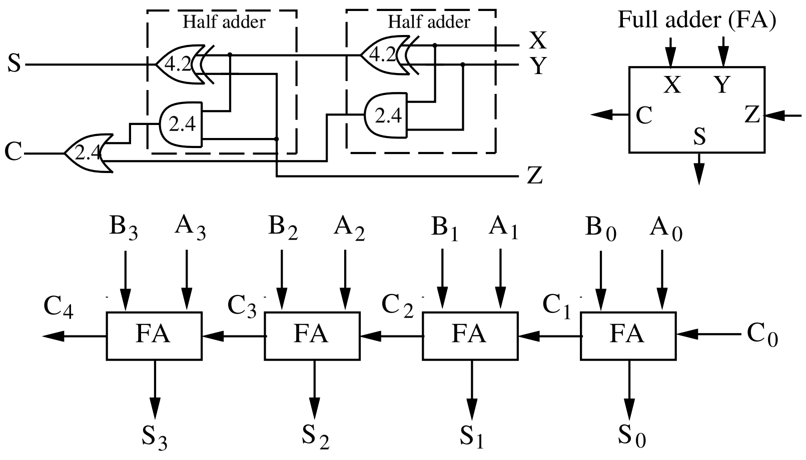
Ripple Carry Adder

- 4-bit ripple carry adder example: Inputs: C_0, $A = A_{n-1} \ldots A_0$, $B = B_{n-1} \ldots B_0$; Outputs: $s_{n-1} \ldots s_0$ and $c_n \ldots c_1$.



- Propagation delays: $C_0(A_0, B_0) \to C_1$: 4.8(9.0) ns; $C_0(A_0, B_0) \to C_2$: 9.6(13.8) ns; $C_0(A_0, B_0) \to C_3$: 14.4(18.6) ns; $C_0(A_0, B_0) \to C_4$: 19.2(23.4) ns.

- **Question:** Can we improve the delays?
  - The carries are the bottleneck!

Carry Lookahead Adder

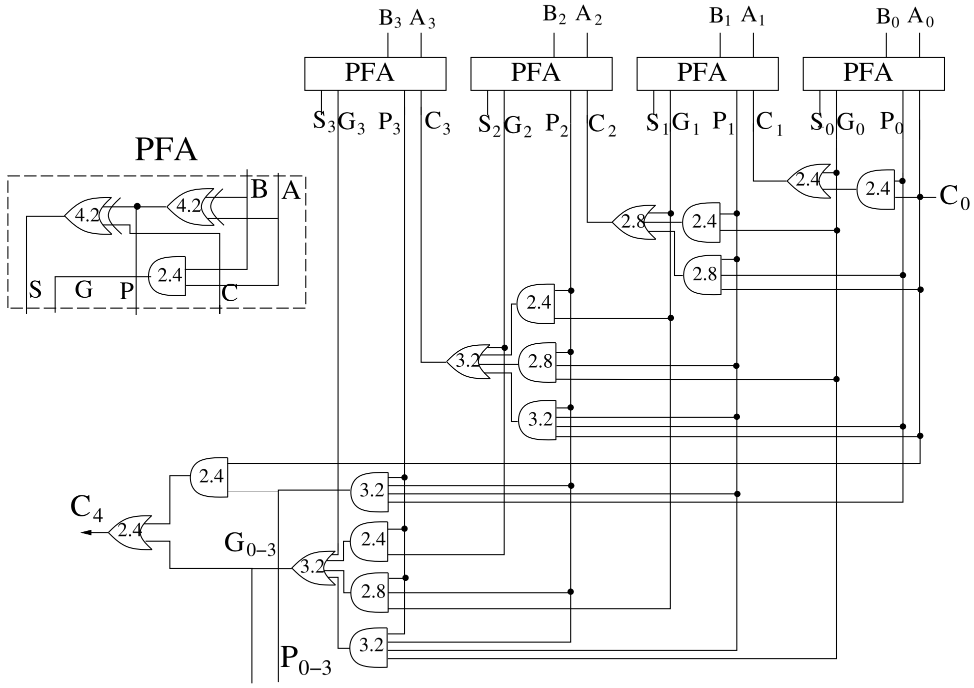
- **Observation:** Major part of the expressions for each carry in the chain can be **precomputed**.
  - Exp: Add $A_3A_2A_1A_0$ and $B_3B_2B_1B_0$.
  - **Carry-generate function:** $G_i = A_iB_i$; **carry-propagate function:** $P_i = A_i \oplus B_i$. Then, the 4 carries C_1, C_2, C_3, C_4 can be expressed as follows:

