Chapter 2  Continuous-Wave Modulation

2.1 Introduction
2.2 Amplitude Modulation

\[ c(t) = A_c \cos(2\pi f_c t) \quad (2.1) \]

\[ A_c : \text{carrier amplitude} \]

\[ f_c : \text{carrier frequency} \]

\[ s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t) \quad (2.2) \]

Where \( m(t) \) is the baseband signal, \( k_a \) is the amplitude sensitivity.

1. \( |k_a m(t)| < 1, \) for all \( t \) \quad (2.3)

2. \( f_c >> W \) \quad (2.4)

where \( W \) is the highest frequency of \( m(t) \)
\[ s(t) = A_c \cos(2\pi f_c t) + A_c k_a m(t) \cos(2\pi f_c t) \]  

Recall

\[
\cos(2\pi f_c t) \iff \frac{1}{2} [\delta(f-f_c) + \delta(f+f_c)]
\]

\[ m(t) \cos(2\pi f_c t) \iff \frac{1}{2} [M(f-f_c) + M(f+f_c)] \]

\[ s(f) = \frac{A_c}{2} [\delta(f-f_c) + \delta(f+f_c)] + \frac{k_a A_c}{2} [M(f-f_c) + M(f+f_c)] \]  

where \( M(f) \) is the Fourier Transform of \( m(t) \)

1. Negative frequency component of \( m(t) \) becomes visible.
2. \( f_c - W < M(f) < f_c \) lower sideband
   \( f_c < M(f) < f_c + W \) upper sideband
3. Transmission bandwidth \( B_T = 2W \)
Virtues and Limitations of Amplitude Modulation

Transmitter

Receiver

Major limitations
1. AM is wasteful of power.
2. AM is wasteful of bandwidth.
2.3 Linear Modulation Schemes

Linear modulation is defined by

\[ s(t) = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t) \quad (2.7) \]

\[ s_I(t) = \text{In-phase component} \]

\[ s_Q(t) = \text{Quadrature component} \]

Three types of linear modulation:
1. Double sideband-suppressed carrier (DSB-SC) modulation
2. Single sideband (SSB) modulation
3. Vestigial sideband (VSB) modulation
### Table 2.1 Different forms of linear modulation

<table>
<thead>
<tr>
<th>Type of Modulation</th>
<th>In-Phase Component $s_1(t)$</th>
<th>Quadrature Component $s_Q(t)$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSB-SC</td>
<td>$m(t)$</td>
<td>0</td>
<td>$m(t) = \text{message signal}$</td>
</tr>
<tr>
<td>SSB:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Upper sideband</td>
<td>$\frac{1}{2}m(t)$</td>
<td>$\frac{1}{2}\hat{m}(t)$</td>
<td>$\hat{m}(t) = \text{Hilbert transform of } m(t)$</td>
</tr>
<tr>
<td>transmitted</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) Lower sideband</td>
<td>$\frac{1}{2}m(t)$</td>
<td>$-\frac{1}{2}\hat{m}(t)$</td>
<td></td>
</tr>
<tr>
<td>transmitted</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VSB:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Vestige of</td>
<td>$\frac{1}{2}m(t)$</td>
<td>$\frac{1}{2}m'(t)$</td>
<td></td>
</tr>
<tr>
<td>lower sideband</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>transmitted</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) Vestige of</td>
<td>$\frac{1}{2}m(t)$</td>
<td>$-\frac{1}{2}m'(t)$</td>
<td>$m'(t) = \text{output of the filter of}$</td>
</tr>
<tr>
<td>upper sideband</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>transmitted</td>
<td></td>
<td></td>
<td>due to $m(t)$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>For the definition of $H_Q(f)$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>see Eq. (2.16)</td>
</tr>
</tbody>
</table>

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**Notes:**

1. $s_1(t)$ is solely dependent on $m(t)$
2. $s_Q(t)$ is a filtered version of $m(t)$.
   
   The spectral modification of $s(t)$ is solely due to $s_Q(t)$.  
   
   [a] For the mathematical description of single sideband modulation, see Problem 2.16.
Double Sideband-Suppressed Carrier (DSB-SC) Modulation

\[ s(t) = A_c m(t) \cos(2\pi f_c t) \]  \hspace{1cm} (2.8)

The Fourier transform of \( S(t) \) is

\[ s(f) = \frac{1}{2} A_c \left[ M(f - f_c) + M(f + f_c) \right] \]  \hspace{1cm} (2.9)
Coherent Detection (Synchronous Detection)

The product modulator output is
\[ v(t) = A_c' \cos(2\pi f_c t + \phi) s(t) \]
\[ = A_c' A_c \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) m(t) \]
\[ = \frac{1}{2} A_c A_c' \cos(4\pi f_c t + \phi) m(t) + \frac{1}{2} A_c A_c' \cos(\phi) m(t) \] (2.10)\]

Let \( V(f) \) be the Fourier transform of \( v(t) \)
\[ v_0(t) = \frac{1}{2} A_c A_c' \cos \phi m(t) \quad \text{(Low pass filtered)} \] (2.11)
Costas Receiver

I-channel and Q-channel are coupled together to form a negative feedback system to maintain synchronization $\phi \approx 0$

$$\frac{1}{4} A_c^2 \cos \phi \sin \phi m^2(t) = \frac{1}{8} A_c^2 m^2(t) \sin(2\phi)$$

$$\approx \frac{1}{4} A_c^2 m^2(t) \phi \quad (\sin 2\phi \approx 2\phi)$$

The phase control signal ceases with modulation.

(multiplier + very narrow band LF)
Quadrature-Carrier Multiplexing (or QAM)

Two DSB-SC signals occupy the same channel bandwidth, where pilot signal (tone) may be needed.

\[ s(t) = A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t) \]
Single-Sideband Modulation (SSB)

The lower sideband and upper sideband of AM signal contain same information.

The frequency-discrimination method consists of a product modulator (DSB-SC) and a band-pass filter. The filter must meet the following requirements:

a. The desired sideband lies inside the passband.
b. The unwanted sideband lies inside the stopband.
c. The transition band is twice the lowest frequency of the message.

To recover the signal at the receiver, a pilot carrier or a stable oscillator is needed (Donald Duck effect).
Vestigial Sideband Modulation (VSB)
When the message contains near DC component

The transition must satisfy

\[ |H(f - f_c)| + |H(f + f_c)| = 1 \]

a. The phase response is linear:

\[ H(f - f_c) + H(f + f_c) = 1 \quad \text{for } -W \leq f \leq W \quad (2.13) \]

\[ B_T = W + f_v \quad (2.14) \]
Consider the negative frequency response:

$$|H(f)|$$

Here, the shift response $$|H(f-f_c)|$$ is
and \( |H(f+fc)| \) is \( |H(f + f_c)| \)
So, we get $|H(f-f_c)| + |H(f+f_c)|$ is
Consider \(-W<f<W\) we get:

\[v_f - W = 0\]

Which is equal to

\[\left| H(f - fc) \right| + \left| H(f + fc) \right| = 1 \quad \text{for} \quad -W<f<W\]
\[ s(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) \pm \frac{1}{2} A_c m'(t) \sin(2\pi f_c t) \]  \hfill (2.15)

± corresponds to upper or lower sideband

\[
H_Q(f) = j \left[ H(f - f_c) - H(f + f_c) \right] \quad \text{for } -W \leq f \leq W \quad (2.16)
\]
Television Signals (NTSC)

(a)

Maximum radiated field strength relative to picture carrier 1.0

0.75 MHz

1.25 MHz

4.5 MHz

0.25 MHz

Picture carrier

Sound carrier

$f(MHz)$

54 56 58 60

(b)

Normalized response

Picture carrier

Sound carrier

Channel bandwidth 6 MHz

$f(MHz)$

54 56 58 60

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2.4 Frequency Translation

Up conversion
\[ f_2 = f_1 + f_l, \quad f_l = f_2 - f_1 \]

Down conversion
\[ f_2 = f_1 - f_l, \quad f_l = f_1 - f_2 \]
2.5 Frequency-Division Multiplexing (FDM)

Transmitter
Carrier frequencies (in kHz) of voice inputs

Carrier frequencies (in kHz) of groups

Receiver

Voice band

Basic group of 12 voice inputs

Supergroup of 5 groups

552 kHz
5
612
504
564
456
516
408
468
360
420
312
60
2.6 Angle Modulation

Basic Definitions:
Better discrimination against noise and interference
(expense of bandwidth).

\[ s(t) = A_c \cos[\theta_i(t)] \quad \text{(2.19)} \]

The instantaneous frequency is

\[ f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t) \]

\[ = \lim_{\Delta t \to 0} \left[ \frac{\theta_i(t + \Delta t) - \theta_i(t)}{2\pi \Delta t} \right] \]

\[ = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} \quad \text{(2.21)} \]

For an unmodulated carrier, \( \theta_i(t) \) is

\[ \theta_i(t) = 2\pi f_c t + \phi_c \quad \text{(2.22)} \]

where \( \phi_c \) is constant
1. Phase modulation (PM)
   \[ \theta_i(t) = 2\pi f_c t + k_p m(t) \]
   
   \( k_p \): phase sensitivity of the modulator

   \[ s(t) = A_c \cos \left[ 2\pi f_c t + k_p m(t) \right] \] (2.23)

2. Frequency Modulation (FM)
   \[ f_i(t) = f_c + k_f m(t) \] (2.24)

   \[ \theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \] (2.25)

   \[ s(t) = A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right] \] (2.26)

   \( k_f \): frequency sensitivity of the modulator

   compare (2.23) and (2.26) \( \Rightarrow k_p m'(t) = 2\pi k_f \int_0^t m(\tau) d\tau \)

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Diagram:

- Modulating wave \( \rightarrow \) Integrator \( \rightarrow \) Phase modulator \( \rightarrow \) FM wave
- Modulating wave \( \rightarrow \) Differentiator \( \rightarrow \) Frequency modulator \( \rightarrow \) PM wave

(a) generating FM signal
(b) generating PM signal
2.7 Frequency Modulation

FM is a nonlinear modulation process, we can not apply Fourier transform to have spectral analysis directly.

