#1491: Introduction to Digital Systems

Instructor: Yao-Wen Chang

National Chiao Tung University
Department of Computer & Information Science

Spring 1998

Administrative Matters

- **Time/Location:** Mondays 3:40–4:30pm, Wednesdays 10:10am–12:00pm; ED 302; #1491.
- **Instructor:** Yao-Wen Chang.
- **E-mail:** ywchang@cis.
- **URL:** http://www.cis.nctu.edu.tw/~ywchang.
- **Office:** EC 125B. (Tel) 5712121 Ext. 56606; (Fax) 5721490.
- **Office Hours:** Mondays 4:40–5:40pm; other times by appointment.
- **Teaching Assistants:** Yue-Tsang Chang (gis86558@cis); Wen-Hao Shiue (gis86569@cis).
- **Prerequisites:** Introduction to Computer Sciences.
- **Goals:** To be able to design and analyze digital logic systems by understanding formal foundations and popular design techniques.

Administrative Matters (Cont'd)

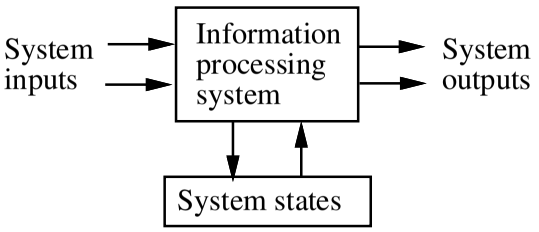
- **Required Text:** M. M. Mano and C. R. Kime, *Logic and Computer Design Fundamentals*, Prentice Hall, 1997. ISBN: 0-13-206780-3.
- **References:**
  - D. D. Gajski, *Principles of Digital Design*, Prentice Hall, 1997.
  - R. H. Katz, *Contemporary Logic Design*, Benjamin/Cummings, 1994.
  - C. H. Roth, Jr, *Fundamentals of Logic Design*, 4th Ed., PWS, 1995.
  - J. F. Wakerley, *Digital Design Principles and Practices*, Prentice Hall, 1990.
- **Course Contents:**
  - Digital computers and information
  - Boolean algebra, combinational (combinatorial) logic
  - Sequential (synchronous & asynchronous) logic
  - Memory and programmable logic devices
  - Register transfers and datapaths
  - Digital systems

Administrative Matters (Cont'd)

- **Grading:**
  - Bi-weekly homework assignments: 20%
  - Two 50-min quizzes: 20% (**March 16th; May 4th**)
  - Two 110-min midterms: 45% (**April 8th ; June 3rd**)
  - Final project & presentation: 15% (due **June 10**, tentative)
- **Homework:** Penalty for late submission: 50% per day.
- **Academic Honesty:** Avoid *cheating* at all cost.
- **WWW:** http://www.cis.nctu.edu.tw/∼ywchang/Courses/Log98/log98.html.
  - Required web browser: **ghostview**
- **Personal Information:** E-mail me your name, student ID, **4–6 character Secret ID**.

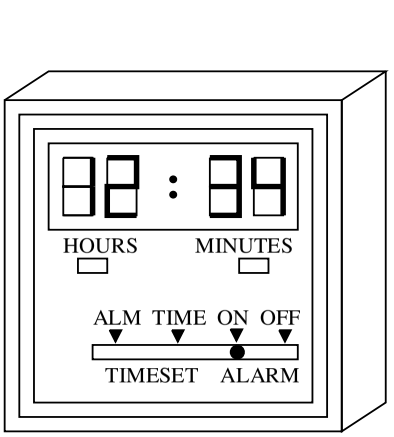
What is a "Digital/Analog System"

- **Digital system:** Takes a set of **discrete** information inputs and **discrete** internal information (system states) and generates a set of **discrete** information outputs.
- **Analog system:** Inputs, outputs, and states can take any value within a given range.
- **Analog-to-digital (A/D) converter:** Takes analog inputs and generates digital outputs.
- **Digital-to-analog (D/A) converter:** Takes digital inputs and generates analog outputs.
- **States:**
  - No state $\Longrightarrow$ Combinational (combinatorial) system: $Output := f(Inputs)$.
  - State present: State updated at **discrete** times $\Longrightarrow$ **Synchronous sequential** system; State updated at **any** time $\Longrightarrow$ **Asynchronous sequential** system.