$$
\begin{aligned}
C_1 &= G_0 + P_0C_0 && (1)\\
C_2 &= G_1 + P_1C_1 && (2)\\
&= G_1 + P_1G_0 + P_1P_0C_0 &&\\
C_3 &= G_2 + P_2C_2 && (3)\\
&= G_2 + P_2G_1 + P_2P_1G_0 + P_2P_1P_0C_0 &&\\
C_4 &= G_3 + P_3C_3 && (4)\\
&= G_3 + P_3G_2 + P_3P_2G_1 + P_3P_2P_1G_0 + P_3P_2P_1P_0C_0 &&
\end{aligned}
$$

- Carries can be computed directly from input bits A_i, B_i and input carry C_0. No ripple effect!

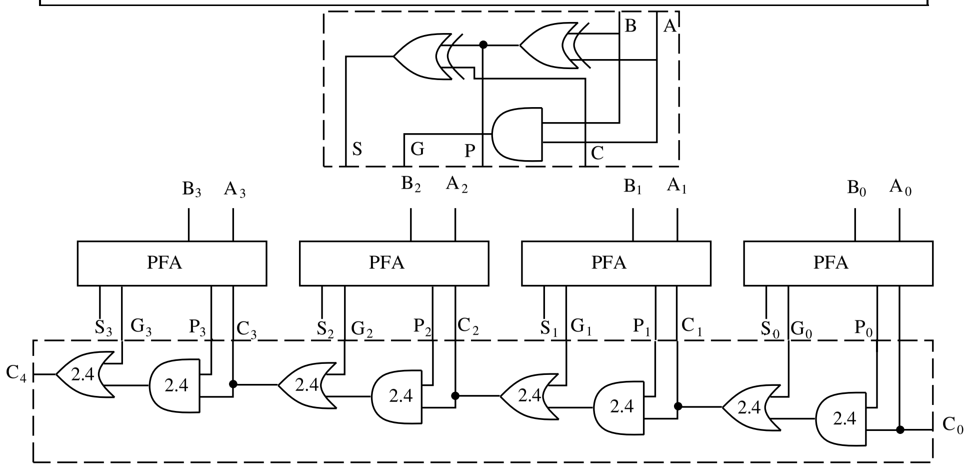
- Extend to n-bit adders?
  - **Problem: Fan-in limitation?**

Carry Lookahead Adder



- **Group propagate function:** $P_{0-3} = P_3P_2P_1P_0$;
  Group generate function: $G_{0-3} = G_3 + P_3G_2 + P_3P_2G_1 + P_3P_2P_1G_0$;
  $C_4 = G_{0-3} + P_{0-3}C_0$.

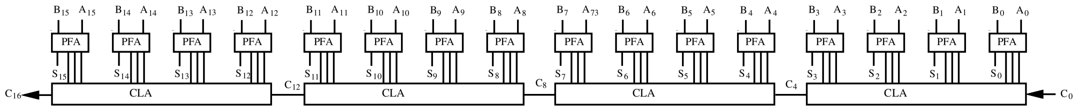
Carry Lookahead vs. Ripple Carry Adders



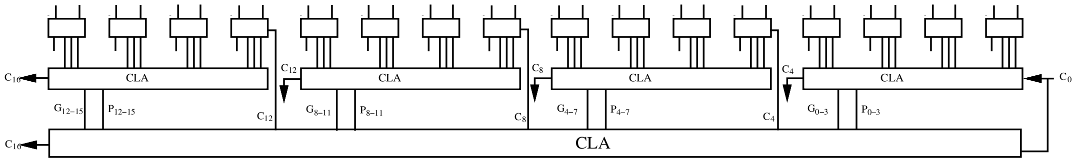
Ripple carry adder

Carry chain	Ripple delay	CLA delay
$C_0(A_0, B_0)$ to C_1	4.8 (9.0)	4.8 (9.0)
$C_0(A_0, B_0)$ to C_2	9.6 (13.8)	5.6 (9.8)
$C_0(A_0, B_0)$ to C_3	14.4 (18.6)	6.4 (10.6)
$C_0(A_0, B_0)$ to C_4	19.2 (23.4)	4.8 (13.0)