1. Consider a single-tone modulation which produces a narrowband FM ($k_f$ is small)

2. Next consider a single-tone and wideband FM ($k_f$ is large)

\[
\text{let } m(t) = A_m \cos(2\pi f_m t) \quad (2.27) \quad \text{(deterministic)}
\]

\[
f_i(t) = f_c + k_f A_m \cos(2\pi f_m t)
\]

\[
= f_c + \Delta f \cos(2\pi f_m t) \quad (2.28)
\]

\[
\Delta f = k_f A_m : \text{frequency deviation}
\]
Recall (2.25), \[ \theta_i(t) = 2\pi \int_0^t f_i(\tau) d\tau \]

\[ = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \] (2.30)

Modulation index \[ \beta = \frac{\Delta f}{f_m} \] (2.31)

\[ \theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t) \] (2.32)

(2.19) => \[ s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \] (2.33)

Narrowband FM, \( \beta \) is smaller than one radian.

Wideband FM, \( \beta \) is larger than one radian.
Narrowband FM

\[ s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \]

\[ = A_c \cos(2\pi f_c t) \cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t) \sin[\beta \sin(2\pi f_m t)] \] (2.34)

Because \( \beta \) is small,

\[ \cos[\beta \sin(2\pi f_m t)] \approx 1 \]

\[ \sin[\beta \sin(2\pi f_m t)] \approx \beta \sin(2\pi f_m t) \]

\[ s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t) \] (2.35)
The output of Fig 2.21 is

\[ s'(t) = A_c \cos(2\pi f_c t) - A_c k_f \int m(\tau)d\tau \sin(2\pi f_c t) \]

\[ s(t) \] differs from ideal condition in two respects:

1. The envelope contains a residual AM.
   (FM has constant envelope)
2. \( \theta_i(t) \) contains odd order harmonic distortions

\[ \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \]

For narrowband FM, \( \beta \leq 0.3 \) radians.
Recall (2.35)
\[ s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t) \]  
\[ \approx A_c \cos(2\pi f_c t) + \frac{1}{2} \beta A_c \left\{ \cos[2\pi (f_c + f_m) t] - \cos[2\pi (f_c - f_m) t] \right\} \] (2.36)

For AM with sinusoidal modulating wave, \( m(t) = \cos(2\pi f_m t) \)
\[ s_{AM}(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t) \] (2.2)
\[ = A_c \cos(2\pi f_c t) + k_a A_c \cos(2\pi f_c t) \cos(2\pi f_m t) \]
\[ = A_c \cos(2\pi f_c t) + \frac{1}{2} \mu A_c \left\{ \cos[2\pi (f_c + f_m) t] + \cos[2\pi (f_c - f_m) t] \right\} \] (2.37)
Wideband FM (large $\beta$)

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \quad (2.33)$$

$\exp(jx) = \cos x + j \sin x$

$$s(t) = \Re[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))] \quad (2.38)$$

where $\Re[\ ]$ denotes the real part and

$\tilde{s}(t)$ is the complex envelope defined by

$$\tilde{s}(t) = A_c \exp[j\beta \sin(2\pi f_m t)] \quad (2.39)$$

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi nf_m t) \quad (2.40)$$

Complex Fourier Transform
\[ c_n = f_m \int_{-f_m}^{f_m} \tilde{s}(t) \exp(-j2\pi nf_m t) dt \]

\[ = f_m A_c \int_{-f_m}^{f_m} \exp \left[ j \beta \sin(2\pi f_m t) - j2\pi nf_m t \right] dt \quad (2.41) \]

Let \( x = 2\pi f_m t \)

\[ c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp \left[ j(\beta \sin x - nx) \right] dx \quad (2.43) \]

Define the \( n \)th order Bessel function of the first kind as

\[ (A3, x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2) y = 0) \]

\[ J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp \left[ j(\beta \sin x - nx) \right] dx \quad (2.44) \]

\[ c_n = A_c J_n(\beta) \]

\[ \tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j2\pi nf_m t) \quad (2.45) \]
s(t) = A_c \text{Re} \left[ \sum_{-\infty}^{\infty} J_n(\beta) \exp[j2\pi(f_c + nf_m)t] \right] \tag{2.47}

= A_c \sum_{-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)t] \tag{2.48}

The Fourier transform of $s(t)$ is

$$S(f) = \frac{A_c}{2} \sum_{-\infty}^{\infty} J_n(\beta) \left[ \delta(f - f_c - nf_m) + \delta(f + f_c + nf_m) \right]$$ \tag{2.49}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{bessel_functions_plot.png}
\caption{Plots of Bessel functions of the first kind for varying order.}
\end{figure}
Properties of $J_n(\beta)$

1. $J_n(\beta) = (-1)^n J_{-n}(\beta)$, for all $n$ \hspace{1cm} (2.50)

2. If $\beta$ is small
   
   $J_0(\beta) \approx 1$
   
   $J_1(\beta) \approx \frac{\beta}{2}$
   
   $J_n(\beta) \approx 0 \hspace{1cm} n > 2$ \hspace{1cm} (2.51)

3. $\sum_{-\infty}^{\infty} J_n^2(\beta) = 1$

Observation of FM

1. An FM signal contains $f_c, f_m, 2f_m, 3f_m, \ldots$ components.

2. For small $\beta$, the FM signal is effectively composed of a carrier and a single pair of side frequencies at $f_c \pm f_m \Rightarrow$ narrowband FM

3. The amplitude of carrier depends on $\beta$

   $$P = \frac{1}{2} A_c^2 = \frac{A_c^2}{2} \sum_{-\infty}^{\infty} J_n^2(\beta)$$ \hspace{1cm} (2.54)
Example 2.2

(a) 

\[ \beta = 1.0 \quad 2\Delta f \]

(b) 

\[ \beta = 2.0 \quad 2\Delta f \]

(c) 

\[ \beta = 5.0 \quad f_c \quad 2\Delta f \quad f_m \]
Transmission Bandwidth of FM signals

With a specified amount of distortion, the FM signal is effectively limited to a finite number of significant side frequencies.

A. Carson’s rule

\[ B_T \approx 2\Delta f + 2f_m = 2\Delta f \left(1 + \frac{1}{\beta}\right) \quad \beta = \frac{\Delta f}{f_m} \quad \Delta f = \beta f_m \quad (2.55) \]
B. \( B_T = 2n_{\text{max}} f_m, \quad |J_{n_{\text{max}}} (\beta)| \gg 0.01 \), \( B_T = 2n_{\text{max}} \frac{\Delta f}{\beta} \)

**Table 2.2** Number of significant side frequencies of a wideband FM signal for varying modulation index

<table>
<thead>
<tr>
<th>Modulation Index</th>
<th>Number of Significant Side Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>( 2n_{\text{max}} )</td>
</tr>
<tr>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>0.3</td>
<td>4</td>
</tr>
<tr>
<td>0.5</td>
<td>4</td>
</tr>
<tr>
<td>1.0</td>
<td>6</td>
</tr>
<tr>
<td>2.0</td>
<td>8</td>
</tr>
<tr>
<td>5.0</td>
<td>16</td>
</tr>
<tr>
<td>10.0</td>
<td>28</td>
</tr>
<tr>
<td>20.0</td>
<td>50</td>
</tr>
<tr>
<td>30.0</td>
<td>70</td>
</tr>
</tbody>
</table>

Universal curve for evaluating the 1 percent bandwidth of an FM wave
Example 2.3

In north America, the maximum value of frequency deviation $\Delta f$ is fixed at 75kHz for commercial FM broadcasting by radio. If we take the modulation frequency $W=15$kHz, which is typically the “maximum” audio frequency of interest in FM transmission, we find that corresponding value of the deviation ratio is

$$D = \frac{75}{15} = 5$$

Using Carson’s rule of Equation (2.55), replacing $\beta$ by $D$, and replacing $f_m$ by $W$, the approximate value of the transmission bandwidth of the FM signal is obtained as

$$B_T = 2(75+15) = 180$$kHz

On the other hand, use of the curve of Figure 2.26 gives the transmission bandwidth of the FM signal to be

$$B_T = 3.2 \Delta f = 3.2 \times 75 = 240$$kHz

In practice, a bandwidth of 200kHz is allocated to each FM transmission. On this basis, Carson’s rule underestimates the transmission bandwidth by 10 percent, whereas the universal curve of Figure 2.26 overestimates it by 20 percent.
Generation of FM signals

