o} - \frac{w_o}{w})}$$

resonant frequency $w_o \equiv \frac{1}{\sqrt{LC}}$

quality factor $Q \equiv w_o \frac{W_m + W_e}{P_{loss}} = \frac{R}{w_o L} = w_o RC$

$$P_{loss} = \frac{1}{2}\frac{|V|^2}{R}, W_m = \frac{1}{4}|I_L|^2 L = \frac{1}{4}\frac{|V|^2}{\omega^2 L}, W_e = \frac{1}{4}|V|^2 C$$

3. Input impedance near resonance

$$Z_{in}(jw) = (\frac{1}{R} + jwC + \frac{1}{jwL})^{-1} = [\frac{1}{R} + jwC(1 - \frac{w_o^2}{w^2})]^{-1} = [\frac{1}{R} + jwC\frac{(w+w_o)(w-w_o)}{w^2}]^{-1}$$

$$\approx (\frac{1}{R} + j2\delta wC)^{-1}$$

$$(1) = (\frac{1}{R} + \frac{j2Q\delta w}{Rw_o})^{-1}$$

$$(2) = [\frac{w_o C}{Q} + j2(w - w_o)C]^{-1} = [j2(w - w_o + \frac{C}{j2Q})C]^{-1}$$

$$= \{j2[w - w_o(1 + j\frac{1}{2Q})]C\}^{-1}$$

Lossy resonator Z_{in}= lossless resonator Z_{in} by letting $w_o \rightarrow w_o(1 + j\frac{1}{2Q})$

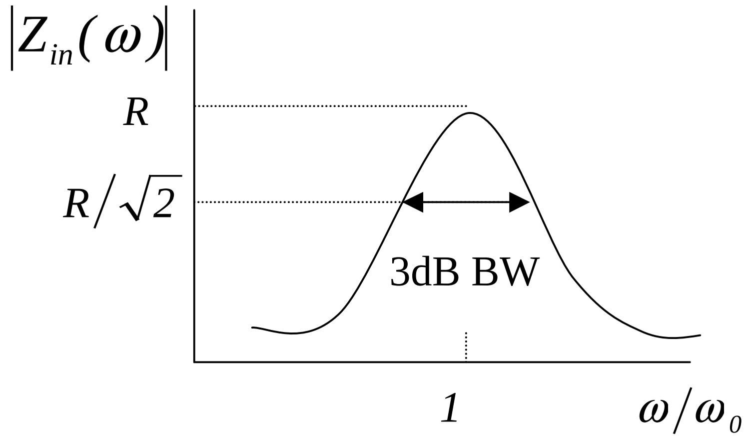
Discussion

1. $Y_{in}(jw)$=G+jB, and G=B at half power frequencies

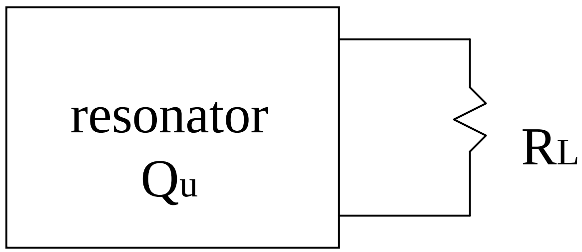
$$Z_{in}(jw_1) = \left(\frac{1}{R} + jw_1 C\,\frac{w_1^2 - w_o^2}{w_1^2}\right)^{-1}$$

$$\xrightarrow{\frac{1}{Q}=\frac{w_1^2-w_o^2}{w_o w_1}} \quad Z_{in}(jw_1) = \left(\frac{1}{R} + j\,\frac{w_o C}{Q}\right)^{-1}$$

$$\xrightarrow{Q=w_o RC} \quad Z_{in}(jw_1) = \frac{R}{1+j}$$



2. Unloaded Q, Q_u, loaded Q, Q_L, external Q, Q_e



$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_U}$$

$$Q_e = \begin{cases} \dfrac{\omega_o L}{R_L} & \text{for series resonant circuit} \\[2ex] \dfrac{R_L}{\omega_o L} & \text{for parallel resonant circuit} \end{cases}$$