Carry Lookahead Adder Construction



Single–level CLA generator



Two–level CLA generator

- Carry chain delays for 16-bit adders:

Carry chain	Ripple delay	One-level CLA	Two-level CLA
$C_0(A_0, B_0)$ to C_4	19.2 (23.4)	4.8 (13.0)	4.8 (13.0)
$C_0(A_0, B_0)$ to C_8	38.4 (42.6)	9.6 (17.8)	5.6 (16.2)
$C_0(A_0, B_0)$ to C_{12}	57.6 (61.8)	14.4 (22.6)	6.4 (17.0)
$C_0(A_0, B_0)$ to C_{16}	76.8 (81.0)	19.2 (27.4)	4.8 (19.4)

- n-bit ripple-carry adder has $2n$ gate delays; n-bit adder with $\lceil \log_4 n \rceil$ levels of CLA needs $2\lceil \log_4 n \rceil$ gate delays to generate top-level carries.

Diminished Radix Complements

- **Diminished radix $(r-1)$'s complement:** Given a number N in based r with n digits, the $(r-1)$'s complement is defined as

$$(r^n - 1) - N. \tag{5}$$

- Exp1: $r = 10, N = 1234_{10}, n = 4 \Longrightarrow$ 9's complement of 1234_{10} is $9999_{10} - 1234_{10} = 8765_{10}$.

- Exp2: $r = 2, N = 01110011_2, n = 8 \Longrightarrow$ 1's complement of 01110011_2 is $11111111_2 - 01110011_2 = 10001100_2$.

- Finding one's complement $\Longrightarrow$ complementing each individual bit.

Radix Complements

- **Radix r's Complement:** Given a number N in based r with n digits, the r's complement is defined as

$$\begin{array}{rl} r^n - N & for \quad N \neq 0; \\ 0 & for \quad N = 0. \end{array}$$

- Exp1: $r = 10, N = 1234_{10}, n = 4 \Longrightarrow$ 10's complement of 1234_{10} is $10000_{10} - 1234_{10} = 8766_{10}$.
- Exp2: $r = 2, N = 01110011_2, n = 8 \Longrightarrow$ 2's complement of 01110011_2 is $100000000_2 - 01110011_2 = 10001101_2$.
- r's complement $= (r-1)$'s complement $+1$.
- The 2's complement of 0 is zero!

Subtraction with Radix Complements

- Subtract two n-digit, unsigned numbers $M - N$ in base r:
  1. Add the minuend M to the r's complement of the subtrahend N:
  $$M + (r^n - N) \;=\; M - N + r^n.$$
  2. If $M \geq N$, the sum will produce an end carry C_n which is discarded; what is left is the result, $M - N$.
  3. If $M < N$, the sum does not produce an end carry C_n and is equal to $r^n - (N - M)$, which is the r's complement of $(N - M)$. We shall take the r's complement of the sum and place a negative sign in front.

Find $543_{10} - 123_{10}$. Step 1: $\begin{array}{r} 1000 \\ -\ 123 \\ \hline 877 \end{array}$ Step 2: $\begin{array}{r} 543 \\ +\ 877 \\ \hline 1420 \end{array}$ Step 3: Since M >= N, discard the carry. Ans: 420	Find $123_{10} - 543_{10}$. Step 1: $\begin{array}{r} 1000 \\ -\ 543 \\ \hline 457 \end{array}$ Step 2: $\begin{array}{r} 123 \\ +\ 457 \\ \hline 580 \end{array}$ Step 3: M < N $\begin{array}{r} 1000 \\ -\ 580 \\ \hline 420 \end{array}$ Ans: –420	Find 1010100 – 1000011 Step 1: 2's complement of 1000011 is 0111101. Step 2: $\begin{array}{r} 1010100 \\ +\ 0111101 \\ \hline 1\ 0010001 \end{array}$ Step 3: Since M >= N, discard the carry. Ans: 0010001	Find 1000011 – 1010100 Step 1: 2's complement of 1010100 is 0101100. Step 2: $\begin{array}{r} 1000011 \\ +\ 0101100 \\ \hline 1101111 \end{array}$ Step 3: Since M < N, complement the result. Ans: (–) 0010001