Baseband signal $m(t)$

Integrator

Narrowband phase modulator

Frequency multiplier

FM signal $s(t)$

Crystal-controlled oscillator

The frequency multiplier output

$\cos 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau$

Frequency Multiplier

$v(t) = a_1 s(t) + a_2 s^2(t) + \cdots + a_n s^n(t)$ \hspace{1cm} (2.56)

$\frac{dv}{dt} = 2\pi \dot{f}_c t + 2\pi k_f \int_0^t m(\tau) d\tau$

FM signal $s(t)$ with carrier frequency $f_c$ and modulation index $\beta$

Memoryless nonlinear device

Band-pass filter with midband frequency $n f_c$

FM signal $s'(t)$ with carrier frequency $n f_c$ and modulation index $n \beta$

$\frac{ds}{dt} = A_c' \cos \left[ 2\pi n f_c t + 2\pi n k_f \int_0^t m(\tau) d\tau \right]$ \hspace{1cm} (2.58)

$f_i'(t) = n f_c + n k_f m(t)$ \hspace{1cm} (2.59)
Varactor diode VCO FM modulator
Crosby Direct FM Transmitter

Modulating signal input → Frequency modulator and master oscillator $f_c = 5.1$ MHz $K_o$ → $f_c$ → $N_1 \times 3$ → $f_1$ → $N_2 \times 2$ → $f_2 = 30.6$ MHz → $N_3 \times 3$ → $f_t = 91.8$ MHz → Power amplifier → To antenna

AFC loop

- LPF
- Discriminator tuned to 2 MHz $K_d$
- BPF
- Mixer
- Crystal reference oscillator 14.3 MHz
- Buffer and $\times 2$ multiplier $N_4$

$f_d = 2$ MHz

$f = 28.6$ MHz
Demodulation of FM signals

The frequency discrimination consists of a slope circuit followed by an envelope detector.

Consider Fig 2.29a, the frequency response of a slope circuit is

\[
H_1(f) = \begin{cases} 
  j2\pi a(f - f_c + \frac{B_T}{2}), & f_c - \frac{B_T}{2} \leq f \leq f_c + \frac{B_T}{2} \\
  j2\pi a(f + f_c - \frac{B_T}{2}), & -f_c - \frac{B_T}{2} \leq f \leq -f_c + \frac{B_T}{2} \\
  0, & \text{elsewhere}
\end{cases} \quad (2.60)
\]
\[ H_1(f) = 2\pi \alpha \left\{ \begin{array}{ll} \frac{B_T}{2} & \text{for } f < -\frac{f_c}{2} - \frac{B_T}{2} \\ \frac{B_T}{2} & \text{for } f > -\frac{f_c}{2} + \frac{B_T}{2} \\ 0 & \text{otherwise} \end{array} \right. \]

Slope = \(2\pi \alpha\)

\[ H_2(f) = -2\pi \alpha \left\{ \begin{array}{ll} \frac{B_T}{2} & \text{for } f < -\frac{f_c}{2} - \frac{B_T}{2} \\ \frac{B_T}{2} & \text{for } f > -\frac{f_c}{2} + \frac{B_T}{2} \\ 0 & \text{otherwise} \end{array} \right. \]

Slope = \(-2\pi \alpha\)

\[ \tilde{H}_1(f - f_c) = 2H_1(f), \quad f > 0 \]

\[ \tilde{H}_2(f - f_c) = 2H_2(f), \quad f > 0 \]
Appendix 2.3 Hilbert Transform

Fourier Transform-frequency-selective

Hilbert Transform-phase-selective

(±90° shift)

Let \( g(t) \leftrightarrow G(f) \)

Denote the \textbf{Hilbert transform} of \( g(t) \) as

\[
\hat{g}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau} d\tau 
\]

\( (A2.31) \)

The inverse Hilbert transform

\[
g(t) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\hat{g}(\tau)}{t-\tau} d\tau 
\]

\( (A2.32) \)
\[
\frac{1}{\pi t} \leftrightarrow -j \text{sgn}(f) \quad \text{(A2.33)}
\]

\[
\text{sgn}(f) = \begin{cases} 
1 & f > 0 \\
0 & f = 0 \\
-1 & f < 0 
\end{cases} \quad \text{(A2.34)}
\]

*The Fourier transform of \( g(t) \) is*

\[
\hat{G}(f) = -j \text{sgn}(f)G(f) \quad \text{(A2.35)}
\]
Properties of the Hilbert Transform
(time domain operation)

If \( g(t) \) is real

1. \( \hat{g}(t) \) and \( g(t) \) have the same magnitude spectrum

2. Hilbert transform of \( \hat{g}(t) \) is \( -g(t) \) (take H.F of \( g(t) \) and compare with A2.32)

3. \( \int_{-\infty}^{\infty} g(t) \hat{g}(t) dt = 0 \implies g(t) \perp \hat{g}(t) \)
For a **band-pass system**, we consider

\[ x(t) \Leftrightarrow X(f) \]

\( X(f) \) is limited within \( \pm W \) Hz

\( W \ll f_c \)

\[ x(t) = x_I(t) \cos(2\pi f_c t) - x_Q(t) \sin(2\pi f_c t) \]  \( \text{(A2.48)} \)

**The complex envelope of** \( x(t) \) **is**

\[ \tilde{x}(t) = x_I(t) + j x_Q(t) \]  \( \text{(A2.49)} \)

\[ h(t) = h_I(t) \cos(2\pi f_c t) - h_Q(t) \sin(2\pi f_c t) \]  \( \text{(A2.50)} \)
Define the complex impulse response
\[ \tilde{h}(t) = h_I(t) + j h_Q(t) \]  
(A2.51)

The complex representation of \( h(t) \)
\[ h(t) = \text{Re}\left[ \tilde{h}(t) \exp(j2\pi f_c t) \right] \]  
(A2.52)

\( h_I(t), h_Q(t) \) and \( \tilde{h}(t) \) are low-pass functions

From (A2.52) we have \( z = v + ju, 2v = z + z^* \)

\[ 2h(t) = \tilde{h}(t) \exp(j2\pi f_c t) + \tilde{h}^*(t) \exp(-j2\pi f_c t) \]  
(A2.53)

Apply Fourier transform to (A2.53)
\[ 2H(f) = \tilde{H}(f - f_c) + \tilde{H}^*(-f - f_c) \]  
(A2.54)

Since \( h(t) \) is real
\[ H^*(f) = H(-f) \]

and \( \tilde{H}(f) \) is limited to \( |f| \leq B \) with \( B < f_c \)
\[ \Rightarrow \tilde{H}(f - f_c) = 2H(f), f > 0 \]  
(A2.55)

We can obtain \( \tilde{H}(f) \) from \( H(f) \), \( \tilde{H}(f') = 2H(f' + f_c) \)
Define the pre-envelope of $h(t)$ as

$$h_+(t) = h(t) + j\hat{h}(t), \quad \hat{h}(t): \text{ Hilbert T. of } h(t)$$

$$H_+(f) = H(f) + \text{sgn}(f)H(f)$$

$$H_+(f) = \begin{cases} 
2H(f) & \text{ if } f > 0 \\
H(0) & \text{ if } f = 0 \\
0 & \text{ if } f < 0 
\end{cases} \quad (A2.37)$$

$$y(t) = \int_{-\infty}^{\infty} \Re[h_+(\tau)]\Re[x_+(t-\tau)]d\tau \quad (A2.59)$$

$$(A2.58) \Rightarrow y(t) = \Re[\tilde{y}(t)\exp(j2\pi f_0 t)]$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$$(A2.57)$$
Recall $h_+(t) = h(t) + j\hat{h}(t)$

$h(t) = \text{Re}[h_+(t)]$

$x(t) = \text{Re}[x_+(t)]$

To prove (A2.60)

\[
\text{Re} \left[ \int_{-\infty}^{\infty} h_+(\tau)x_+(t-\tau)d\tau \right]
\]

\[
= \text{Re} \left[ \int_{-\infty}^{\infty} [h(\tau) + j\hat{h}(\tau)][x(t-\tau) + j\hat{x}(t-\tau)]d\tau \right]
\]

\[
= \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau - \int_{-\infty}^{\infty} \hat{h}(\tau)\hat{x}(t-\tau)d\tau
\]

\[
= \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau - \int_{-\infty}^{\infty} \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{\tau-u} h(u)\hat{x}(t-\tau)d\tau du,
\]

\[
= \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau + \int_{-\infty}^{\infty} \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{-1}{t-h(u)} \hat{x}(\nu)d\nu h(u)du
\]

\[
= \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau + \int_{-\infty}^{\infty} h(u)x(t-u)du
\]

\[
= 2\int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau
\]

\[
= 2\int_{-\infty}^{\infty} \text{Re}[h_+(\tau)] \text{Re}[x_+(t-\tau)]d\tau
\]
(A2.58) becomes

\[ y(t) = \int_{-\infty}^{\infty} \text{Re}[h_+(\tau)] \text{Re}[x_+(t-\tau)] \, d\tau \quad (A2.59) \]

\[ = \frac{1}{2} \text{Re} \left[ \int_{-\infty}^{\infty} h_+(\tau) x_+(t-\tau) \, d\tau \right] \]