3. Resonator relations

	Series resonator	Parallel resonator
Z_{in}	$R + jwL + \frac{1}{jwC} \approx R + j2\delta wL$ $\approx R + j\frac{2RQ\delta w}{w_o}$	$(\frac{1}{R} + jwC + \frac{1}{jwL})^{-1} \approx (\frac{1}{R} + j2\delta wC)^{-1}$ $\approx (\frac{1}{R} + j\frac{2Q\delta w}{Rw_o})^{-1}$
w_o	$\frac{1}{\sqrt{LC}}$	
Q_u	$\frac{w_o L}{R} = \frac{1}{w_o RC}$	$w_o RC = \frac{R}{w_o L}$
Q_e	$\frac{w_o L}{R_L}$	$\frac{R_L}{w_o L}$
Q_L	$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_U}$	

4. Ex.4.2 A parallel resonator with R=10kΩ, L=10uH, C=10pF and its load R_L=100kΩ→wo, Q_u, Q_L

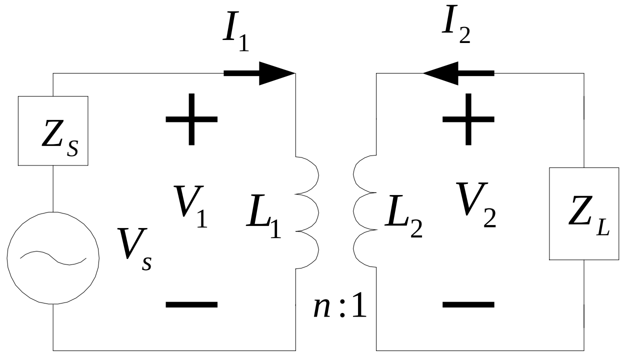
$$w_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-5} \times 10^{-11}}} = 10^8 rad/\sec$$

$$Q_u = \frac{R}{w_o L} = \frac{10^5}{10^8 \times 10^{-5}} = 100, Q_e = \frac{R_L}{w_o L} = 100, Q_L = 50$$

4.3 Transformer-coupled circuits

Basics

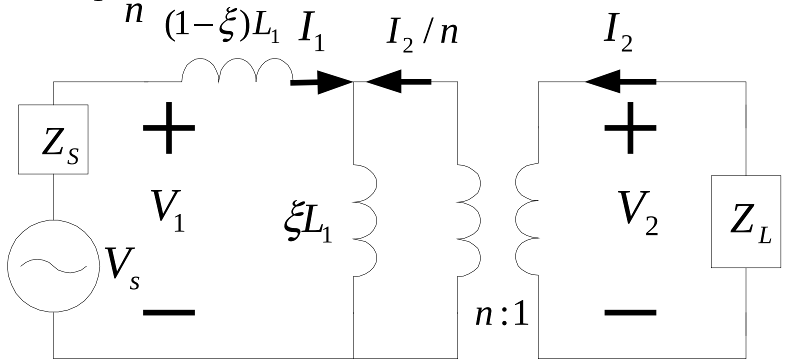
1. Equivalent circuit



$$V_1 = nV_2, I_1 = -\frac{I_2}{n}$$

$$\begin{cases} V_1 = jwL_1 I_1 + jwMI_2 \\ V_2 = jwMI_1 + jwL_2 I_2 \end{cases}$$

$$\begin{cases} V_1 = jw(1-\xi)L_1 I_1 + jw\xi L_1 (I_1 + \frac{I_2}{n}) = jwL_1 I_1 + jw\xi L_1 \frac{I_2}{n} \\ V_2 = \frac{1}{n} jw\xi L_1 (I_1 + \frac{I_2}{n}) \end{cases}$$



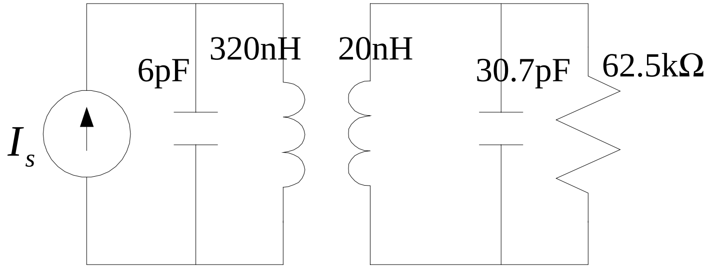
$$\rightarrow \frac{\xi L_1}{n} = M, \frac{\xi L_1}{n^2} = L_2$$

$$\rightarrow n = \sqrt{\frac{\xi L_1}{L_2}}, \xi = \frac{nM}{L_1} = \frac{M\sqrt{\xi}}{\sqrt{L_1 L_2}}$$

