National Taiwan University
Department of Electrical Engineering
Algorithms, Fall 2009

Handout #6
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Sample Solutions to Quiz #1

1. $\sqrt{n}^{\lg n} > \sqrt{n}(\lg n)^3 > n^{1/3} > (\lg n)^{30}$
   (Method: applying lg operation to each functions)

2. (a) $T(n) = \sqrt{n} \cdot T(n/2) + \sqrt{n} = \Theta(n^{1/2} \cdot \lg n)$.
   Since $f(n) = \sqrt{n} = \Theta(n^{\log_b a}) = \Theta(n^{\log_2 \sqrt{2}}) = \Theta(n^{1/2})$, Case 2 of master theorem applies.

   (b) According to Case 1 of the Master Theorem, $a = 3, b = 2, f(n) = n/\lg n$
   $f(n) = O(n^{\lg 3-\epsilon})$ for $0 < \epsilon < \lg 5 - 2$
   Therefore, $f(n) = \Theta(n^{\lg 3})$

   (c) Construct the recurrence tree as Figure 1.
   $O(n) = \sum_{k=0}^{\lg n}(n \cdot \frac{5}{6}^k) = O(n)$,
   $\Omega(n) = \sum_{k=0}^{\log_3 n}(n \cdot \frac{5}{6}^k) = \Omega(n)$.
   Thus, $T(n) = \Theta(n)$.

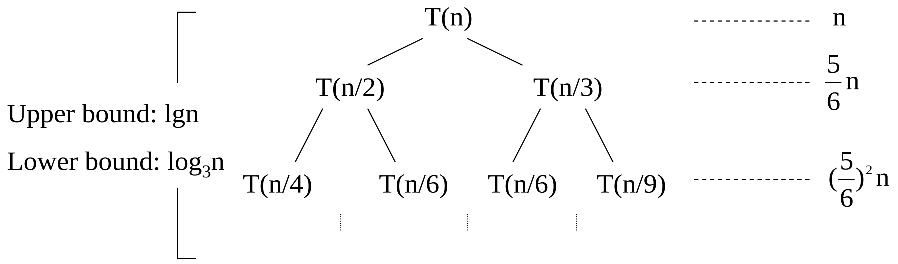

Figure 1: The recursion tree for Problem 2.

3. Correct!
   $f(n) = 2n^2 + 5n - 8$
   $0 \leq f(n) \leq cn^2$, for all $n \geq n_0 \Longrightarrow$ the smallest integer constant: $c = 3$.
   $0 \leq 2n^2 + 5n - 8 \leq 3n^2$, for all $n \geq n_0 \Longrightarrow$ the smallest integer constant: $n_0 = 2$.
   Hence, $2n^2 + 5n - 8 = O(n^2)$ is correct.