\[ = \frac{1}{2} \text{Re} \left[ \int_{-\infty}^{\infty} \tilde{h}(\tau) \exp(j2\pi f_c \tau) \tilde{x}(t-\tau) \exp(j2\pi f_c(t-\tau)) \, d\tau \right] \]

\[ = \frac{1}{2} \text{Re} \left[ \exp(j2\pi f_c t) \int_{-\infty}^{\infty} \tilde{h}(\tau) \tilde{x}(t-\tau) \, d\tau \right] \]
\[ F\{e^{j2\pi nf_c t}\} = \int_{-\infty}^{\infty} e^{j2\pi nf_c t} e^{-j2\pi f_c t} dt \]

\[ = \int_{-\infty}^{\infty} e^{-j2\pi (f-nf_c) t} dt \]

\[ = \delta(f - nf_c) \]
(2) \[ F\{e^{j2\pi mf_c t}\} = \int_{-\infty}^{\infty} e^{j2\pi f_c k} e^{-j2\pi \frac{f}{n}} \frac{dk}{n}, \text{令 } nt = k, \ dt = \frac{dk}{n} \]

\[ = \left\{ \begin{array}{ll}
\int_{-\infty}^{\infty} e^{j2\pi f} e^{-j2\pi \frac{f}{n}} \frac{dk}{n}, & n > 0 \\
\int_{-\infty}^{\infty} e^{j2\pi f} e^{-j2\pi \frac{f}{n}} \frac{dk}{n}, & n < 0
\end{array} \right. \]

\[ = \left\{ \begin{array}{ll}
\frac{1}{n} \int_{-\infty}^{\infty} e^{-j2\pi \left(\frac{f}{n} - f_c\right)k} dk, & n > 0 \\
\frac{-1}{n} \int_{-\infty}^{\infty} e^{-j2\pi \left(\frac{f}{n} - f_c\right)k} dk, & n < 0
\end{array} \right. \]

\[ = \left\{ \begin{array}{ll}
\frac{1}{n} \delta\left(\frac{f}{n} - f_c\right), & n > 0 \\
\frac{-1}{n} \delta\left(\frac{f}{n} - f_c\right), & n < 0
\end{array} \right. \]

\[ = \frac{1}{|n|} \delta\left(\frac{f}{n} - f_c\right) = \delta(f - nf_c) \]
Comparing (A2.57) and (A2.61) we have

\[ 2\tilde{y}(t) = \int_{-\infty}^{\infty} \tilde{h}(\tau)\tilde{x}(t - \tau) \, d\tau \]  \hspace{1cm} (A2.62)

or \[ 2\tilde{y}(t) = \tilde{h}(t) \ast \tilde{x}(t) \]  \hspace{1cm} (A2.63)

We can represent bandpass signals and systems by the equivalent lowpass functions \( \tilde{x}(t), \tilde{y}(t) \) and \( \tilde{h}(t) \) without the factor \( \exp(j2\pi f_c t) \)
\[ 2\tilde{y}(t) = \left[ hI(t) + jhQ(t) \right] \ast \left[ xI(t) + jxQ(t) \right] \quad (A2.64) \]
\[ = \left[ hI(t) \ast xI(t) - hQ(t) \ast xQ(t) \right] \]
\[ + j \left[ hQ(t) \ast xI(t) + hI(t) \ast xQ(t) \right] \quad (A2.65) \]

let \( \tilde{y}(t) = \tilde{y}_I(t) + j\tilde{y}_Q(t) \) \quad (A2.66)

\[ 2y_I(t) = hI(t) \ast xI(t) - hQ(t) \ast xQ(t) \quad (A2.67) \]
\[ 2y_Q(t) = hQ(t) \ast xI(t) + hI(t) \ast xQ(t) \quad (A2.68) \]
Procedure for evaluating the response of a band-pass system

1. Replace $x(t)$ by $\tilde{x}(t)$
   $$x(t) = \text{Re}[\tilde{x}(t)\exp(j2\pi f_c t)]$$

2. $h(t) = \text{Re}[\tilde{h}(t)\exp(j2\pi f_c t)]$

3. Obtain $2\tilde{y}(t) = \tilde{h}(t) * \tilde{x}(t)$

4. $y(t) = \text{Re}[\tilde{y}(t)\exp(j2\pi f_c t)]$
To simplify the analysis

1. shift $\tilde{H}_1(f)$ to the right by $f_c$ to align to the band-pass frequency

2. set $\tilde{H}_1(f - f_c) = 2H_1(f)$, for $f > 0$ \hspace{1cm} (2.61)

Recall

$$H_1(f) = \begin{cases} 
  j2\pi a(f - f_c + \frac{B_T}{2}) & f_c - \frac{B_T}{2} \leq f \leq f_c + \frac{B_T}{2} \\
  j2\pi a(f + f_c - \frac{B_T}{2}) & -f_c - \frac{B_T}{2} \leq f \leq -f_c + \frac{B_T}{2} \\
  0 & \text{elsewhere}
\end{cases} \hspace{1cm} (2.60)$$

From (2.60) and (2.61), we get

$$\tilde{H}_1(f) = \begin{cases} 
  j4\pi a(f + \frac{B_T}{2}) & -\frac{B_T}{2} \leq f \leq \frac{B_T}{2} \\
  0 & \text{elsewhere}
\end{cases} \hspace{1cm} (2.62)$$
Recall FM signal \( s(t) \)

\[
s(t) = A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right]
\]

The complex envelope is

\[
\tilde{s}(t) = A_c \exp \left[ j2\pi k_f \int_0^t m(\tau) d\tau \right] \quad (2.63)
\]

Let \( \tilde{s}_1(t) \) denote the complex envelope of the slope ckt. response output.

Recall (A2.63) \( 2\tilde{y}(t) = \tilde{h}(t) \ast \tilde{x}(t) \), we have

\[
\tilde{S}_1(f) = \frac{1}{2} \tilde{H}(f) \tilde{S}(f) \quad \text{(upper arm of Fig 2.30 in text)}
\]

\[
= \begin{cases} 
  j2\pi a(f + \frac{B_T}{2}) \tilde{S}(f) & -\frac{B_T}{2} \leq f \leq \frac{B_T}{2} \\
  0 & \text{elsewhere}
\end{cases} \quad (2.64)
\]

\[
\Rightarrow \tilde{s}_1(t) = a \left[ \frac{d}{dt} \tilde{s}(t) + j\pi B_T \tilde{s}(t) \right] \quad (2.65)
\]

From (2.63) and (2.65), we have

\[
\tilde{s}_1(t) = j\pi B_T aA_c \left[ 1 + \frac{2k_f}{B_T} m(t) \right] \exp \left[ j2\pi k_f \int_0^t m(\tau) d\tau \right] \quad (2.66)
\]
\[ s_1(t) = \text{Re}\left[ \tilde{s}_1(t) \exp(j2\pi f_c t) \right] \]
\[ = \pi B_T a A_c \left[ 1 + \frac{2k_f}{B_T} m(t) \right] \cos \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau + \frac{\pi}{2} \right] \]
\[ - \sin \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right] \]

\( s_1(t) \) is a hybrid-modulated signal (amplitude, frequency)

However, provided that we choose \( \left| \frac{2k_f}{B_T} m(t) \right| < 1 \), for all \( t \)

using an envelope detector, we have

\[ \left| \tilde{s}_1(t) \right| = \pi B_T a A_c \left[ 1 + \frac{2k_f}{B_T} m(t) \right] \]

(2.68)

The bias term \( \pi B_T a A_c \) can be removed by a second frequency discriminator with \( H_2(f) \), where \( \tilde{H}_2(f) = \tilde{H}_1(-f) \).
Balanced Frequency Discriminator

Let the transfer function of the second branch of Fig 2.30 be (complementary slope circuit)

\[ \tilde{H}_2(f) = \tilde{H}_1(-f) \]  \hspace{1cm} (2.69)

\[ |\tilde{s}_2(t)| = \pi B_T a A_c \left[ 1 - \frac{2k_f}{B_T} m(t) \right] \] \hspace{1cm} (2.70)

\[ s_0(t) = |\tilde{s}_1(t)| - |\tilde{s}_2(t)| \]

\[ = 4\pi k_f a A_c m(t) \] \hspace{1cm} (2.71)
FM Stereo Multiplexing

Two factors which influence FM stereo standards
1. Operation within the allocated FM channels.
2. Compatible with monophonic radio receiver.

\[ m(t) = [m_l(t) + m_r(t)] + [m_l(t) - m_r(t)] \cos(4\pi f_c t) + K \cos(2\pi f_c t) \quad (2.72) \]
Figure 9-40. FM stereo generation block diagram.
In Figure 9-40, audio signals from both left and right microphones are combined in an linear matrixing network to produce an $L+R$ signal and an $L-R$ signal.

Both $L+R$ and $L-R$ are signals in the audio band and must be separated before modulating the carrier for transmission. This is accomplished by translating the $L-R$ audio signal up in the spectrum.