$$\rightarrow \xi = \frac{M}{\sqrt{L_1 L_2}} : \text{coefficient of coupling}$$

Discussion

1. Ex.4.3 A tightly coupled transformer circuit → equivalent resonator circuit, w_o, Q



$$\xi \approx 1, n = \sqrt{\frac{\xi L_1}{L_2}} = \sqrt{\frac{320}{20}} = 4$$

$$Z_1 = n^2 Z_2 \rightarrow Y_1 = \frac{1}{n^2}\left(\frac{1}{62.5k} + jw \times 30.7p\right)$$

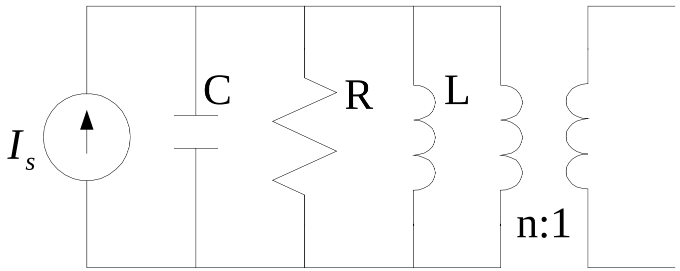
$$R = 16 \times 62.5 \times 10^3 = 1M\Omega$$

$$C = 6 + \frac{30.7}{16} = 7.9pF$$

$$L = 320nH$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = 99.97MHz$$

$$Q = \frac{R}{w_o L} = \frac{10^6}{2\pi \times 99.7 \times 10^6 \times 320 \times 10^{-9}} = 4975$$

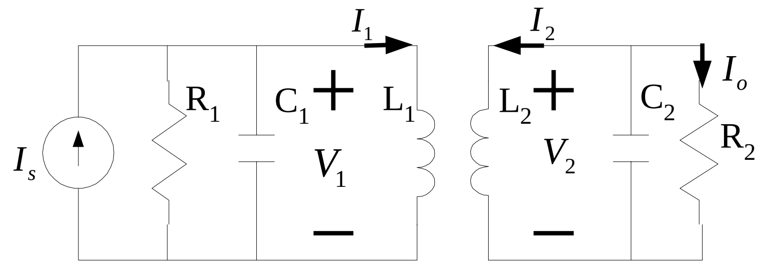


2. Ex.4.4 A double-tuned transformer-coupled circuit → equivalent circuit, (a) w_o and Z_{in} as R_1, C_1 removed (b) I_o/I_s with R_1, C_1

(a) without R_1, C_1

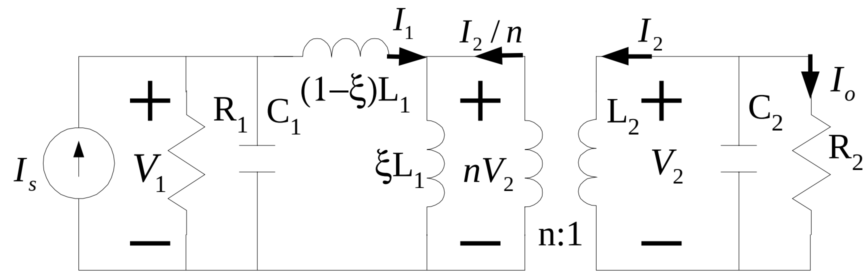
$$w_o = \frac{1}{\sqrt{L_2 C_2}}, Z_{in}(jw_o) = n^2 R_2$$

$$Z_{in}(jw) = jwL_1(1-\xi) + \frac{jwn^2 L_2}{-w^2 L_2 C_2 + jw\frac{L_2}{R_2} + 1}$$