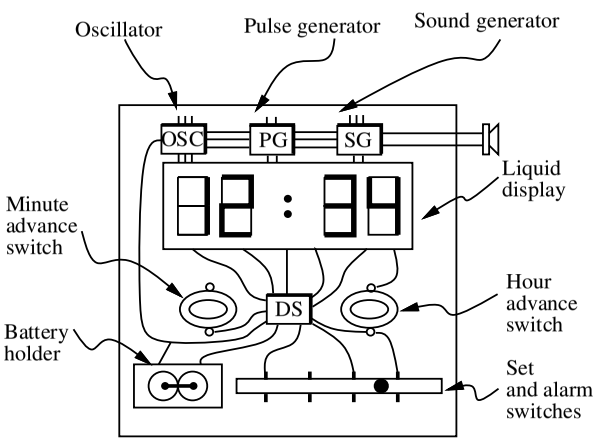


Mixed Digital/Analog System Example: Alarm Clock

- Outputs: visual display, sound; Inputs? States?
- Alarm clock **physical** representation:



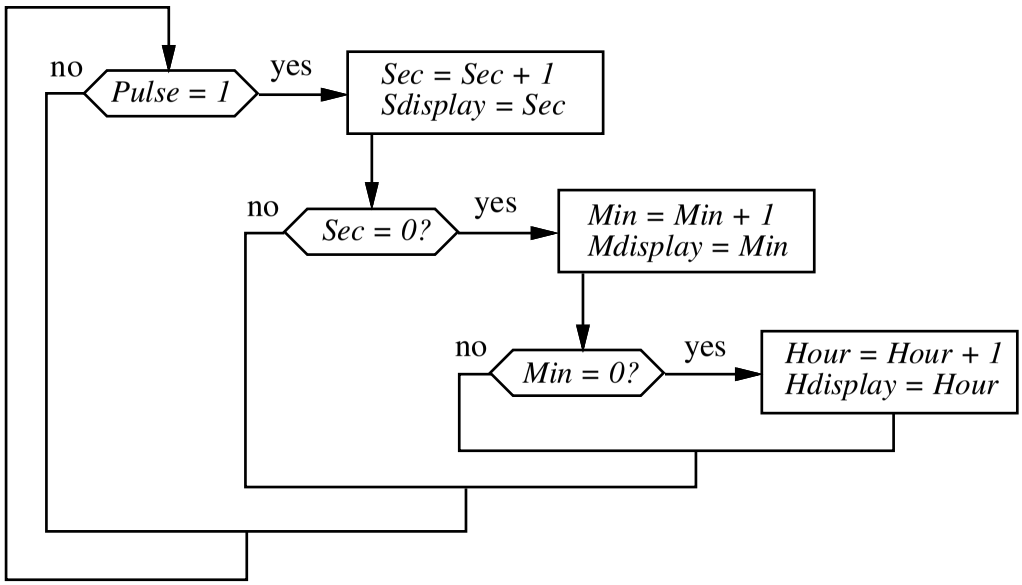
Front view



Printed circuit board

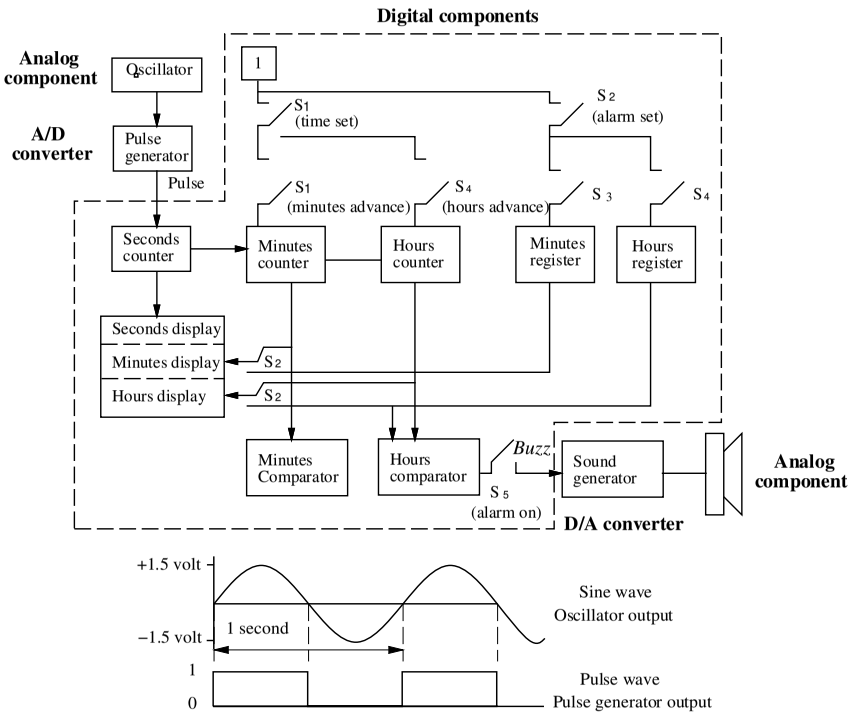
Mixed Digital/Analog System Example: Alarm Clock