As seen in Figure 9-40, the frequency translation is achieved by amplitude-modulating a 38-kHz subsidiary carrier in a balanced modulator to produce DSB-SC.
Stereo FM transmitter using frequency-division multiplexing.
Stereo FM transmitter: (a) block diagram; (b) resulting spectrum. SAC: Subsidiary Communication Authorization
Stereo FM

- The stereo receiver will need a frequency-coherent 38-kHz reference signal to demodulate the DSB-SC.

- To simplify the receiver, a frequency- and phase-coherent signal is derived from the subcarrier oscillator by frequency division (÷2) to produce a pilot.

- The 19-kHz pilot fits nicely between the L+R and DSB-SC L-R signals in the baseband frequency spectrum.
Stereo FM

- As indicated by its relative amplitude in the baseband composite signal, the pilot is made small enough so that its FM deviation of the carrier is only about 10% of the total 75-kHz maximum deviation.

- After the FM stereo signal is received and demodulated to baseband, the 19-kHz pilot is used to phase-lock an oscillator, which provides the 38-kHz subcarrier for demodulation of the $L-R$ signal.

- A simple example using equal frequency but unequal amplitude audio toned in the $L$ and $R$ microphones is used to illustrate the formation of the composite stereo (without pilot) in Figure 9-41.
Figure 9-41. Development of composite stereo signal. The 38 kHz alternately multiplies $L-R$ signal by +1 and $-1$ to produce the DSB-SC in the balanced AM modulator (part d). The adder output (shown in e without pilot) will be filtered to reduce higher harmonics before FM modulation.
Stereo FM

Spectrum of stereo FM signal.

SCA: Subsidiary communication authorization (commercial-free program)
2.8 Nonlinear Effects in FM Systems

1. Strong nonlinearity, e.g., square-law modulators, hard limiter, frequency multipliers.
2. Weak nonlinearity, e.g., imperfections

Nonlinear input-output relation

\[ v_0(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) \]  \hspace{1cm} (2.73)
For FM signal

\[ v_i(t) = A_c \cos[2\pi f_c t + \phi(t)] \]

\[ \phi(t) = 2\pi k_f \int_0^t m(\tau)d\tau \]

\[ v_0(t) = a_1 A_c \cos[2\pi f_c t + \phi(t)] + a_2 A_c^2 \cos^2[2\pi f_c t + \phi(t)] \]
\[ + a_3 A_c^3 \cos^3[2\pi f_c t + \phi(t)] \quad (2.74) \]

\[ = \frac{1}{2} a_2 A_c^2 + (a_1 A_c + \frac{3}{4} a_3 A_c^3) \cos[2\pi f_c t + \phi(t)] \]

\[ + \frac{1}{2} a_2 A_c^2 \cos[4\pi f_c t + 2\phi(t)] \]

\[ + \frac{1}{4} a_3 A_c^3 \cos[6\pi f_c t + 3\phi(t)] \quad (2.75) \]
Carson's rule, $B_T = 2\Delta f + 2f_m = 2\Delta f + 2W$

In order to separate the desired FM signal from the second harmonic, we have

$$2f_c - (2\Delta f + W) > f_c + \Delta f + W$$

$$f_c > 3\Delta f + 2W$$ \hspace{1cm} (2.76)$$

The output of the band-pass filter is

$$v_0'(t) = (a_1A_c + \frac{3}{4}a_3A_c^3)\cos[2\pi f_c t + \phi(t)] \hspace{0.5cm} \text{(no effect to } m(t))$$

An FM system is extremely sensitive to phase nonlinearities. Common type of source: AM-to-PM conversion.
2.9 Super Heterodyne Receiver

(Carrier-frequency tuning, filtering, amplification, and demodulation)

\[ f_{IF} = f_{LO} - f_{RF} \]  \hspace{1cm} (2.78)

A FM system may use a limiter to remove amplitude variations.

<table>
<thead>
<tr>
<th>Table 2.3 Typical frequency parameters of AM and FM radio receivers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AM Radio</strong></td>
</tr>
<tr>
<td>RF carrier range</td>
</tr>
<tr>
<td>Midband frequency of IF section</td>
</tr>
<tr>
<td>IF bandwidth</td>
</tr>
</tbody>
</table>
Commercial FM Broadcast allocations and Sidebands

![Diagram showing FM broadcast allocations and guard bands.](image.png)
2.10 Noise in CW modulation System

1. **Channel model**: additive white Gaussian noise (AWGN)

2. **Receiver model**: a band-pass filter followed by an ideal demodulator

The PSD of $w(t)$ is denoted by $\frac{N_0}{2}$. 

![Diagram showing a CW modulation system with channel and receiver models, and the PSD of noise]
The filtered noise in narrowband noise representation:
\[ n(t) = n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t) \]  \hspace{1cm} (2.79)

The filtered signal for demodulation is
\[ x(t) = s(t) + n(t) \] \hspace{1cm} (2.80)

The channel signal-to-noise ratio
\[ (\text{SNR})_c = \frac{\text{average power of } s(t)}{\text{average power of } n(t)} \]

The output signal-to-noise ratio
\[ (\text{SNR})_o = \frac{\text{average power of the demodulated signal}}{\text{average power of noise at the output}} \]

Figure of merit \[= \frac{(\text{SNR})_o}{(\text{SNR})_c} \] \hspace{1cm} (2.81)
2.11 Noise in Linear Receiver Using Coherent Detection

The DSB-SC system

\[ s(t) = CA_c \cos(2\pi f_c t)m(t) \quad m(t) \Leftrightarrow S_M(f) \]

\[ P = \int_{-W}^{W} S_M(f)df \]  \hspace{2cm} (2.83)

\[ (SNR)_{C,DSB} = \frac{C^2 A_c^2 P}{2WN_0} \]

\[ = \frac{C^2 A_c^2 P}{2WN_0} \quad (baseband) \]  \hspace{2cm} (2.84)

\(C\): system dependent scaling factor
\[ x(t) = s(t) + n(t) \]
\[ = CA_c \cos(2\pi f_c t) m(t) + n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t) \quad (2.85) \]

\[ v(t) = x(t) \cos(2\pi f_c t) \]
\[ = \frac{1}{2} CA_c m(t) + \frac{1}{2} n_I(t) \]
\[ + \frac{1}{2} [CA_c m(t) + n_I(t)] \cos(4\pi f_c t) - \frac{1}{2} n_Q(t) \sin(4\pi f_c t) \]

\[ \text{high frequency components} \]

Low - pass filter \( \Rightarrow \)
\[ y(t) = \frac{1}{2} CA_c m(t) + \frac{1}{2} n_I(t) \quad (2.86) \]

(2.86) indicates :

1. \( m(t) \) and \( n_I(t) \) are additive at the receiver output.

2. \( n_Q(t) \) is completely rejected by the coherent detector.
The average output signal \( \left( \frac{1}{2} CA_c m(t) \right) \) power = \( C^2 A_c^2 \frac{P}{4} \)

Let \( B_T = 2W \)

The average noise \( \left( \frac{1}{2} n_I(t) \right) \) power = \( \left( \frac{1}{2} \right)^2 2WN_0 = \frac{1}{2} WN_0 \)

\[
(SNR)_{O,DSB-SC} = \frac{C^2 A_c^2 P}{W N_0^2/2} = \frac{C^2 A_c^2 P}{2WN_0} \quad (2.87)
\]

\[
\frac{(SNR)_O}{(SNR)_C}_{DSB-SC} = 1 \quad (2.88)
\]

1. Coherent SSB has the same figure of merit of DSB-SC
2. No trade-off between performance and bandwidth.

    Serious problem!
2.12 Noise in AM Receivers Using Envelope Detection

\[ s(t) = A_c \left[ 1 + k_a m(t) \right] \cos(2\pi f_c t) \]  
\[ = A_c \cos(2\pi f_c t) + A_c k_a m(t) \cos(2\pi f_c t) \]  

\[ (\text{SNR})_{C, \text{AM}} = \frac{A_c^2 (1 + k_a^2 P)}{2WN_0} \]  

At the output of the filter:

\[ x(t) = s(t) + n(t) \]  
\[ = \left[ A_c + A_c k_a m(t) + n_I(t) \right] \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t) \]
\[ y(t) = \text{envelope of } x(t) \]
\[ = \left\{ \left[ A_c + A_ck_a m(t) + n_l(t) \right]^2 + n_Q^2(t) \right\}^{1/2} \quad (2.92) \]

Assume \(|A_c + A_ck_a m(t) + n_l(t)| \gg |n_Q(t)|\)

\[ y(t) = A_c + A_ck_a m(t) + n_l(t) \]

1. \(\frac{A_c^2}{2} \gg WN_0\) (carrier power > noise power)

2. \(|k_a| < 1\)

\[ (SNR)_{O, AM} \approx \frac{A_c^2 k_a^2 P}{2WN_0} \quad (2.94) \]

\[ \left(\frac{SNR}{SNR}_C\right)_{AM} \approx \frac{k_a^2 P}{1 + k_a^2 P} \quad (2.95) \]
Supplements

Define the pre-demodulation SNR as

\[
\text{SNR pre-de} = \frac{\text{The average power of the modulated signal}}{\text{The average noise power at the input of the demodulator}}
\]

The Bandwidth of the bandpass filter is \( B_T \) then the average noise power at the input of the demodulator is \( N_o B_T \)