(b) with R_1, C_1

$$\begin{cases} nV_2 = jw\xi L_1 (I_1 + \frac{I_2}{n}) \\ V_2 = -I_2 \frac{R_2}{1 + jwR_2 C_2} = I_o R_2 \\ I_s = I_1 + (\frac{1}{R_1} + jwC_1)[nV_2 + jw(1-\xi)L_1 I_1] \end{cases}$$



$$\rightarrow \frac{I_o}{I_s} = \frac{\frac{jw\xi L_1}{nR_2}}{(-w^2 L_2 C_2 + jw\frac{L_2}{R_2} + 1)[1 + jw(1-\xi)L_1(\frac{1}{R_1} + jwC_1)] + jw\xi L_1(\frac{1}{R_1} + jwC_1)} : \text{four poles}$$

as $\xi \to 0$

$$\frac{I_o}{I_s} \approx \frac{\dfrac{jw\xi L_1}{nR_2}}{(-w^2 L_2 C_2 + jw\dfrac{L_2}{R_2} + 1)[1 + jwL_1(\dfrac{1}{R_1} + jwC_1)] + jw\xi L_1(\dfrac{1}{R_1} + jwC_1)}$$

for $w_1 = w_2 = w_o, Q_1 = Q_2 = Q_o$

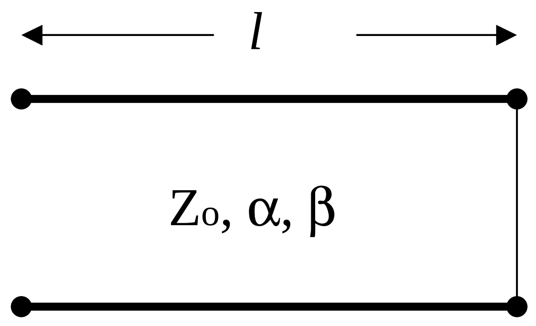
$$L_1 C_1 = L_2 C_2 = \frac{1}{w_o^2}, \frac{R_1}{L_1} = \frac{R_2}{L_2} = w_o Q_o$$

$$\to \frac{I_o}{I_s} \approx \frac{\dfrac{jw\xi L_1}{nR_2}}{(-\dfrac{w^2}{w_o^2} + \dfrac{jw}{w_o Q} + 1)^2 + \xi(-\dfrac{w^2}{w_o^2} + \dfrac{jw}{w_o Q})}$$

4.4 Transmission line resonant circuits

Basics

1. Short-circuited line



$$Z_{in} = Z_o \frac{Z_L + Z_o \tanh \gamma l}{Z_o + Z_L \tanh \gamma l}\bigg|_{Z_L = 0} = Z_o \tanh \gamma l$$

$$= Z_o \tanh(\alpha + j\beta)l = Z_o \frac{\tanh \alpha l + j \tan \beta l}{1 + j \tanh \alpha l \tan \beta l}$$

lowloss line $\tanh \alpha l \approx \alpha l$

$$\beta l = \frac{wl}{v_p} = \frac{(w_o + \delta w)l}{v_p} = \beta_r l + \frac{\delta w l}{v_p} = \begin{cases} \pi + \dfrac{\pi \delta w}{w_o} & l = \dfrac{\lambda}{2} \\ \dfrac{\pi}{2} + \dfrac{\pi \delta w}{2 w_o} & l = \dfrac{\lambda}{4} \end{cases}$$

$$\tan \beta l = \begin{cases} \tan(\pi + \dfrac{\pi \delta w}{w_o}) = \tan \dfrac{\pi \delta w}{w_o} \approx \dfrac{\pi \delta w}{w_o} & l = \dfrac{\lambda}{2} \\ \tan(\dfrac{\pi}{2} + \dfrac{\pi \delta w}{2 w_o}) = -\cot \dfrac{\pi \delta w}{2 w_o} \approx -\dfrac{2 w_o}{\pi \delta w} & l = \dfrac{\lambda}{4} \end{cases}$$

2. $\lambda/2$ short-circuited line→series resonator

$$Z_{in} = Z_o \frac{\tanh \alpha l + j \tan \beta l}{1 + j \tanh \alpha l \tan \beta l} \approx Z_o \frac{\alpha l + j \frac{\pi \delta w}{w_o}}{1 + j \alpha l \frac{\pi \delta w}{w_o}} \approx Z_o \alpha l + j \frac{Z_o \pi \delta w}{w_o}$$

for series resonator $Z_{in} \approx R + j2\delta w L$

→ equivalent circuit parameters

$$\mathrm{R} \approx Z_o \alpha l = Z_o \alpha \frac{\lambda_r}{2}, L \approx \frac{\pi Z_o}{2 w_o}, C = \frac{1}{w_o^2 L} \approx \frac{2}{\pi w_o Z_o}, Q = \frac{w_o L}{R} \approx \frac{\pi}{2\alpha l} = \frac{\beta_r}{2\alpha}$$