- Alarm clock **behavioral** (**functional**) representation: clock process



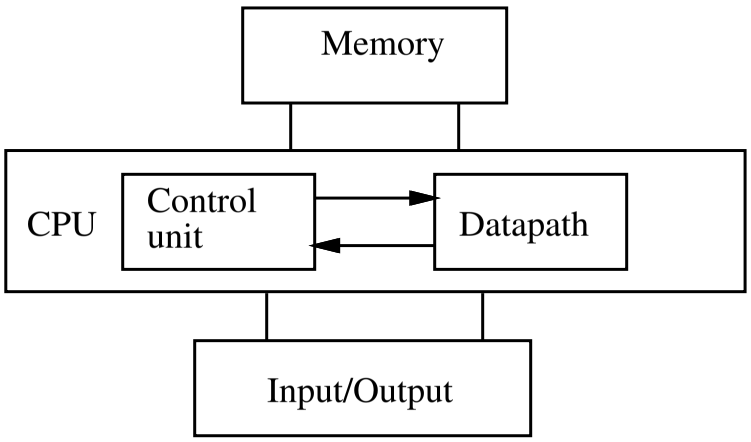
Mixed Digital/Analog System Example: Alarm Clock

- Alarm clock **structural** representation.

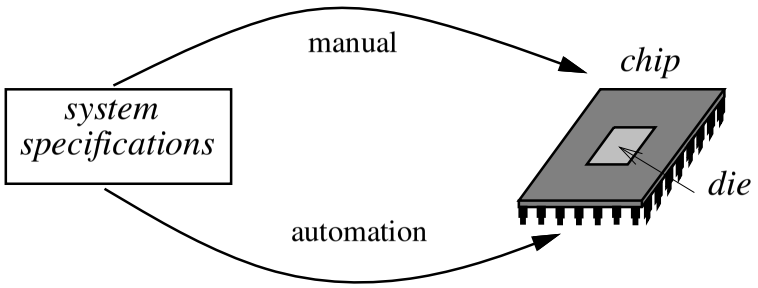


Digital Computer Example

- Inputs: Data from keyboard, disk drive, floppy, microphone, etc.
- Outputs: Data from screen, disk drive, floppy, sound card, etc.
- State: Data in memory, registers.
- Synchronous or asynchronous?



Design Considerations



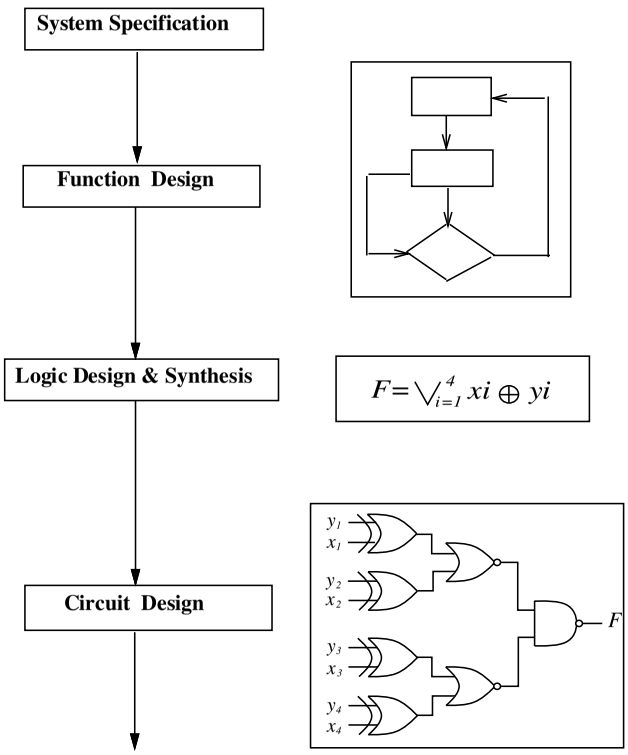
- Several conflicting considerations
  - **Design complexity:** large number of devices/transistors
  - **Performance:** optimization requirements for high performance
  - **Time-to-market:** about a 15% gain for early birds
  - **Cost:** die area, packaging, testing, etc