For an AM system

\[
\text{SNR}_{\text{pre-de}}^{\text{AM}} = \frac{A_c^2 (1 + k_a^2 p)}{2N_o B_T} = \frac{A_c^2 (1 + k_a^2 p)}{2N_o B_T}
\]

If \( B_T = 2W \)

\[
\text{SNR}_{\text{pre-de}}^{\text{AM}} = \frac{A_c^2 (1 + k_a^2 p)}{4N_o W}
\]
Supplements

For a DSB-SC system,

\[
\text{SNR} = C^2 \frac{A_c^2 \cdot P}{N_o B_T} = \frac{C^2 A_c^2 \cdot p}{4N_o W}
\]

For an FM system

\[
\text{SNR} = C^2 \frac{A_c^2}{N_o B_T} = \frac{A_c^2}{2N_o B_T}
\]

If using Carson’s rule, we have

\[
B_T = 2\Delta f + 2f_m >> f_m = w
\]

For the purpose of comparing different CW modulation systems, we define

\[
\text{(SNR)}_c = \frac{\text{The average power of the modulated signal}}{\text{The average power of channel noise in the message band}}
\]

Message signal with
the same power as
modulated wave

LP filter

with bandwidth w

output

noise

n(t)

The equivalent baseband transmission model.
Supplements

More precisely, we may express the DSB-SC as

\[ \cos(2\pi f_c t + \theta) \]

\( \theta \) is uniformly distributed over \([0, 2\pi]\)

\[ S'(t) = A_c m(t) \cos(2\pi f_c t + \theta) \]

At the receiver we may write

\[ S(t) = C A_c m(t) \cos(2\pi f_c t + \theta) \]

\[ P_s = E[S^2(t)] = R_s(O) \]

\[ = \int_{-\infty}^{\infty} S_x(f) df \]

\[ = E[(C A_c m(t) \cos(2\pi f_c t + \theta))^2] \]

\[ = C^2 A_c^2 E[\cos^2(2\pi f_c t + \theta)] E[m^2(t)] \]

\[ = C^2 A_c^2 R_m(O) / 2 = C^2 A_c^2 P / 2 \]

\[ R_m(O) = P = \int_{-w}^{w} S_m(f) df \]

The average noise power in \(-w < f < w\)

\[ P_n = \int_{-w}^{w} \frac{N_0}{2} df = N_o W \]
Supplements

\[ \text{SNR}_c = \frac{\text{The average power of } S(t)}{\text{The average power of channel noise in the message band}} \]

\[ = \frac{\text{The average power of the modulated signal}}{\text{The average power of channel noise in the message band}} \]

\[ = \frac{P_s}{P_n} = \frac{C^2 A_c^2 P}{2N_0 W} \]

For convenience we write the modulated signal as

\[ S(t) = CA_c m(t) \cos(2\pi f_c t) \]

Since \( \cos(2\pi f_c t + \theta) \) is ergodic and we take \( \cos(2\pi f_c t) \) as a sample function

\[ P_s = C^2 A_c^2 R_m(0) \quad [\text{time average of } [\cos^2(2\pi f_c t)]] \]

\[ = C^2 A_c^2 P / 2 \]

\[ \text{SNR}_c = \frac{C^2 A_c^2 P / 2}{N_0 W} = \frac{C^2 A_c^2 P}{2N_0 W} \]
Example 2.4 Single-Tone Modulation

Consider the special case of a sinusoidal wave of frequency $f_m$ and amplitude $A_m$ as the modulating wave, as shown by

$$m(t) = A_m \cos(2\pi f_m t)$$

The corresponding AM wave is

$$s(t) = A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

where $\mu = k_a A_m$ is the modulation factor. The average power of the modulating wave $m(t)$ is (assuming a load resistor of 1 ohm)

$$P = \frac{1}{2} A_m^2$$

Therefore, using Equation (2.95), we get

$$\left. \frac{(SNR)_O}{(SNR)_c} \right|_{AM} = \frac{\frac{1}{2} k_a^2 A_m^2}{1 + \frac{1}{2} k_a^2 A_m^2}$$

$$= \frac{\mu^2}{2 + \mu^2}$$

When $\mu = 1$, which corresponds to 100 percent modulation, we get a figure of merit equal to 1/3. This means that, other factors being equal, an AM system (using envelope detection) must transmit three times as much average power as a suppressed-carrier system (using coherent detection) to achieve the same quality of noise performance.
Threshold Effect

\[(\text{SNR})_O \approx \rho \]

\[(\text{SNR})_O \approx 0.91\rho^2 \]

noise power > carrier power
2.13 Noise in FM Receivers

The discriminator consists of a slope network and an envelope detector.

Let \( n(t) = n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t) \)

\[
n(t) = r(t) \cos[(2\pi f_c t) + \psi(t)]
\]

(2.130)

The envelope is \( r(t) = \left[ n_I^2(t) + n_Q^2(t) \right]^{1/2} \)

(2.131)

The phase is \( \psi(t) = \tan^{-1} \left[ \frac{n_Q(t)}{n_I(t)} \right] \)

(2.132)

where \( r(t) \) is Rayleigh distributed, and \( \Psi(t) \) is uniform distributed over \( 2\pi \).

\[
f_R(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad r \geq 0
\]

(1.115)

\[
f_{\psi}(\psi) = \frac{1}{2\pi}, \quad 0 < \psi < 2\pi
\]

(1.114)
The incoming FM signal $s(t)$ is defined by

$$s(t) = A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right]$$

(2.133)  

$$= A_c \cos \left[ 2\pi f_c t + \phi(t) \right]$$

(2.135)

where $\phi(t) = 2\pi k_f \int_0^t m(\tau) d\tau$  

(2.134)

At the bandpass filter output

$$x(t) = s(t) + n(t)$$

$$= A_c \cos \left[ 2\pi f_c t + \phi(t) \right] + r(t) \cos \left[ 2\pi f_c t + \psi(t) \right]$$

(2.136)

where $A_c \gg |r(t)|$

$$\theta(t) = \phi(t) + \tan^{-1} \left\{ \frac{r(t) \sin [\psi(t) - \phi(t)]}{A_c + r(t) \cos [\psi(t) - \phi(t)]} \right\}$$

(2.137)
Note that the envelope of \( x(t) \) is of no interest to us (limiter)

Because \( A_c >> |r(t)| \)

\[
\theta(t) \approx \phi(t) + \frac{r(t)}{A_c} \sin[\psi(t) - \phi(t)] \tag{2.138}
\]

\[
= 2\pi k_f \int_0^t m(\tau) d\tau + \frac{r(t)}{A_c} \sin[\psi(t) - \phi(t)] \tag{2.139}
\]

The discriminator output is (Fig 2.40)

\[
v(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = k_f m(t) + n_d(t) \tag{2.140}
\]

where

\[
n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} \left\{ r(t) \sin[\psi(t) - \phi(t)] \right\} \tag{2.141}
\]
Assume $\psi(t) - \phi(t)$ is uniformly distributed over $(0, 2\pi)$, then $n_d(t)$ is independent of message signal.

We may simplify $n_d(t)$ as

$$n_d(t) \approx \frac{1}{2\pi A_c} \frac{d}{dt} \{ r(t) \sin[\psi(t)] \} \quad (2.142)$$

From definition of $r(t)$ and $\psi(t)$, we have

$$n_Q(t) = r(t) \sin[\psi(t)] \quad (2.143)$$

$$n_d(t) \approx \frac{1}{2\pi A_c} \frac{dn_Q(t)}{dt} \quad (2.144)$$

The quadrature component appears
From (2.140)
The average output signal power = $k_f^2P$
Recall
\[
\frac{d}{dt} \iff j2\pi f
\]

\[
\begin{array}{c}
n_Q(t) \quad S_{N_Q}(f) \quad \frac{1}{2\pi A_c} \frac{d}{dt} \\
\downarrow & \downarrow & \downarrow \\
n_d(t) \quad S_{N_d}(f)
\end{array}
\]

\[
S_{N_d}(f) = \frac{f^2}{A_c^2} S_{N_Q}(f) \quad (2.145)
\]

noise is enhanced at high frequency
Assume that $n_Q(t)$ has ideal low-pass characteristic with bandwidth $B_T$

$$S_{N_d}(f) = \frac{N_0 f^2}{A_c^2}, \quad |f| \leq \frac{B_T}{2} \quad (2.146)$$

If $\frac{B_T}{2} > W$

At the receiver output

$$S_{N_0}(f) = \frac{N_0 f^2}{A_c^2}, \quad |f| \leq W \quad (2.147)$$
Average power of $n_0(t) = \frac{N_0}{A_c^2} \int_{-W}^{W} f^2 df$

$$= \frac{2N_0W^3}{3A_c^2} \quad (2.148)$$

$$\propto \frac{1}{A_c^2} \text{ noise quieting effect}$$

$$\left(\text{SNR}\right)_{o, FM} = \frac{3A_c^2k_f^2P}{2N_0W^3}$$

The average power of $s(t)$ is $\frac{A_c^2}{2}$,

the average noise power in message bandwidth is $WN_0$

$$\Rightarrow \left(\text{SNR}\right)_{C, FM} = \frac{A_c^2}{2WN_0} \quad (2.150)$$

$$\Rightarrow \frac{\left(\text{SNR}\right)_{o}}{\left(\text{SNR}\right)_{C, FM}} = \frac{3k_f^2P}{W^2} \quad (2.151)$$

$$\Delta f = k_f A_m \quad (2.29) \quad (\text{SNR})_{o, FM} \propto (\Delta f)^2$$
Example 2.5 Single-Tone Modulation

\[ s(t) = A_c \cos \left[ 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \right] \]