3. $\lambda/4$ short-circuited line→parallel resonator

$$Z_{in} = Z_o \frac{\tanh \alpha l + j \tan \beta l}{1 + j \tanh \alpha l \tan \beta l} \approx Z_o \frac{\alpha l - j \frac{2 w_o}{\pi \delta w}}{1 - j \alpha l \frac{2 w_o}{\pi \delta w}} \approx Z_o \frac{-j \frac{2 w_o}{\pi \delta w}}{1 - j \alpha l \frac{2 w_o}{\pi \delta w}} \times \frac{j \frac{\delta w \pi}{2 w_o}}{j \frac{\delta w \pi}{2 w_o}} = \frac{\frac{Z_o}{\alpha l}}{1 + j \frac{\pi \delta w}{\alpha l 2 w_o}}$$

for parallel resonator $Z_{in} \approx (\frac{1}{R} + j2\delta w C)^{-1} = \frac{R}{1 + j2\delta w RC}$

→ equivalent circuit parameters

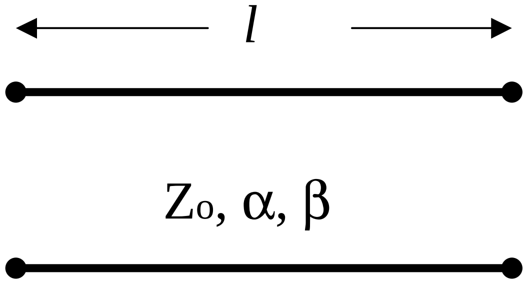
$$\mathrm{R} \approx \frac{Z_o}{\alpha i} = \frac{4 Z_o}{\alpha \lambda_r}, C \approx \frac{\pi}{4 w_o Z_o}, L = \frac{1}{w_o^2 L} \approx \frac{4 Z_o}{\pi w_o}, Q = \frac{R}{w_o L} \approx \frac{\pi}{4 \alpha l} = \frac{\beta_r}{2\alpha}$$

4. Open-circuited line

$$Z_{in}=Z_o\left.\frac{Z_L+Z_o\tanh\gamma l}{Z_o+Z_L\tanh\gamma l}\right|_{Z_L=\infty}=\frac{Z_o}{\tanh\gamma l}=\frac{Z_o}{\tanh(\alpha+j\beta)l}=Z_o\frac{1+j\tanh\alpha l\tan\beta l}{\tanh\alpha l+j\tan\beta l}$$

lowloss line $\tanh\alpha l\approx\alpha l$

$$\tan\beta l\approx\begin{cases}\dfrac{\pi\delta w}{w_o} & l=\dfrac{\lambda}{2}\\ -\dfrac{2w_o}{\pi\delta w} & l=\dfrac{\lambda}{4}\end{cases}$$



5. $\lambda/2$ open-circuited line→parallel resonator

$$Z_{in}=Z_o\frac{1+j\tanh\alpha l\tan\beta l}{\tanh\alpha l+j\tan\beta l}\approx Z_o\frac{1+j\alpha l\dfrac{\pi\delta w}{w_o}}{\alpha l+j\dfrac{\pi\delta w}{w_o}}\approx\frac{Z_o}{\alpha l+j\dfrac{\pi\delta w}{w_o}}$$

for parallel resonator $Z_{in}\approx(\frac{1}{R}+j2\delta wC)^{-1}=\frac{R}{1+j2\delta wRC}$

→ equivalent circuit parameters

$$\mathrm{R}\approx\frac{Z_o}{\alpha l}=\frac{2Z_o}{\alpha\lambda_r},C\approx\frac{\pi}{2w_oZ_o},L\approx\frac{2Z_o}{\pi w_o},Q=\frac{\pi}{2\alpha l}=\frac{\beta_r}{2\alpha}$$

6. λ/4 open-circuited line→series resonator

$$Z_{in} = Z_o \frac{1 + j \tanh \alpha l \tan \beta l}{\tanh \alpha l + j \tan \beta l} \approx Z_o \frac{1 - j\alpha l \frac{2w_o}{\pi \delta w}}{\alpha l - j \frac{2w_o}{\pi \delta w}} \approx Z_o \frac{1 - j\alpha l \frac{2w_o}{\pi \delta w}}{- j \frac{2w_o}{\pi \delta w}} = Z_o (\alpha l + j \frac{\pi \delta w}{2w_o})$$

for series resonator $Z_{in} \approx R + j2\delta wL$

$\rightarrow$ equivalent circuit parameters

$$\text{R} \approx Z_o \alpha l = Z_o \alpha \frac{\lambda_r}{4}, L \approx \frac{\pi Z_o}{4w_o}, C \approx \frac{4}{\pi w_o Z_o}, Q \approx \frac{\pi}{4\alpha l} = \frac{\beta_r}{2\alpha}$$