Traditional Design Cycle

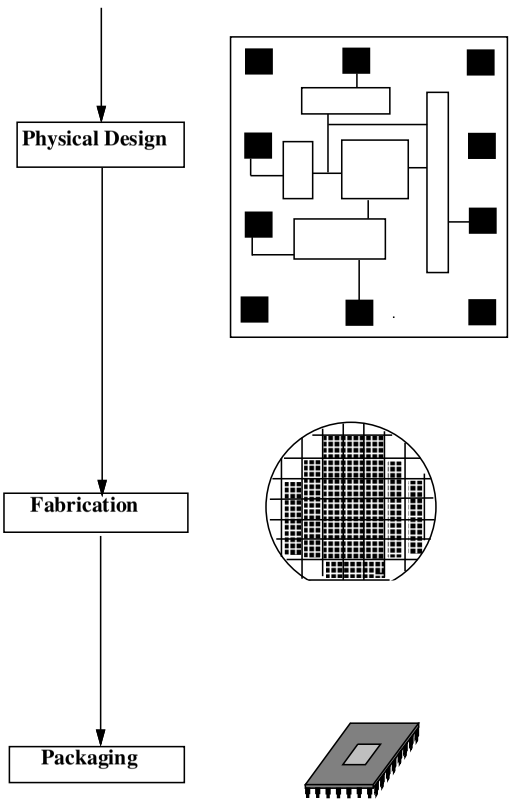
1. System specification
2. Functional design
3. **Logic design and synthesis**
4. Circuit design
5. Physical Design
6. Fabrication
7. Packaging

- Other tasks involved: Design verification, Testing, simulation

Traditional Design Flow

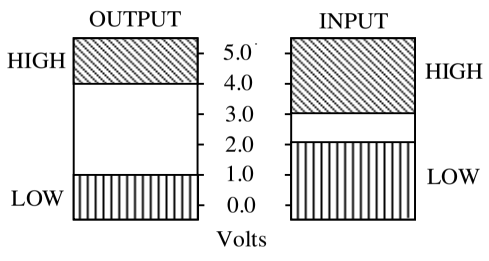


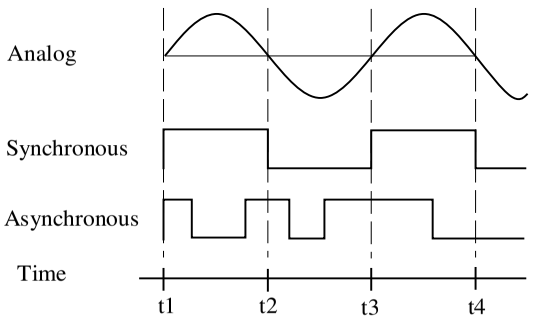
Traditional Design Flow (Cont'd)



Signals

- A collection of information variables mapped to some physical quantity.
- For digital systems, the quantities take on discrete values.
- Two level (**binary**) values are the most popular values in digital systems.





Number Systems

- Decimal (base-10) numbers: $(9876.5)_{10} = 9 \times 10^3 + 8 \times 10^2 + 7 \times 10^1 + 6 \times 10^0 + 5 \times 10^{-1}$.
- Binary (base-2) numbers: Only the digits 0 and 1 are needed; $(11010.1)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} = (26.5)_{10}$.

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096	20	1,048,576
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

- K (kilo): 2^{10}; M (mega): 2^{20}; G (giga): 2^{30}; T (tera): 2^{40}.