We may write, \( 2\pi k_f \int_0^t m(\tau) d\tau = \frac{\Delta f}{f_m} \sin(2\pi f_m t) \)

\[ \frac{d}{dt} \text{ both side} \Rightarrow m(t) = \frac{\Delta f}{k_f} \cos(2\pi f_m t) \]

The average power of \( m(t) \) (across 1Ω load) is

\[ P = \frac{(\Delta f)^2}{2k_f^2} \]

From (2.149), \( (SNR)_{o,FM} = \frac{3A_c^2(\Delta f)^2}{4N_0W^3} = \frac{3A_c^2\beta^2}{4N_0W} \), \( \beta = \frac{\Delta f}{W} \)

\[ \Rightarrow \left. \frac{(SNR)_o}{(SNR)_c} \right|_{FM} = \frac{3}{2} \left( \frac{\Delta f}{W} \right)^2 = \frac{3}{2} \beta^2 \quad (2.152) \]

compare to AM, \( \left. \frac{(SNR)_o}{(SNR)_c} \right|_{AM} = \frac{1}{3} \) (from Example 2.4)

When \( \frac{3}{2} \beta^2 > \frac{1}{3} \), FM has better performance.

\[ \Rightarrow \beta > \frac{\sqrt{2}}{3} = 0.471 \]

Define \( \beta = 0.5 \) as the transition between narrowband FM and wideband FM.
FM Threshold Effect (When CNR is low)

The composite signal at the frequency discriminator input

\[ x(t) = [A_c + n_I(t)]\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \quad (2.153) \]

\[ \theta(t) = \tan^{-1} \left( \frac{n_Q(t)}{A_c + n_I(t)} \right) \]

Occasionally, \( P_1 \) may sweep around the origin, \( (r(t) > A_c) \)
\( \theta(t) \) increases or decreases \( 2\pi \)

The discriminator output is equal to \( \frac{\theta'(t)}{2\pi} \)
Figure 2.44 Illustrating impulselike components in $\theta'(t) = d\theta(t)/dt$ produced by changes of $2\pi$ in $\theta(t)$; (a) and (b) are graphs of $\theta(t)$ and $\theta'(t)$, respectively.
A positive-going click occurs, when

\[ r(t) > A_c, \quad \psi(t) < \pi \leq \psi(t) + d\psi(t) , \quad \frac{d\psi(t)}{dt} > 0 \]

A negative-going click occurs when

\[ r(t) > A_c, \quad \psi(t) > -\pi > \psi(t) + d\psi(t) , \quad \frac{d\psi(t)}{dt} < 0 \]

The carrier-to-noise ratio is defined by

\[ \rho = \frac{A_c^2}{2B_T N_0} \quad (2.154) \]

The output signal-to-noise ratio is calculated as

1. The average output signal power is calculated assuming

   a sinusoidal modulation which produces \( \Delta f = \frac{B_T}{2} \). (noise free)

2. The average output noise power is calculated when no
   signal is present (The carrier is unmodulated).
\[ (\text{SNR})_o = 3\rho \left( \frac{B_T}{2W} \right)^3 \]

\[ \frac{B_T}{2W} = 5 \]

**Figure 2.45** Dependence of output signal-to-noise ratio on input carrier-to-noise ratio for FM receiver. In curve I, the average output noise power is calculated assuming an unmodulated carrier. In curve II, the average output noise power is calculated assuming a sinusoidally modulated carrier. Both curves I and II are calculated from theory.

When \[ \rho = \frac{A_c^2}{2B_T N_0} \geq 20 \] or \[ \frac{A_c^2}{2} \geq 20B_T N_0 \] (2.155), threshold effects may be avoided
The procedure to calculate minimum $A_c \ (\rho \geq 20)$

1. Given $\beta$ and $W$, determine $B_T$
   (using Figure 2.26 or Carson's rule)

2. Given $N_0$, we have $\frac{A_c^2}{2} \geq 20B_TN_0$

Capture Effect:

The receiver locks onto the stronger signal
and suppresses the weaker one.
FM Threshold Reduction (tracking filter)

- FM demodulator with negative feedback (FMFB)
- Phase locked loop

**Figure 2.46**
FM threshold extension.

**Figure 2.47**
FM demodulator with negative feedback.
Pre-emphasis and De-emphasis on FM

Figure 2.48 (a) Power spectral density of noise at FM receiver output. (b) Power spectral density of a typical message signal.

Figure 2.49 Use of pre-emphasis and de-emphasis in an FM system.
\[ H_{de}(f) = \frac{1}{H_{pe}(f)} , \quad -W \leq f \leq W \]  

(2.156)

The PSD at the discriminator output is

\[ S_{N_d}(f) = \frac{N_0f^2}{A_c^2} , \quad |f| \leq \frac{B_T}{2} \]  

(2.146)

\[ |H_{de}(f)|^2 S_{N_d}(f) = \frac{N_0f^2}{A_c^2} |H_{de}(f)|^2 , \quad |f| \leq \frac{B_T}{2} \]  

(2.157)

\[
\left( \text{Average output noise power with de-emphasis} \right) = \frac{N_0}{A_c^2} \int_{-W}^{W} f^2 |H_{de}(f)|^2 \, df
\]  

(2.158)

The improvement factor \( I \) is

\[ I = \frac{2W^3}{3 \int_{-W}^{W} f^2 |H_{de}(f)|^2 \, df} \]  

(2.162)
Example 2.6  

**Figure 2.50** (a) Pre-emphasis filter.  
(b) De-emphasis filter.

A simple pre-emphasis filter response is

\[ H_{pe}(f) = 1 + \frac{jf}{f_0} \]

A de-emphasis filter response is

\[ H_{de}(f) = \frac{1}{1 + \frac{jf}{f_0}} \]

\[ I = \frac{2W^3}{3 \int_{-w}^{w} \frac{f^2 df}{1 + (\frac{f}{f_0})^2}} = \frac{(W/f_0)^3}{3 \left[ (W/f_0) - \tan^{-1}(W/f_0) \right]} \quad (2.161) \]
The main difference between FM and PM is in the relationship between frequency and phase.

\[ f = \frac{1}{2\pi} \cdot \frac{d\theta}{dt} \]

A PM detector has a flat noise power (and voltage) output versus frequency (power spectral density). This is illustrated in Figure 9-38a.

However, an FM detector has a parabolic noise power spectrum, as shown in Figure 9-38b. The output noise voltage increases linearly with frequency.

If no compensation is used for FM, the higher audio signals would suffer a greater S/N degradation than the lower frequencies. For this reason compensation, called emphasis, is used for broadcast FM.
Preemphasis for FM

Figure 9-38. Detector noise output spectra for (a). PM and (b). FM.
A preemphasis network at the modulator input provides a constant increase of modulation index $m_f$ for high-frequency audio signals.

Such a network and its frequency response are illustrated in Figure 9-39.

Fig. 9-39. (a) Preemphasis network, and (b) Frequency response.
Preemphasis for FM

- With the RC network chosen to give $\tau = R_1C = 75\mu s$ in North America ($150\mu s$ in Europe), a constant input audio signal will result in a nearly constant rise in the VCO input voltage for frequencies above 2.12 kHz. The larger-than-normal carrier deviations and $m_f$ will preemphasize high-audio frequencies.

- At the receiver demodulator output, a low-pass RC network with $\tau = RC = 75\mu s$ will not only decrease noise at higher audio frequencies but also deemphasize the high-frequency information signals and return them to normal amplitudes relative to the low frequencies.