Discussion

1. Ex.4.5 A coaxial λ/2 short-circuited line has inner conductor radius a=0.455mm, outer conductor radius b=1.499mm→Q at 5GHz for air dielectric and Teflon dielectric (ε_r=2.08, tan δ =0.0004)

$$\text{copper } R_s = \sqrt{\frac{w\mu_o}{2\sigma}} = \sqrt{\frac{2\pi\times5\times10^9\times4\pi\times10^{-7}}{2\times5.813\times10^7}} = 0.0184\,\Omega$$

$$\alpha_c = \frac{R_s}{2\sqrt{\frac{\mu_o}{\varepsilon\varepsilon_{ro}}}\ln\frac{b}{a}}\left(\frac{1}{a}+\frac{1}{b}\right) = \begin{cases} 0.0588\,Np/m & \text{air} \\ 0.085\,Np/m & \text{Teflon} \end{cases}$$

$$\alpha_d = \frac{w}{2}\sqrt{\mu_o\varepsilon_o\varepsilon_r}\tan\delta = \begin{cases} 0 & \text{air} \\ 0.03\,Np/m & \text{Teflon} \end{cases}$$

$$\beta = \frac{2\pi f\sqrt{\varepsilon_r}}{c} = \begin{cases} 104.7\ rad/m & \text{air} \\ 151.03\ rad/m & \text{Teflon} \end{cases}$$

$$Q = \frac{\beta}{2\alpha} = \begin{cases} 891 & \text{air} \\ 657 & \text{Teflon} \end{cases}$$

2. Ex.4.6 A 50Ω microstrip λ/2 short-circuited line has Teflon substrate (ε_r=2.08, tan δ =0.0004) with thickness h=0.159cm, t=0.159um →Q and *l* at 2GHz

$$\varepsilon_r = 2.08, Z_o = 50\Omega, h = 0.159cm \begin{cases} \xrightarrow{A2.29,30} w = 0.51cm \\ \xrightarrow{A2.21,22} \varepsilon_{re} = 1.79 \end{cases}$$

$$l = \frac{\lambda}{2} = \frac{c}{2f\sqrt{\varepsilon_{re}}} = 5.6cm$$

$$\left.\begin{array}{l} w = 0.51cm, h = 0.159cm, t = 0.159um \\ \varepsilon_r = 2.08, \sigma = 5.813\times10^7 S/m,\ f = 2GHz \end{array}\right\} \xrightarrow{A2.26,27} \alpha_c = 0.0428 Np/m$$

$$\left.\begin{array}{l} w = 0.51cm, h = 0.159cm, \varepsilon_r = 2.08 \\ \tan\delta = 0.0004,\ f = 2GHz \end{array}\right\} \xrightarrow{A2.28} \alpha_d = 0.0095 Np/m$$

$$\beta = \frac{2\pi}{\lambda} = \frac{\pi}{l} = 56 rad/m$$

$$Q = \frac{\beta}{2\alpha} = \frac{56}{2\times(0.0428+0.0095)} = 535.4$$

4.5 Microwave resonators

Basics

1. Rectangular cavities dimension a, b,c

TE_{mnp}, TM_{mnp} mode resonant frequency $f_r = \frac{3\times10^8}{2\pi\sqrt{\mu\varepsilon}}\sqrt{(\frac{m\pi}{a})^2+(\frac{n\pi}{b})^2+(\frac{p\pi}{c})^2}$

TE_{10p} mode $Q_c = \frac{2W_e}{P_l} = \frac{60b(acw\sqrt{\mu\varepsilon})^3}{\pi R_s(2p^2a^3b+2bc^3+pa^3c+ac^3)}\sqrt{\frac{\mu_r}{\varepsilon_r}}$