Binary-Decimal Conversion

- Binary $\rightarrow$ Decimal: Use decimal arithmetic to sum the weighted powers of two.
  - $(11010.1)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} = 16 + 8 + 0 + 2 + 0 + 0.5 = (26.5)_{10}$.
- Decimal $\rightarrow$ Binary:
  - **S1:** Find the largest power of two that you can subtract and still be positive.
  - **S2:** Subtract it and note the power.
  - **S3:** Repeat until the remainder is zero.
  - $(625)_{10} = 2^9 + 2^6 + 2^5 + 2^4 + 2^0 = (1001110001)_2$.

$$
\begin{array}{rrcrrl}
2^9 = & 512 & \Rightarrow & 625\text{-}512 = & 113 = & N_1 \\
2^6 = & 64 & \Rightarrow & 113\text{-}64 = & 49 = & N_2 \\
2^5 = & 32 & \Rightarrow & 49\text{-}32 = & 17 = & N_3 \\
2^4 = & 16 & \Rightarrow & 17\text{-}16 = & 1 = & N_4 \\
2^0 = & 1 & \Rightarrow & 1\text{-}1 = & 0 = & N_5
\end{array}
$$

Common Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Base Conversion

- Given a number of **radix** r (base-r) with
  - n **integer** digits, $a_{n-1}a_{n-2}\ldots a_0$ and m **fraction** digits $a_{-1}a_{-2}\ldots a_{-m}$
  - Written so: $a_{n-1}a_{n-2}\ldots a_0.a_{-1}a_{-2}\ldots a_{-m}$
- $(N)_r = \underbrace{\sum_{i=0}^{n-1} a_i r^i}_{Integer\ portion} + \underbrace{\sum_{j=-m}^{-1} a_j r^j}_{Fraction\ portion}$
  - $(9876.54)_{10} \Longrightarrow a_3a_2a_1a_0.a_{-1}a_{-2} = 9\times10^3 + 8\times10^2 + 7\times10^1 + 6\times10^0 + 5\times10^{-1} + 4\times10^{-2}$.
  - $(11010.1)_2 \Longrightarrow a_4a_3a_2a_1a_0.a_{-1} = 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0 + 1\times2^{-1}$.
- Base conversion
  - **S1:** Convert the integer portion.
  - **S2:** Convert the fraction portion.
  - **S3:** Join the two with a radix point.

Base Conversion

- **S1:** Convert the integer portion.
  - Repeatedly **divide** the integer by the radix r you want to convert to, and save the **remainders**.
  - The new radix digits are the remainders in **reverse order** of computation.
  - Exp: $(41)_{10} = (101001)_2$.

```
41 ÷ 2 => 20 rem = 1   ↑ Least significant digit
20 ÷ 2 => 10 rem = 0   |
10 ÷ 2 =>  5 rem = 0   |
 5 ÷ 2 =>  2 rem = 1   |
 2 ÷ 2 =>  1 rem = 0   |
 1 ÷ 2 =>  0 rem = 1   | Most significant digit
```

- **S2:** Convert the fraction portion.
  - Repeatedly **multiply** the fraction by the radix r, and save the **integer digits**.
  - The new radix fraction digits are the integer digits in **computed order**.
  - Exp: $(0.6875)_{10} = (.1011)_2$.

```
0.6875 * 2 = 1.3750   Integer = 1   | Most significant digit
0.3750 * 2 = 0.7500   Integer = 0   |
0.7500 * 2 = 1.5000   Integer = 1   |
0.5000 * 2 = 1.0000   Integer = 1   ↓ Least significant digit
```

- **S3:** Join the two with a radix point. (So, $(41.6875)_{10} = (101001.1011)_2$.)

Binary, Octal, and Hexadecimal Conversion

- Binary to Hexadecimal
  - Group the binary digits into **four-bit** groups, starting at the radix point and going both ways—integer and fraction—(padding with zeros as needed).
  - Convert each group of four bits to an hexadecimal digit.
  - Exp: $(1011100101.0111101101)_2 = (2E5.7B4)_{16}$.
- Hexadecimal to Binary
  - Restate the hexadecimal as **four** binary digits, starting at the radix point and going both ways (padding with zeros as needed).
  - Exp: $(2E5.7B4)_{16} = (1011100101.0111101101)_2$.

```
0010 1110 0101 . 0111 1011 0100
  ↕    ↕    ↕   .   ↕    ↕    ↕
  2    E    5   .   7    B    4
```