- The overall result will be nearly constant $S/N$ across the 15-kHz audio baseband and a noise performance improvement of about 12dB over no preemphasis. Phase modulation systems do not require emphasis.
Pre-emphasis and De-emphasis on FM

Pre-emphasis and deemphasis: (a) schematic diagrams; (b) attenuation curves
Pre-emphasis and De-emphasis on FM

Example of S/N without preemphasis and deemphasis.
Example of S/N with preemphasis and deemphasis.
Dolby dynamic preemphasis
Figure 2.55 Comparison of the noise performance of various CW modulation systems. Curve I: Full AM, $\mu = 1$. Curve II: DSB-SC, SSB. Curve III: FM, $\beta = 2$. Curve IV: FM, $\beta = 5$. (Curves III and IV include 13-dB pre-emphasis, de-emphasis improvement.)
In making the comparison, it is informative to keep in mind the transmission bandwidth requirement of the modulation systems in question. Therefore, we define normalized transmission bandwidth as

\[ B_n = \frac{B_T}{W} \]

Table 2.4 Values of \( B_n \) for various CW modulation schemes

|                | AM, DSB-SC | SSB | FM  
|----------------|------------|-----|-----
| \( B_n \)      | 2          | 1   | \( \beta = 2 \)  
|                |            |     | \( \beta = 5 \)  
|                |            |     | 8   | 16  |
李家同教授—我的恩师

【聯合報／李家同】

2011.11.03 02:23 am

我虽然学问普通，但的确有一点值得骄傲的地方，我虽然已年过七十三，仍然一直在学新东西。我过去对通讯完全不懂，后来因为在暨南大学的校长职务垮台了，驾驶也没有了，我找到了两位教授一齐开车去南投，在路上，老是听到他们讲有关通讯的东西，我本来就对通讯好奇，就一直问他们，问到后来，居然还写了一本有关通讯的教科书。

我发现我之所以到老了，仍能吸收新知识，乃是因为我好奇心很重，好奇心使我对很多事情有兴趣，当然我对电机和资讯的领域最有兴趣。两位教授教我了我通讯以后，我对类比线路线设计非常好奇，也就靠我的东问西问，现在也能教类比线路设计了。
可是我並不是一直都有好奇心的，小的時候，我根本是一個糊裡糊塗的小孩，老師教什麼，我並不完全懂，可是我並不想真正搞懂這門學問，我的哲學是：我要知道如何應付老師的考試，這一點最重要，而我對如何應付考試是很有經驗的。

可是，我的運氣很好，碰到了一位好老師。

我從大學畢業以後，就想找一個安穩的工作，以度一生。我運氣很好，可以到一所國中（當初叫作初中）去教理化，孩子們很乖，也很用功，該背的都會去背，該做的練習也都會去做。所以我這個老師感覺真好。
没有想到的是，我碰到了一位好奇心非常重的孩子，他叫陳義明，他第一次發問，是我教他公制的時候，我說我們長度用公尺，他馬上發問：「公尺是根據什麼制定的？」我去大學圖書館查了資料，上課時告訴他公尺是根據法國的鄧刻爾刻到西班牙巴塞隆納的距離制定的。他的問題又來了：「如此長距離的測量是怎麼完成的？」我又被他問倒了，研究了好久才搞清楚。但這一次我告訴他這種測量牽涉到很多初中生不懂的數學，他就算了。

不久，我教到元素，照教科書上的定義，元素是用一般化學方法無法分解的物質，這位好奇的孩子的問題又來了，他說：「什麼是一般化學方法？」我只好舉了一些例子講給他聽，他的問題仍然不斷：(1) 古人怎麼知道一個物質是元素的？(2) 誰給元素下定義的？(3) 是誰最早將所知的元素整理出來？說實話，我花了很大的功夫才將這些問題搞清楚。
等到我教到質量和重量的時候，就知道陳義明一定會問何謂「重量」，當然我們還沒有教過力學，要解釋「重量」差一點把我的半條命送掉。

自從陳義明一直好奇地問問題，他的同學們也慢慢地被傳染到了，他們的問題越來越多，我也越來越有成就。不能說我的學生們考試時一定會得心應手，而是說顯然他們對科學越來越好奇。因為好奇，他們變得很會思考，在過去，這批孩子們全部都是背書的機器，也難怪，背多分也。

就在我教得興致高昂的時候，問題來了，有些家長發現他們的孩子回家以後會和他們談一些他們聽都沒有聽過的玩意兒，比方說，很多孩子會解釋給爸媽會道耳中如何能提出原子說。家長們聊天時發現這些都是教科書上沒有提到的，他們非常擔憂，因為教科書裡沒有的學問，不論多有意義，都是不會考的。他們認為我做老師的人應該教聯招考試的項目，那些不考的，就不該教，教了反而浪費孩子的時間，也影響到孩子們未來考試的成績。
校長將我抓去罵了一頓，他說，學校的績效就在於有多少學生考上明星高中。我問他使學生對科學產生興趣，重不重要？他不願正面回答我的問題，但警告我絕對不可以再在上課期間教那些不考的東西。我問他學生如果提問題，怎麼辦？他說一定要在課後解釋給他聽，上課期間不要理他。

從此以後，我們上課時變成了單行道，可是下課以後，孩子們的問題就來了，上課時雖然是一攤死水，下課以後就活了起來。但我覺得我自己好像在做一件見不得人的事，每天都鬼鬼祟祟的。而且我也有一種不舒服的感覺，因為同事們有點將我看成異類。
陳義明的好奇心不限在科學上，他在歷史課上也問題不斷。有一次，他問何謂元朝，其實是不是我們已經亡國了？又有一次，他問唐朝時代，是不是人人都有工作可做？有多少人失業？他另一個問題更有趣了，漢朝時，中國有多少人口？最精采的問題是：「中國」這個名字是什麼時候來的？因為他知道清朝時，我們國家叫作「大清帝國」，並非「中國」。

陳義明使同事们分裂成兩派，一派很喜歡他，另一派討厭他。討厭他的老師占大多數，他們感到陳義明對他們是一種威脅，但又不敢直接了當地說出來。他們都想過安穩日子，陳義明使他們感到日子不好過。

陳義明畢業了，他沒有考上任何一所公立學校，他的家境使他無法念學費高的私立學校，於是他就結束了學業，四處去找工作做，總不能遊手好閒啊。
陳義明事件對我是一個很大的打擊，第一，我覺得自己的學問實在太差，第二，我很不喜歡我們不鼓勵學生有好奇心的教育。我決定繼續念書，希望能使自己變得有學問一點，將來能教一批有好奇心的學生。

我得到了博士學位，也做到了教授。我開始指導研究生，其中有很多都是博士生，他們當然都是好問之徒也。唯一和過去不同之處，在於他們從不問我，他們的那些問題，我如何能回答？好在他們都能靠自己的努力找到答案。對他們而言，「死背」是一點用都沒有的。我感覺到我總算如願以償。

有一天，我的中古汽車有些毛病，友人告訴我，有一家修車廠技術很高，價格也算公道，我就開車去了。這家修車廠規模相當大，而且一塵不染，管理也井然有序，我當然要等一下，他們也有顧客休息室。我在裡面休息的時候，忽然進來了一個人，他親熱地叫我李老師，我一看，就認出了他，他就是陳義明。
陳義明告訴我，他初中畢業以後，就去到汽車修理廠的學徒，由於他好奇心很重，學得非常快，他也去念了補校。高中念完了，他決定自己開修車廠，果真生意很好，有的時候，汽車公司的修理廠還會偷偷地來求救於他，他笑著說，可惜他沒有資金，否則他也可以設計汽車了。

我問他是不是仍然十分好奇，他反而覺得難為情起來，他將我請到了他的辦公室，在他的電腦裡找出了一份檔案，裡面記錄了所有他要問的問題，如果有答案，他會記錄在裡面，我發現大多數都仍是懸案。我現在將他的幾個懸案寫在下面：

（1）電影裡面動物的嘴巴如何會動？

陳義明說他常看電影，很多電影裡有動物明星，這些電影明星往往會講話，這不稀奇，只要配音就可以了，但為什麼這些動物的嘴巴會動呢？如果嘴巴不動，就不像動物在講話了。他猜動物是真的，嘴巴是動畫，但他沒有把握，所以仍是懸案。
（2）森林小孩如何能和獅子搏鬥？

陳義明顯然是個愛看電影的人，他說他常看到電影裡有一位英勇強壯的小男孩，他居然敢和獅子搏鬥，最後獅子被他殺死。他知道這頭獅子是被馴服的，牠只是在表演而已，但獅子畢竟是獅子，小男孩是個明星，雖然身體還不錯，但他怎麼敢和獅子打架呢？萬一獅子兇性大發，他怎麼辦？

（3）九一一事件的謎

九一一事件是大家有興趣的事，對陳義明來說，他說他有眾多疑點：1、為什麼美國機場會在幾個小時之內，同時讓十幾位恐怖分子進入飛機，他們都帶有武器，一個恐怖分子成為漏網之魚已經很怪，這麼多人同時順利過關，這簡直不可思議；2、為什麼幾架飛機的飛行員沒有立刻報告有人劫機？3、飛行員的座艙門是打不開的，他們是如何進入的？4、飛機當時一定已在自動飛行的狀態之下，恐怖分子怎麼會有如此大的本領能將自動飛行改為手動？他們並非真正有執照的飛行員，只受過幾個月的飛行訓練而已。
（4）花為什麼同時開、同時謝？

陳義明有時到一座公園去散步，他看見兩個花壇種了同樣的花，令他百思不解的是同一個花壇的花會同時開、同時謝。但不同花壇的花開和謝的時間是不一樣的，要注意的是這些都是同樣的合歡花。

我只能舉這些例子，因為陳義明的問題太多了。他看到了我，好高興，他說我是最鼓勵他發問的老師，而且也認為他的好奇心對他日後幫助很大，他一再稱我為恩師。

我要在這裡告訴陳義明，你才是我的恩師，我雖然老了，仍對很多事情極有好奇心，這應該歸功於你，沒有你的話，我雖然會看到應該感到困惑的事，卻不會感到困惑。
我也希望年輕人知道：好奇心是十分重要的，我們都知道外國跑車的引擎好厲害，但如果沒有一個人想知道這種引擎是怎樣製造的，那我們就永遠不可能有厲害的引擎了。

我更希望老師們知道，如果你們的學生學了庫倫定律，而不過問庫倫如何測量電量，我們國家是不太可能有偉大的科學家的。

【2011/11/03 聯合報】@ http://udn.com/

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