$Q_d = \frac{1}{\tan\delta}$

2. Circular cylindrical cavities

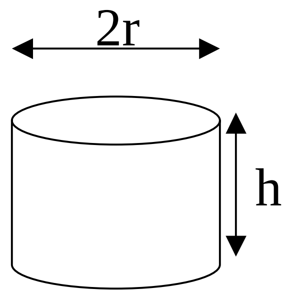
$$f_r = \frac{c}{2\pi\sqrt{\mu\varepsilon}}\sqrt{(\frac{\chi_{nm}}{r})^2+(\frac{p\pi}{h})^2},\ p = \begin{cases} p_{nm} & TM_{nmp} \text{ mode} \\ p'_{nm} & TE_{nmp} \text{ mode} \end{cases}$$

p_{nm} : mth zero of the Bessel function $J_n(x)$ of the first kind and order n

p'_{nm} : mth zero of $J'_n(x)$: Table 4.3

$$Q_c = \begin{cases} 4.5.10 & TE_{nmp} \text{ mode} \\ 4.5.11 & TM_{nmp} \text{ mode} \end{cases},\ Q_d = \frac{1}{\tan\delta}$$

3. Dielectric resonator (DR): high ε, low tan δ, low temperature coefficient, $TE_{01\delta}$ mode



$$f_r = \frac{34}{r\sqrt{\varepsilon_r}}(3.45 + \frac{r}{h})GHz \quad TE_{01\delta} \text{ mode}$$

<u>Discussion</u>

1. Ex.4.7 A rectangular cavity made of WR-90 resonates at 9.379GHz in TE_{101} mode → length l and Q

$WR-90$ waveguide a = 0.9in = 2.286cm, b = 0.4in = 1.016cm

$$TE_{101} \text{ mode } 9.379\times 10^9 = \frac{3\times 10^8}{2\pi\sqrt{\mu\varepsilon}}\sqrt{(\frac{\pi}{2.286})^2 + (\frac{\pi}{l})^2} \to l = 2.238cm$$

$$(4.5.2)Q_c = 7858$$

2. Ex.4.8 A rectangular cavity with a=1.6cm, b=0.71cm, c=1.56cm is filled with Teflon → TE_{101} mode f_r and Q

$$f_r = \frac{3\times 10^8}{2\pi\sqrt{\mu\varepsilon}}\sqrt{(\frac{\pi}{1.6})^2 + (\frac{\pi}{1.56l})^2} = 9.375GHz$$

$$(4.5.2)Q_c = 5489, Q_d = \frac{1}{\tan\delta} = 3417, \frac{1}{Q} = \frac{1}{Q_c} + \frac{1}{Q_d} \to Q = 2106$$

3. Ex.4.9 A cylindrical cavity is operated at 5GHz in TE_{011}mode → Q and height h (h=2r)

$$\chi_{01} = p'_{01} = 3.832$$

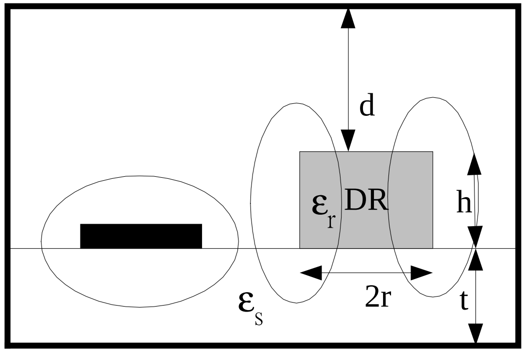
$$5\times 10^9 = \frac{c}{2\pi\sqrt{\mu\varepsilon}}\sqrt{(\frac{3.831}{r})^2 + (\frac{\pi}{2r})^2} \rightarrow r = 0.0395\,cm, h = 0.079\,cm$$

$$Q = 39984.6$$

4. DR selection in the operation with microstrip line

$$\frac{1.2892\times 10^8}{f_r\sqrt{\varepsilon_s}} > r > \frac{1.2892\times 10^8}{f_r\sqrt{\varepsilon_r}}$$

$$(4.5.17) \sim (4.5.23) \rightarrow h$$



5. Ex.4.10 DR er=36, substrate es=9.9, t=0.25mm→DR dimensions for 35GHz

$$1.17\,mm > r > 6.139, \rightarrow r = 0.835\,mm$$

$$(4.5.17) \sim (4.5.23) \rightarrow h = 0.668\,mm$$

Hw #3(due 2 weeks) 5, 7, 12, 15, 18